

signal, i.e., the integral of the pulse with respect to time. Assuming that the shape of the pulse does not change from one event to the other, however, this integral is directly proportional to the amplitude or *pulse height* of the signal, so that this characteristic may be used instead. The relation between the radiation energy and the total charge or pulse height of the output signal is referred to as the *response* of the detector.

Ideally, of course, one would like this relation to be linear although it is not absolutely necessary. It does, however, simplify matters greatly when transforming the measured pulse height to energy. For many detectors, the response is linear or approximately so over a certain range of energies. In general, however, the response is a function of the particle type and energy, and it does not automatically follow that a detector with a linear response for one type of radiation will be linear for another. A good example is organic scintillator. As will be seen later, the response is linear for electrons down to a very low energies but is nonlinear for heavier particles such as the proton, deuteron, etc. This is due to the different reaction mechanisms which are triggered in the medium by the different particles.

5.3 Energy Resolution. The Fano Factor

For detectors which are designed to measure the energy of the incident radiation, the most important factor is the energy resolution. This is the extent to which the detector can distinguish two close lying energies. In general, the resolution can be measured by sending a monoenergetic beam of radiation into the detector and observing the resulting spectrum. Ideally, of course, one would like to see a sharp delta-function peak. In reality, this is never the case and one observes a peak structure with a finite width, usually Gaussian in shape. This width arises because of fluctuations in the number of ionizations and excitations produced.

The resolution is usually given in terms of the *full width at half maximum* of the peak (FWHM). Energies which are closer than this interval are usually considered unresolvable. This is illustrated in Fig. 5.1. If we denote this width as ΔE , then the relative resolution at the energy E is

$$\text{Resolution} = \Delta E/E. \quad (5.1)$$

Equation (5.1) is usually expressed in percent. A NaI detector has about a 8% or 9% resolution for γ -rays of about 1 MeV, for example, while germanium detectors have resolutions on the order of 0.1%!

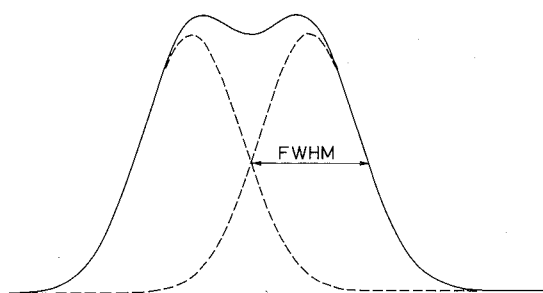


Fig. 5.1. Definition of energy resolution. Two peaks are generally considered to be resolved if they are separated by a distance greater than their full widths at half maximum (FWHM). The *solid line* shows the sum of two identical Gaussian peaks separated by just this amount

In general, the resolution is a function of the energy deposited in the detector, with the ratio (5.1) improving with higher energy. This is due to the Poisson or Poisson-like statistics of ionization and excitation. Indeed, it is found that the average energy required to produce an ionization is a fixed number, w , dependent only on the material. For a deposited energy E , therefore, one would expect on the average, $J = E/w$ ionizations. Thus as energy increases, the number of the ionization events also increases resulting in smaller relative fluctuations.

To calculate the fluctuations it is necessary to consider two cases. For a detector in which the radiation energy is not totally absorbed, for example, a thin transmission detector which just measures the dE/dx loss of a passing particle, the number of signal-producing reactions is given by a Poisson distribution. The variance is then given by

$$\sigma^2 = J, \quad (5.2)$$

where J is the mean number of events produced. The energy dependence of the resolution can then be seen to be

$$R = 2.35 \frac{\sqrt{J}}{J} = 2.35 \sqrt{\frac{w}{E}}, \quad (5.3)$$

where the factor 2.35 relates the standard deviation of a Gaussian to its FWHM. Thus the resolution varies inversely as the square root of the energy.

If the full energy of the radiation is absorbed as is the case for detectors used in spectroscopy experiments, the naive assumption of Poisson statistics is incorrect. And indeed, it is observed that the resolution of many such detectors is actually smaller than that calculated from Poisson statistics. The difference here is that the total energy deposited is a fixed, constant value, while in the previous case, the energy deposited can fluctuate. The total number of ionizations which can occur and the energy lost in each ionization are thus constrained by this value. Statistically, this means that the ionization events are not all independent so that Poisson statistics is not applicable. Fano [5.1] was the first to calculate the variance under this condition and found

$$\sigma^2 = FJ, \quad (5.4)$$

where J is the mean ionization produced and F is a number known as the Fano factor.

