BI Praktikum
Theory 1

Theory:

Pion lifetime and decay

π - Meson

- Lightest hadronic state
- Three charge states π^+, π^0, π^-
- $m_{\pi^\pm} \approx 139\ MeV/c^2$, $m_{\pi^0} \approx 135\ MeV/c^2$

  m : rest mass.

  From now on: $c = 1$

- Charged π discovered 1947 by Occhialini and Powell in photoemulsion exposed to cosmic rays:

  $$\pi^+ \to \mu^+ (\nu_\mu) \to e^+ (\nu_\mu \bar{\nu}_\mu \nu_e)$$

  Predicted by Yukawa 1935

- Rare decay of π^+: $\pi^+ \to e^+ \nu_e$ $\quad B(\pi^+ \to e^+ \nu_e) \approx 1.2 \cdot 10^{-4}$

- Both decays $\pi^+ \to e^+ \nu_e$ and $\pi^+ \to \mu^+ \nu_\mu$ are weak decays

  $$\tau(\pi^\pm) = 2.6 \cdot 10^{-8}\, s \qquad (c\tau = 7.8\, m)$$

- In contrast $\pi^0 \to \gamma\gamma$ is an electromagnetic decay

  $$\tau = 8.4 \cdot 10^{-17}\, s$$

- Spin of π $\quad J(\pi) = 0$
- Parity of π $\quad P|\pi\rangle = -|\pi\rangle$

  parity operator P: discrete transformation $\vec{x}' = -\vec{x}$

  $\Rightarrow$ Particles with $J^P = 0^-$ are <u>pseudoscalars</u>

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Quark model

- Mesons consist of quark-antiquark pair:

$q\bar{q}$ ~ valence quarks

Pion: $|q\,\bar{q}\rangle$

$$\left.\begin{array}{ll} \pi^+ & |u\,\bar{d}\rangle \\ \pi^0 & \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \\ \pi^- & |\bar{u}\,d\rangle \end{array}\right\} \text{triplet (isospin)}$$

u-quarks $Q = +\frac{2}{3}$
d-quark $Q = -\frac{1}{3}$

- Quarks interact via strong force, described by Quantum-Chromodynamics (QCD)

Forces:

electromagnetic IA electric charges/currents (Photon)
weak IA weak charges/currents (W/Z-Boson)
strong IA colour charges (Gluon)

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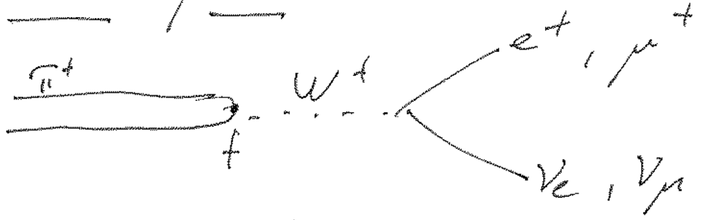
Decay of charged pions

Hadron picture:

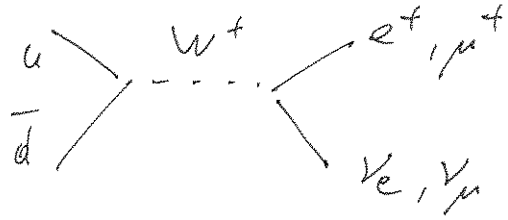


$m_\mu = 105\,\text{MeV} < m_\pi$

$\pi^+ \not\to \tau^+ \nu_\tau$ not possible because $m_\tau = 1777\,\text{MeV} > m_\pi$

Neutrino masses $m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau} \ll 1\,\text{MeV}$

Quark picture



(Feynman diagram)

Kinematics of π - decay

Rest frame of π^+:

$\mu^+ \longleftarrow \pi^+ \longrightarrow \nu_\mu$

- Energy conservation:

$$E_{\pi^+} = E_{\mu^+} + E_{\nu_\mu}$$

- Momentum conservation: $\vec{p}_{\pi^+} = 0$

$$\vec{p}_{\mu^+} = -\vec{p}_{\nu_\mu}$$

- Energy - momentum - mass relation

$$E^2 = p^2 + m^2$$

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For the neutrinos: $m_\nu \approx 0 \Rightarrow E_\nu = p_\nu$

Energy conservation:

$$E_{\pi^+} = \sqrt{m_\mu^2 + p_\mu^2} + E_\nu = m_{\pi^+} \qquad p = |\vec{p}| \qquad E_\nu = p_\nu = p_\mu$$

$$\sqrt{m_\mu^2 + p_\mu^2} = m_\pi - p_\mu$$

$$m_\mu^2 + p_\mu^2 = (m_\pi - p_\mu)^2 = m_\pi^2 - 2m_\pi p_\mu + p_\mu^2$$

$$p_\mu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \approx 30 \text{ MeV}$$ 'mono-energetic'

