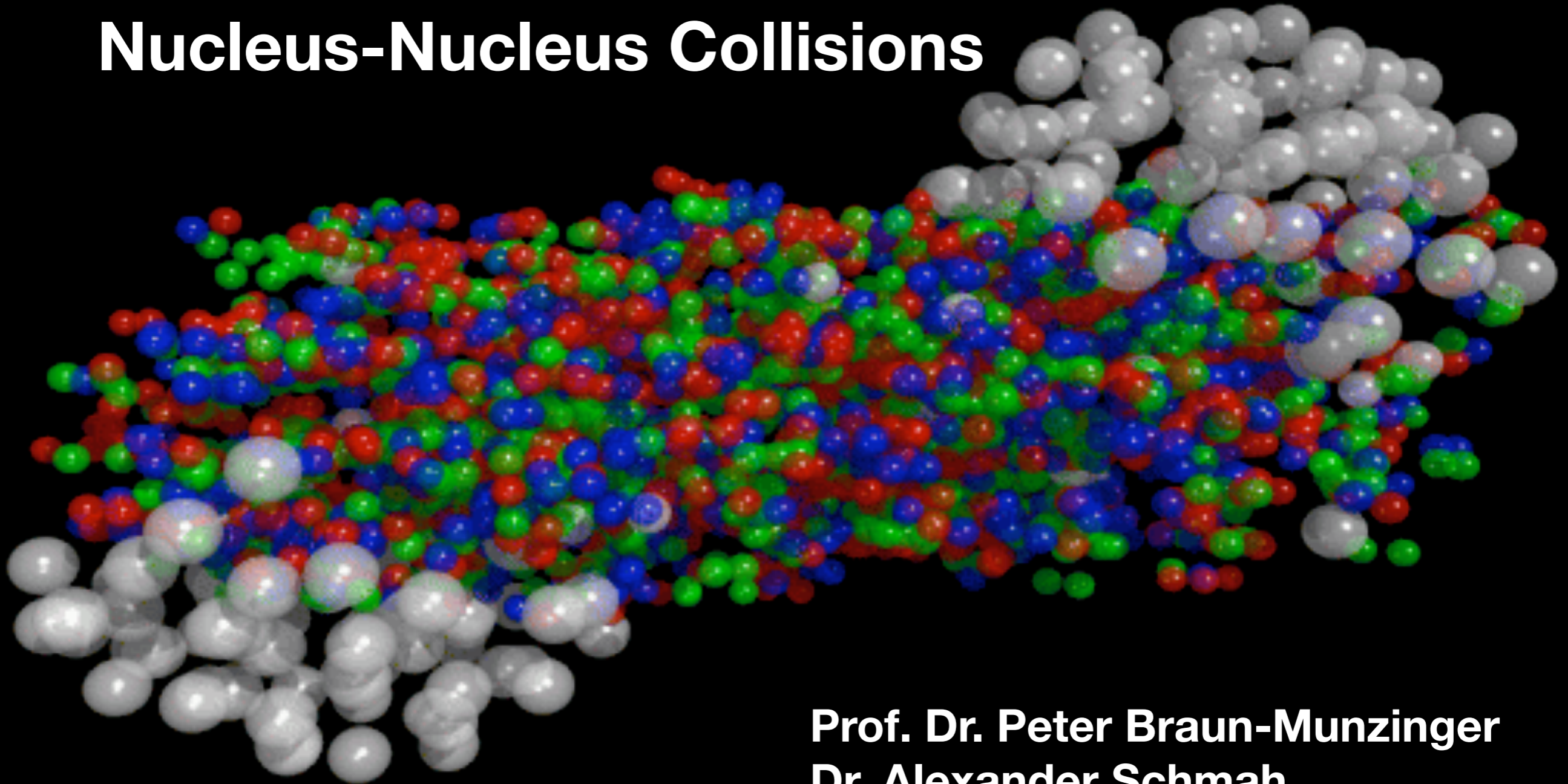


Quark-Gluon Plasma Physics

2. Basics of Nucleon-Nucleon and Nucleus-Nucleus Collisions

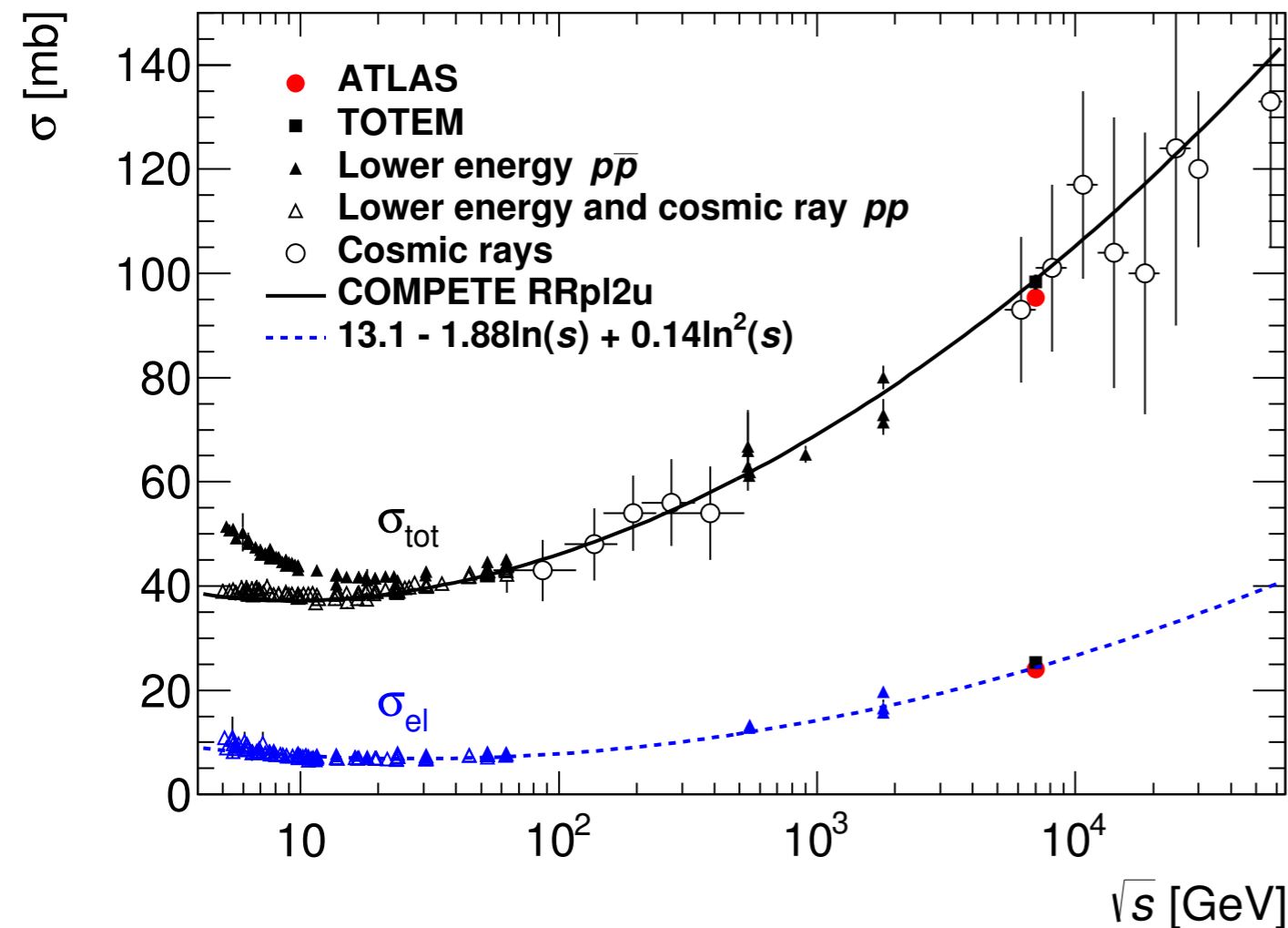


Prof. Dr. Peter Braun-Munzinger
Dr. Alexander Schmah
Heidelberg University
SS 2021

Part I: proton-proton collisions

Total p+p(pbar) Cross Section (I)

ATLAS, arXiv:1408.5778



Above $\sim \sqrt{s} = 20$ GeV all hadronic cross sections rise with increasing \sqrt{s}

Data show that

$$\sigma_{\text{tot}}(h + X) = \sigma_{\text{tot}}(\bar{h} + X)$$

(in line with Pommeranchuk's theorem)

Soft processes:

hard to calculate $\sigma_{\text{tot}}(\sqrt{s})$ in QCD

Modeling based on Regge theory: exchange of color-neutral object called *pomeron*

parameterization from Regge theory:

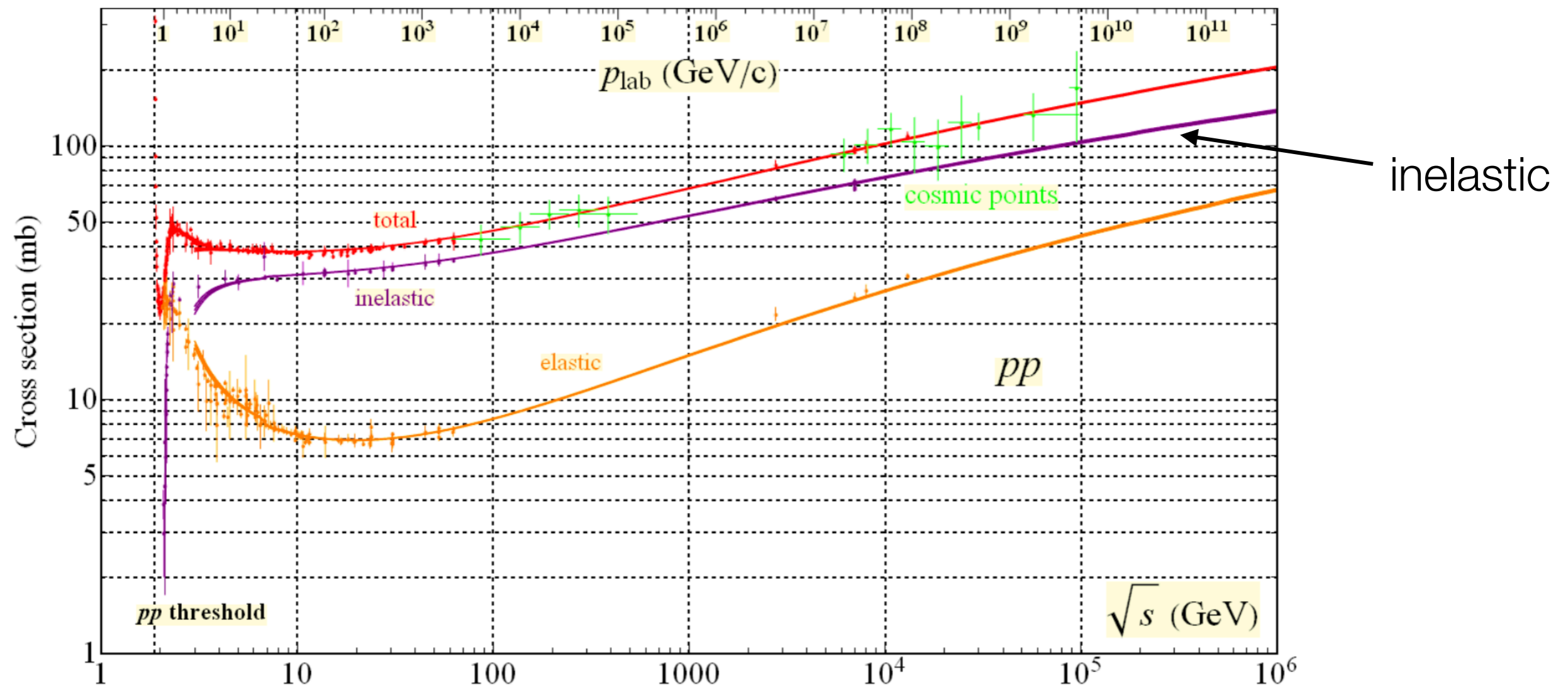
$$\sigma_{\text{tot}} = Xs^{\epsilon} + Ys^{\epsilon'}$$

$$\epsilon = 0.08 - 0.1, \quad \epsilon' \approx -0.45$$

“According to Regge theory, the strong interaction is due not to the exchange of particles with a definite spin, but rather to the exchange of a Regge trajectory, i.e., of a whole family of resonances.” Vincenzo Barone, Enrico Predazzi Pages 83-121

Total p+p(pbar) Cross Section (II)

<https://pdg.lbl.gov/2019/reviews/rpp2019-rev-cross-section-plots.pdf>



Diffraction collisions (I)

(Single) diffraction in p+p:

“Projectile” proton is excited to a hadronic state X with mass M

$$p_{\text{proj}} + p_{\text{targ}} \rightarrow X + p_{\text{targ}}$$

The excited state X fragments, giving rise to the production of (a small number) of particles in the forward direction

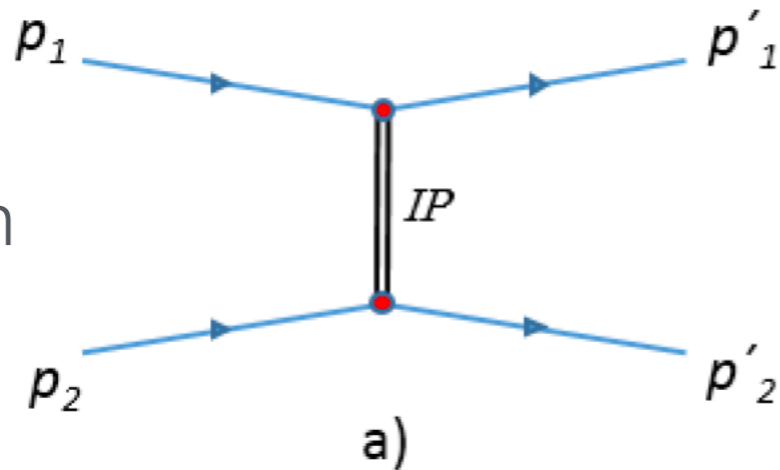
Theoretical view:

- Diffractive events correspond to the exchange of a Pomeron
- The Pomeron carries the quantum numbers of the vacuum ($J^{PC} = 0^{++}$)
- Thus, there is no exchange of quantum numbers like color or charge
- In a QCD picture the Pomeron can be considered as a two- or multi-gluon state, see, e.g., O. Nachtmann ([→ link](#))

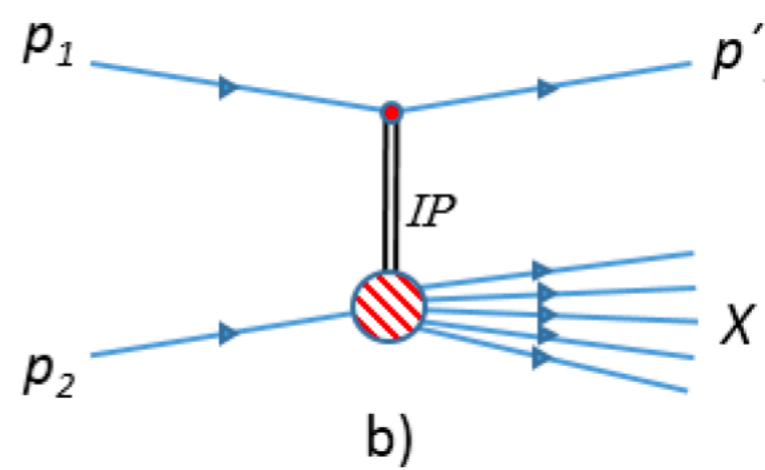
Diffractive collisions (II)

Diffractive collision = no color charge exchanged = “pompon exchange”

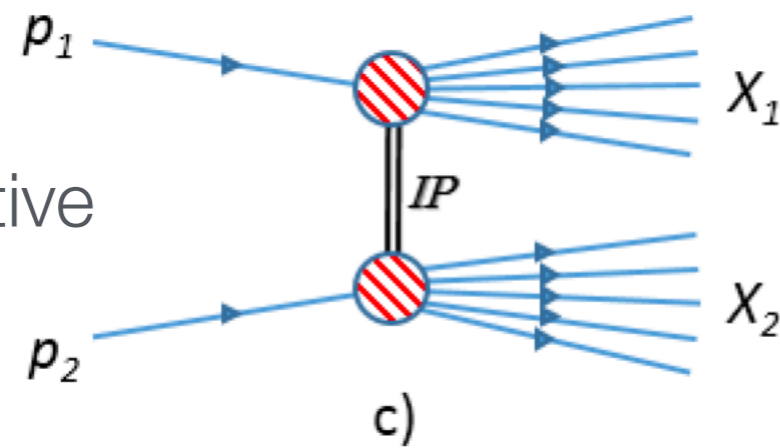
elastic collision



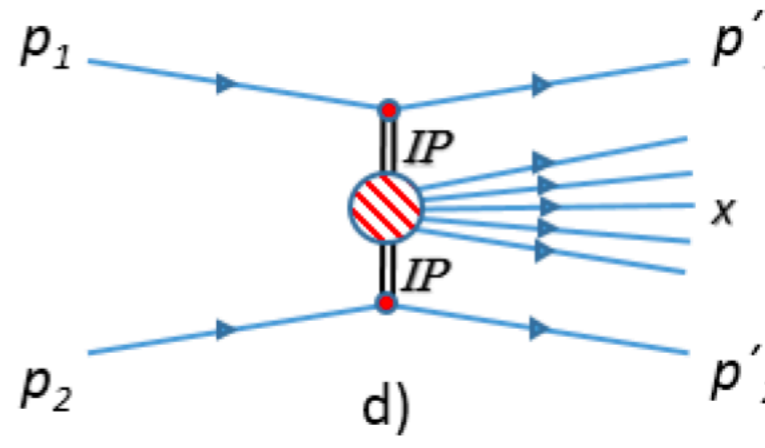
single-diffractive



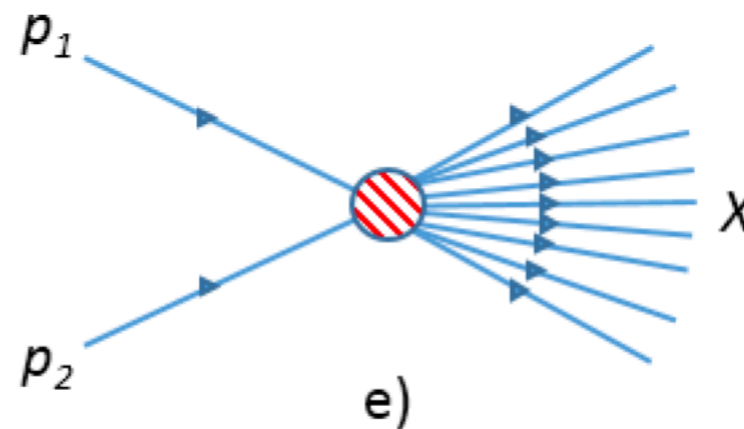
double-diffractive



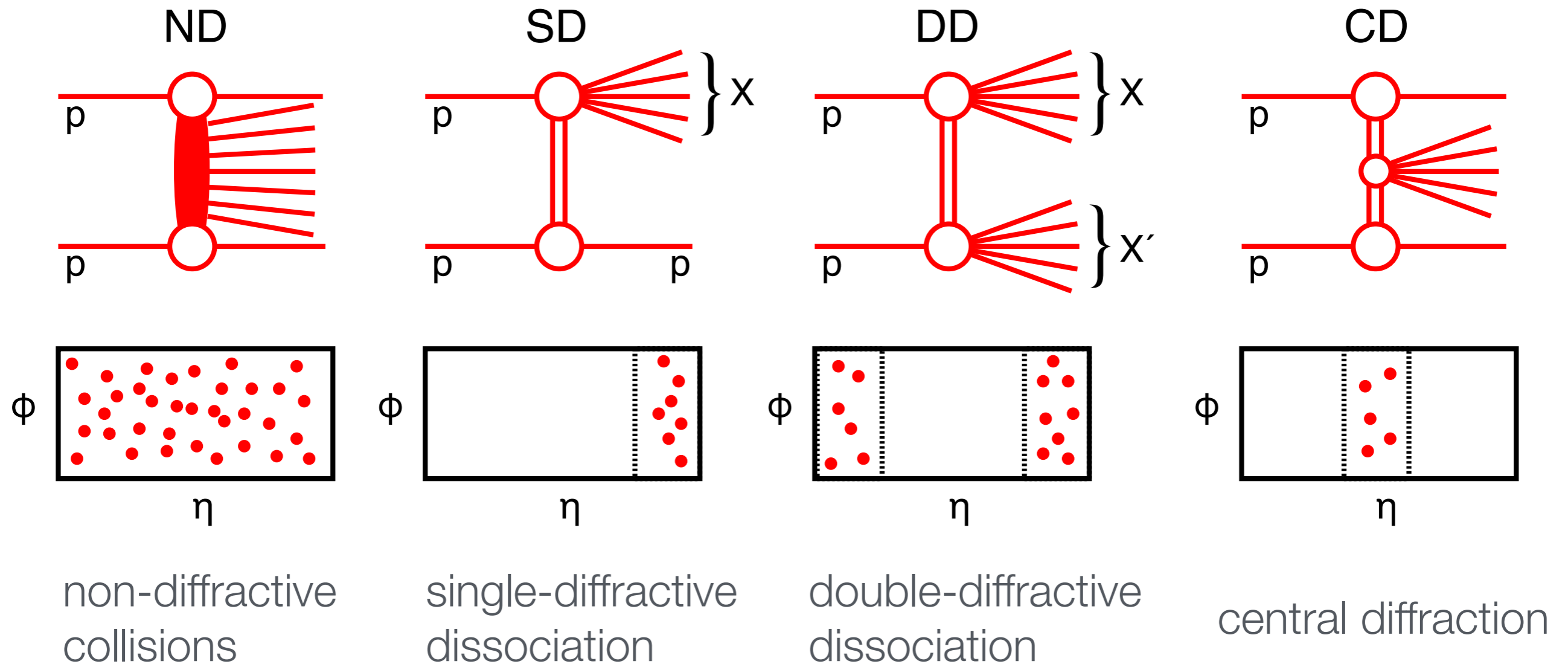
central-diffraction



non-diffractive collisions = exchange of color charge



Diffractive collisions (II)