The factor F is a function of all the various fundamental processes which can lead to an energy transfer in the detector. This includes all reactions which do not lead to ionization as well, for example, phonon excitations, etc. It is thus an intrinsic constant of the detecting medium. Theoretically, F is very difficult to calculate accurately as it requires a detailed knowledge of all the reactions which can take place in the detector. From (5.4), the resolution is then given by

$$R = 2.35 \frac{\sqrt{FJ}}{J} = 2.35 \sqrt{\frac{Fw}{E}}. \quad (5.5)$$

If $F = 1$, the variance is the same as that for a Poisson distribution and (5.5) reduces to (5.3). This seems to be the case for scintillators, however, for many detectors such as semiconductors or gases, $F < 1$. This, of course, greatly increases the resolution of these types of detectors.

In addition to the fluctuations in ionization, a number of external factors can affect the overall resolution of a detector. This includes effects from the associated electronics such as noise, drifts, etc. Assuming all these sources are independent and distributed as Gaussians, the total resolution E is then given by (4.64), i.e.,

$$(\Delta E)^2 = (\Delta E_{\text{det}})^2 + (\Delta E_{\text{elect}})^2 + \dots \quad (5.6)$$

5.4 The Response Function

For the measurement of energy spectra, an important factor which must be considered is the *response function* of the detector for the type of radiation being detected. This is the spectrum of pulse heights observed from the detector when it is bombarded by a monoenergetic beam of the given radiation. Up to now, we have assumed that this response spectrum is a Gaussian peak. If we ignore the finite width for a moment, this essentially corresponds to a Dirac delta function, i.e., for a fixed incident energy the output signal has a single, fixed amplitude. Then, if the response is linear, the spectrum of pulse heights measured from the detector corresponds directly to the energy spectrum of the incident radiation. This is the ideal case. Unfortunately, a Gaussian peak response is not always realized particularly in the case of neutral radiation.

The response function of a detector at a given energy is determined by the different interactions which the radiation can undergo in the detector and its design and geometry. To take an example, consider monoenergetic charged particles, say electrons, incident on a detector thick enough to stop the particles. Assuming all the electrons lose their energy by atomic collisions, it is clear that the spectrum of pulse heights will be a Gaussian peak. In reality, however, some of the electrons will scatter out of the detector before fully depositing their energy. This produces a low energy tail. Similarly some electrons will emit bremsstrahlung photons which may escape from the detector. This again gives rise to events at a lower energy than the peak. The response function thus consists of a Gaussian peak with a low energy tail determined by the amount of scattering and bremsstrahlung energy loss. If the tail is small, however, this can still be a reasonable approximation to the ideal Gaussian response depending on the precision desired. Moreover, the response function can be improved by changing the design and geometry of the detector. A material of lower atomic number Z can be chosen, for example, to minimize backscattering and bremsstrahlung. Similarly if the detector is made to surround the source, backscattering electrons will be captured thus decreasing the escape of these particles, etc.

To see how the response function can change with radiation type, consider the same detector with gamma rays instead. As we have already mentioned, gamma rays must first convert into charged particles in order to be detected. The principal mechanisms for this are the photoelectric effect, Compton scattering and pair production. In the photoelectric effect, the gamma ray energy is transferred to the photoelectron which is then stopped in the detector. Since the energy of all the photoelectrons is the same, this results in a sharp peak in the pulse height spectrum, which is the desired Gaussian response. However, some gamma rays will also suffer Compton scatterings. As given by (2.109), the Compton electrons are distributed continuously in energy so that a distribution, similar to Fig. 2.24, also appears in the response function. This, of course,

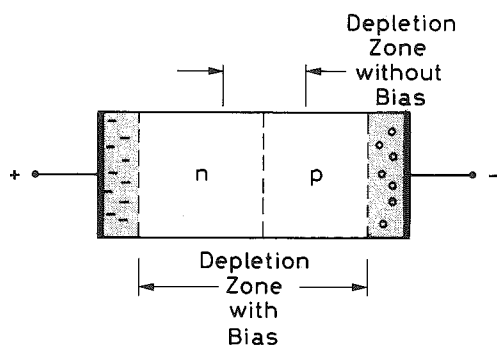


Fig. 10.7. Reversed-bias junction

enlarge the depletion zone and thus the sensitive volume for radiation detection – the higher the external voltage, the wider the depletion zone. Moreover, the higher external voltage will also provide a more efficient charge collection. The maximum voltage which can be applied, however, is limited by the resistance of the semiconductor. At some point, the junction will breakdown and begin conducting.

Under a reverse bias, the width of the depletion layer can be calculated from (10.20) by replacing V_0 with $V_0 + V_B$, where V_B is the bias voltage. In general $V_0 \ll V_B$, so that V_0 may usually be neglected and equations (10.20) to (10.23) used directly with the substitution $V_0 = V_B$. This is also true for the capacitance in (10.25). An interesting point to note is that because of the difference in mobility between electrons and holes, the same bias voltage V_B will yield a somewhat larger depletion depth if the material is n-type rather than p-type.

If we now calculate the depth for n-type silicon, we can see that a depletion layer greater than 1 mm can be obtained if a reverse bias of about $V_B = 300$ V is applied. This, of course, is a great improvement over the few tens of microns in an unbiased junction. With current high resistivity silicon, depletion depths up to 5 mm can be obtained before breakdown occurs. In order to obtain greater depletion widths, even higher resistivity material is necessary which means using higher purity semiconductors or compensated material. These will be discussed in a later section.

10.4 Detector Characteristics of Semiconductors

Having reviewed some of the basic properties of semiconductor materials and junctions, we now turn to some of the characteristics of semiconductors as detectors of radiation.

Figure 10.8 shows the basic configuration used for operating a junction diode as a radiation detector. In order to be able to collect the charges produced by radiation, electrodes must be fitted onto the two sides of the junction. With semiconductors, however, an *ohmic* metal contact cannot, in general, be formed by directly depositing the metal onto the semiconductor material. Indeed, as we shall see in Sect. 10.5.2, contact between many metals and semiconductors results in the creation of a rectifying junction with a depletion zone extending into the semiconductor. To prevent this formation, a heavily doped layer of n^+ and p^+ material is used between the semiconductor and the metal leads. Because of the high dopant concentrations, the depletion depth is

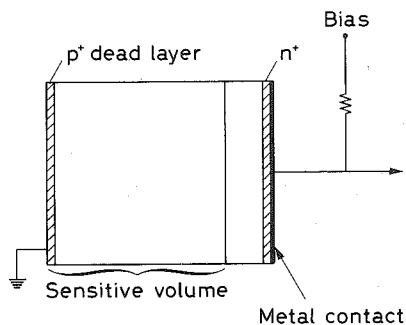


Fig. 10.8. Basic layout of a junction diode detector

then essentially zero as can be seen from (10.19). This then forms the desired *ohmic* contact.

For signal isolation purposes, the bias voltage to the detector is supplied through a series resistor rather than directly. To collect the charge signal from the detector, a preamplifier of the charge-sensitive type is generally used. Because of the low-level of the signal, this preamplifier must have low-noise characteristics. Signal processing after the preamplifier also requires pulse shaping in order to obtain the best signal-to-noise characteristics as explained in Chap. 14.

10.4.1 Average Energy per Electron-Hole Pair

The primary advantage of semiconductors over other detectors is the very small average energy needed to create an electron-hole pair. Like gases, the average energy at a given temperature is found to be independent of the type and energy of the radiation and only dependent on the type of material. Table 10.2 summarizes these values for various semiconductors at normal and liquid nitrogen temperatures.

Table 10.2. Average energy for electron-hole creation in silicon and germanium

	Si	Ge
300 K	3.62 eV	—
77 K	3.81 eV	2.96 eV

For the same radiation energy, the number of charge carriers created will therefore be almost an order of magnitude greater in these materials than in gases. Compared to the number of photoelectrons created in a scintillation counter, this increase approaches two orders of magnitude. Not surprisingly, semiconductors should provide a greatly improved energy resolution.

As small as the average energies are it is interesting to compare these values to the band gaps in Table 10.1. Since the gaps are only on the order 1 eV wide, it is clear that less than a third of the energy deposited by passing radiation is actually spent on the production of electron-hole pairs. The other two thirds, in fact, go into exciting lattice vibrations.

10.4.2 Linearity

Assuming that the depletion region is sufficiently thick to completely stop all particles, the response of semiconductors should be perfectly linear with energy. If E is the energy of the radiation then E/w electron-hole pairs should be created, where w is the average energy from Table 10.2. Assuming a collection efficiency n , then a charge $Q = nE/w$ is collected on the electrodes. Since the depletion region has a capacitance C , as we saw in Sect. 10.3.2, the observed voltage on the electrodes is then

$$V = \frac{Q}{C} = n \frac{E}{wC} \quad (10.26)$$

which varies linearly with E . Moreover, since w is independent of particle type, the response is, in principle, also independent of the type of radiation. This turns out to be true only for lightly ionizing radiation such as electrons and protons. For heavier ions, *plasma* effects occur which affect the collection efficiency and lead to differences in the pulse height between different particles with the same energy. This is discussed further in Sect. 10.9.5.

If the depletion zone is smaller than the range of the radiation, it is clear then that a nonlinear response should be expected since the full energy is not totally deposited in the sensitive volume. What is measured instead is the energy loss ΔE which is a nonlinear function of energy. For a given depletion depth, the response is therefore linear up until the range of the particles exceeds this depth.

10.4.3 The Fano Factor and Intrinsic Energy Resolution

The intrinsic energy resolution, as we saw in Chap. 5, is dependent on the number of charge carriers and the Fano Factor. Despite numerous experiments, the Fano factor for both silicon and germanium is still not well determined. However, it is clear that F is small and on the order of 0.12. This, of course, contributes greatly to enhancing the resolution of semiconductors which already profit greatly from the small average energy required to create an electron hole pair.

From (5.6), the expected resolution is

$$R = 2.35 \sqrt{\frac{F}{J}} = 2.35 \sqrt{\frac{Fw}{E}}, \quad (10.27)$$

where w is the average energy for electron-hole creation and $J = E/w$. For a 5 MeV alpha particle, the intrinsic resolution expected for silicon is therefore $R \approx 0.07\%$ or 3.5 keV. Typical measured resolutions are about 18 keV, however, which indicates that contributions from other sources, e.g., electronics, play important roles.

10.4.4 Leakage Current

Although a reversed biased diode is ideally nonconducting, a small fluctuating current nevertheless flows through semiconductor junctions when voltage is applied. This current appears as noise at the detector output and sets a limit on the smallest signal pulse height which can be observed.

The leakage current has several sources. One is the movement of minority carriers, i.e., holes from the n-region which are attracted across the junction to the p-side and electrons from the p-region which are similarly attracted to the opposite side. This current is generally quite small and in the range of nanoamperes per cm^2 . A second source is thermally generated electron-hole pairs originating from recombination and trapping centers in the depletion region. These centers cannot capture electrons or holes since they have all been swept out, but they can catalyze the creation of electrons and holes from the valence band by serving as intermediate states. The contribution from this source depends on the absolute number of traps in the depletion region and thus on their concentration and the volume of the zone. In general, current densities on the order of a few $\mu\text{A}/\text{cm}^2$ can be expected. The third and by far the largest source of leakage current is through surface channels. This component is complex and depends on very many factors including the surface chemistry, the existence of contaminants, the surrounding atmosphere, the type of mounting, etc. Clean encapsulation is generally required here to minimize this component.

10.4.5 Sensitivity and Intrinsic Efficiency

For charged particles, the intrinsic detection efficiency of semiconductors is close to 100% as very few particles will fail to create some ionization in the sensitive volume. The limiting factor on sensitivity is the noise from leakage currents in the detector and the associated electronics which set a lower limit on the pulse amplitude which can be detected. To ensure an adequate signal, the depletion depth must therefore be chosen sufficiently thick such that enough ionization will be produced to form a signal larger than the noise level. These events are then detected with almost 100% efficiency. If energy measurements are being made, however, the depletion depth must also be larger than the range of the particles. Figure 10.9 shows the range of various particles in silicon.

For gamma-ray detection, germanium is preferred over silicon because of its high atomic number. However, because of the smaller band gap, the leakage current in germanium at normal temperatures is too high to be acceptable and must be cooled to liquid nitrogen temperatures.

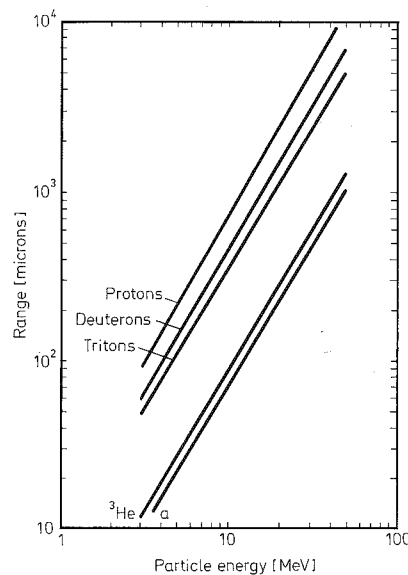


Fig. 10.9. Range of various particles in silicon (from Skyrme [10.10])

and nitrogen temperatures. For low energy x-rays below ~ 30 keV, silicon detectors are preferred because of the K -edge in germanium which is located at ~ 11 keV. Photon absorption at these energies, of course, would yield almost zero energy photoelectrons.

10.4.6 Pulse Shape. Rise Time

Because the collection time for electrons and holes depends on the location of the charges with respect to the electrodes, pulse shapes from semiconductors vary in form and rise time.

As in gas detectors, the electrical pulse on the electrodes arises from induction caused by the movement of the charges rather than the actual collection of the charge itself. Assuming two parallel electrodes, (6.29) gives the change in potential energy for a charge q which moves a distance dx . Since we are interested in the charge collected we can reexpress this as

$$dQ = \frac{q dx}{d}, \quad (10.28)$$

where d is the distance between electrodes. Although (10.28) is derived for the case of empty space between the electrodes, it can be shown that this is also valid [10.11] in the presence of a space-charge as well.

Let us take the example of a pn-junction detector. These are generally formed from a p-type material which is heavily doped on one side with n-type donors. As we saw in Sect. 10.3.1, the depletion zone then extends almost entirely into the p-side. This is illustrated in Fig. 10.10 which also shows the electric field in the zone as given by (10.13). Using the coordinate system shown in the figure, (10.13) is rewritten as

$$E = -\frac{eN_A}{\epsilon}x. \quad (10.29)$$

From (10.9), we have the result that the conductivity, $\sigma \approx eN_A\mu_h$. Substituting into (10.29) then gives us

$$E = -\frac{x}{\mu_h\tau}, \quad (10.30)$$

where we have defined $\tau = \epsilon/\sigma = \rho\epsilon$ and ρ is the resistivity.

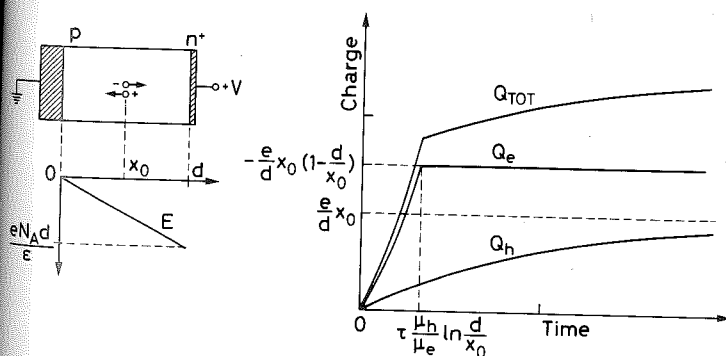


Fig. 10.10. Signal pulse shape due to a single electron-hole pair in an np junction

Assume now that an electron-hole pair is created at a point x in the depletion zone. The electron will thus begin to drift towards the n^+ layer and the hole towards the p electrode. From the definition of mobility, we have for electrons

$$v = \frac{dx}{dt} = -\mu_e E = \frac{\mu_e}{\mu_h} \frac{x}{\tau} \quad (10.31)$$

Assuming that the mobilities are independent of E , this yields the solution

$$x(t) = x_0 \exp \frac{\mu_e t}{\mu_h \tau} \quad (10.32)$$

The time it takes for the electron to reach the electrode at $x = d$ is then

$$t = \tau \frac{\mu_h}{\mu_e} \ln \frac{d}{x_0} \quad (10.33)$$

The charge induced as a function of t during this period is thus

$$Q_e(t) = -\frac{e}{d} \int \frac{dx}{dt} dt = \frac{e}{d} x_0 \left(1 - \exp \frac{\mu_e t}{\mu_h \tau} \right) \quad (10.34)$$

Similarly for the hole, we have the equation,

$$v_h = \frac{dx}{dt} = \mu_h E = -\frac{x}{\tau} \quad (10.35)$$

which yields the solution,

$$x(t) = x_0 \exp \frac{-t}{\tau} \quad (10.36)$$

The collection time in this case is infinite and the induced charge,

$$Q_h(t) = -\frac{e}{d} x_0 \int \exp \frac{-t}{\tau} \frac{dt}{\tau} = -\frac{e}{d} x_0 \left(1 - \exp \frac{t}{\tau} \right) \quad (10.37)$$

The pulse shape is now given by the total charge induced and this is diagrammed in Fig. 10.10 along with the contributions Q_e and Q_h . The total charge collected is $Q_{\text{tot}} = -e$ as can be shown by taking the maximum limits on (10.34) and (10.37).

The parameter τ , as can be seen, determines the rise time of the signal. In silicon, this is given roughly by $\tau = \rho \cdot 10^{-12}$ s, where ρ is in Ω cm. For typical 1000 Ω cm material, τ is thus on the order of a nanosecond.

The above calculation, of course, was only for a single electron-hole pair. To calculate the pulse shape due to incident radiation, it is necessary to know the particle trajectory, the density of ionization along the track, the variation of mobilities, the electric field distribution, etc. and integrate all these factors – a complicated task!