For the decay $\pi^+ \to e^+ \nu_e$

$$p_e = \frac{m_\pi^2}{2} \approx 70 \text{ MeV}$$

· Topology: "Back-to-back", isotropic decay $(J=0)$

Charged π decay rate

- $\tau = \frac{\hbar c}{\Gamma} = \frac{1}{\Gamma}$ $(\hbar = 1)$

- With N pions at the beginning

$$\Gamma = -\frac{1}{N}\frac{dN}{dt} \quad \text{(relative decrease)}$$

- Calculate using Fermi's golden rule

$$T_{i\to f} = 2\pi\, \rho(E_f)\, |T_{fi}|^2 \leftarrow \text{transition probability}$$

density of final states

$T_{fi} = \langle f|H|i\rangle$ matrix element

for decay

$$d\Gamma = \frac{1}{2M}|T_{fi}|^2 \int d\Phi$$

phase space integration

- Two-body decay: Phase space given by decay angle of one of the decay particles

$$\frac{d\Gamma}{d\Omega_1} = \frac{1}{32\pi^2}\frac{|\vec{p}_1|}{M}|T_{fi}|^2$$

Ω_1: solid angle of particle 1 (μ, e)

Note: Phase space predicts $\Gamma(\pi^+ \to e^+\nu_e) > \Gamma(\pi^+ \to \mu^+\nu_\mu)$

Now we need the matrix element...

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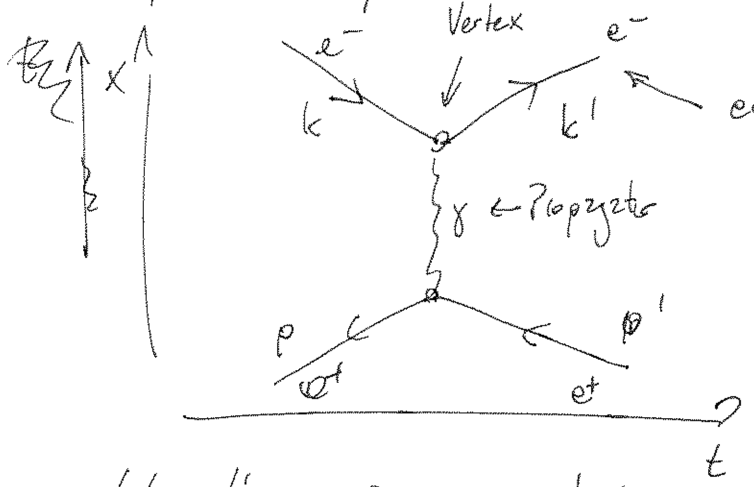
Theory N1

- Matrix Element T_{fi}

Simple example: e^+e^- scattering $e^+e^- \to e^+e^-$



p, p', k, k': four-vector

Interaction energy electromagnetic IA:

$$H^{em} = Qej^{\mu}A_{\mu}$$

j^{μ} = (vector) current (4-vector)

A^{μ} = electromagnetic field

Q = electric charge (1 for e^+/e^-)

Matrix element:

$$T_{fi} = -e^2 \int j^{\mu}(x_1) A_{\mu}(x_1) j^{\nu}(x_2) A_{\nu}(x_2) \, dx_1 \, dx_2$$

integration over all photon paths (path integral)