$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}, \quad \sigma_{\text{inel}} = \sigma_{\text{SD}} + \sigma_{\text{DD}} + \sigma_{\text{CD}} + \sigma_{\text{ND}}$$

Diffractive collisions (III)

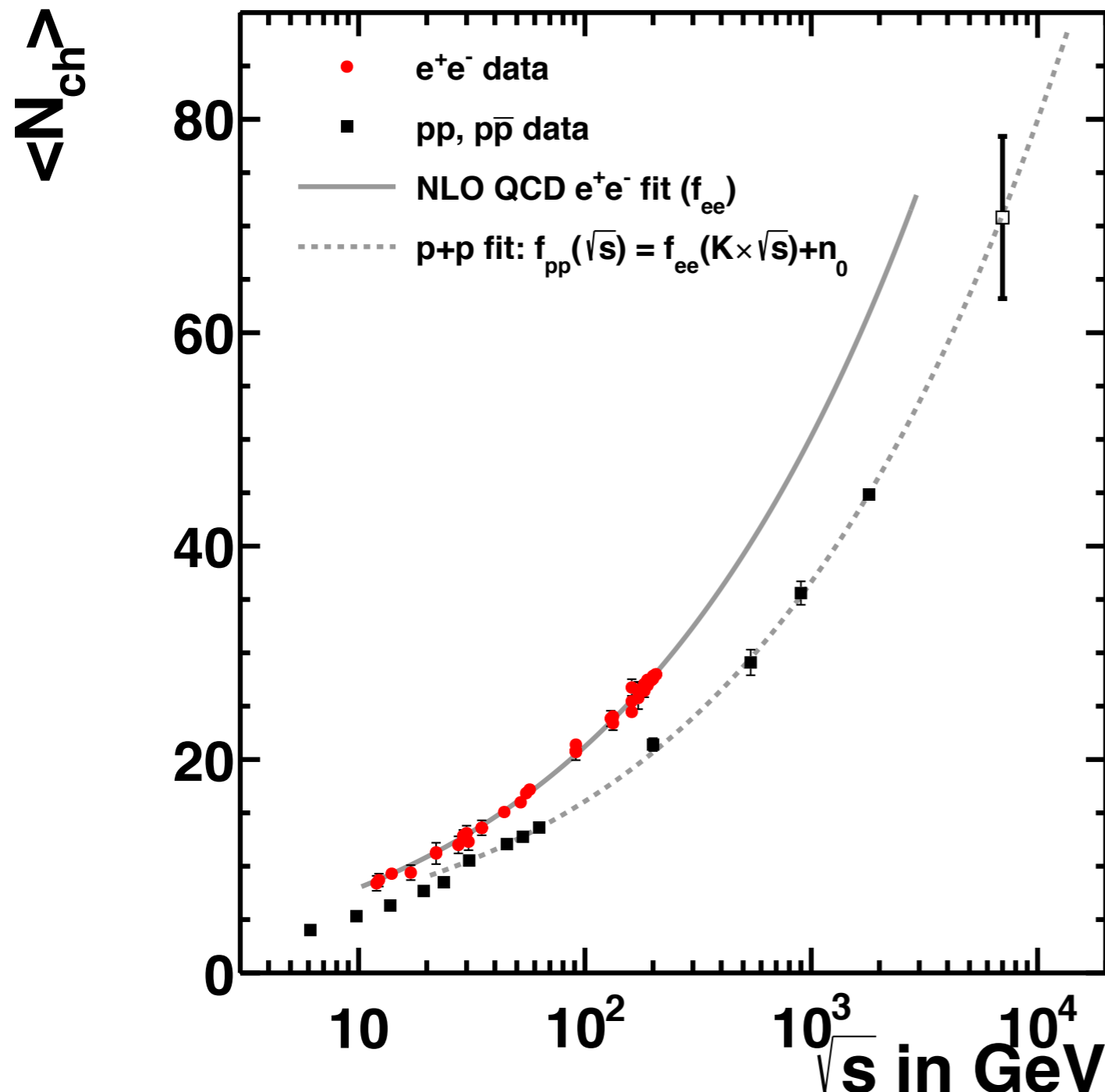
UA5, Z. Phys. C33, 175, 1986

$p + \bar{p}$	$\sqrt{s} = 200 \text{ GeV}$	$\sqrt{s} = 900 \text{ GeV}$
Total inelastic	$(41.8 \pm 0.6) \text{ mb}$	$(50.3 \pm 0.4 \pm 1.0) \text{ mb}$
Single-diffractive	$(4.8 \pm 0.5 \pm 0.8) \text{ mb}$	$(7.8 \pm 0.5 \pm 1.8) \text{ mb}$
Double-diffractive	$(3.5 \pm 2.2) \text{ mb}$	$(4.0 \pm 2.5) \text{ mb}$
Non-diffractive	$\approx 33.5 \text{ mb}$	$\approx 38.5 \text{ mb}$

Fraction of diffractive dissociation events with respect to all inelastic collisions is about 20–30% (rather independent of \sqrt{s})

See also ATLAS, arXiv:1201.2808

Charged-particle Multiplicity as a fct. of \sqrt{s} : Similarities between pp and e^+e^-



The increase of N_{ch} with \sqrt{s} looks rather similar in p+p and e^+e^-

Roughly speaking, the energy available for particle production in p+p seems to be $\sim 30\text{--}50\%$:

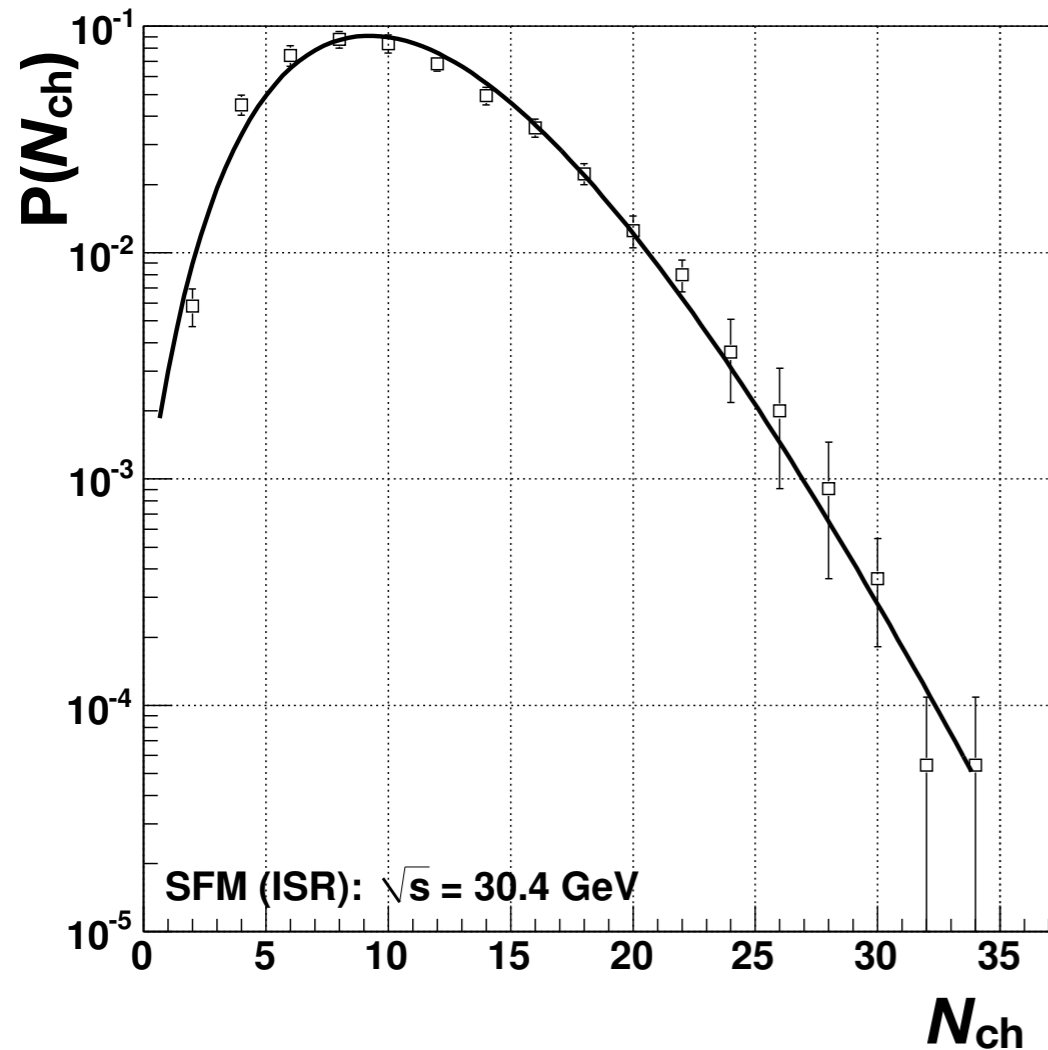
$$f(\sqrt{s}) := N_{ch}^{e^+e^-}(\sqrt{s})$$

$$\rightarrow N_{ch}^{p+p} = f(K\sqrt{s_{pp}}) + n_0$$

A fit yields: $K \approx 0.35$, $n_0 \approx 2.2$

What is the distribution of the number of produced particles per collision?

Independent sources: Poisson distribution



Observation:

Multiplicity distributions in pp, e^+e^- , and lepton-hadron collisions well described by a Negative Binomial Distribution (NBD)

Deviations from the NBD were discovered by UA5 at $\sqrt{s} = 900$ GeV and later confirmed at the Tevatron at $\sqrt{s} = 1800$ GeV (shoulder structure at $n \approx 2 \langle n \rangle$)

Signature of stochastic particle production

$$P_{\mu,k}^{\text{NBD}}(n) = \frac{(n+k-1) \cdot (n+k-2) \cdot \dots \cdot k}{\Gamma(n+1)} \left(\frac{\mu/k}{1+\mu/k} \right)^n \frac{1}{(1+\mu/k)^k}$$

$$\langle n \rangle = \mu, \quad D := \sqrt{\langle n^2 \rangle - \langle n \rangle^2} = \sqrt{\mu \left(1 + \frac{\mu}{k} \right)}$$

Limits of the NBD:

$k \rightarrow \infty$: Poisson distribution

integer $k, k < 0$: Binomial distribution

($N = -k, p = -\langle n \rangle/k$)

π^0 transverse momentum distributions at different \sqrt{s}

Low p_T ($< \sim 2$ GeV/c):
"soft processes"

$$E \frac{d^3\sigma}{d^3p} = A(\sqrt{s}) \cdot e^{-\alpha p_T}, \quad \alpha \approx 6/(\text{GeV}/c)$$

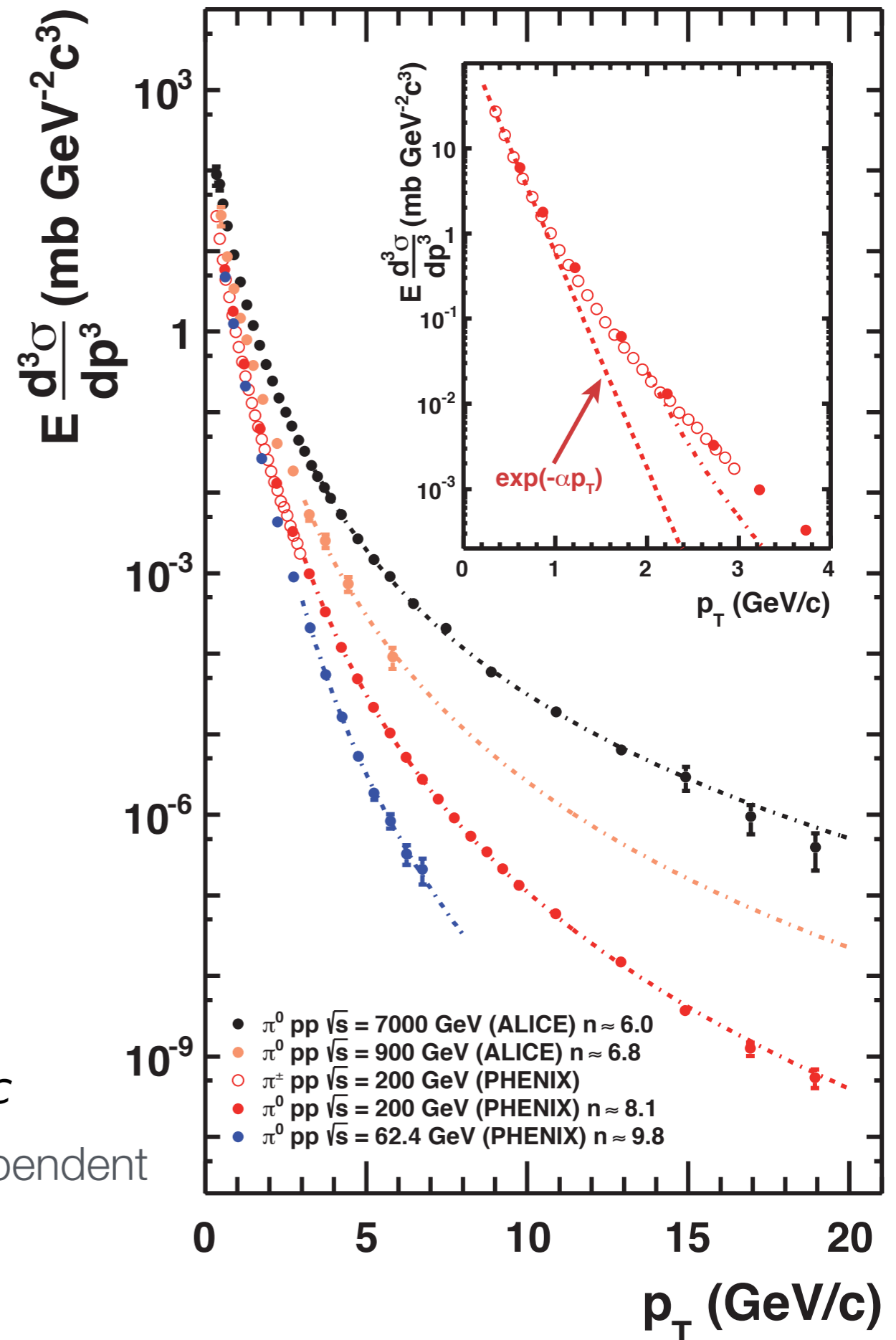
High p_T ("hard scattering"):

$$E \frac{d^3\sigma}{d^3p} = B(\sqrt{s}) \cdot \frac{1}{p_T^{n(\sqrt{s})}}$$

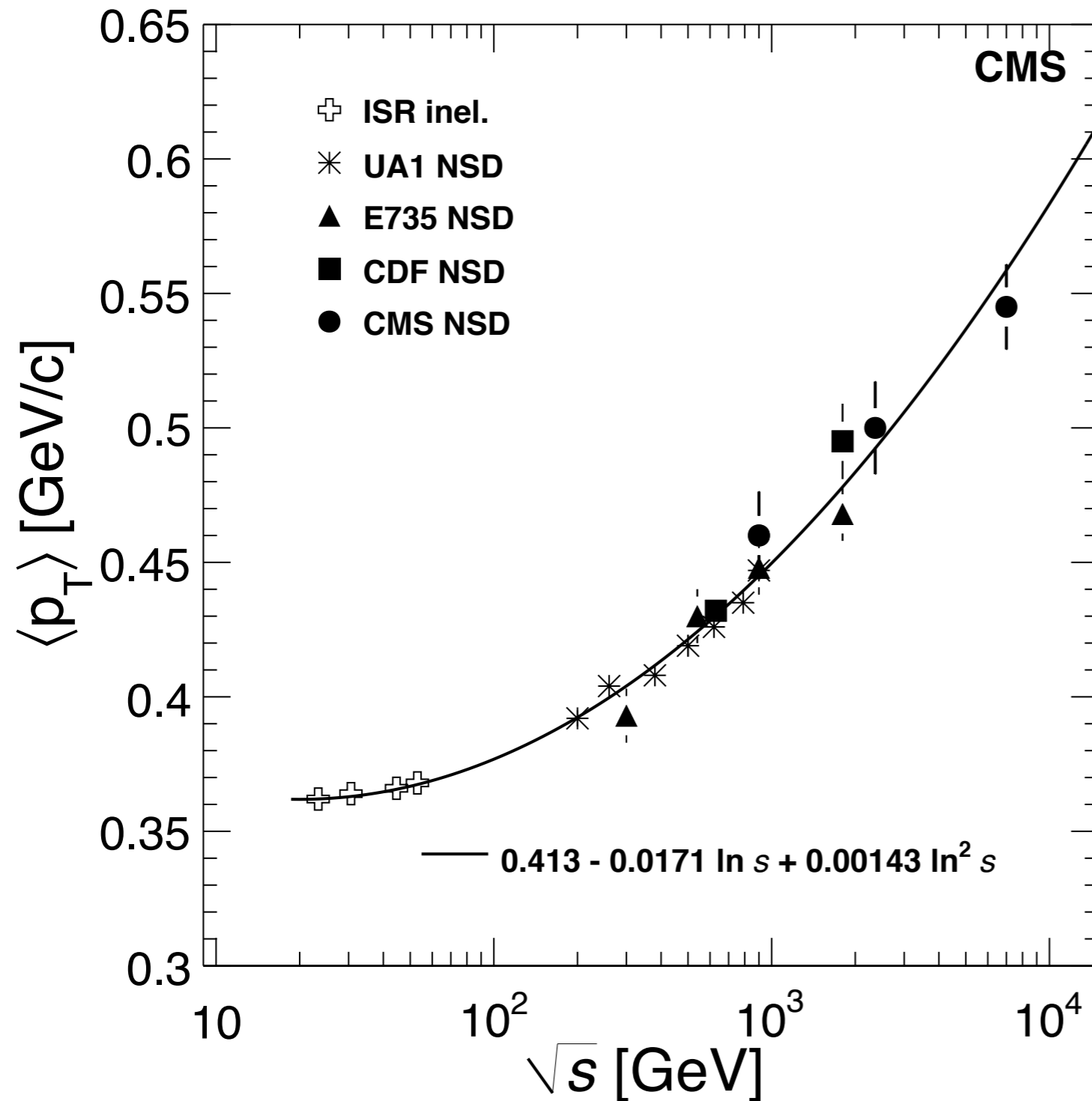
Average p_T :

$$\langle p_T \rangle = \frac{\int_0^\infty p_T \frac{dN_x}{dp_T} dp_T}{\int_0^\infty \frac{dN_x}{dp_T} dp_T} \approx 300 - 400 \text{ MeV}/c$$