Result: $$T_{fi} = -e^2 j_1^{\mu} \frac{g_{\mu\nu}}{q^2} j_2^{\nu}$$

$g^{\mu\nu}$ = metric tensor

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

q: four-momentum transfer

$q = p - p' = k' - k$

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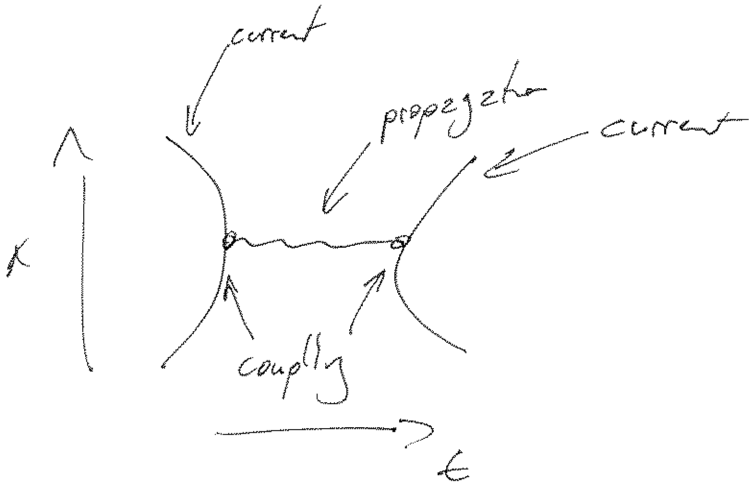
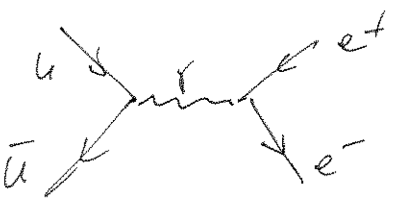


- if more than one diagram: Interference

$$T_{fi} = T_{fi}^{(1)} + T_{fi}^{(2)} + T_{fi}^{(3)}$$

Example: π^0 Decay (electromagnetic)



$\pi^0 \to e^+ e^-$ $\qquad B = 6.5 \cdot 10^{-8}$

couplings $\qquad$ propagator

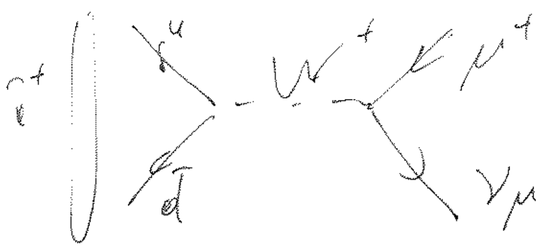
$$T_{fi} = -e^2 \cdot q_u \, j_u^{\mu} \, \frac{g_{\mu\nu}}{s} \, j^{\nu}_{e} \quad \text{(for free quarks)}$$

$q_u = \frac{2}{3}$ $\qquad s = m_{\pi^0}^2 = q^2$ $\qquad$ quark current $\qquad$ electron current

How to get from π^0 to π^+ decay?

Problems:

- Electromagnetic $\longrightarrow$ weak interaction (Fermi Theory)
- Quarks are not free (Pion Form-Factor)
- Quarks are mixed (CKM matrix)



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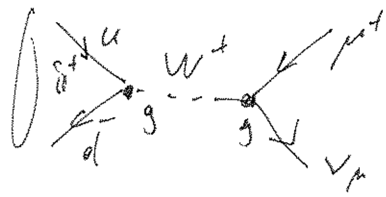
Weak Interactions



$\pi^+ \to \mu^+ \nu_\mu$ charged current

	QED		weak
coupling	e	$\to$	g
propagator	$\frac{1}{q^2}$	$\to$	$\frac{1}{q^2+m_W^2}$
current	j^μ_{em}	$\to$	j^μ_{weak}

massive propagator $m_W = 80.4$ GeV

weak current $\to$ later

Comments

1. g: weak coupling $g = \frac{e}{\sin\vartheta_W}$ $e = 0.3$ $\sin^2\vartheta_W = 0.23$ $g \simeq 0.6$

   weak coupling is not weak

2. $q^2 = m_\pi^2 \ll m_W^2$

   propagator $\frac{1}{q^2+m_W^2} \to \frac{1}{m_W^2}$

3. $|m$ matrix element squared: $\frac{g^2}{m_W^2+q^2} \to \frac{g^2}{m_W^2} = \frac{8 G_F}{\sqrt{2}}$

   G_F: Fermi constant $= 1.17 \cdot 10^{-5}\ \mathrm{GeV}^{-2} \simeq 10^{-5}/m_p^2$

   $T_{fi} = -g^2 \cos\vartheta_c\, j^\mu_{weak}(\text{quark})\, j_{weak,\mu}(\text{lepton})$

   ϑ_c: Cabbibo angle from quark mixing

   For 6 Quarks: CKM, here only consider 4

   $d' = d \cos\vartheta_c + s \sin\vartheta_c$

   $s' = -d \sin\vartheta_c + s \cos\vartheta_c$

weak states couple to W

mass states form hadrons in QCD

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Theory
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Weak current

Matrix elements have to be Lorentz-Invariant and fullfill discrete symmetries (parity, charge conjugation)

$\Rightarrow$ only 5 currents possible

spinors ↓ ↙ γ-matrices

vector current	(V)	QED: $j^\mu_V = \psi^+ \gamma^\mu \psi$ (electromagnetic IA)
axial-vector current	(A)	$j^\mu_A = \psi^+ \gamma^5 \gamma^\mu \psi$
scalar current	(S)	$\psi^+ \psi$
pseudoscalar current	(P)	$\psi^+ \gamma^5 \psi$
tensor current	(T)	$\psi \gamma^\mu \gamma^\nu \psi$

- Difference between vector and axialvector (and scalar and pseudoscalar)

Parity: $P(j_V) = -j_V$

$P(j_A) = j_A$

Parity and helicity of fermions $J = 1/2$

Particle: $\longrightarrow \vec{p}$, $\quad \vec{p}\,(0, 0, p_z)$

$\Rightarrow$ spin $\quad J = \frac{1}{2} \quad j_z = +\frac{1}{2}$

$\longrightarrow z$

Helicity $\lambda = \frac{\vec{J} \cdot \vec{p}}{|p|}$ in QM $\lambda = \pm\frac{1}{2}$

Particles can have positive (right) or negative (left) helicity

$P(\vec{p}) = -\vec{p}$

$P(\vec{J}) = \vec{J}$

$P(\lambda) = -\lambda$

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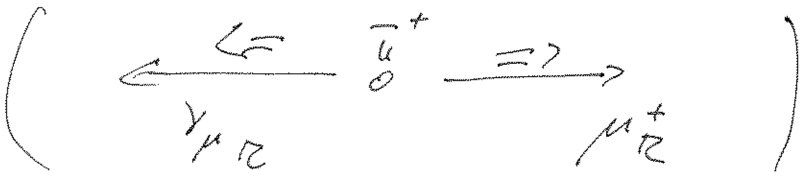
Left / Right-handed currents

$$\vec{j}_R = \frac{1}{2}\left(\vec{j}_V + \vec{j}_A\right) \qquad \vec{j}_L = \frac{1}{2}\left(\vec{j}_V - \vec{j}_A\right)$$

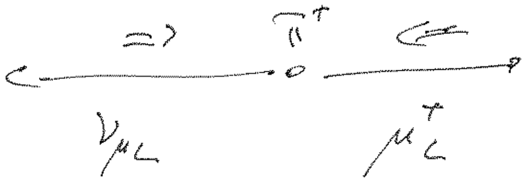
$$P(\vec{j}_R) = \frac{1}{2}\left(P(\vec{j}_V) + P(\vec{j}_A)\right) = \frac{1}{2}\left(-\vec{j}_V + \vec{j}_A\right) = -\vec{j}_L$$

$$\vec{j}_V = \frac{1}{2}\left(\vec{j}_L + \vec{j}_R\right) \qquad \vec{j}_A = \frac{1}{2}\left(j_R - \vec{j}_L\right)$$

- Pion decay and helicities

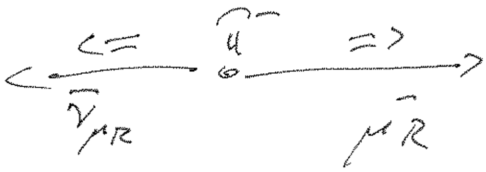


(V+A) coupling
not seen
(no right-handed neutrinos)

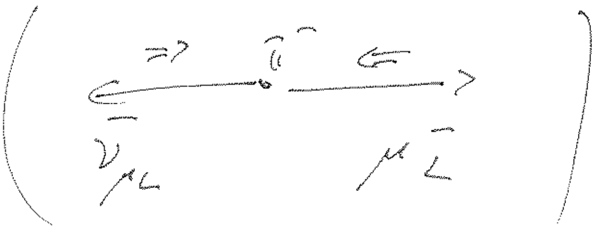


(V-A) coupling

$\Downarrow$ charge conjugation C



(V-A) coupling



(V+A) coupling

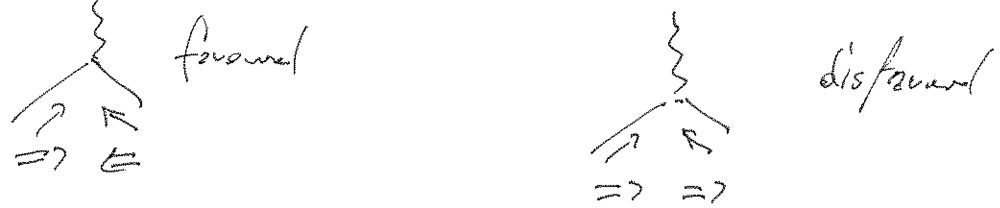
not seen → no left-handed antineutrinos

- Is V-A coupling the only one giving left-handed neutrinos? (Goldhaber experiment)