pretty energy-independent
for $\sqrt{s} < 100$ GeV



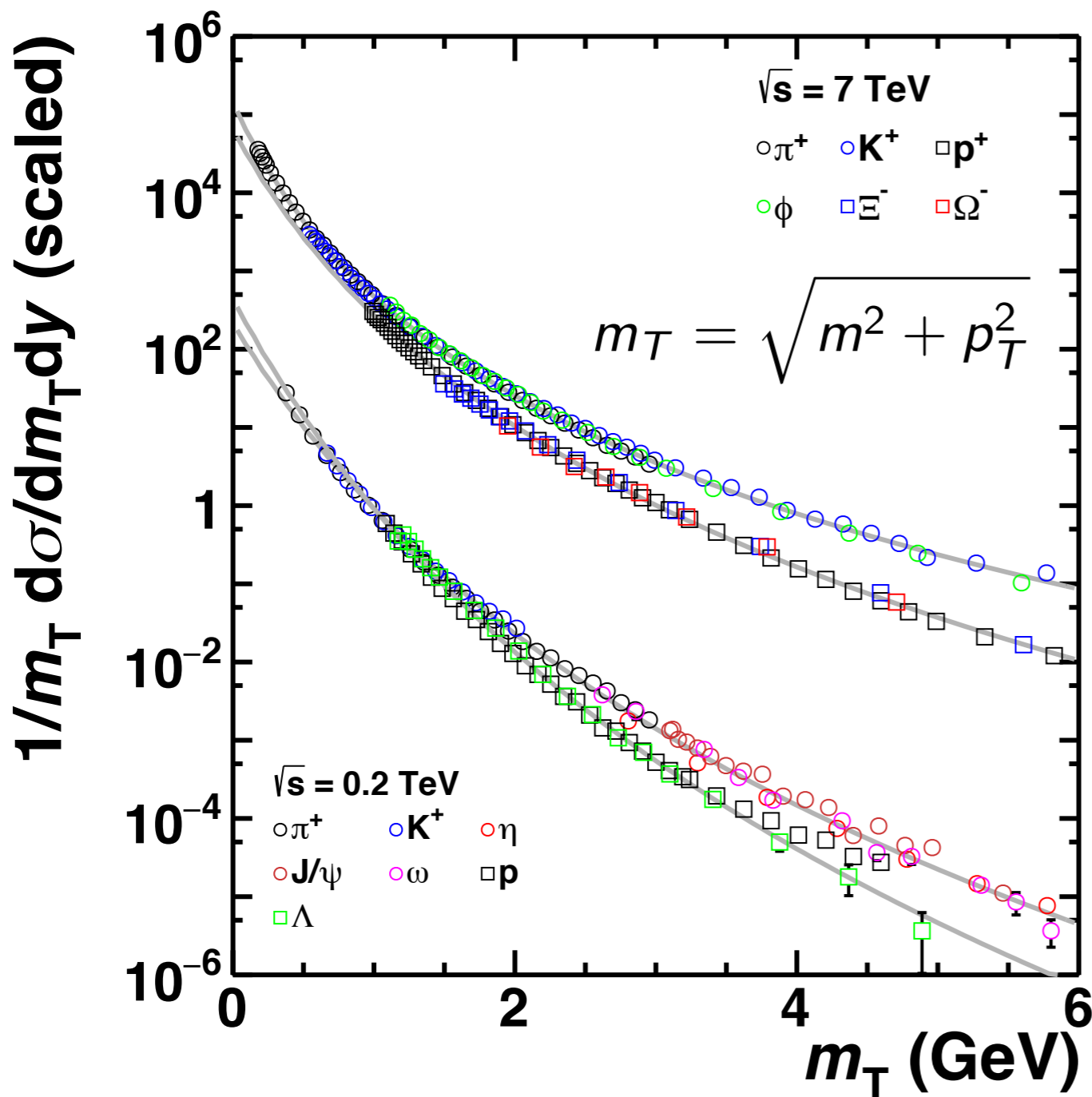
Mean p_T increases with \sqrt{s}



Increase of $\langle p_T \rangle$ with \sqrt{s} (most likely) reflects increase in particle production from hard parton-parton scattering

CMS, PRL 105, 022002 (2010)
CDF, PRL 61, 1819 (1988)

m_T scaling in pp collisions



m_T scaling (early ref's):
 Nucl. Phys. B70, 189–204 (1974)
 Nucl. Phys. B120 (1977) 14–22

m_T scaling:
 shape of m_T spectra the same
 for different hadron species

example: $\frac{dN/dm_T|_{\eta}}{dN/dm_T|_{\pi^0}} \approx 0.45$

possible interpretation:
 thermodynamic models

$$E \frac{d^3 n}{d^3 p} \propto E e^{-E/T}$$

$$\rightarrow \frac{1}{m_T} \frac{dn}{dm_T} \propto K_1 \left(\frac{m_T}{T} \right)$$

RHIC/LHC:
 m_T scaling (approximately)
 satisfied, different universal
 function for mesons and baryons

Do deviations from m_T scaling in pp at
 low p_T indicate onset of radial flow?
 (1312.4230)

Theoretical modeling: General considerations

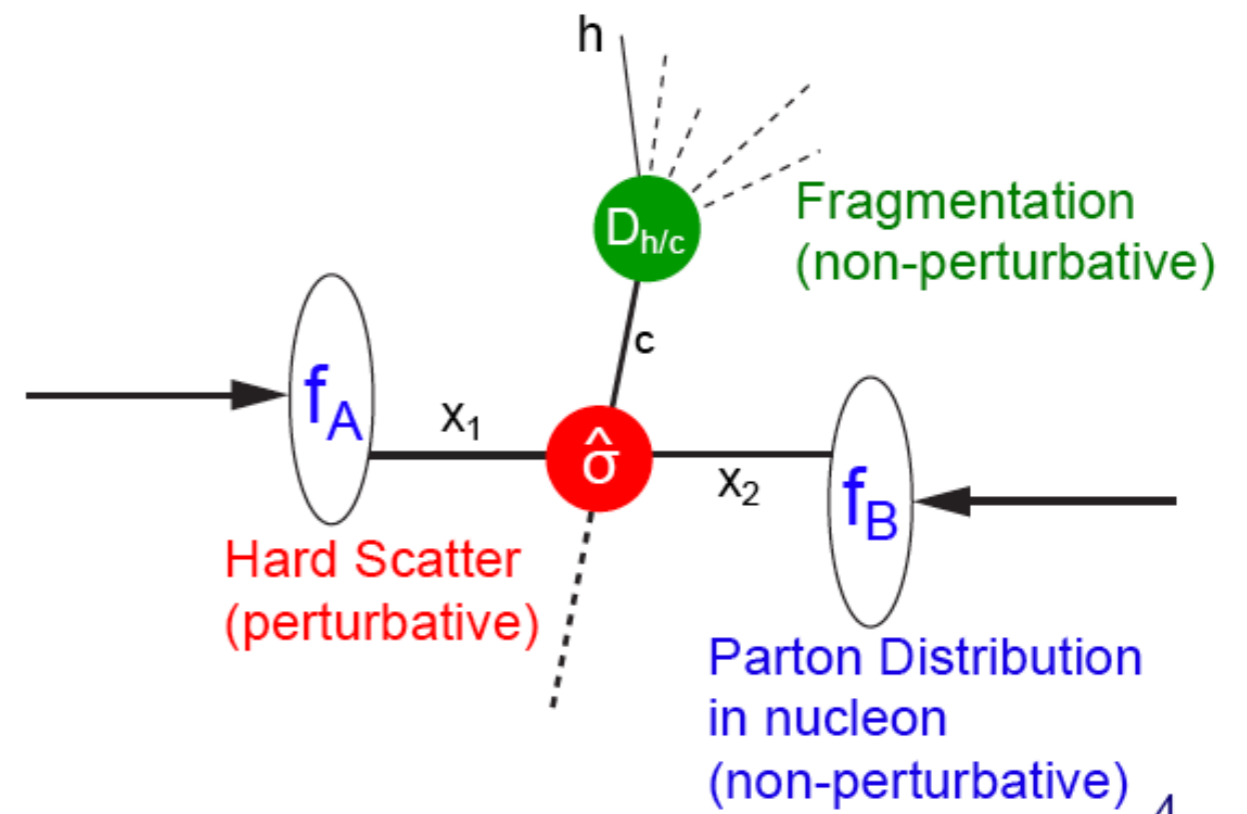
- Description of particle production amenable to perturbative methods only at sufficiently large p_T (so that α_s becomes sufficiently small)

- ▶ parton distributions (PDF)
- ▶ parton-parton cross section from perturbative QCD (pQCD)
- ▶ fragmentation functions (FF)

- Low- p_T :
Need to work with (QCD inspired) models, and confront them with data

- ▶ e.g. Lund string model

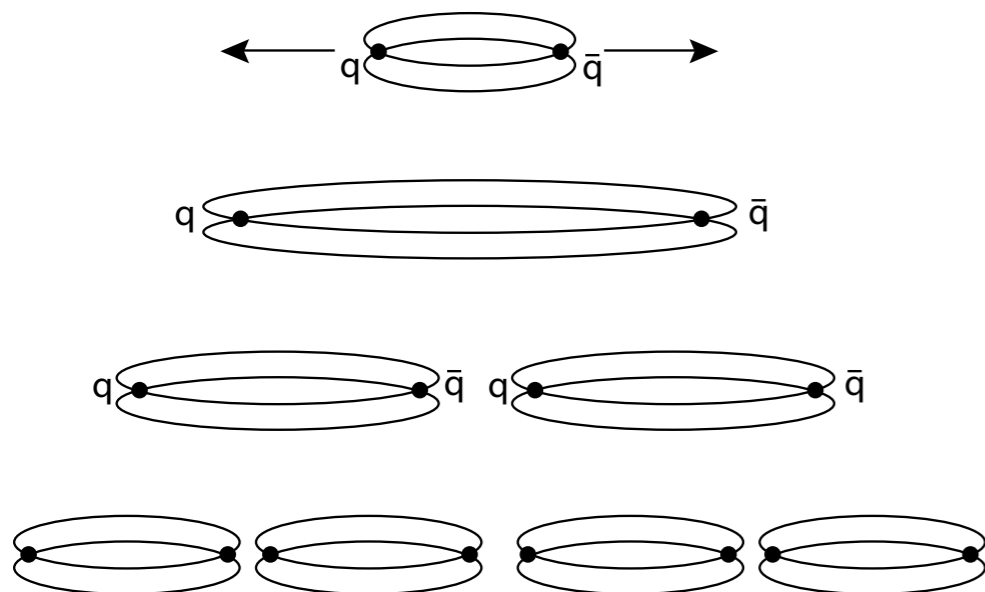
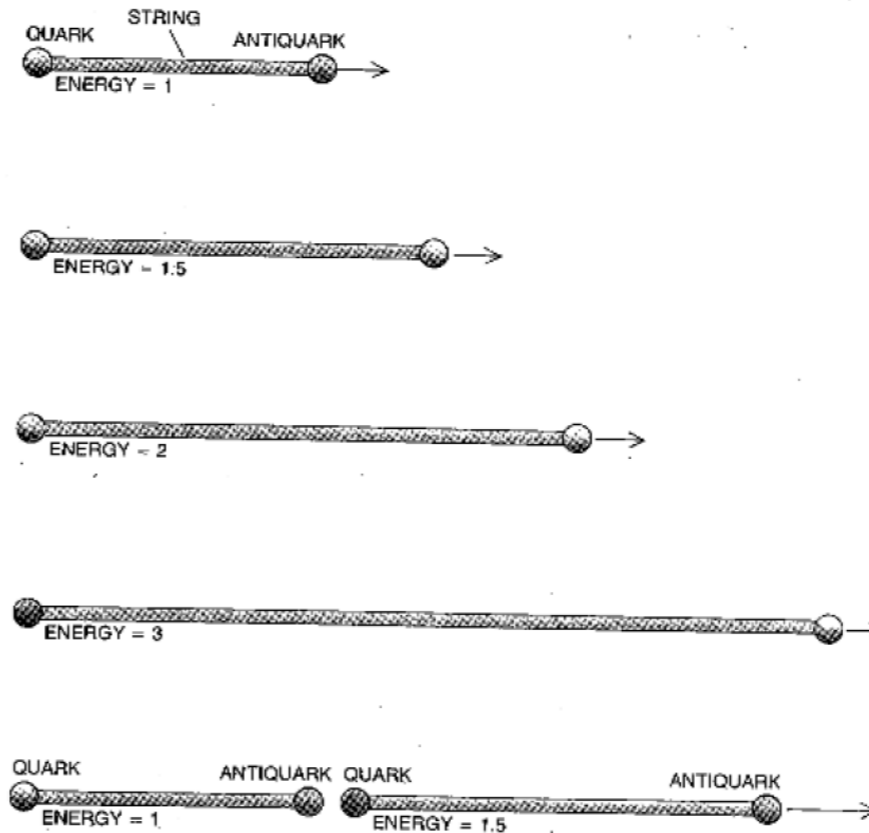
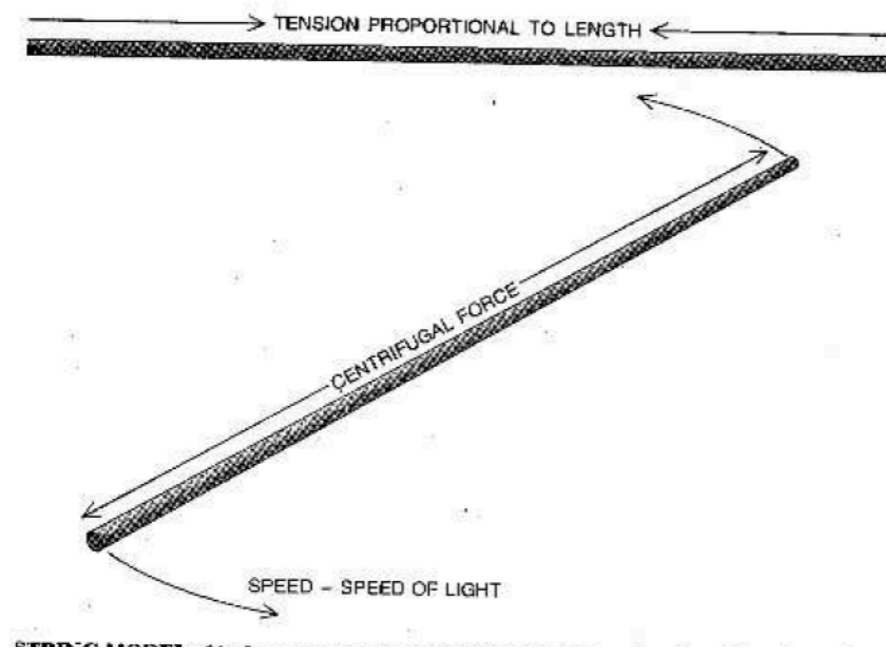
$$E \frac{d^3\sigma}{d^3p} = \int \text{PDF} \otimes \text{pQCD} \otimes \text{FF}$$



Modeling particle production as string breaking (I)

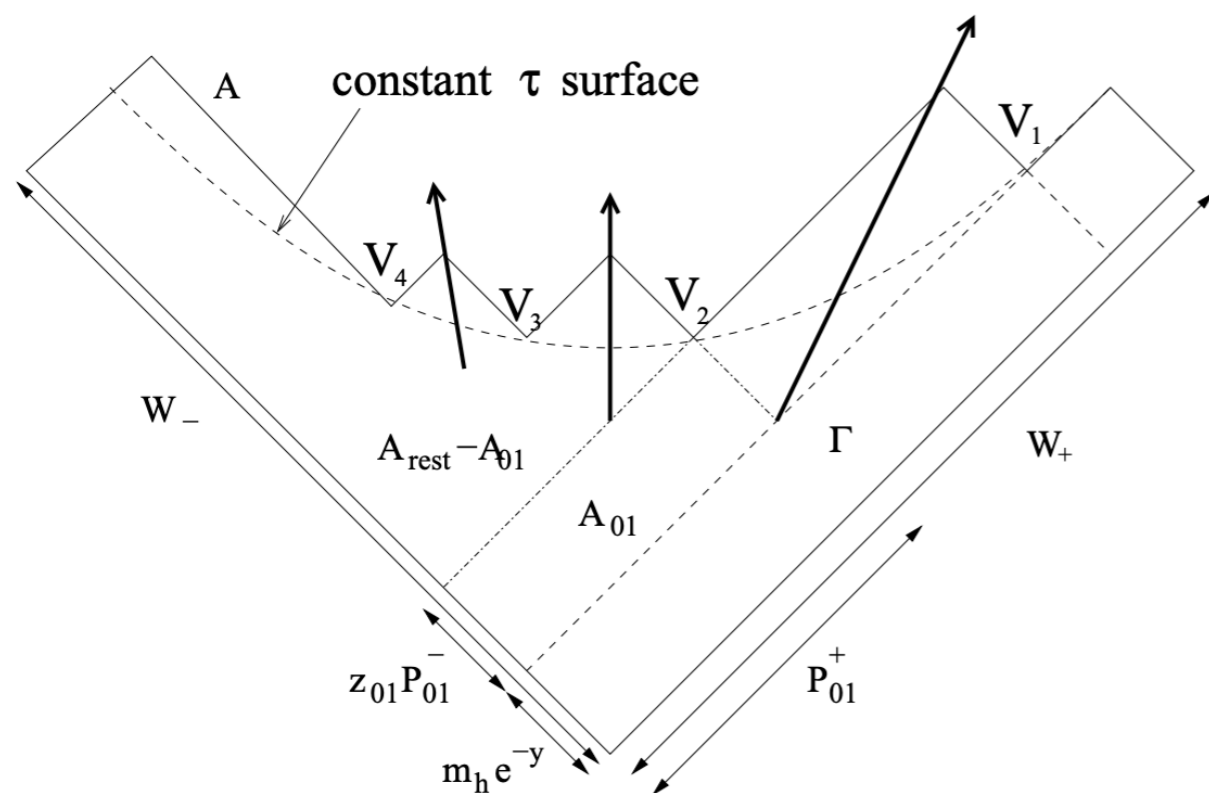
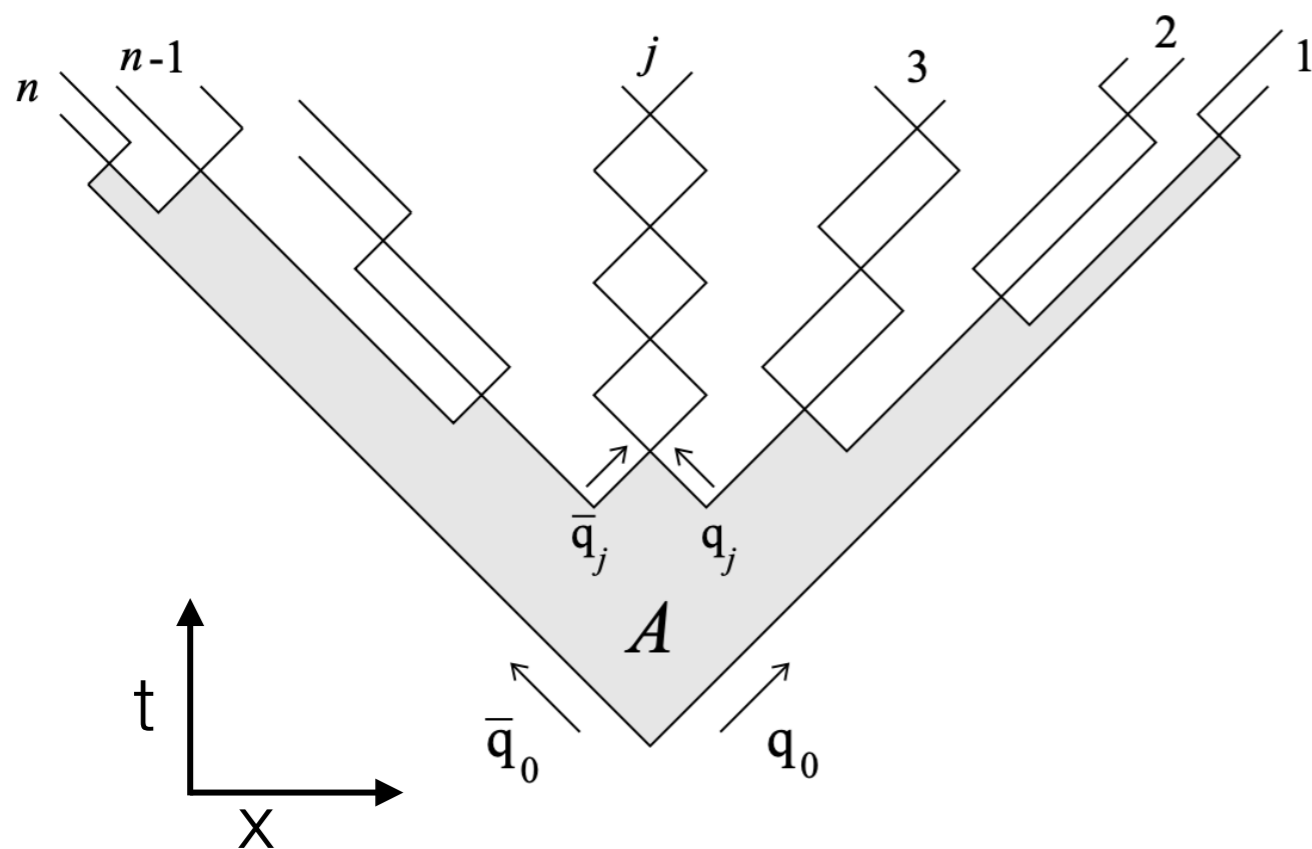
by Yoichiro Nambu

1976



- Color flux tube between two quarks breaks due to quark-antiquark pair production in the intense color field
- String tension increases linearly with distance

Modeling particle production as string breaking (II)



- Lund model:
The basic assumption of the symmetric Lund model is that the vertices at which the quark and the antiquark are produced lie approximately on a curve on constant proper time
- Result: flat rapidity distribution of the produced particles

Modeling particle production as string breaking (III)



$$m_T := \sqrt{m^2 + p_T^2} \quad m_{\perp q'} = 0$$

$$d = m_{\perp q'} / \kappa$$

$$m_{\perp q'} > 0$$

In terms of the transverse mass of the produced quark ($m_{T,q'} = m_{T,q'\text{bar}}$) the probability that the break-up occurs is:

$$P \propto \exp\left(-\frac{\pi m_{\perp q'}^2}{k}\right) = \exp\left(-\frac{\pi p_{\perp q'}^2}{k}\right) \exp\left(-\frac{\pi m_{q'}^2}{k}\right)$$

Result of the Schwinger equation

This leads to a transverse momentum distribution for the quarks of the form:

$$\frac{1}{p_T} \frac{dN_{\text{quark}}}{dp_T} = \text{const.} \cdot \exp(-\pi p_T^2/k) \quad \rightsquigarrow \quad \sqrt{\langle p_T^2 \rangle_{\text{quark}}} = \sqrt{k/\pi}$$

For pions (two quarks) one obtains: $\sqrt{\langle p_T^2 \rangle_{\text{pion}}} = \sqrt{2k/\pi}$

With a string tension of 1 GeV/fm this yields $\langle p_T \rangle_{\text{pion}} \approx 0.37$ GeV/c, in approximate agreement with data

Modeling particle production as string breaking (IV)

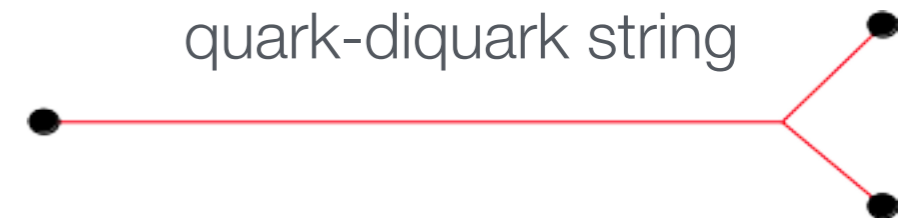
Convolution of the string breaking mechanism with fluctuations of the string tension described by a Gaussian give rise to exponential p_T spectra

Phys. Lett. B466, 301–304 (1999)

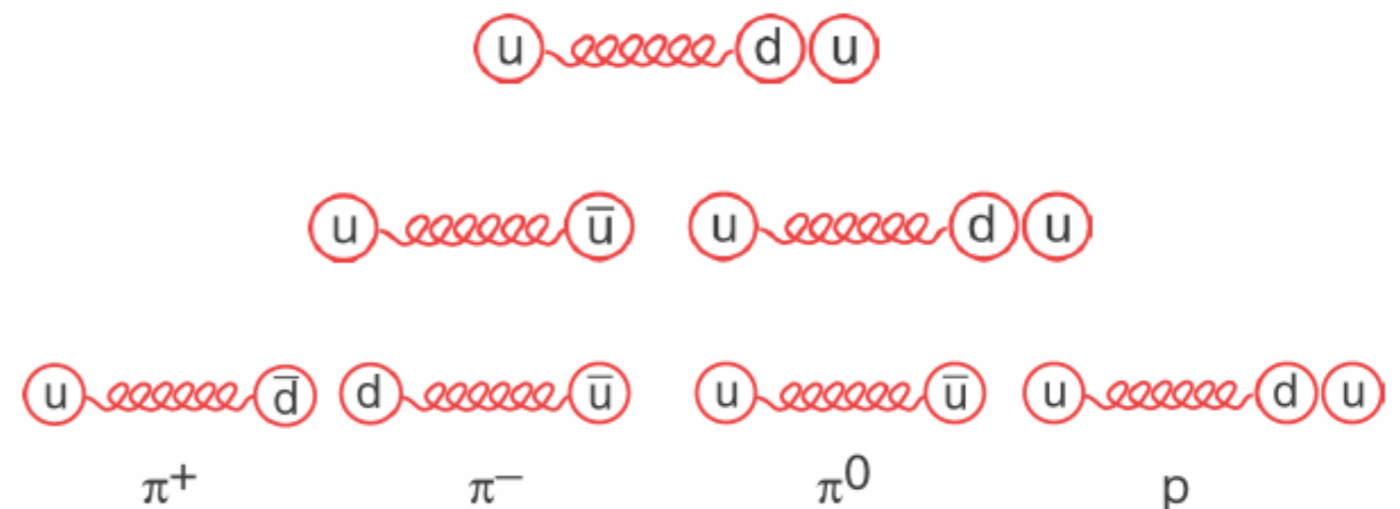
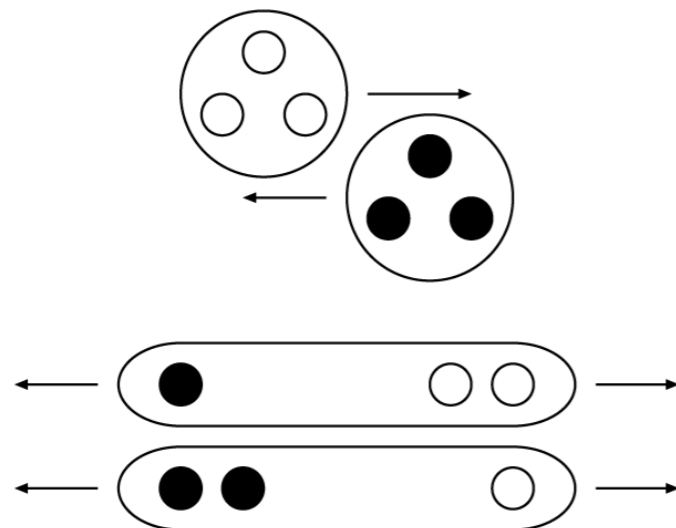
The tunneling process implies heavy-quark suppression:

$$u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$$

The production of baryons can be modeled by replacing the q-qbar pair by an quark-diquark pair



Collisions of hadrons described as excitation of quark-diquarks strings:

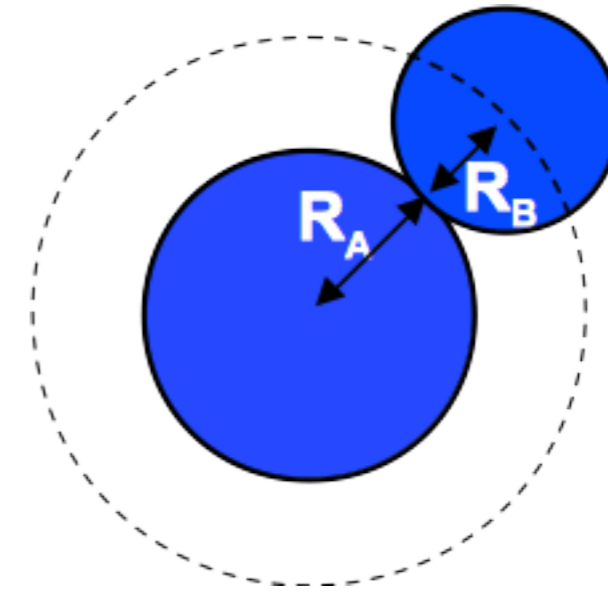


Part II: nucleus-nucleus collisions

Ultra-Relativistic Nucleus-Nucleus Collisions: Importance of Nuclear Geometry

■ Ultra-relativistic energies

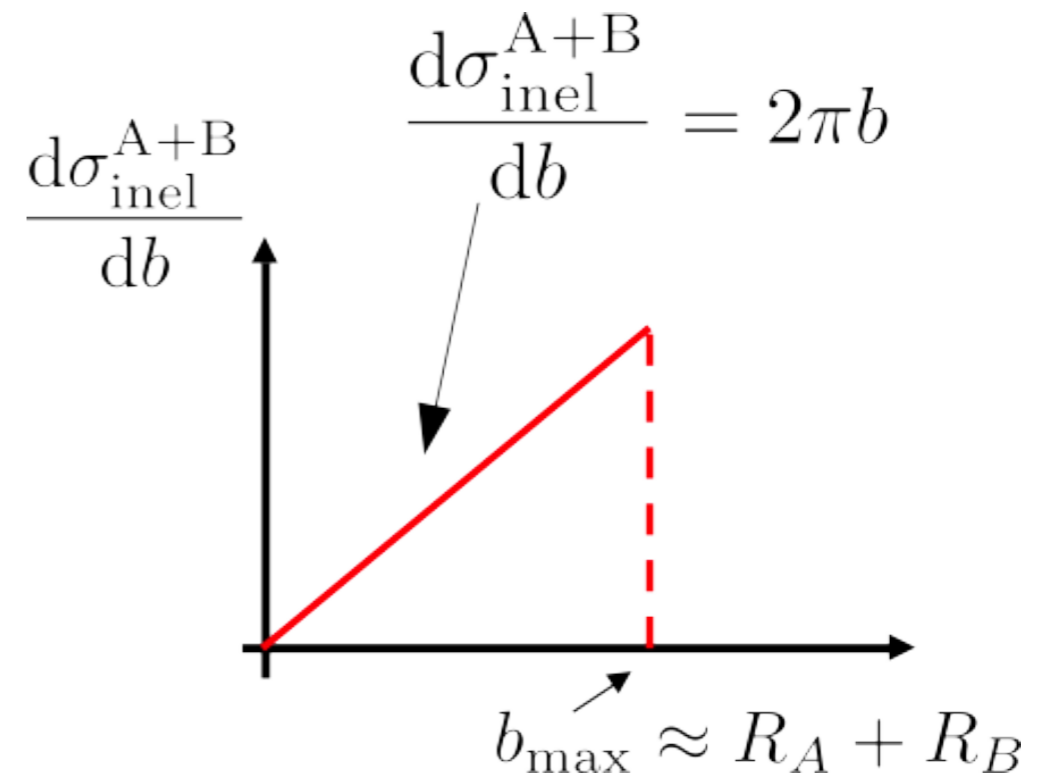
- ▶ De Broglie wave length much smaller than size of the nucleon
- ▶ Wave character of the nucleon can be neglected for the estimation of the total cross section



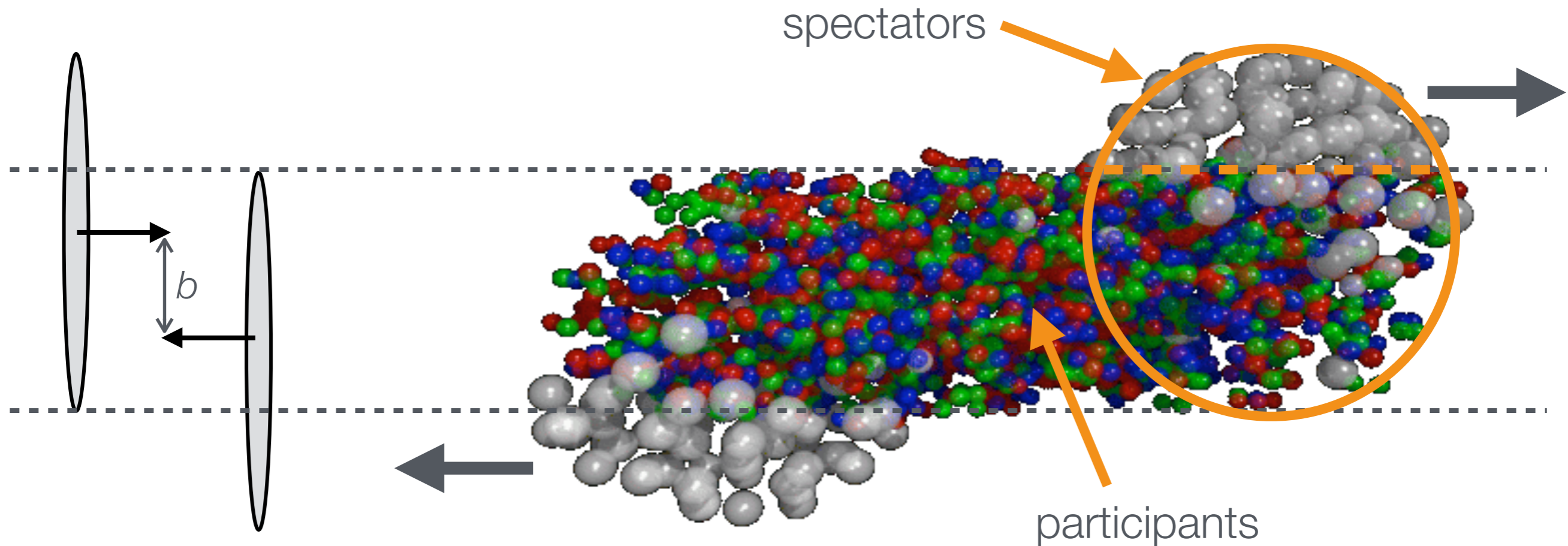
■ Nucleus-Nucleus collision can be considered as a collision of two black disks

$$R_A \approx r_0 \cdot A^{1/3}, \quad r_0 = 1.2 \text{ fm}$$

$$\sigma_{\text{inel}}^{A+B} \approx \sigma_{\text{geo}} \approx \pi r_0^2 (A^{1/3} + B^{1/3})^2$$



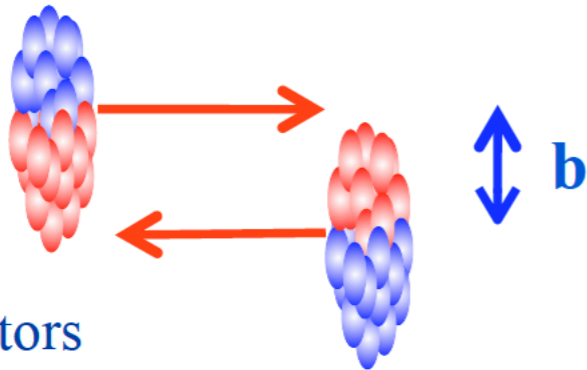
Participants and spectators. (I)



- N_{coll} : number of (binary) inelastic nucleon-nucleon collisions (important for hard processes)
- N_{part} : number of nucleons which underwent at least one inelastic nucleon-nucleon collision (important for soft processes)

Participants and spectators (II)

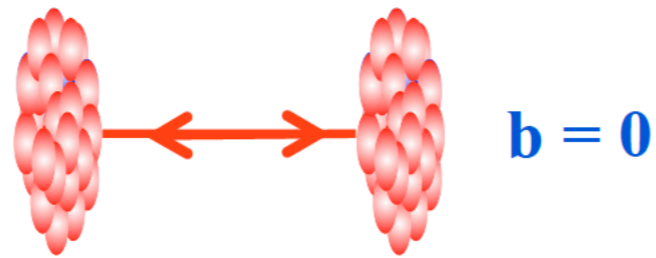
semi-central collision



Spectators
Participants

b impact parameter

central collision



$b = 0$



$$N_{\text{part}} = 2$$

$$N_{\text{coll}} = 1$$



$$N_{\text{part}} = 5$$

$$N_{\text{coll}} = 6$$

Pb-Pb cent.

$$N_{\text{part}} = 360$$

$$N_{\text{coll}} = 1500$$

p-Pb cent.

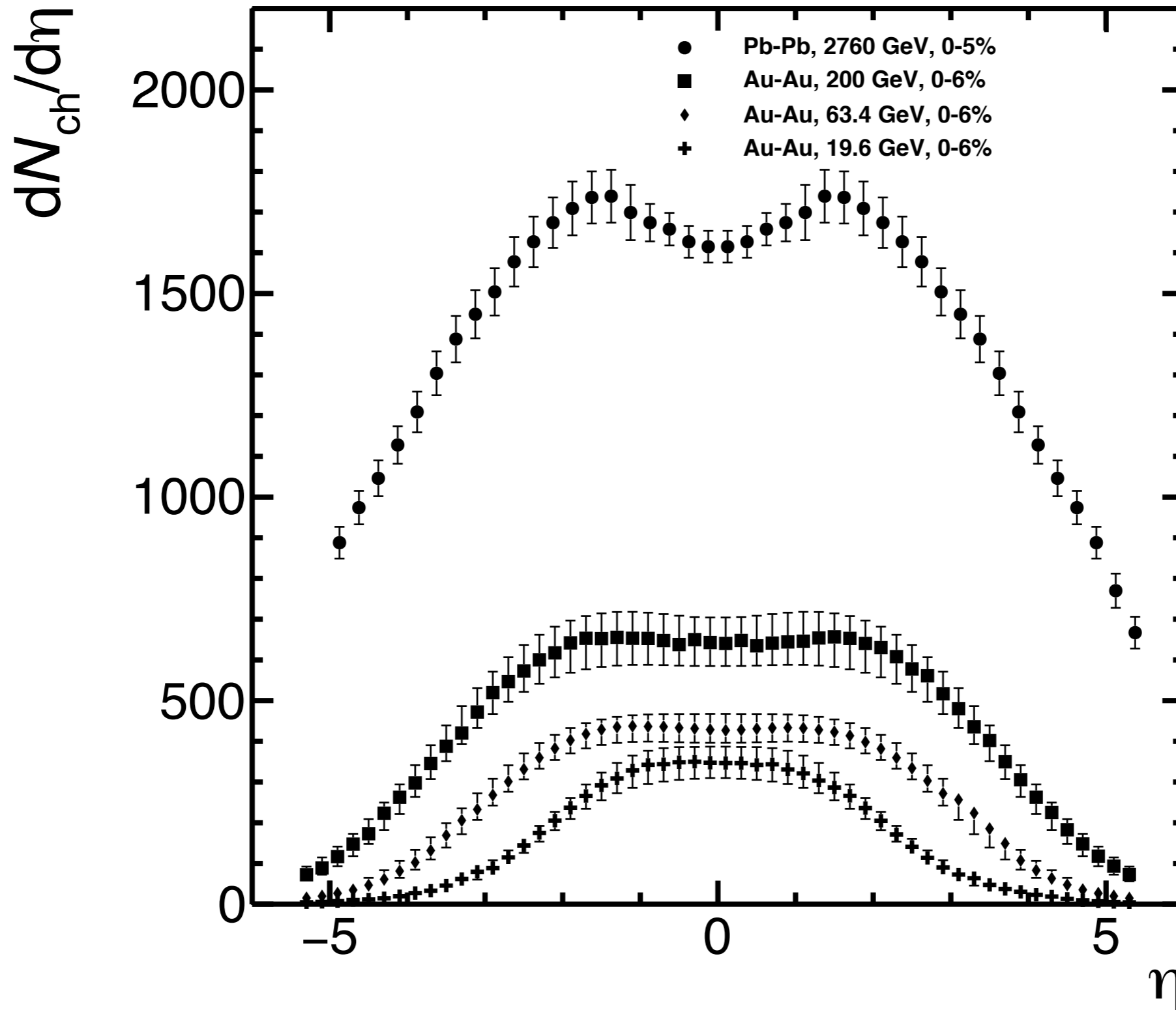
$$N_{\text{part}} = 16$$

$$N_{\text{coll}} = 15$$

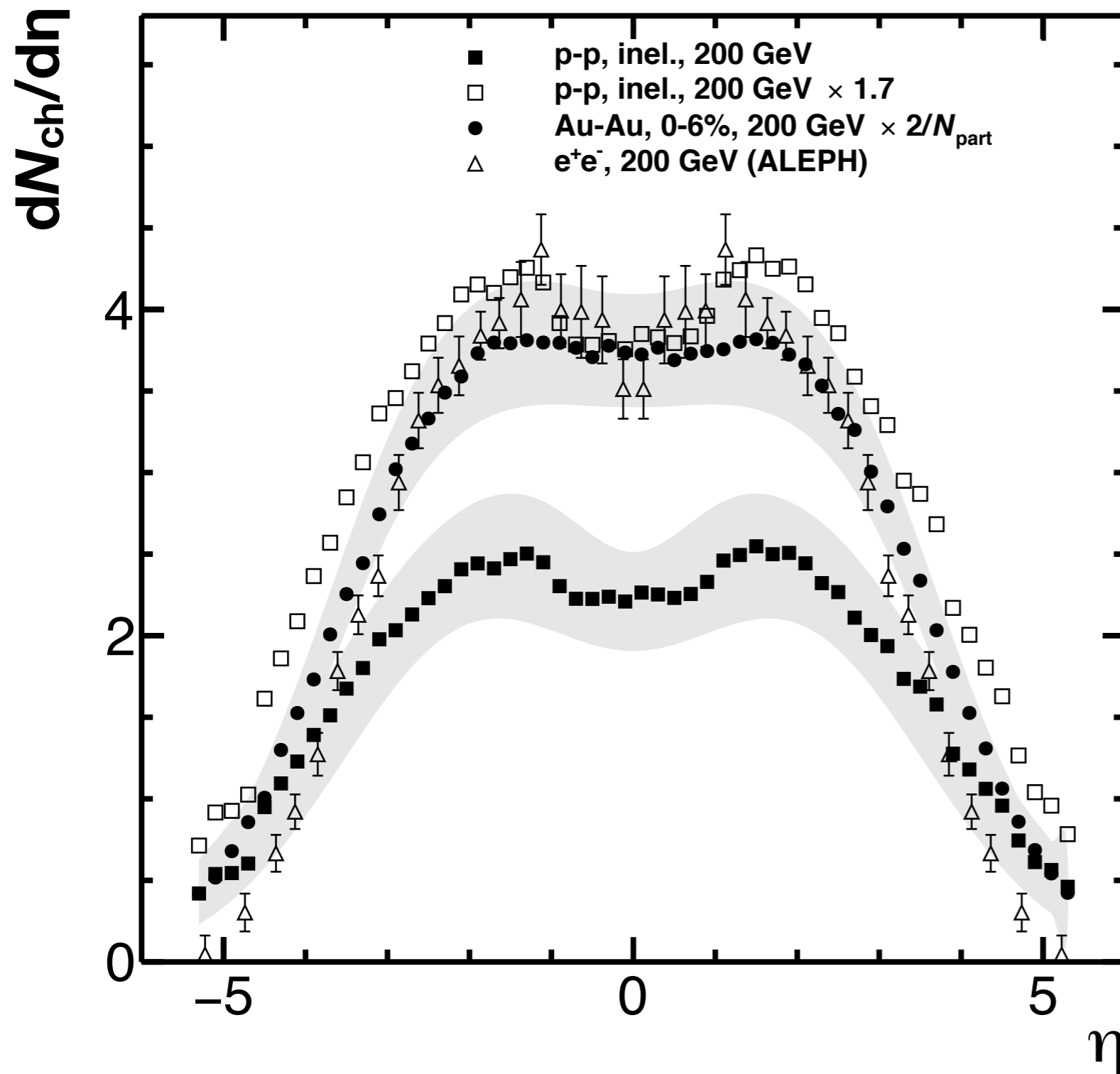
Example shows that for heavy ions (usually)

$$N_{\text{coll}} > N_{\text{part}}$$

Charged particle pseudorapidity distributions for different $\sqrt{s_{NN}}$

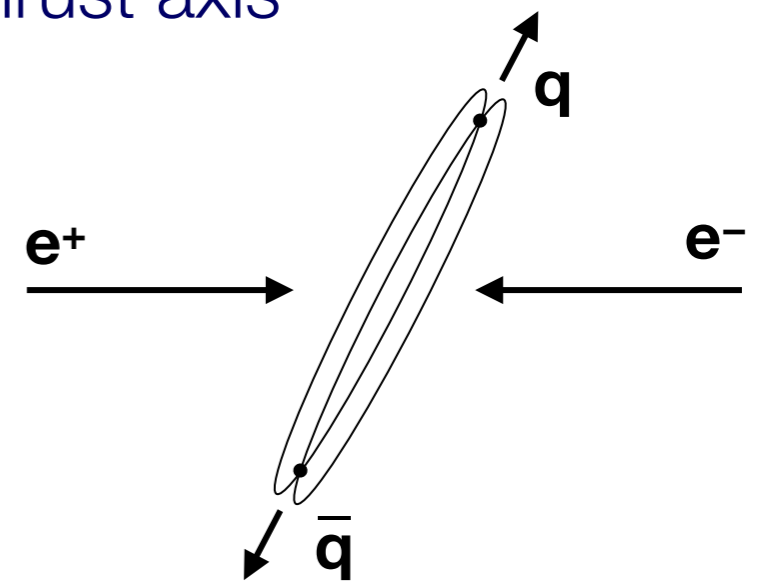


Charged-particle Pseudorapidity Distributions: Comparison e^+e^- , pp, and AA



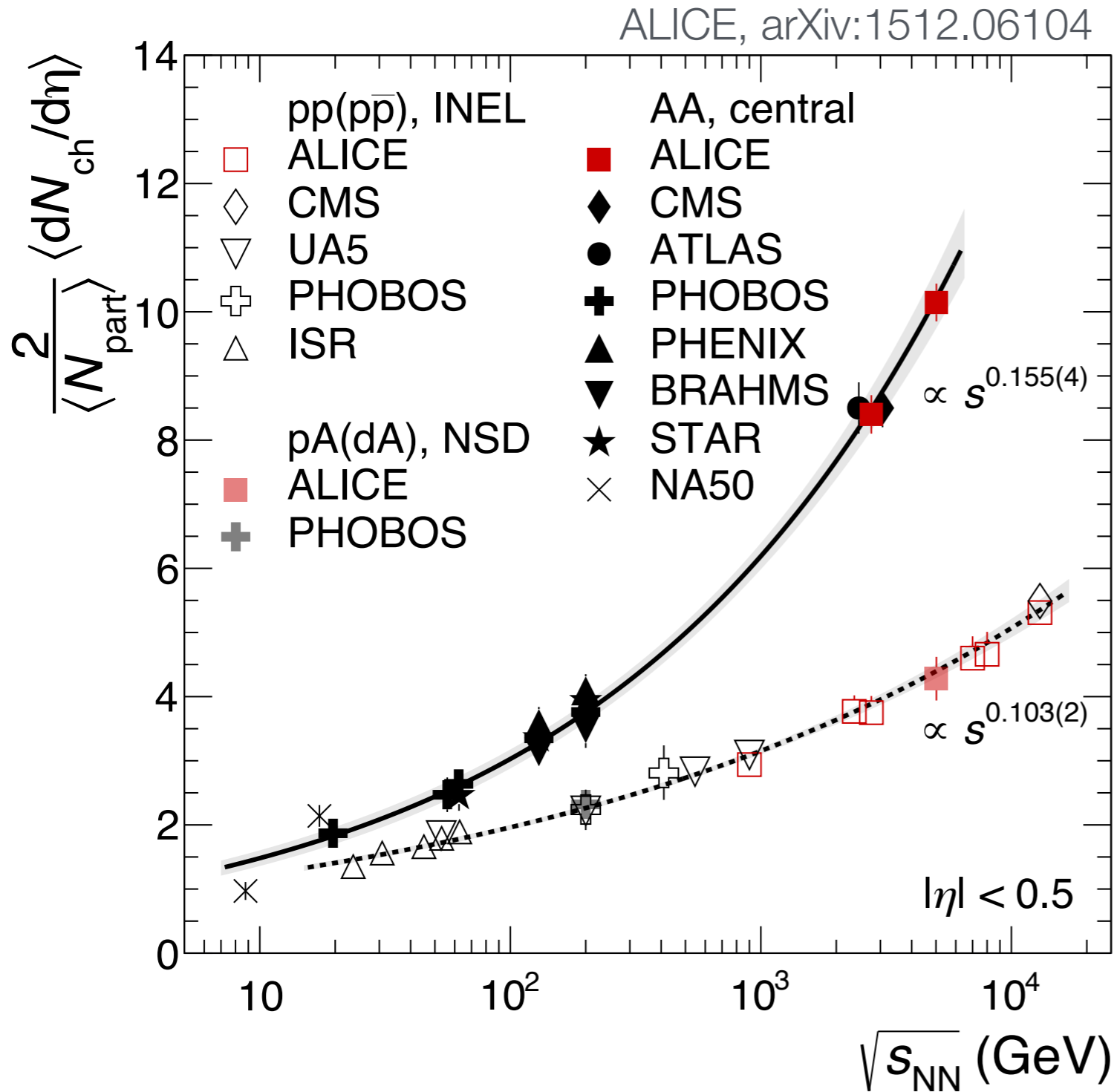
Multiplicity per participant
higher in AA than in pp

e^+e^- :
pseudorapidity along the
thrust axis



AA and e^+e^- η distributions
strikingly similar

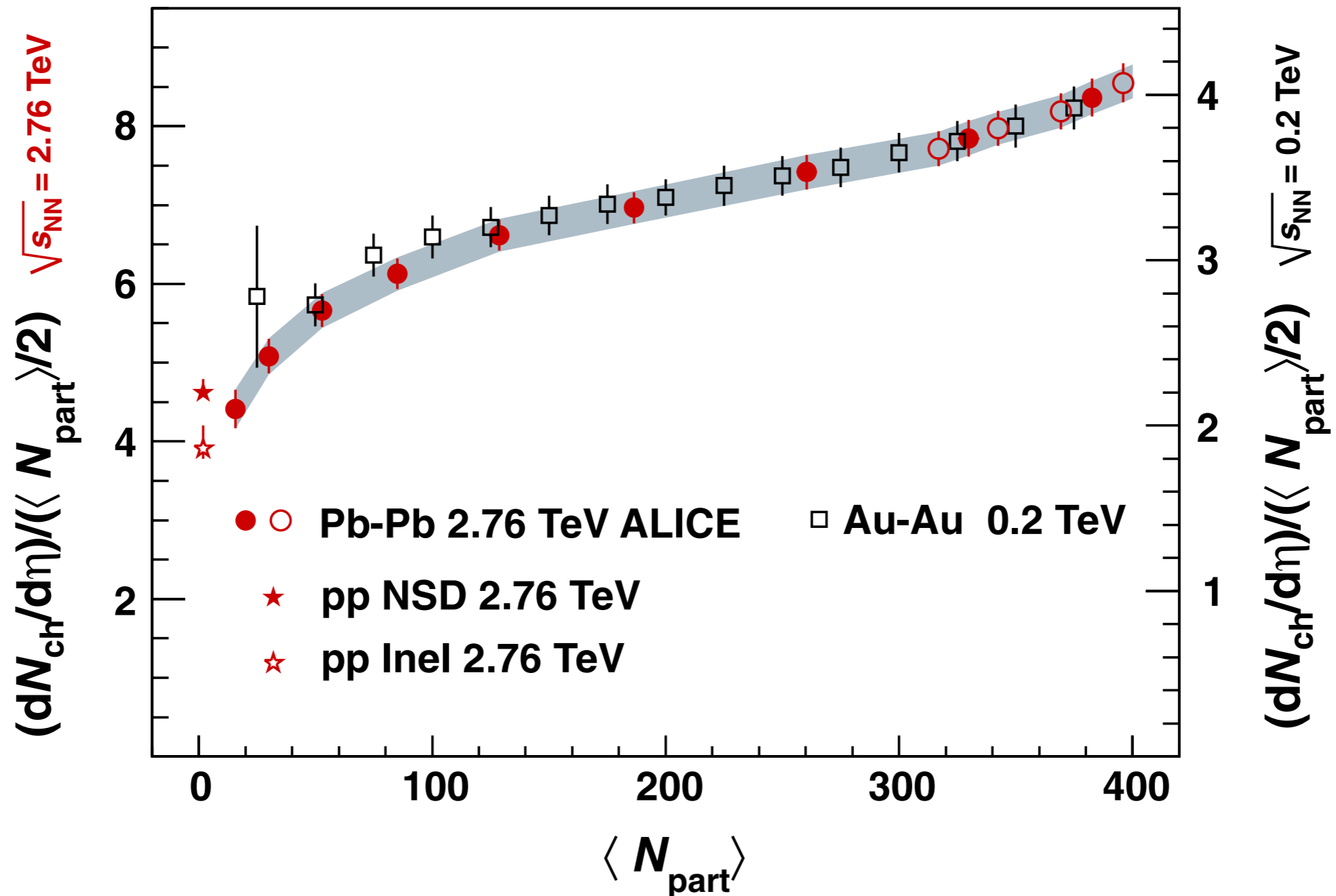
$dN_{ch}/d\eta$ vs $\sqrt{s_{NN}}$ in pp and central A-A collisions



- $dN_{ch}/d\eta$ scales with s^α
- Increase in central A+A stronger than in p+p

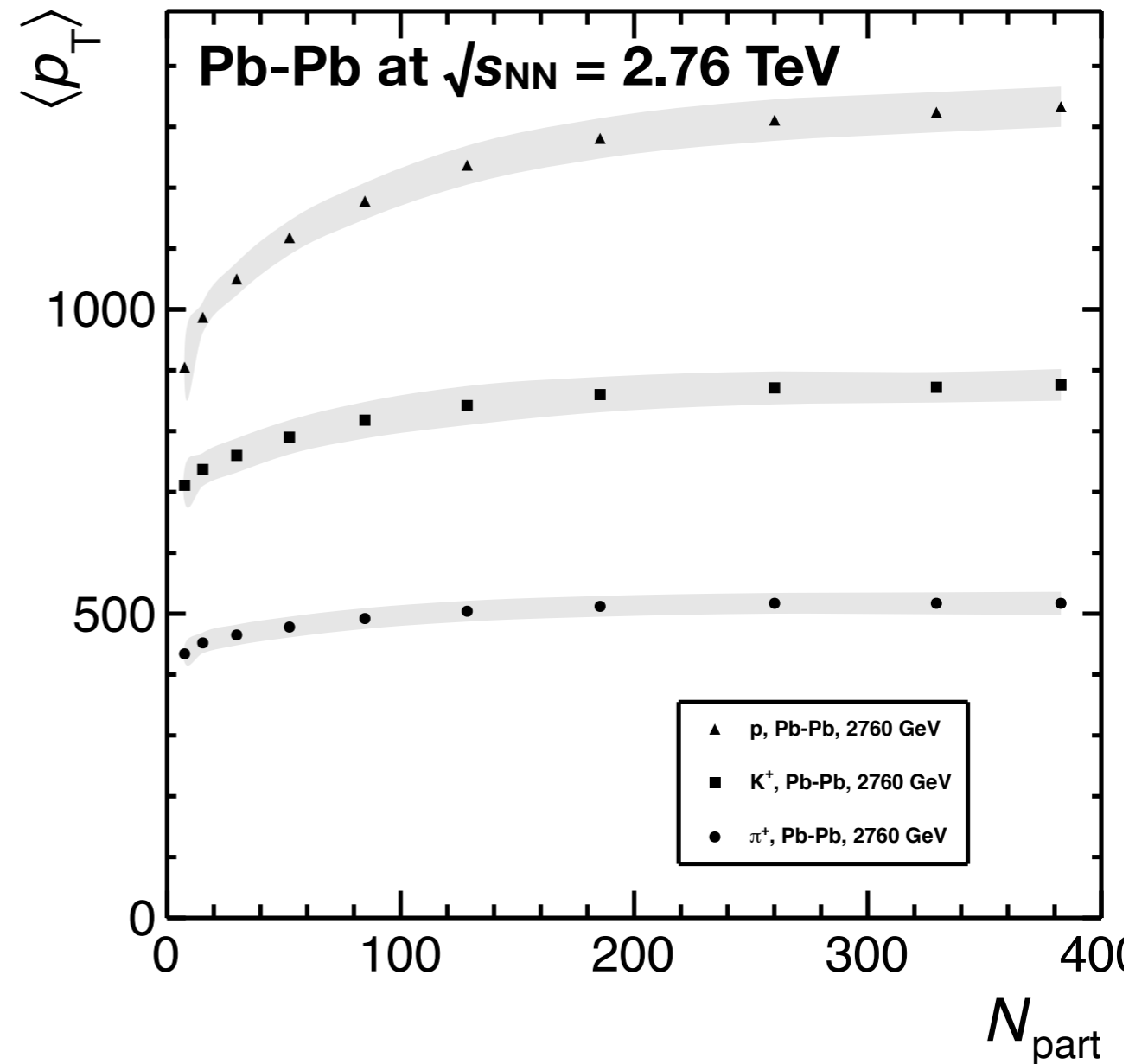
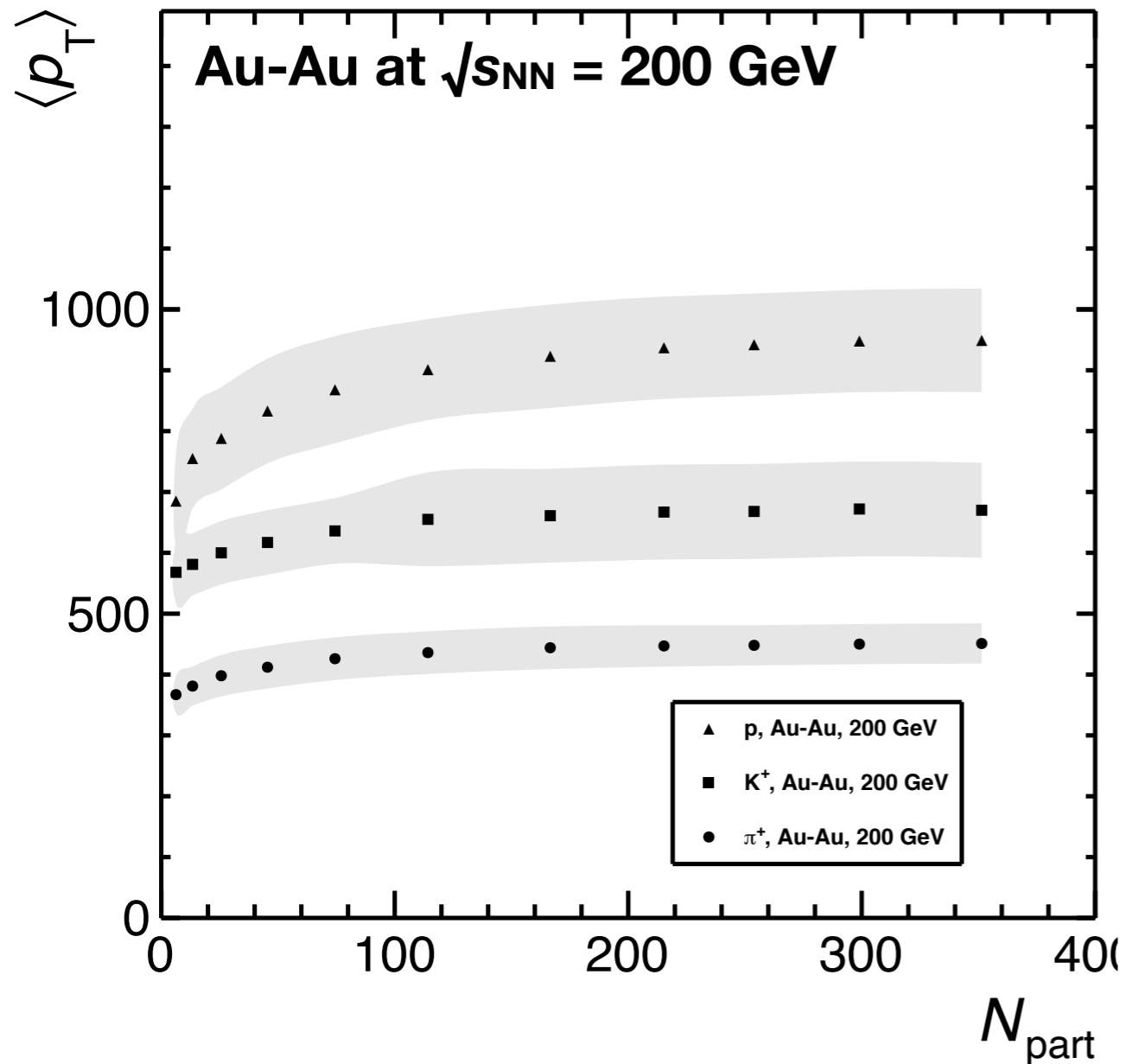
Centrality dependence of $dN_{ch}/d\eta$

ALICE, arXiv:1012.1657



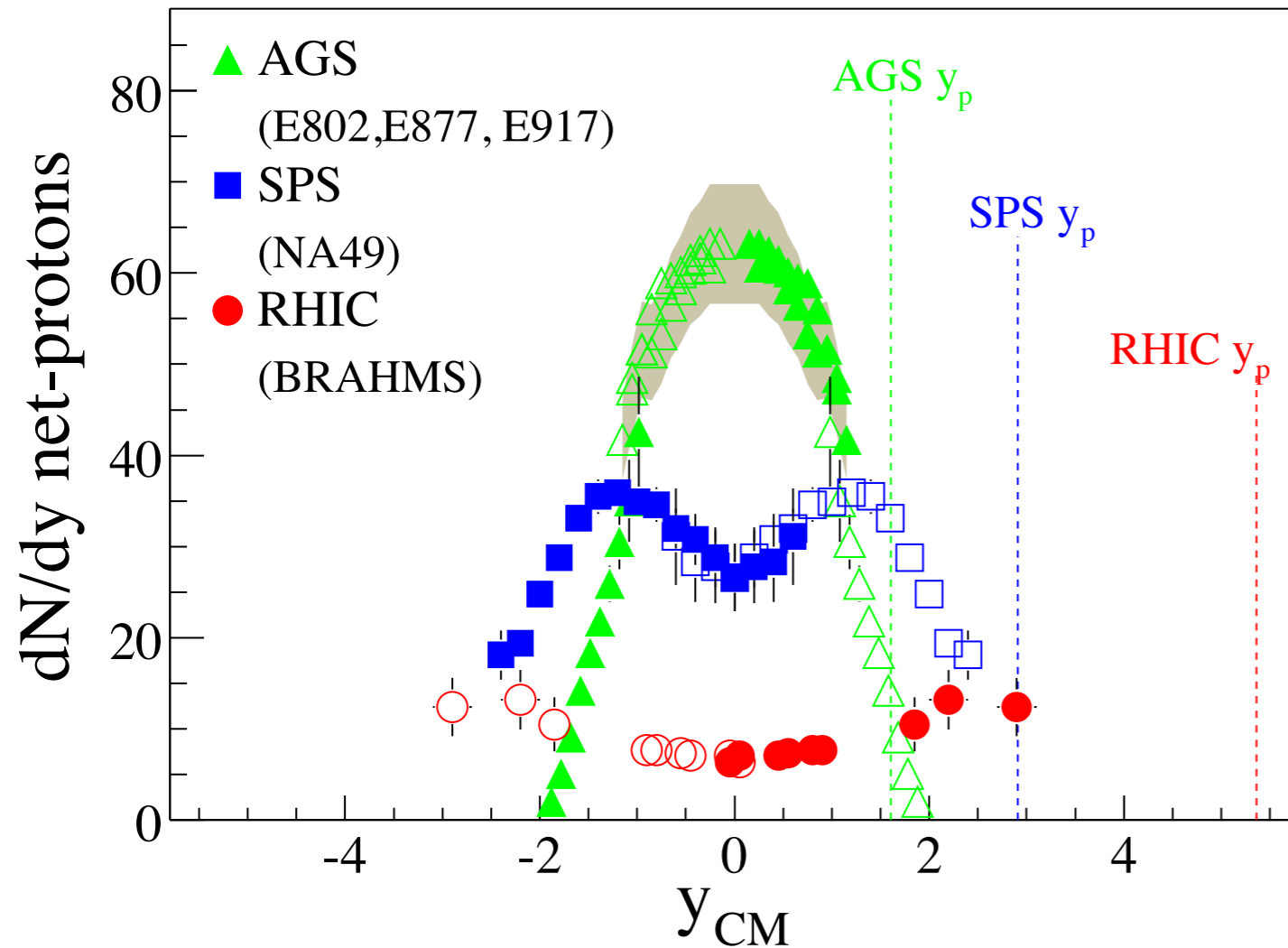
- $dN_{ch}/d\eta / N_{part}$ increases with centrality
- Relative increase similar at RHIC and the LHC: Importance of geometry!

Average p_T of pions, kaons, and protons in Au-Au@200 GeV and Pb-Pb@2.76 TeV



Nuclear stopping power (Au-Au at $\sqrt{s_{NN}} = 200$ GeV)

Brahms, PRL 93:102301, 2004



Average rapidity loss:

Initial rapidity:

$$y_p = 5.36$$

Net baryons after the collision:

$$\langle y \rangle = \frac{2}{N_{\text{part}}} \int_0^{y_p} y \frac{dN_{B-\bar{B}}}{dy} dy$$

Average rapidity loss:

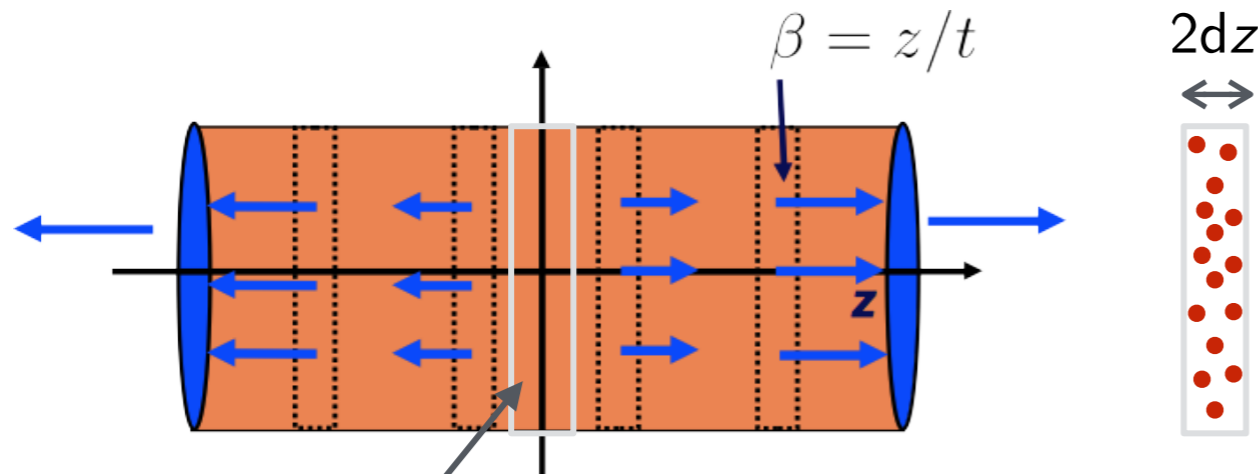
$$\langle \delta y \rangle = y_p - \langle y \rangle \approx 2$$

Average energy per (net) baryon:

$$E_p = 100 \text{ GeV}, \quad \langle E \rangle = \frac{1}{N_{\text{part}}} \int_{-y_p}^{y_p} \underbrace{\langle m_T \rangle \cosh y}_E \frac{dN_{B-\bar{B}}}{dy} dy \approx 27 \pm 6 \text{ GeV}$$

Average energy loss of a nucleon in central Au+Au@200GeV is 73 ± 6 GeV

Bjorken's formula for the initial energy density



Consider total energy in slice at $z = 0$ at time τ_0

Assumptions:

- Particles (quarks and gluons) materialize at proper time τ_0
- Position z and longitudinal velocity (i.e. rapidity) are correlated
 - ▶ As if particles streamed freely from the origin

$$V = A \cdot z$$

$$\beta\gamma = \sinh(y)$$

$$z = \tau \sinh y$$

$$E_T = \langle m_T \rangle \cdot N$$

$$\varepsilon = \frac{E}{V} = \frac{1}{A} \frac{dE}{dz} \Big|_{z=0} = \frac{1}{A} \frac{dE}{dy} \Big|_{y=0} \frac{dy}{dz} \Big|_{z=0} = \frac{1}{A} \frac{dE}{dy} \Big|_{y=0} \frac{1}{\tau} = \frac{\langle m_T \rangle}{A \cdot \tau} \frac{dN}{dy} \Big|_{y=0}$$

A = transverse area

$$\varepsilon = \frac{1}{A \cdot \tau_0} \frac{dE_T}{dy} \Big|_{y=0}, \quad \tau_0 \approx 1 \text{ fm}/c$$

However, this formula neglects longitudinal work:

- ▶ dE/dy drops as a fct. of time
- ▶ Bjorken formula underestimates ε

Energy density in central Pb-Pb collisions at the LHC

$$\begin{aligned}\varepsilon &= \frac{1}{A \cdot \tau_0} \left. \frac{dE_T}{dy} \right|_{y=0} \\ &= \frac{1}{A \cdot \tau_0} J(y, \eta) \left. \frac{dE_T}{d\eta} \right|_{\eta=0} \\ &\quad \text{with } J(y, \eta) \approx 1.09\end{aligned}$$

Transverse area:

$$A = \pi R_{Pb}^2 \quad \text{with } R_{Pb} \approx 7 \text{ fm}$$

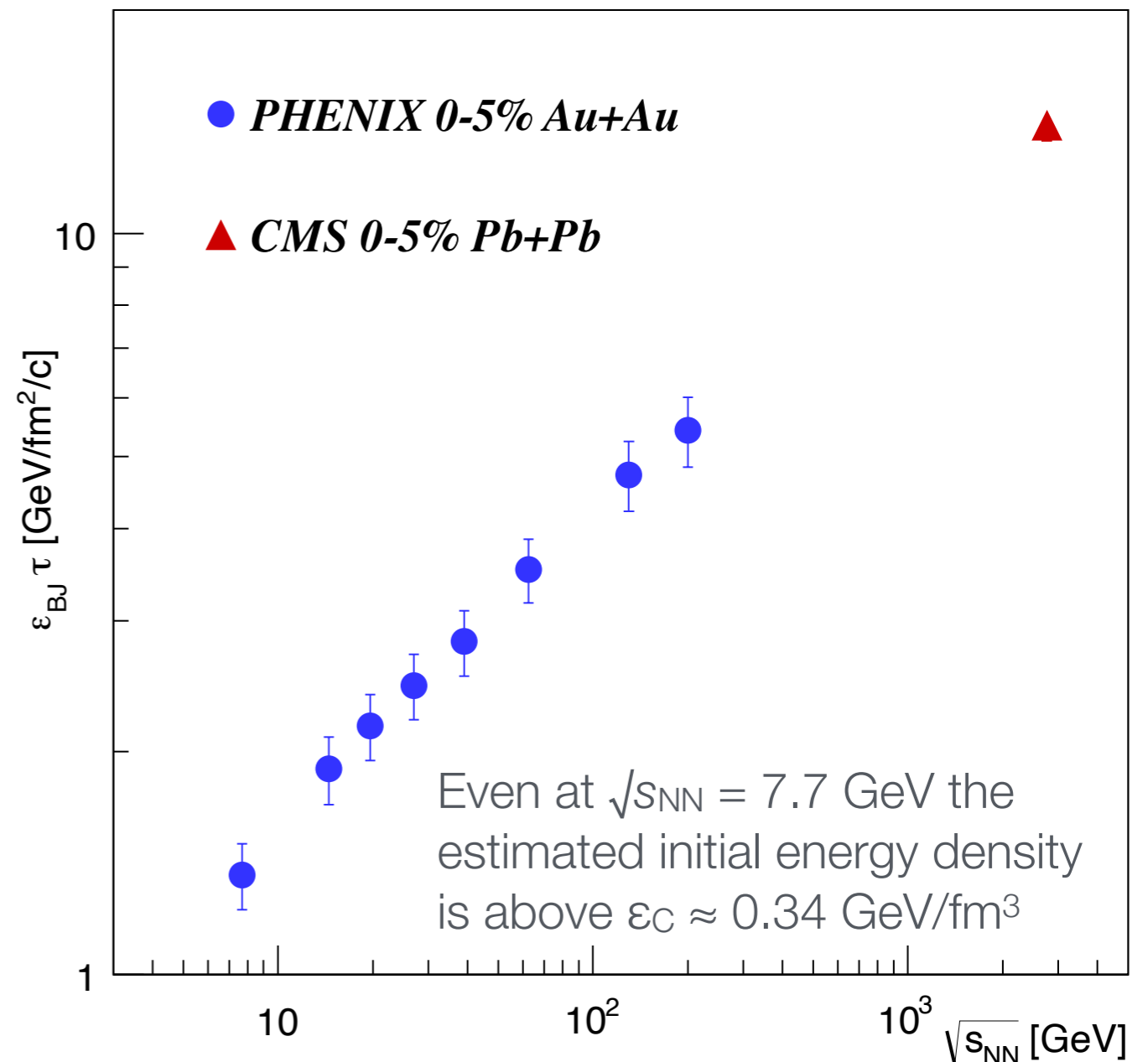
Central Pb-Pb at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$:

$$dE_T/d\eta = 2000 \text{ GeV}$$

Energy density:

$$\begin{aligned}\varepsilon_{LHC} &= 14 \text{ GeV/fm}^3 \\ &\approx 2.6 \times \varepsilon_{RHIC} \text{ for } \tau_0 = 1 \text{ fm}/c\end{aligned}$$

PHENIX, arXiv:1509.06727

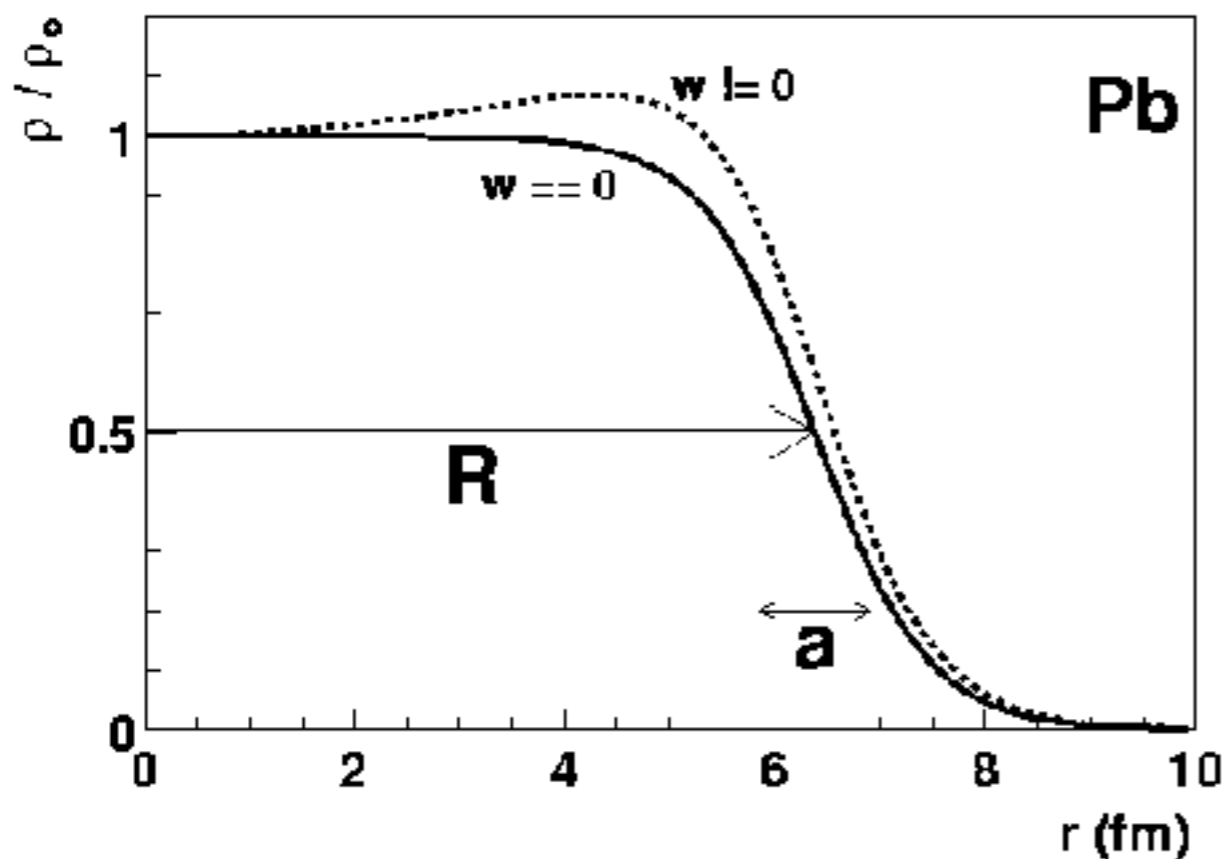


Glauber modeling: An interface between theory and experiment

Starting point: nucleon density

$$\rho(r) = \frac{\rho_0 (1 + wr^2/R^2)}{1 + \exp((r - R)/a)}$$

w = "wine bottle" parameter



H. De Vries, C.W. De Jager, C. De Vries,
Nuclear charge-density-distribution parameters from elastic
electron scattering,
Atomic Data and Nuclear Data Tables, Volume 36, Issue 3, 1987

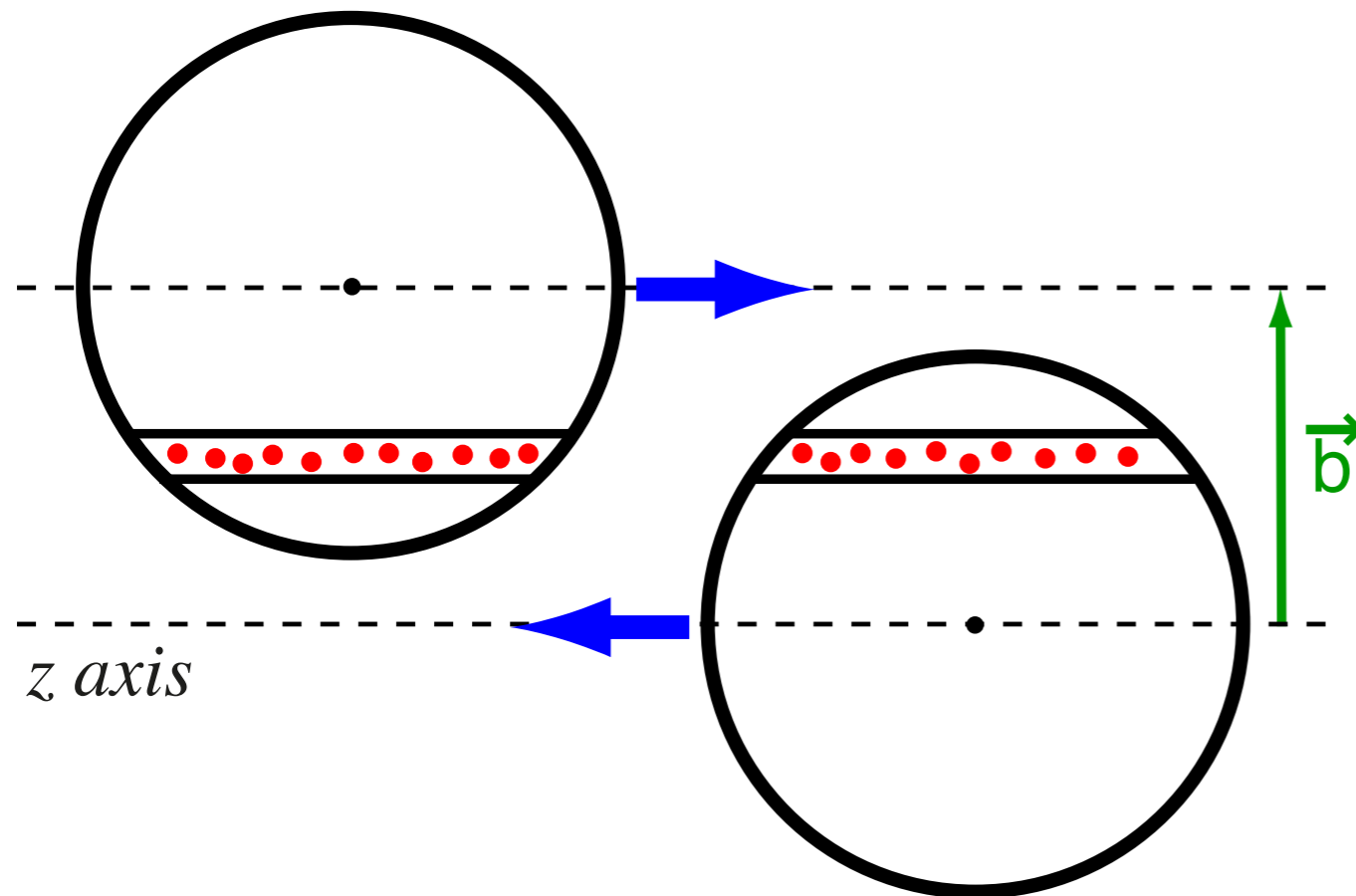
Nucleus	A	R (fm)	a (fm)	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	197	6.38	0.535	0
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

Woods-Saxon parameters typically
from e⁻-nucleus scattering (sensitive to
charge distribution only)

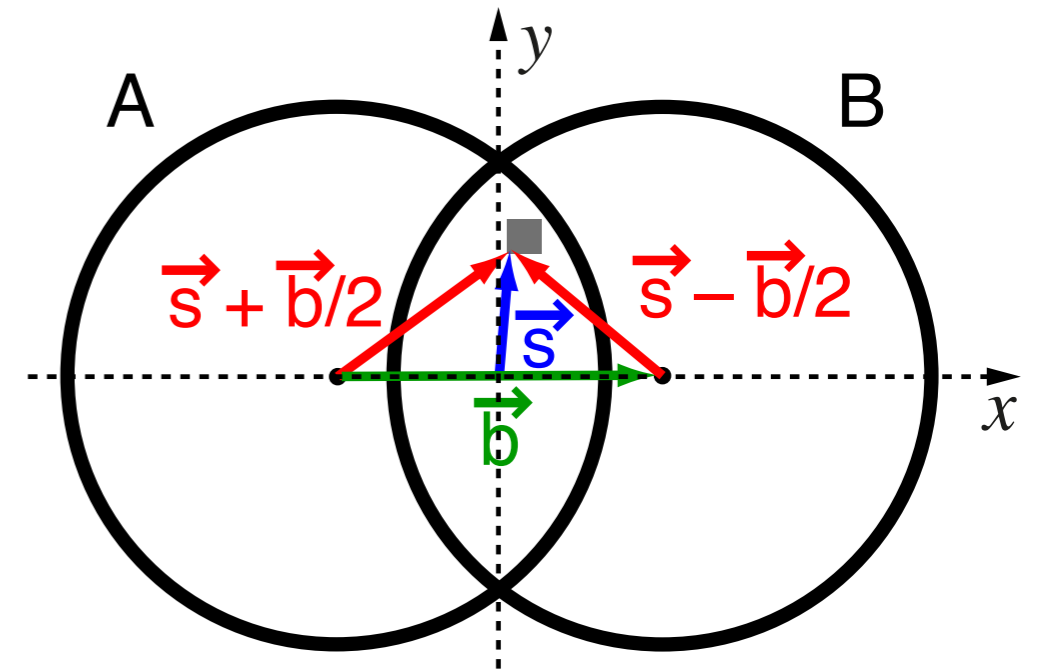
Difference between neutron and
proton distribution small and typically
neglected

Nuclear Thickness Function

side view:



transverse plane:



Projection of nucleon density on the transverse plane ("nuclear thickness fct.):

$$T_A(\vec{s}') = \int dz \rho_A(z, \vec{s}')$$

(analogous for nucleus B)

Number of nucleon-nucleon encounters per transverse area element:

$$dT_{AB} = T_A(\vec{s} + \vec{b}/2) \cdot T_B(\vec{s} - \vec{b}/2) d^2s$$

Nuclear Overlap function and the number of nucleon-nucleon collisions

Nuclear overlap function:

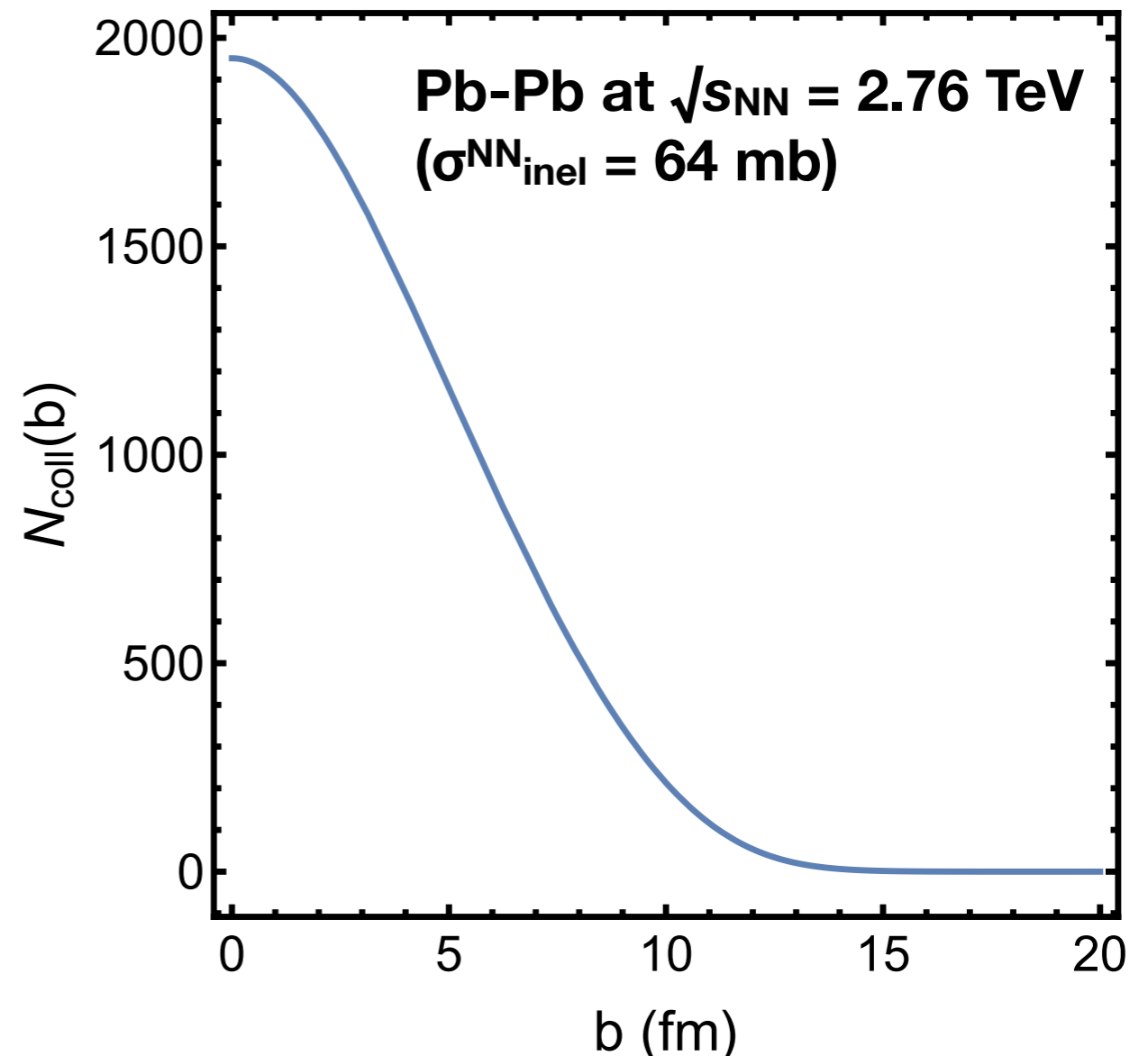
$$T_{AB}(\vec{b}) = \int T_A(\vec{s} + \vec{b}/2) \cdot T_B(\vec{s} - \vec{b}/2) d^2s$$

Nuclear overlap function resembles integrated luminosity of a collider:

$$N_{\text{coll}}(b) = T_{AB}(b) \cdot \sigma_{\text{inel}}^{\text{NN}}$$

Or, more generally for a process with cross section σ_{int} :

$$N_{\text{int}}(b) = T_{AB}(b) \cdot \sigma_{\text{int}}$$



Probability for an Inelastic A+B collision

Def's (different normalization of the thickness functions):

$$\hat{T}_A(\vec{s}') = T_A(\vec{s}')/A \quad \hat{T}_B(\vec{s}') = T_B(\vec{s}')/B \quad \hat{T}_{AB}(\vec{b}) = T_{AB}(\vec{b})/(AB)$$

We can then write:

$$N_{\text{coll}}(b) = AB \hat{T}_{AB}(b) \cdot \sigma_{\text{inel}}^{\text{NN}}$$

$$p_{\text{NN}} = \hat{T}_{AB}(\vec{b}) \cdot \sigma_{\text{inel}}^{\text{NN}}$$

\ probability for a certain nucleon from nucleus A to collide with a certain nucleon from nucleus B

Probability for k nucleon-nucleon coll.:

$$P(k, \vec{b}) = \binom{AB}{k} p_{\text{NN}}^k (1 - p_{\text{NN}})^{AB-k}$$

Probability for $k = 0$ is $(1 - p_{\text{NN}})^{AB}$. Thus:

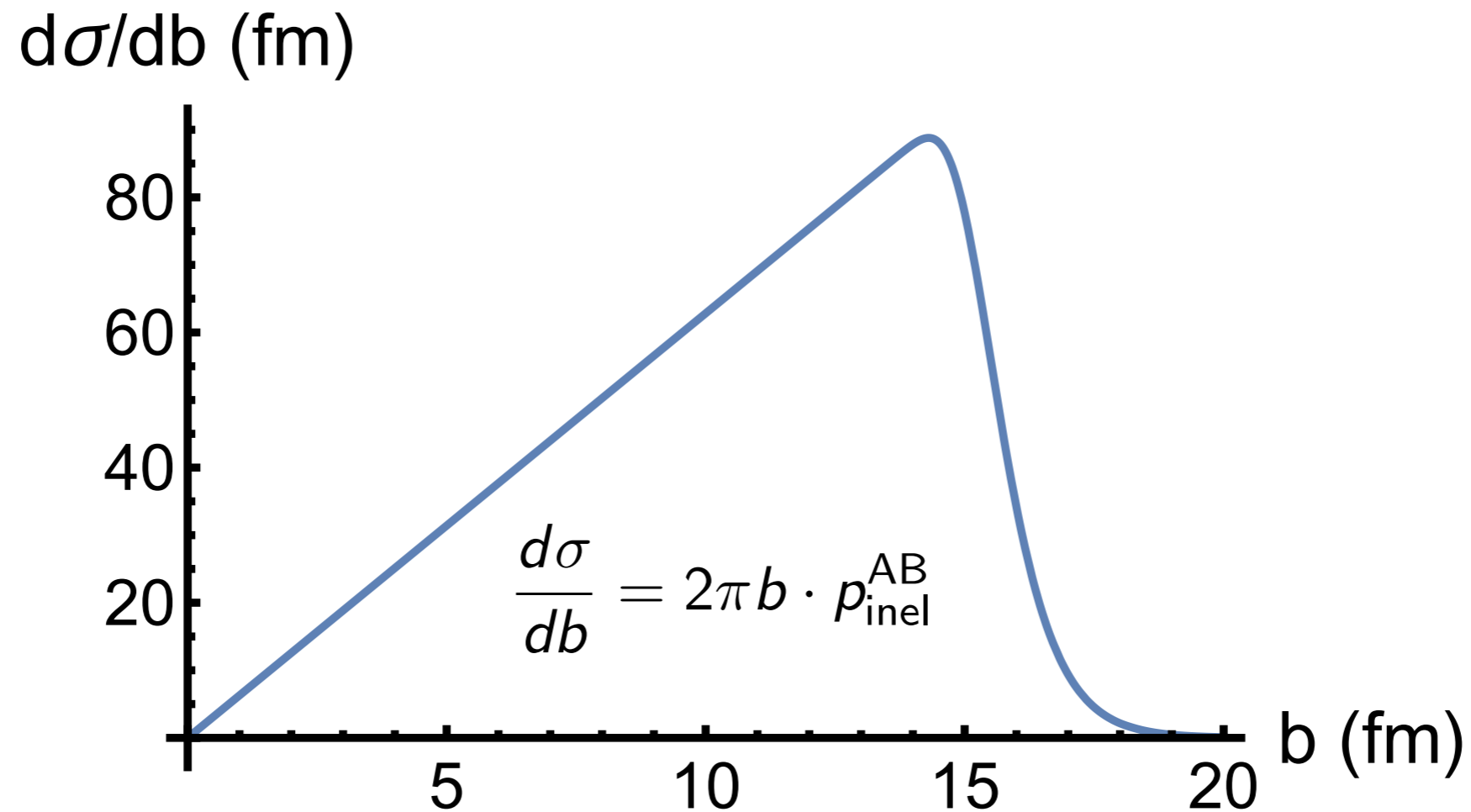
$$p_{\text{inel}}^{\text{AB}}(\vec{b}) = 1 - (1 - \hat{T}_{AB}(\vec{b}) \cdot \sigma_{\text{inel}}^{\text{NN}})^{AB} \approx 1 - \exp(-AB \hat{T}_{AB}(\vec{b}) \cdot \sigma_{\text{inel}}^{\text{NN}})$$

$$(1 - x)^n = \exp(n \ln(1 - x))$$

$$\stackrel{x \rightarrow 0}{\approx} \exp(-nx)$$

\ Poisson limit of the binomial distribution

$d\sigma/db$ for Pb-Pb



Total cross section:
$$\sigma_{\text{inel}}^{\text{AB}} = \int_0^{\infty} \frac{d\sigma}{db} db \approx 784 \text{ fm}^2 = 7.84 \text{ b}$$

Number of Participants

Probability that a test nucleon of nucleus A interacts with a certain nucleon of nucleus B:

$$p_{\text{NN,A}}(\vec{s}) = \hat{T}_{\text{B}}(\vec{s} - \vec{b}/2) \sigma_{\text{inel}}^{\text{NN}}$$

Probability that the test nucleon does not interact with any of the B nucleons of nucleus B:

$$(1 - p_{\text{NN,A}}(\vec{s}))^B$$

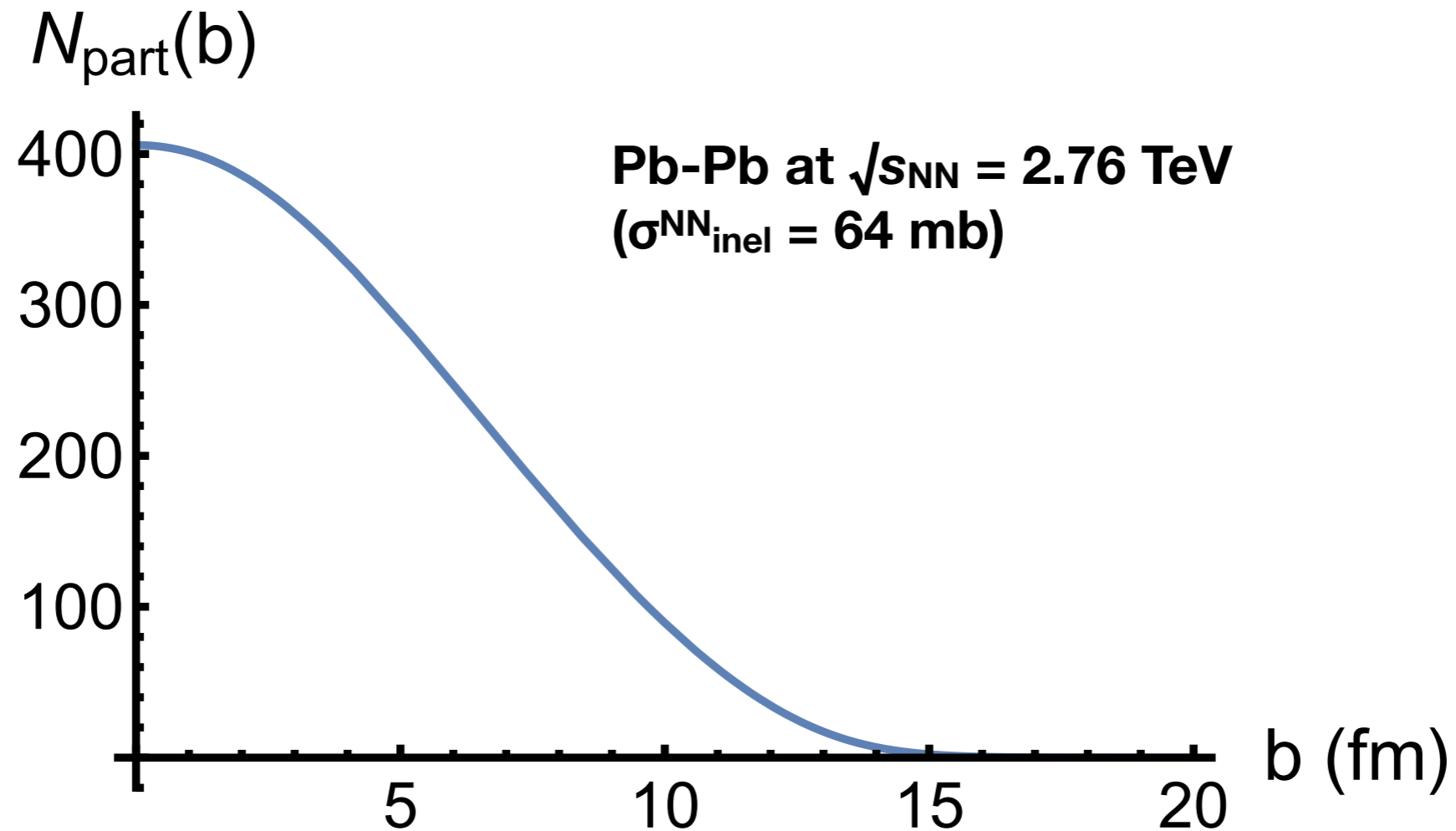
Probability that the test nucleon makes at least one interaction:

$$1 - (1 - p_{\text{NN,A}}(\vec{s}))^B \approx 1 - \exp(-B p_{\text{NN,A}}(\vec{s}))$$

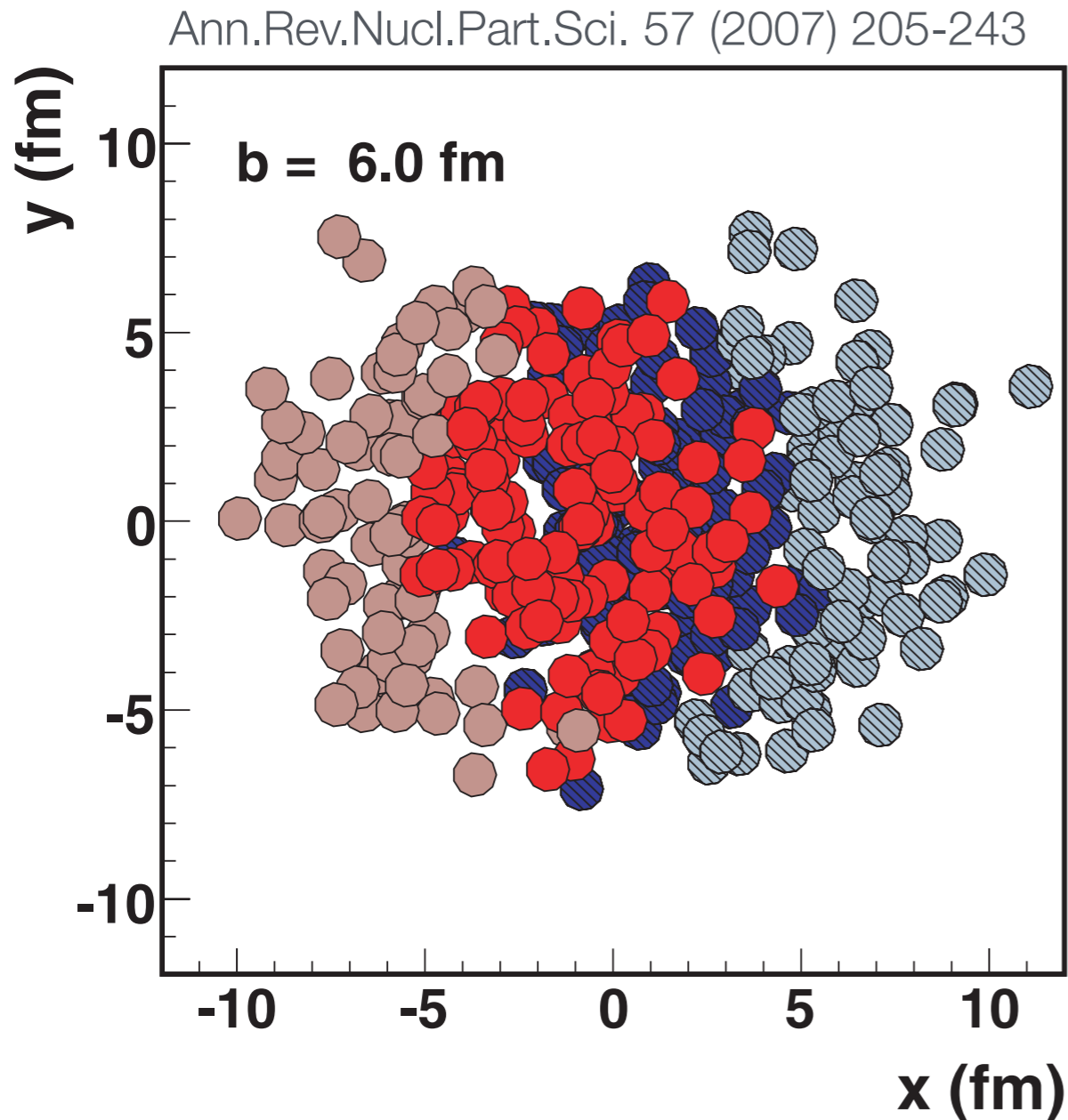
Number of participants:

$$\begin{aligned} N_{\text{part}}(\vec{b}) &= N_{\text{part}}^{\text{A}}(\vec{b}) + N_{\text{part}}^{\text{B}}(\vec{b}) \\ &= \int T_{\text{A}}(\vec{s} + \vec{b}/2) \cdot \left[1 - \exp(-T_{\text{B}}(\vec{s} - \vec{b}/2) \sigma_{\text{inel}}^{\text{NN}}) \right] d^2s \\ &\quad + \int T_{\text{B}}(\vec{s} - \vec{b}/2) \cdot \left[1 - \exp(-T_{\text{A}}(\vec{s} + \vec{b}/2) \sigma_{\text{inel}}^{\text{NN}}) \right] d^2s \end{aligned}$$

N_{part} vs Impact Parameter b



Glauber Monte Carlo Approach



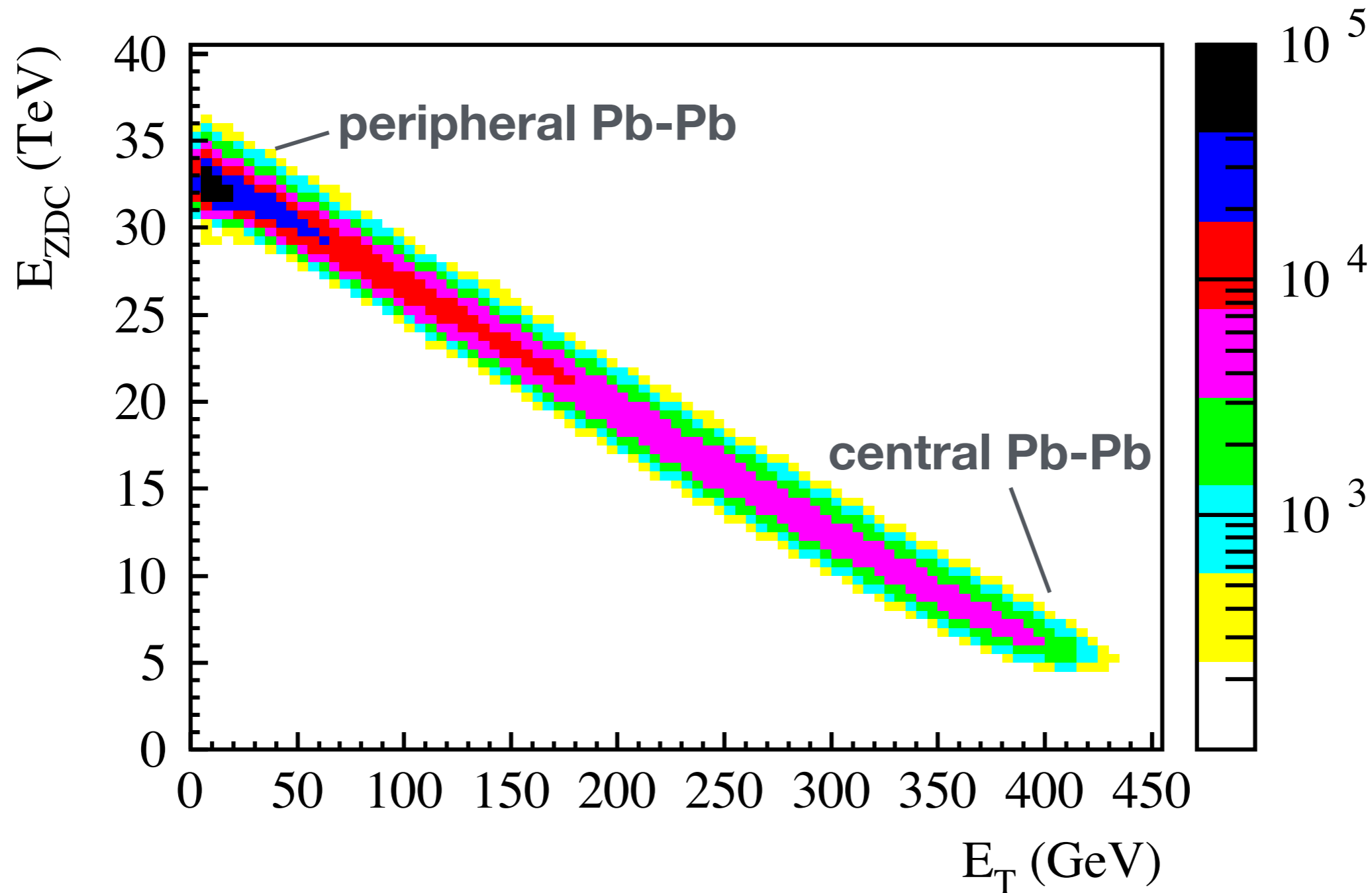
- Randomly select impact parameter b
- Distribute nucleons of two nuclei according to nuclear density distribution
- Consider all pairs with one nucleon from nucleus A and the other from B
- Count pair as inel. n-n collision if distance d in x - y plane satisfies:

$$d < \sqrt{\sigma_{\text{inel}}^{\text{NN}} / \pi}$$

- Repeat many times:
 $\langle N_{\text{part}} \rangle(b)$ $\langle N_{\text{coll}} \rangle(b)$

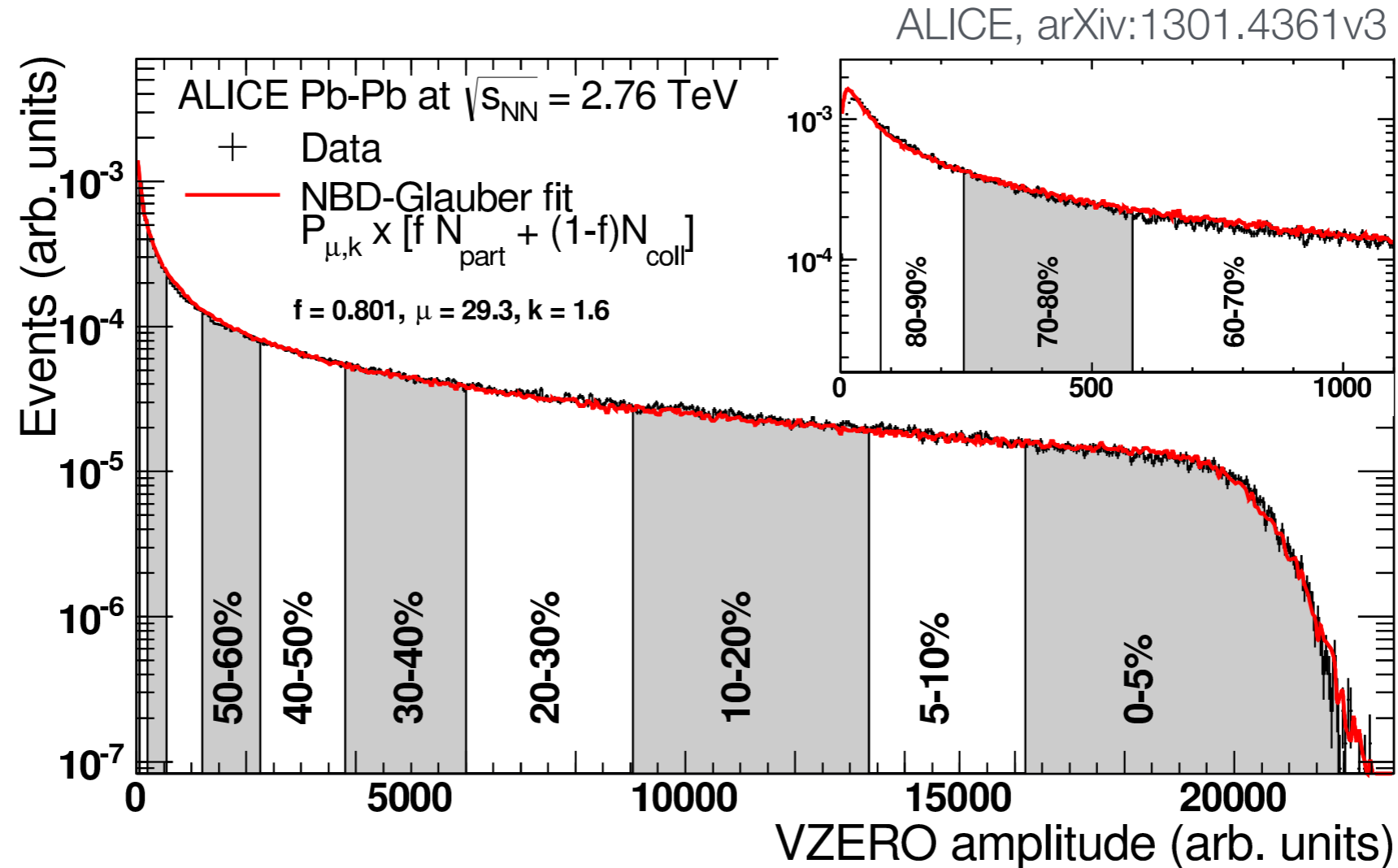
Centrality selection: Forward and transverse energy

Example: Pb-Pb, fixed-target experiment (WA98, CERN SPS)



Both E_T and E_{ZDC} can be used to define centrality classes

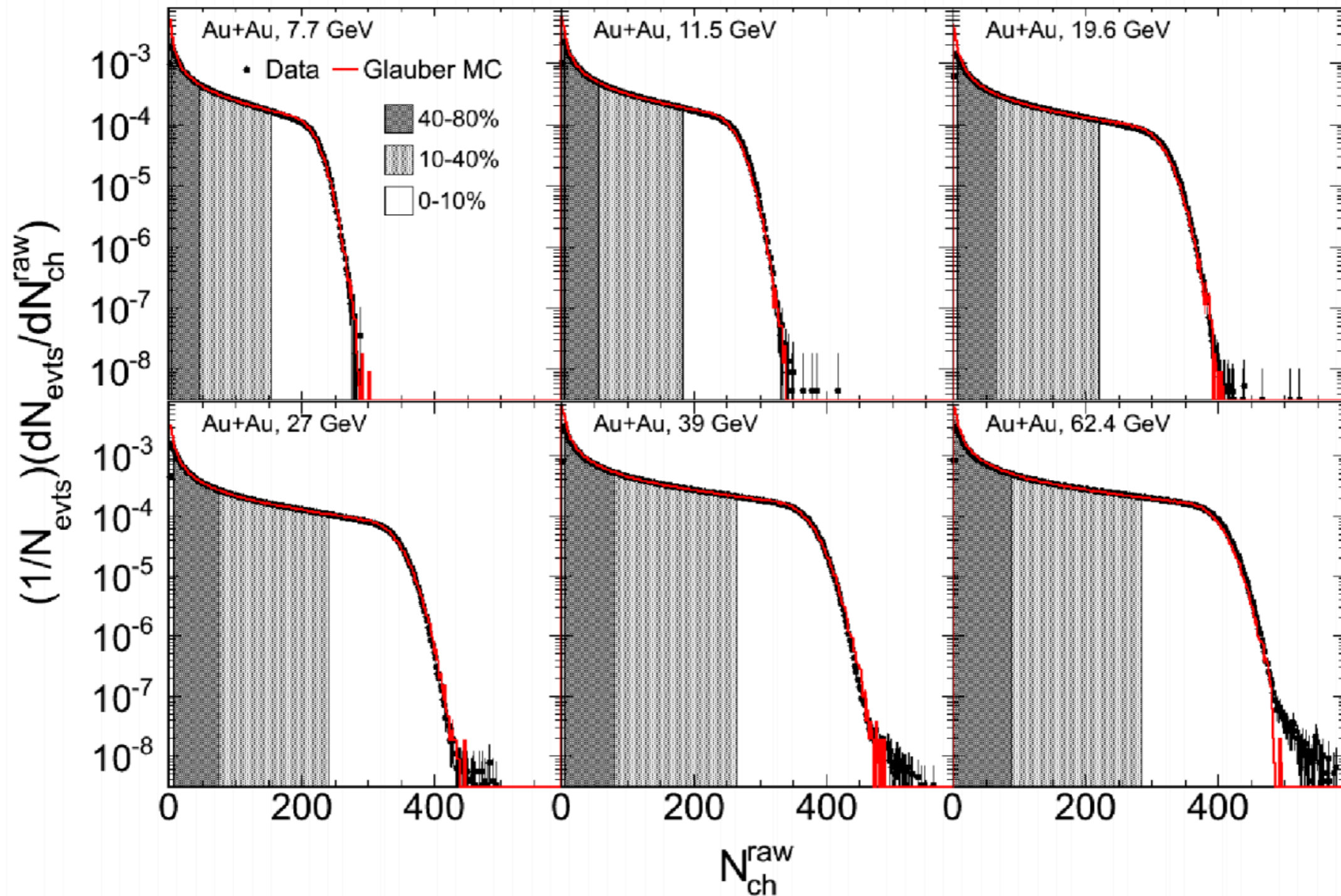
Centrality Selection: Charged-Particle Multiplicity



- Measure charged particle multiplicity
 - ▶ ALICE: VZERO detectors ($2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$)
 - ▶ Assumption: $\langle N_{ch} \rangle(b)$ increases monotonically with decreasing b
- Define centrality class by selecting a percentile of the measured multiplicity distribution (e.g. 0-5%)
 - ▶ Need Glauber fit to define “100%” (background at low multiplicities)

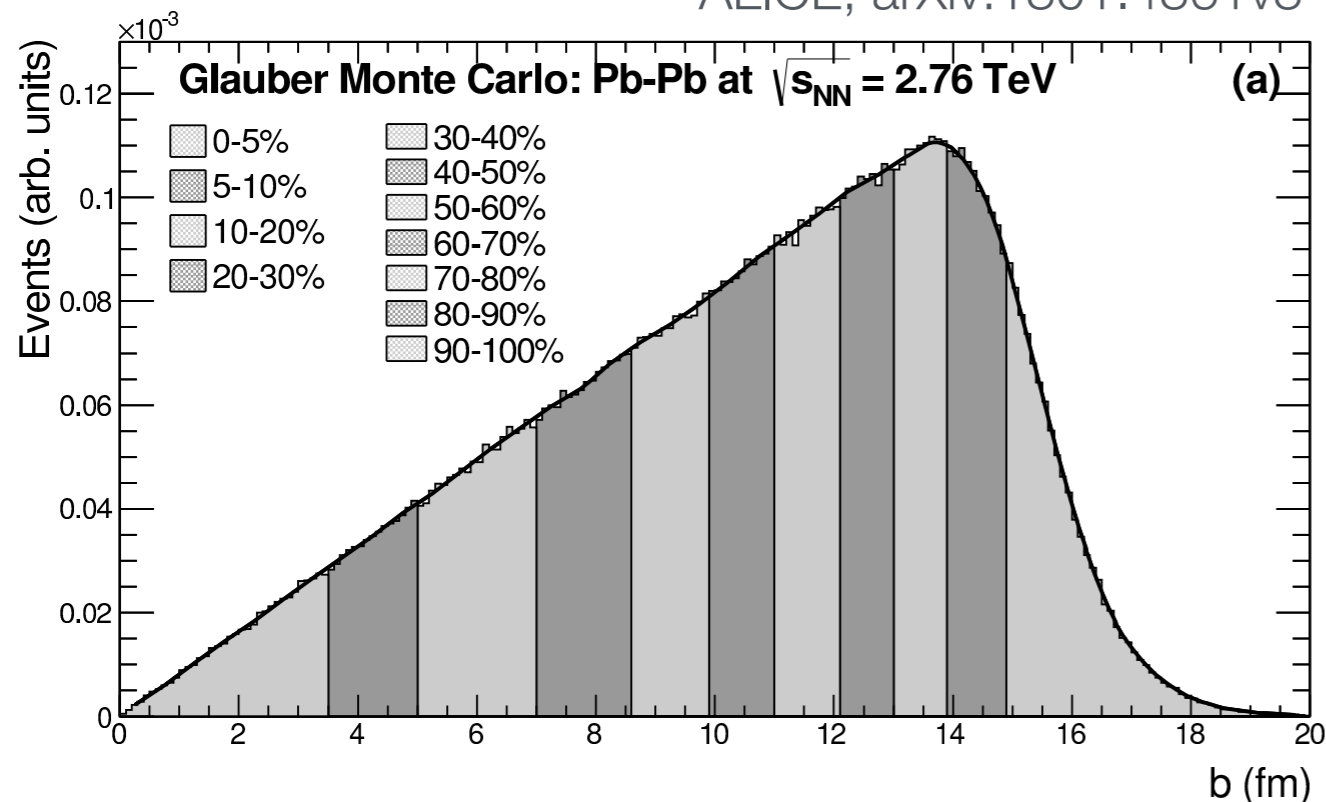
Energy dependence of charged particle multiplicity

Phys.Rev.C 88 (2013) 014902 • e-Print: [1301.2348](https://arxiv.org/abs/1301.2348)



How $\langle N_{\text{part}} \rangle$, $\langle N_{\text{coll}} \rangle$, and $\langle b \rangle$ are Assigned to an Experimental Centrality Class?

ALICE, arXiv:1301.4361v3



■ Glauber Monte Carlo

- ▶ Find impact parameter interval $[b_1, b_2]$ which corresponds to the same percentile
- ▶ Average $N_{\text{part}}(b)$, $N_{\text{coll}}(b)$, etc over this interval

■ Example:

Pb-Pb at $\sqrt{s_{\text{NN}}} = 2.76$ TeV

- ▶ $\sigma_{\text{NN}}(\text{inel}) = (64 \pm 5)$ mb

Centrality	b_{min} (fm)	b_{max} (fm)	$\langle N_{\text{part}} \rangle$	RMS (<i>sys.</i>)	$\langle N_{\text{coll}} \rangle$	RMS (<i>sys.</i>)	$\langle T_{\text{AA}} \rangle$ 1/mbarn	RMS 1/mbarn	(<i>sys.</i>) 1/mbarn		
0-5%	0.00	3.50	382.7	17	3.0	1685	140	190	26.32	2.2	0.85
5-10%	3.50	4.94	329.4	18	4.3	1316	110	140	20.56	1.7	0.67
10-20%	4.94	6.98	260.1	27	3.8	921.2	140	96	14.39	2.2	0.45
20-40%	6.98	9.88	157.2	35	3.1	438.4	150	42	6.850	2.3	0.23
40-60%	9.88	12.09	68.56	22	2.0	127.7	59	11	1.996	0.92	0.097
60-80%	12.09	13.97	22.52	12	0.77	26.71	18	2.0	0.4174	0.29	0.026
80-100%	13.97	20.00	5.604	4.2	0.14	4.441	4.4	0.21	0.06939	0.068	0.0055