  No: S-P also possible

- V,A interaction favour opposite helicity states:

- S,P interaction favour same helicity states



• $\pi^+ \to \mu^+\nu, e^+\nu$

Neutrinos are forced to be left-handed

μ^+, e^+ " " " " right-handed (spin conservation)

favoured $\dfrac{(1+\frac{v}{c})}{2}$ depends on velocity of e, μ

disfavoured $\dfrac{(1-\frac{v}{c})}{2}$ ← helicity suppression

• it can be shown $\dfrac{v}{c} = \dfrac{p}{E} = \dfrac{m_\pi^2 - m^2}{m_\pi^2 + m^2}$

$\Rightarrow \frac{v}{c} \approx 1$ for electron $\quad 1-\frac{v}{c} \approx 0 \quad 1+\frac{v}{c} \approx 2$

$\frac{v}{c} \approx \frac{1}{3}$ for muons $\quad 1-\frac{v}{c} \approx \frac{2}{3} \quad 1+\frac{v}{c} \approx \frac{4}{3}$

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- Calculation of ratio:

$$R = \frac{B(\pi \to e\nu)}{B(\pi \to \mu\nu)}$$

phase space, helicity

V, A coupling: $B \propto p \cdot \left(1 - \frac{v}{c}\right) \Rightarrow R = \frac{m_e^2}{m_\mu^2} \frac{1}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}$
$= 1.275 \cdot 10^{-4}$

S, P coupling: $B \propto p \cdot \left(1 + \frac{v}{c}\right) \Rightarrow R = \frac{1}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} = 5.5$

from experiment we know: V, A couplings, not S, P
+ Goldhaber $\to$ V-A !

- Assume V-A coupling $\pi^+ \to \mu^+ \nu$

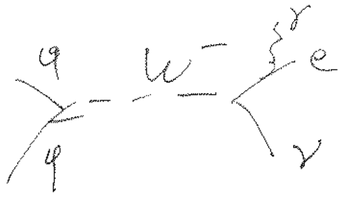
$$|T_{fi}^{free}|^2 = 12 G_F^2 \cos^2\vartheta_c \, m_\mu^2 \cdot m_\pi^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$\Rightarrow \Gamma = \frac{G_F^2 f_\pi^2}{8\pi} \cos^2\vartheta_c \, m_\mu^2 \, m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

with $f_\pi \simeq 130$ MeV ~ about the pion mass.

Also: There are radiative decays

$\pi^+ \to e^+ \gamma \nu$



Ignore for now

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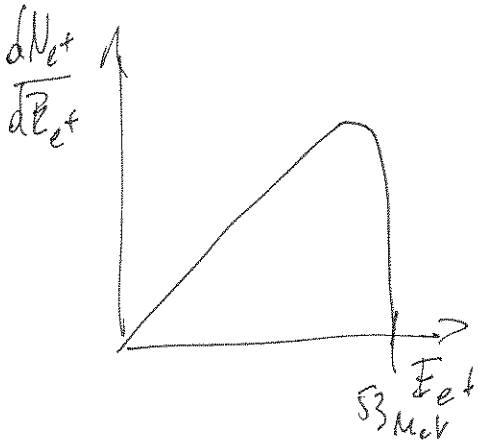
- Muon decay

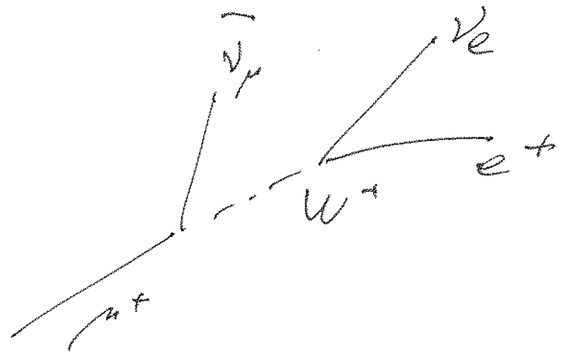
$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$ 3-body decay (Michel decay)

muon lifetime $\tau \simeq 2\,\mu s$ → muon has time to stop in material before the decay

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Positron energy given by Michel spectrum





$$\frac{d\Gamma}{dx\,d\cos\vartheta} \sim x^2\Big((3-2x) + P_\mu \cos\vartheta\,(1-2x)\Big)$$

$x = \frac{2E_e}{m_\mu}$

$\cos\vartheta$: angle w.r.t. μ spin

Maximum energy $x=1 \hat{=} E_e = 53$ MeV

Energy spectrum: