High-Energy Collisions with ALICE at the LHC

3. Jets in e^+e^- and p+p(pbar) Collisions

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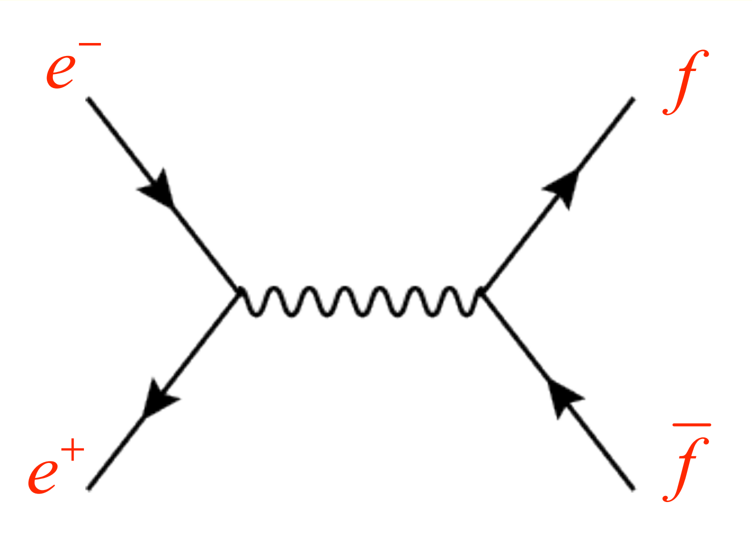
3.1 Jets in e^+e^--Collisions

Experimental Evidence for Color Charge (I)

Total cross section for the production of a fermion-antifermion pair (to order $O(\alpha_s^0)$):

charge

$$\sigma(e^+e^- \to f\,\bar{f}) = \frac{4\pi}{3}(\hbar c)^2 \cdot \frac{Q_f^2 \alpha^2}{s}$$



f : Muon (μ) or quark with certain color (q_{color})

Total cross section for the production of a $u\bar{u}$– pair:

$Q_f = \frac{2}{3}$, 3 colors $\Rightarrow$ factor 3

$$\sigma(e^+e^- \to u\,\bar{u}) = 3 \cdot \sigma(e^+e^- \to u_{\text{color}}\,\bar{u}_{\text{anti-color}}) = \frac{16\pi}{9}(\hbar c)^2 \cdot \frac{\alpha^2}{s}$$

Experimental Evidence for Color Charge (II)

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= \frac{\sigma(e^+e^- \to \text{all } q,\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

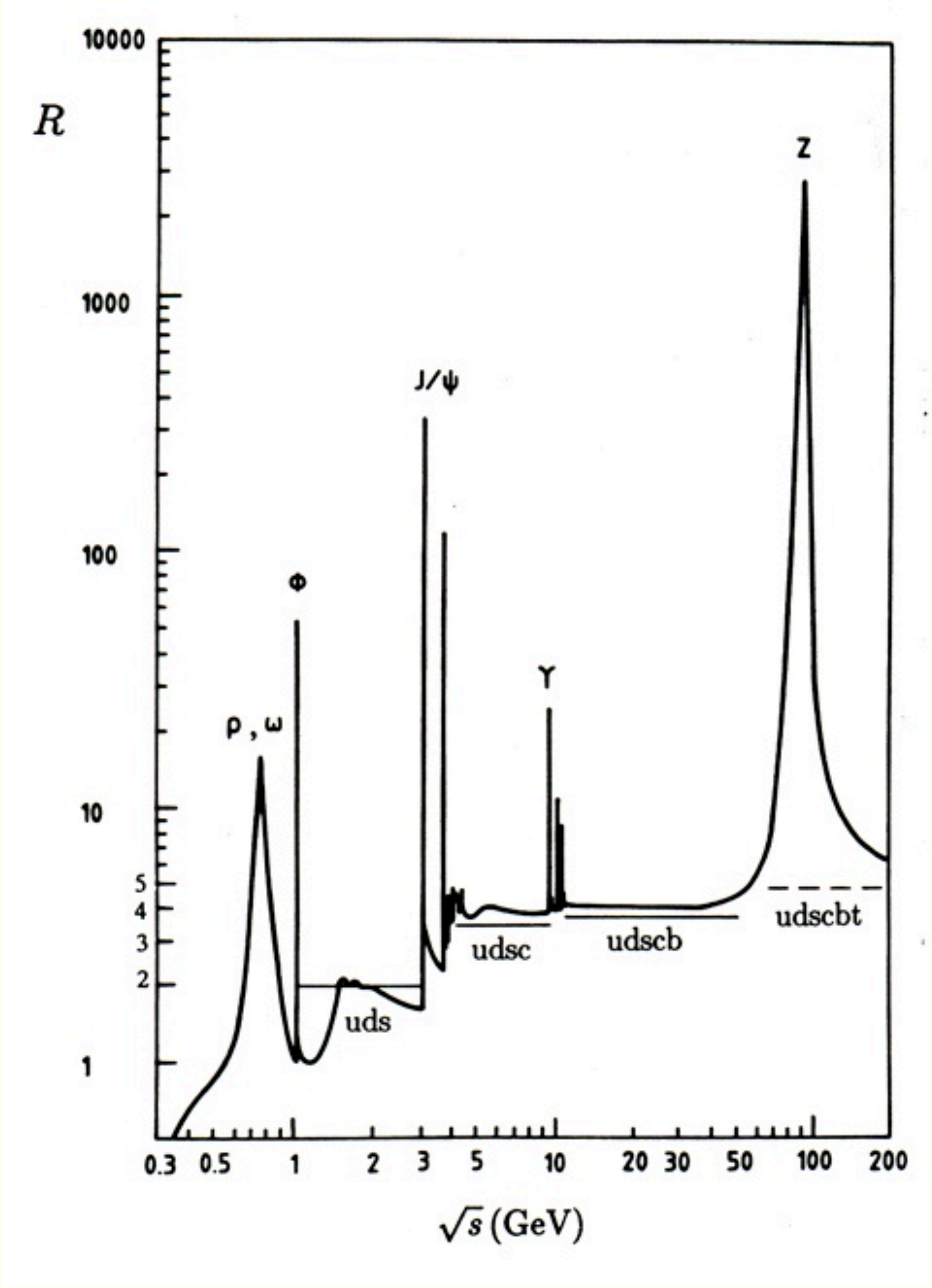
$$= 3 \cdot \sum_q Q_q^2$$

Sum over all quark flavors whose production is energetically allowed

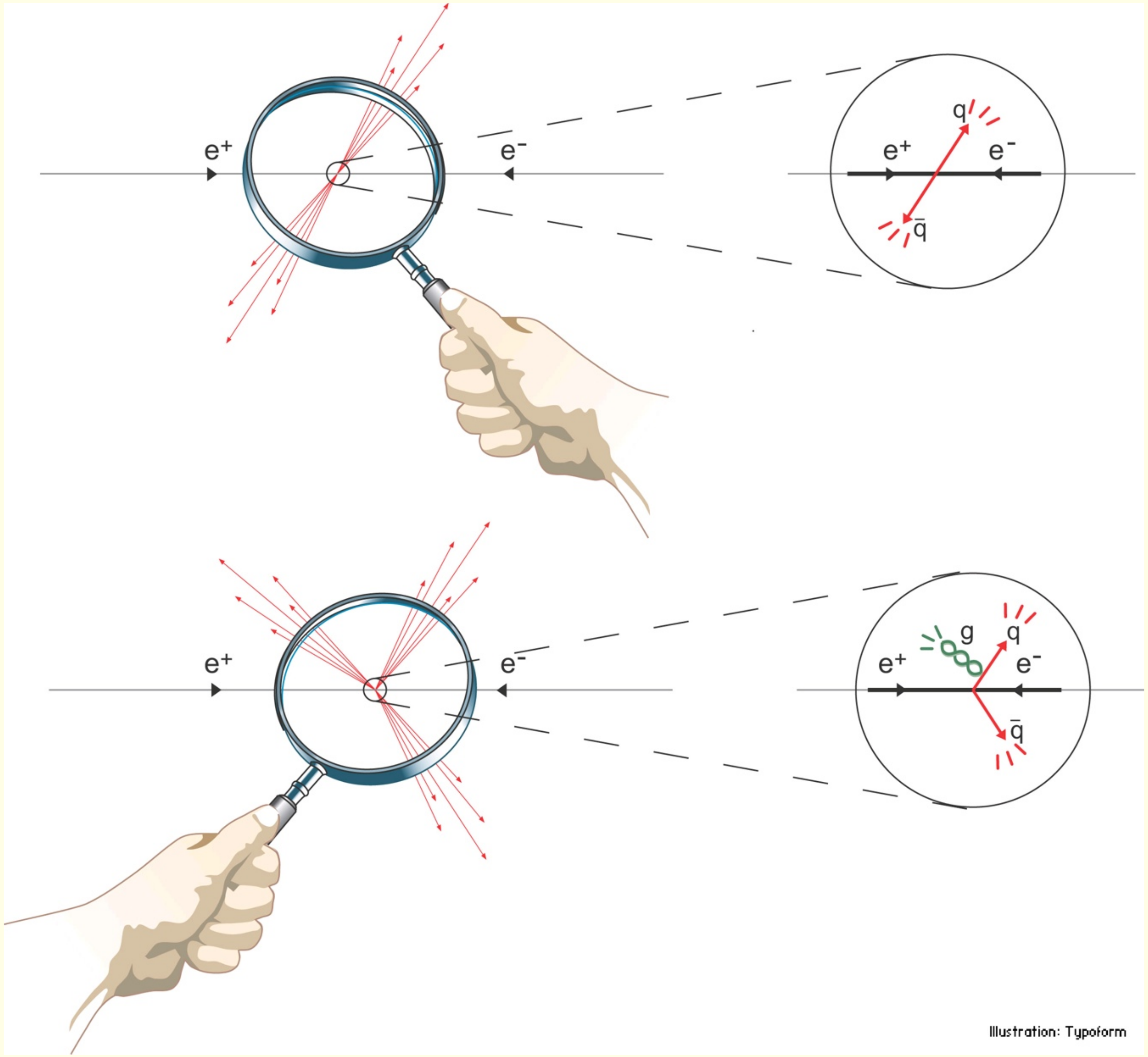
u, d, s: $R = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] = 2$

u, d, s, c: $R = 2 + 3\left(\frac{2}{3}\right)^2 = \frac{10}{3}$

u, d, s, c, b: $R = \frac{10}{3} + 3\left(\frac{1}{3}\right)^2 = \frac{11}{3}$



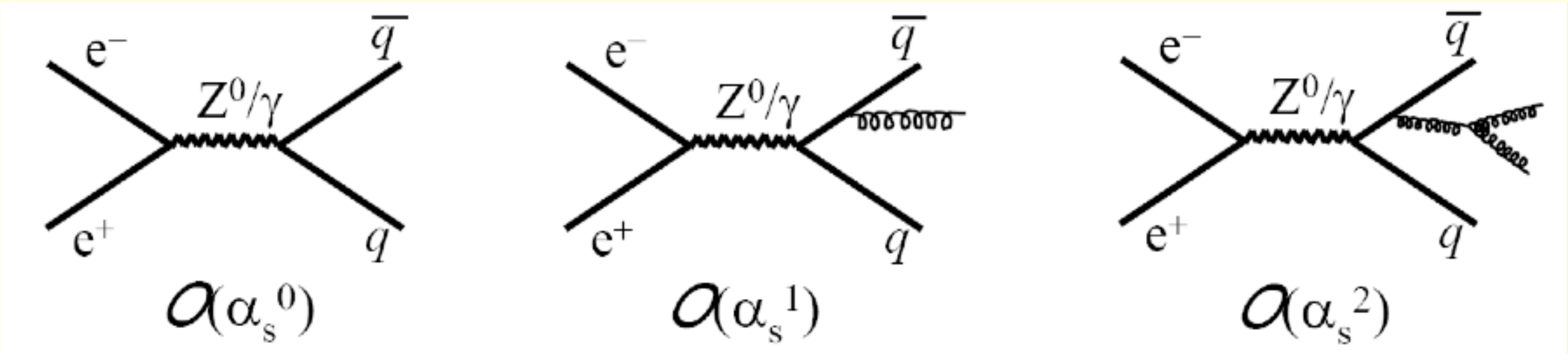
Two- and Three-Jet Events in e^+e^-

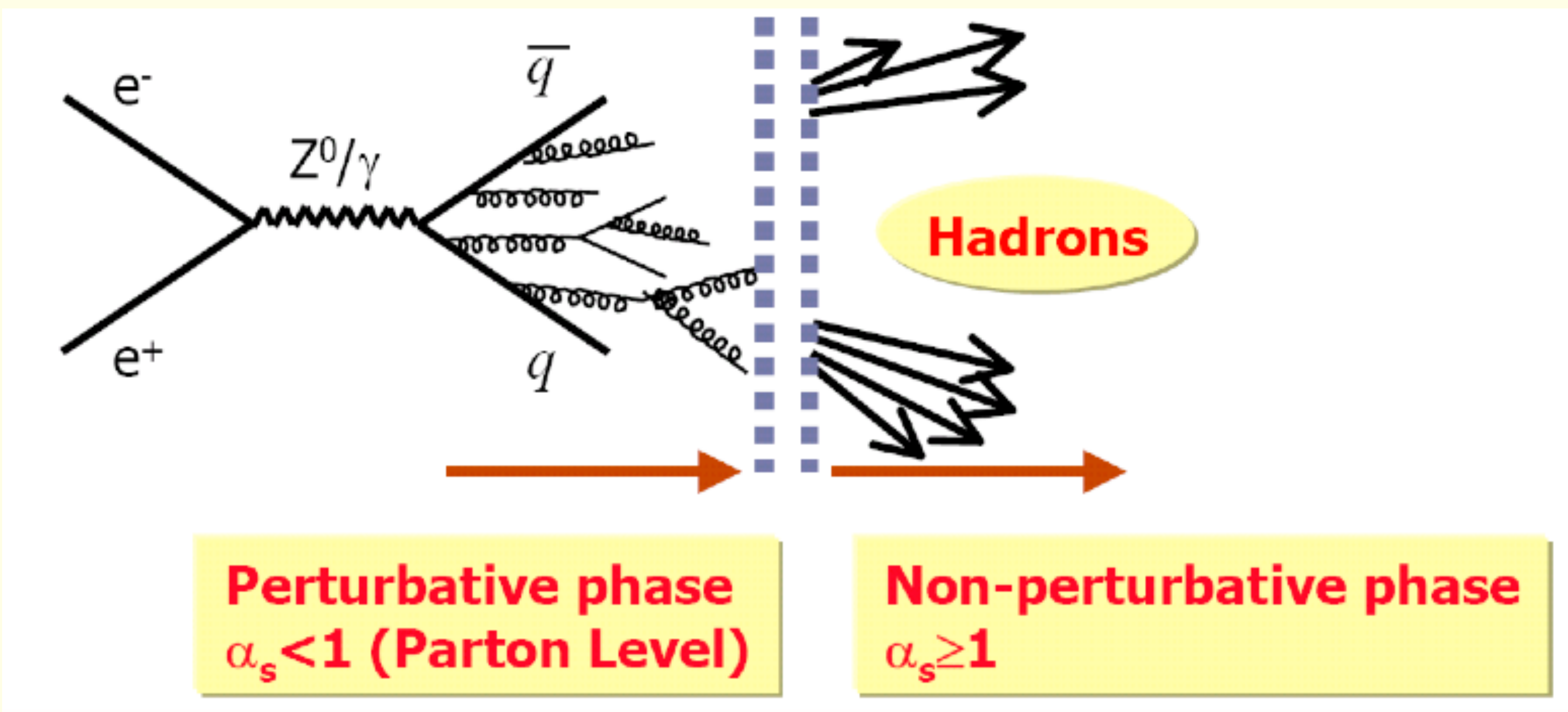


The two jets reflect the direction and the energy of the primordial quark-antiquark pair

3-Jet event
Direct experimental evidence for gluons

Feynman Diagrams: Higher Order Corrections

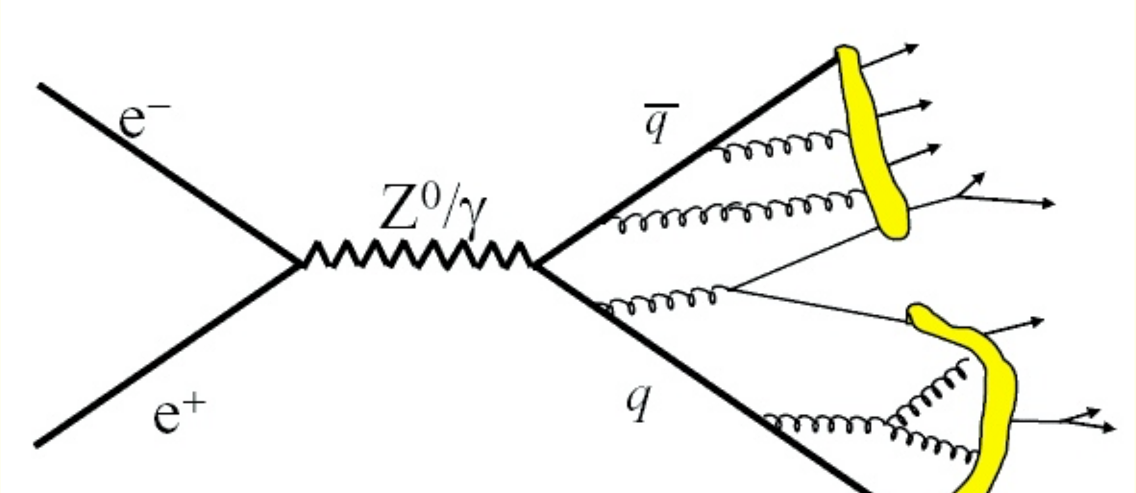




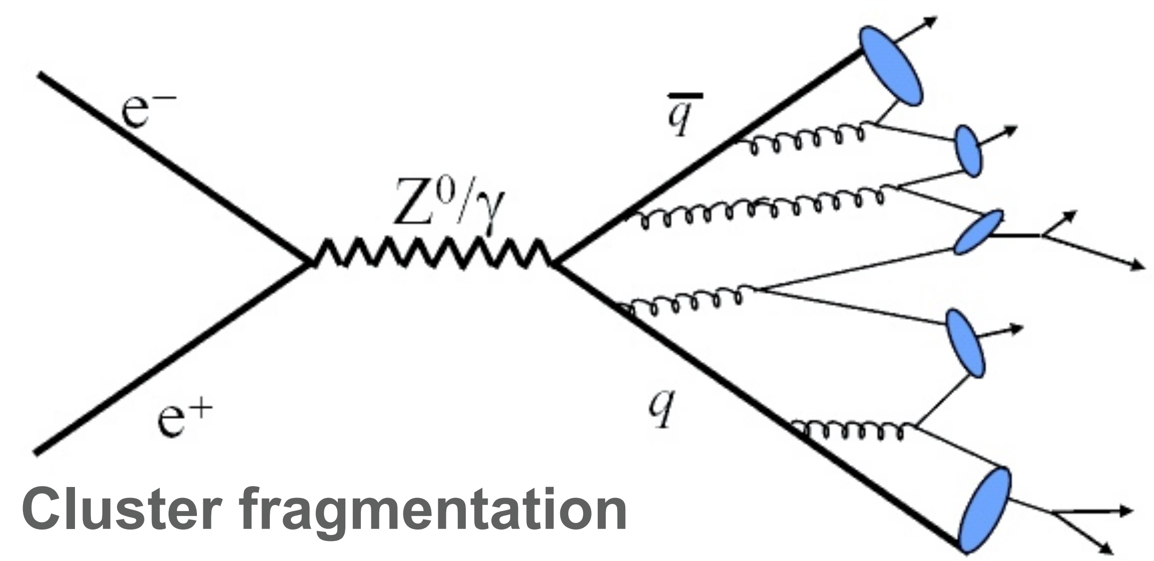
Digression: Models for Jet Hadronization

Nicos Varelas, CTEQ summer school

- Independent fragmentation
  - Each parton fragments independently
- String fragmentation
  - Separating partons connected by color string
  - Used in Pythia/Jetset
- Cluster fragmentation
  - Pairs of neighboring partons combine to color singlets
  - Then apply recipe for cluster decay, e.g., isotropic decay in rest frame into hadrons



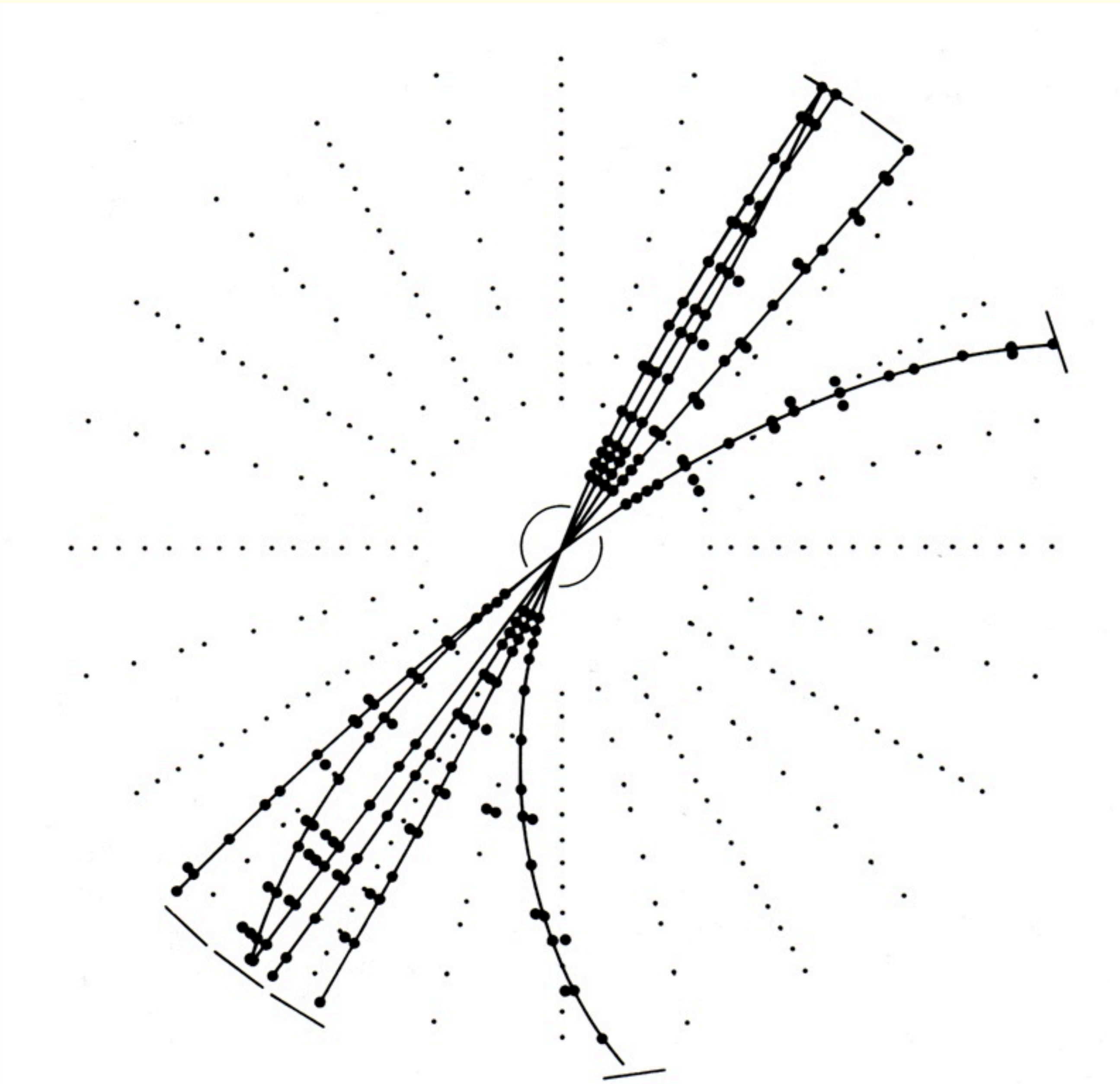
String fragmentation



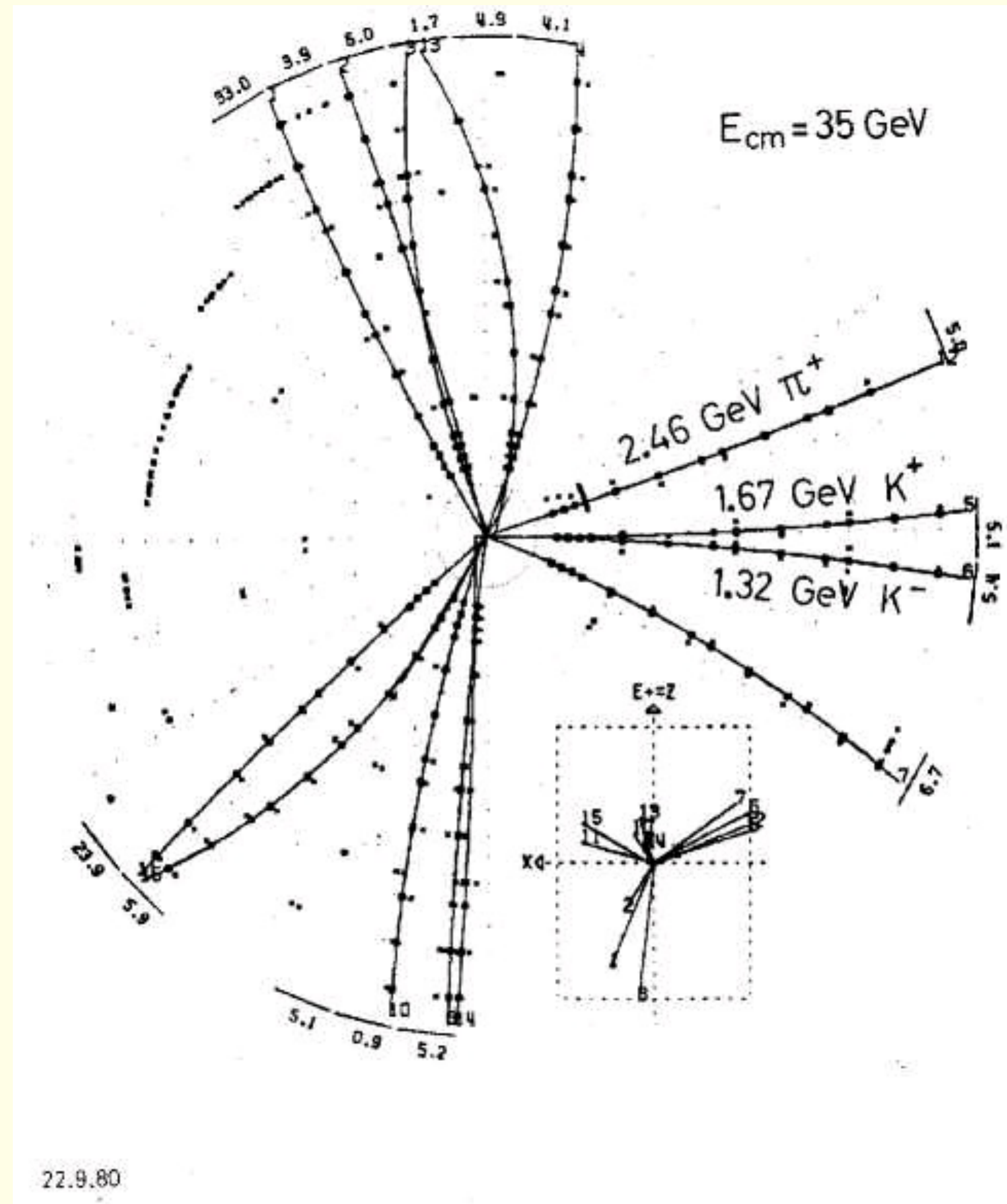
Cluster fragmentation

Experimental Evidence for Gluons from Three-Jet Events

e^+e^- - collision at PETRA storage ring at DESY (ca. 1980):

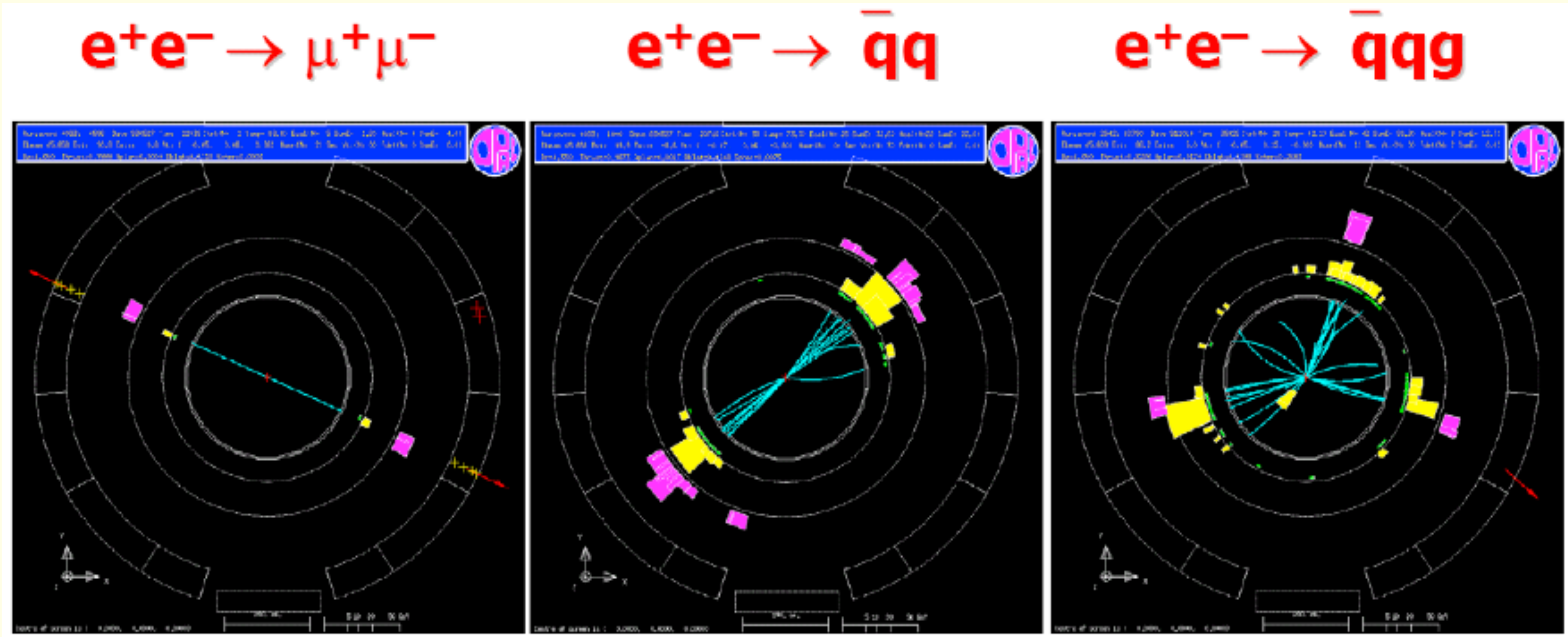


2-jet event

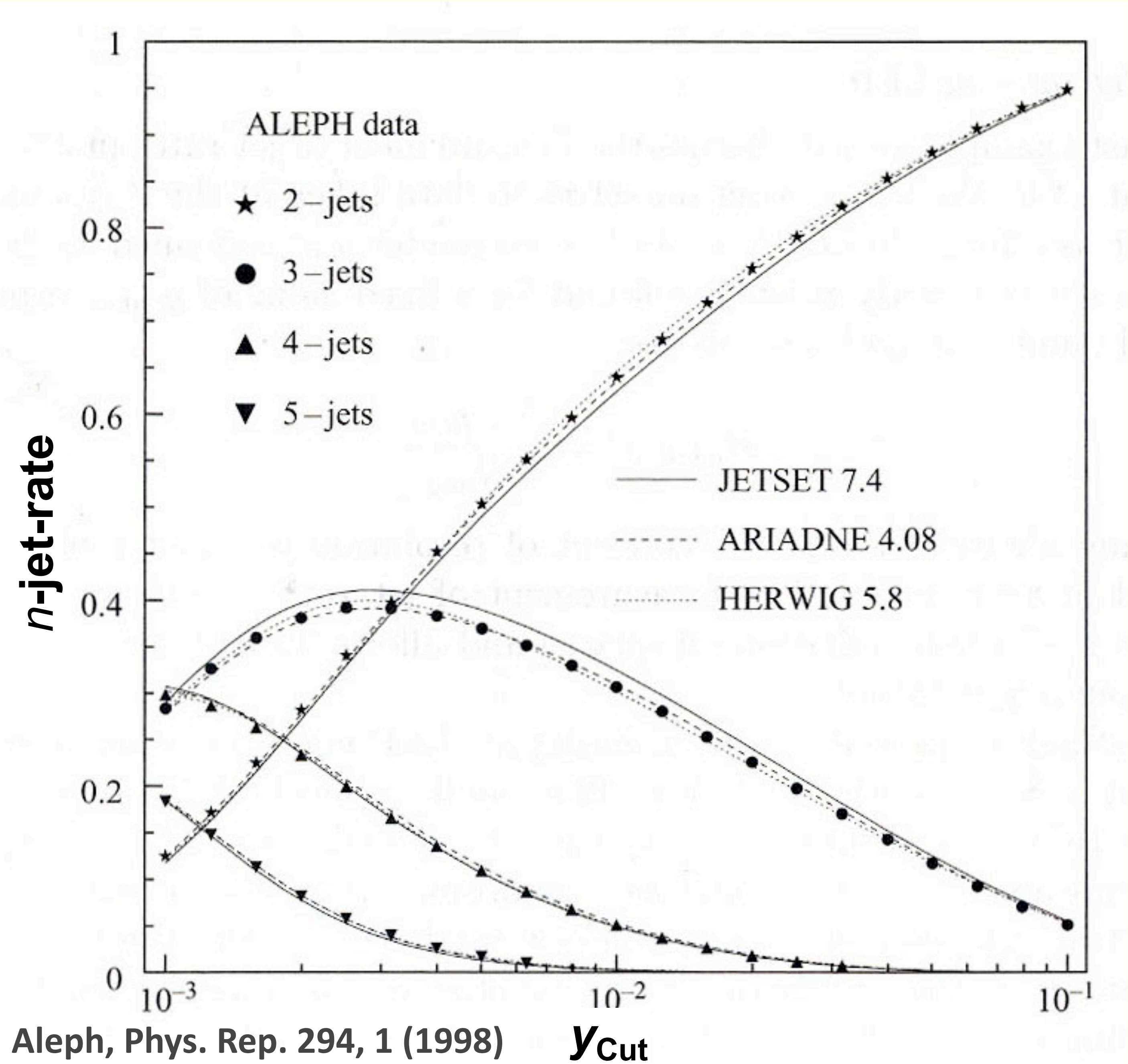


3-jet event

Jets at LEP (OPAL Experiment)



What a Jet is Depends on the Jet Reconstruction: Rate of *n*-jet Events



Aleph, Phys. Rep. 294, 1 (1998)

$$R_3(y_{\mathrm{Cut}}, \sqrt{s}) = \frac{\sigma_{\text{3-Jet}}}{\sum_n \sigma_{n\text{-Jet}}}$$

$$= C_{3,1}(y_{\mathrm{Cut}}) \frac{\alpha_s(\sqrt{s})}{2\pi}$$

$$\approx 0{,}1 \quad \text{for large } y_{\mathrm{Cut}}$$

y_{Cut} : **Threshold in jet reconstruction algorithm (Durham or k_T algorithm, will be discussed later)**

Example of a Jet Variable: Thrust

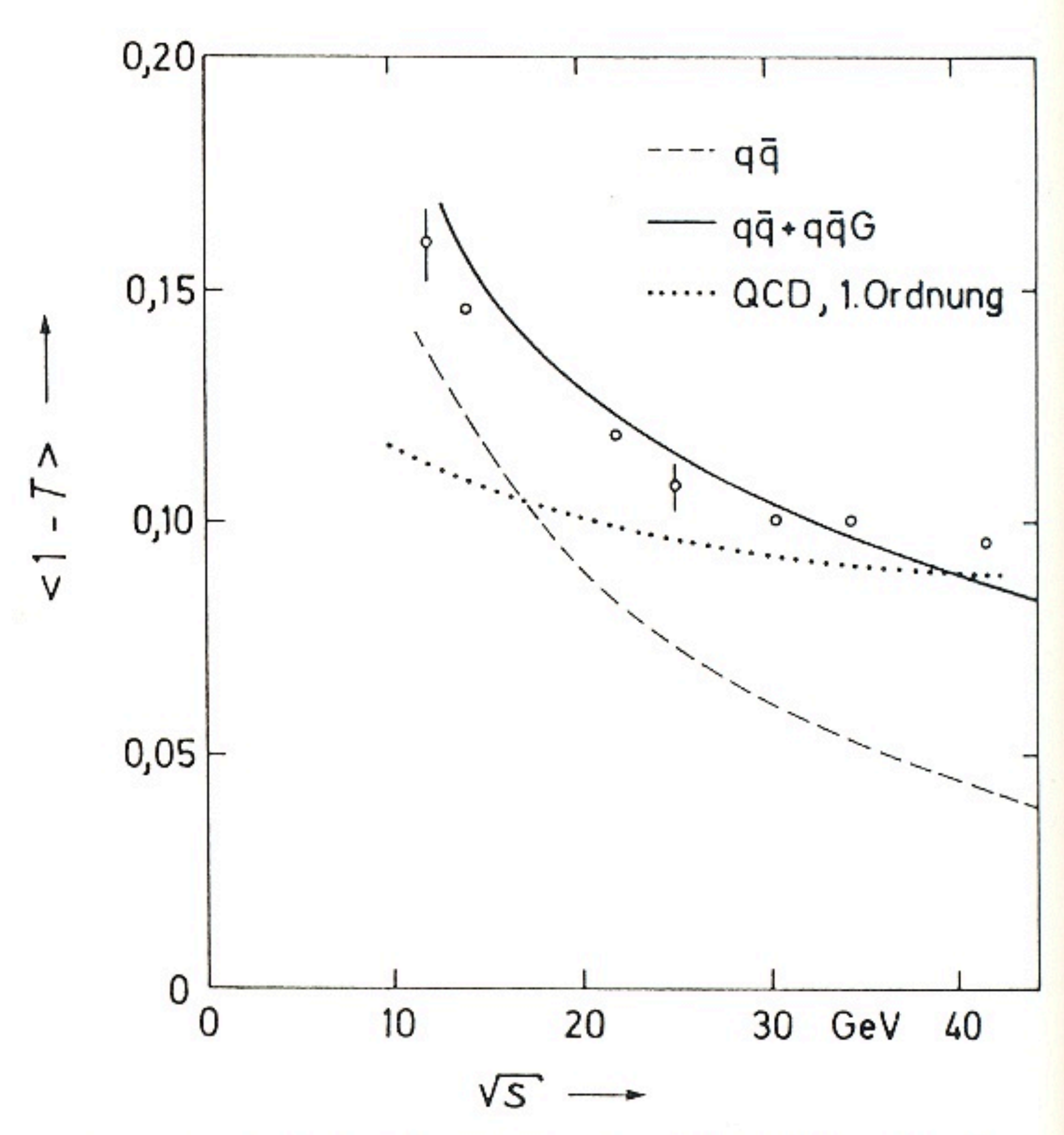
Denoting the particle momenta in an event as $\vec{p}_i$ and a given a unit vector $\vec{n}$ one can calculate

$$t(\vec{n}) = \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

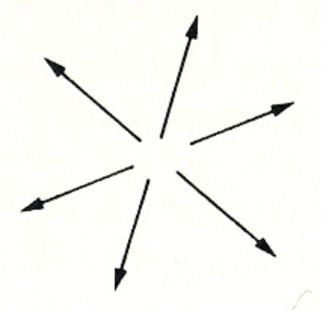
The vector $\vec{n}$ that maximizes $\sum_i |\vec{p}_i \cdot \vec{n}|$ is called the thrust axis.

The thrust variable is defined as

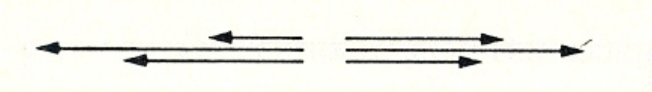
$$T = \max t(\vec{n}) \qquad (0.5 \le T \le 1)$$



Isotropic event: $T = \frac{1}{2}$

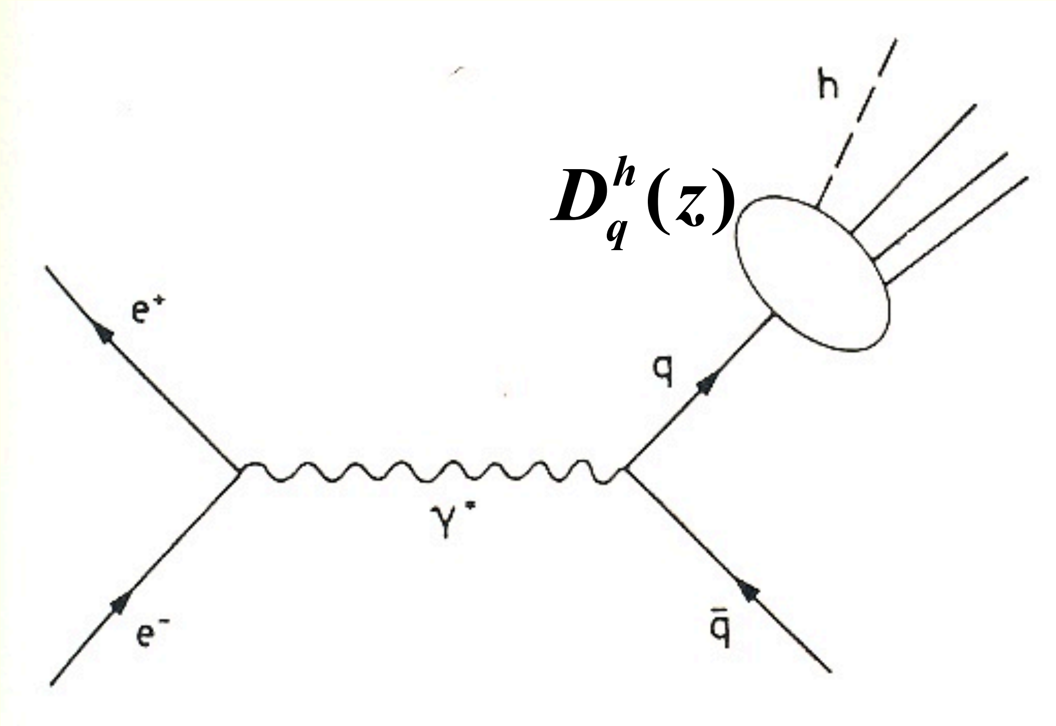


Ideal jet event: $T = 1$



Jet Fragmentation (I)

- **Jet fragmentation is independent of the creation process of the mother parton**
- **Hadron transverse momentum p_T with respect to parton direction independent of parton momentum (p_T < 300 MeV/c)**
- **Distribution of longitudinal hadron momenta depends only on the fraction z of the parton momentum (scaling)**



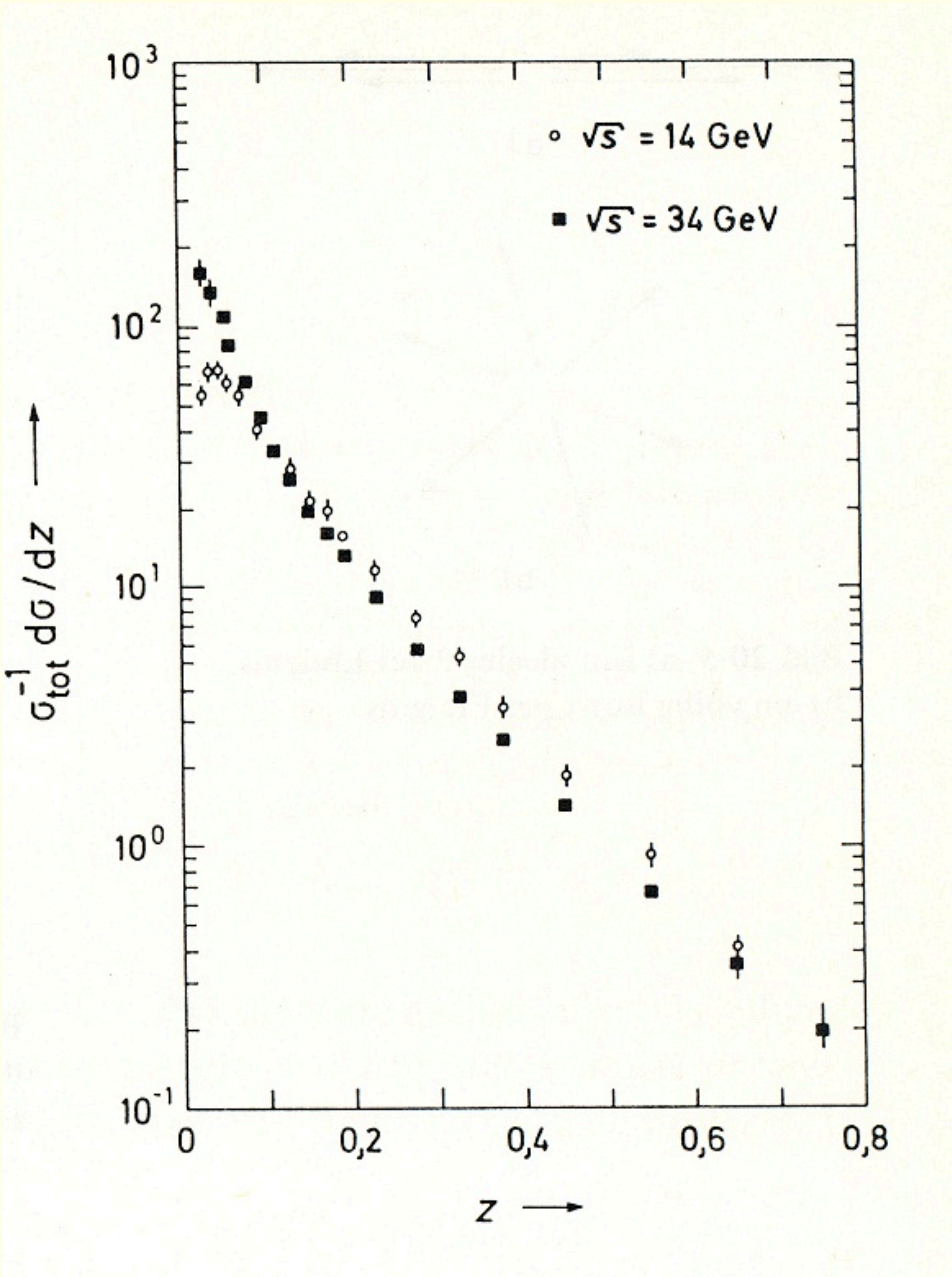
$$\frac{d\sigma}{dz}(e^- + e^- \to h + X) = \sum_q \sigma(e^- + e^- \to q + \bar{q})\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

$$z = \frac{E_h}{E_q} = \frac{E_h}{E_{beam}} = \frac{2E_h}{\sqrt{s}}$$

Fragmentation function:

$D_q^h(z)\,dz$: number of hadrons of type h with momentum fraction between $z \ldots z + dz$

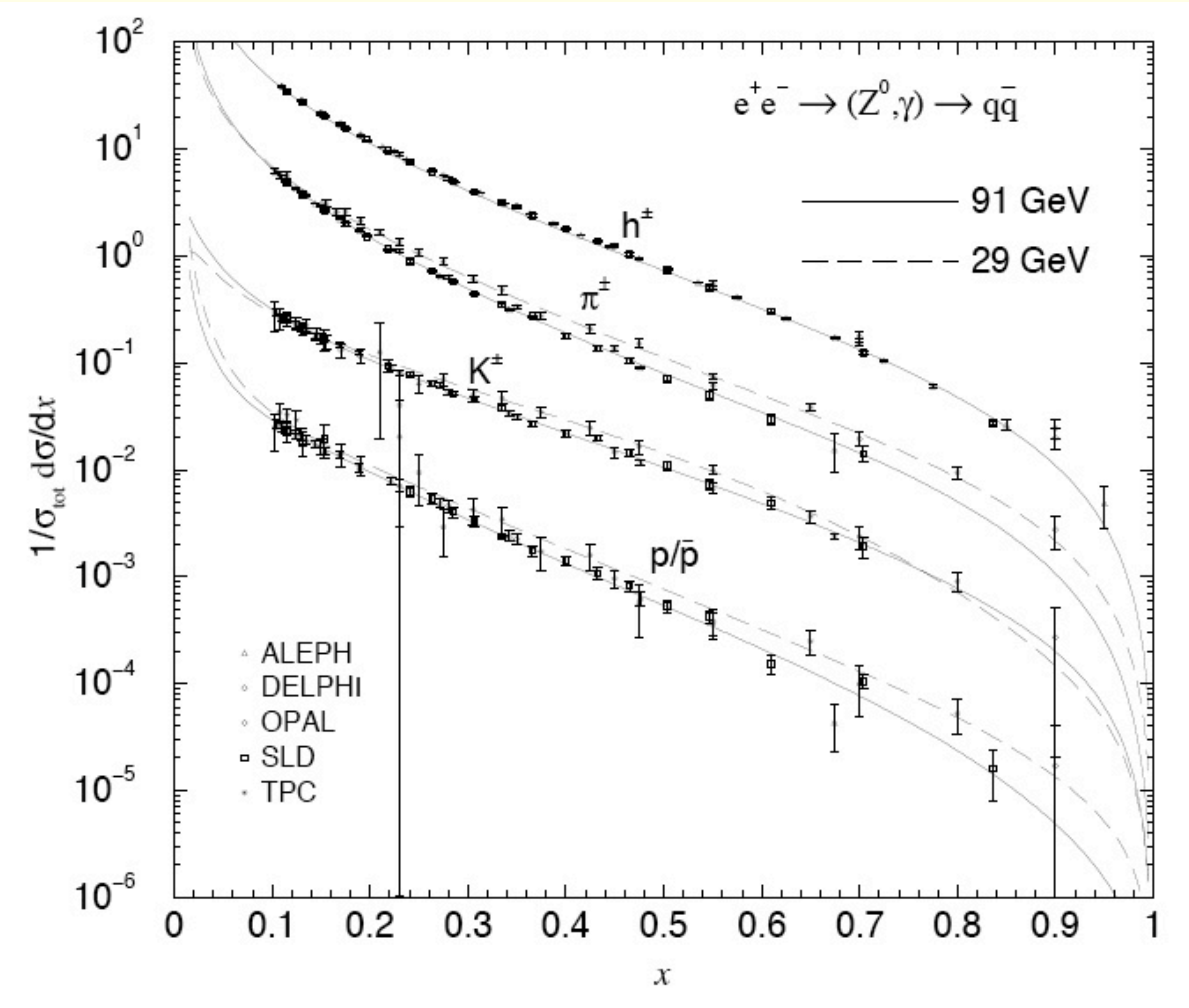
Jet Fragmentation (II)



$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dz}(e^- + e^- \to h + X) = \frac{\sum_q Q_q^2\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]}{\sum_q Q_q^2} \equiv f(z)$$

The expected scaling behavior is indeed approximately observed experimentally

Fragmentation Functions (FF) from e+e- Data



NLO fragmentation functions from Albino, Kniehl und Kramer (hep-ph/0502188v2)

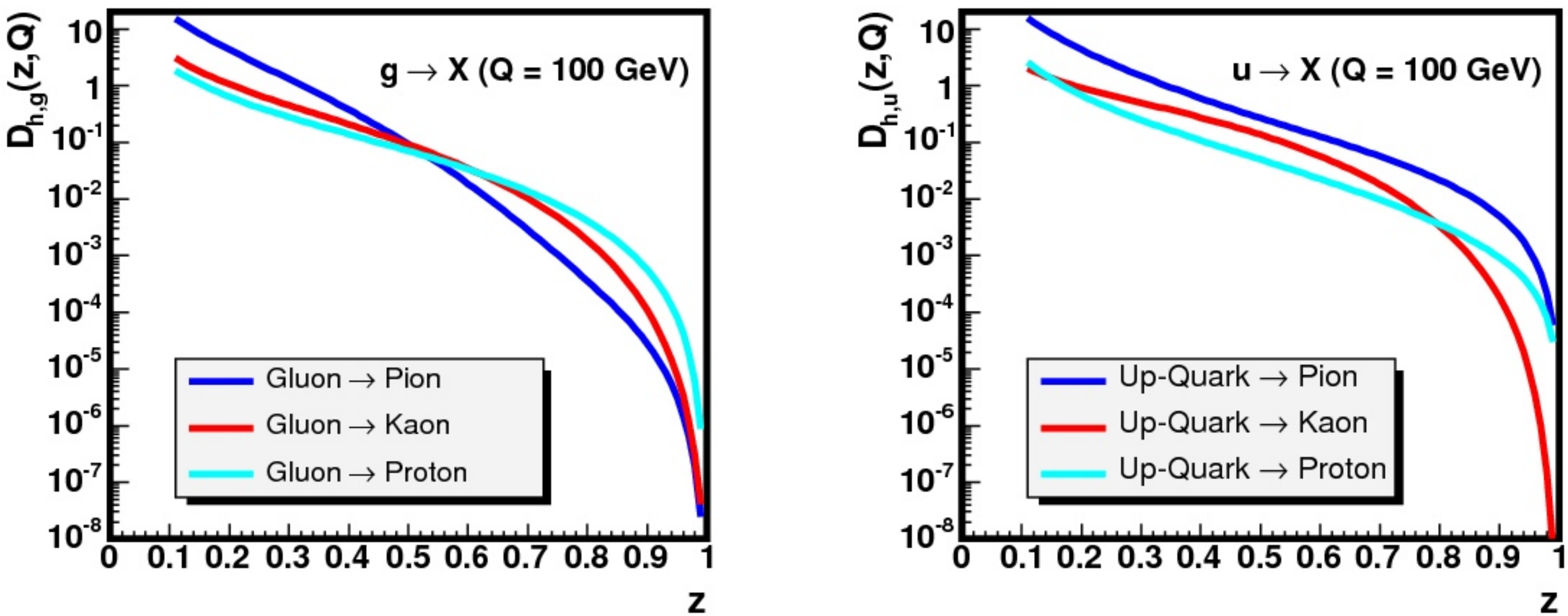
- **Usual ansatz for functional form of the fragmentation function:**

$$D_a^h(x, M_0^2) = N x^\alpha (1-x)^\beta$$

- **Small differences between FF‘s can be seen („scaling violation“)**

Example: Gluon and u-Quark Fragmentation Functions

Albino, Kniehl, Kramer, Nucl. Phys. B 725 (2005), 181



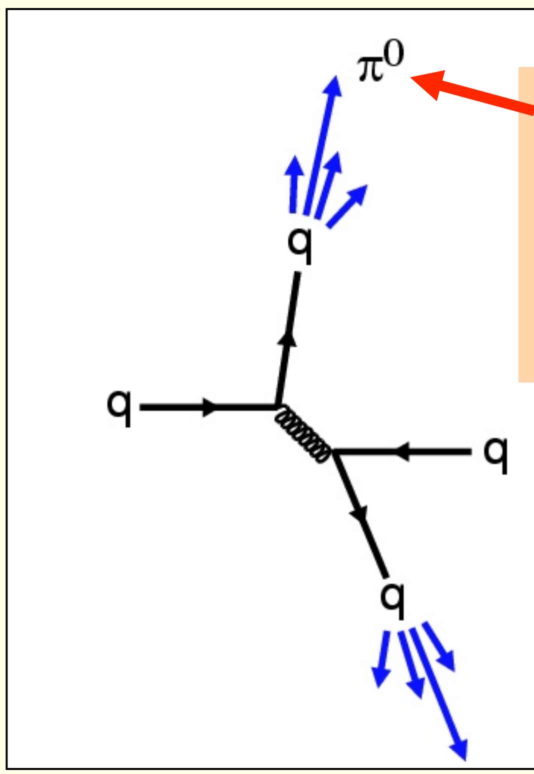
$$z = \frac{p_{\text{Hadron}}}{p_{\text{Parton}}}$$

Fragmentation functions:
Number density for the production of a hadron h
with fractional energy *z* in the fragmentation of a parton
(e.g. determined from $e^+e^- \to Z^0 \to q\bar{q}$)

3.2 Hard Scattering and Particle Yields at High p_T in p+p(bar p) Collisions

Theoretical Description of High-p_T Particle Production

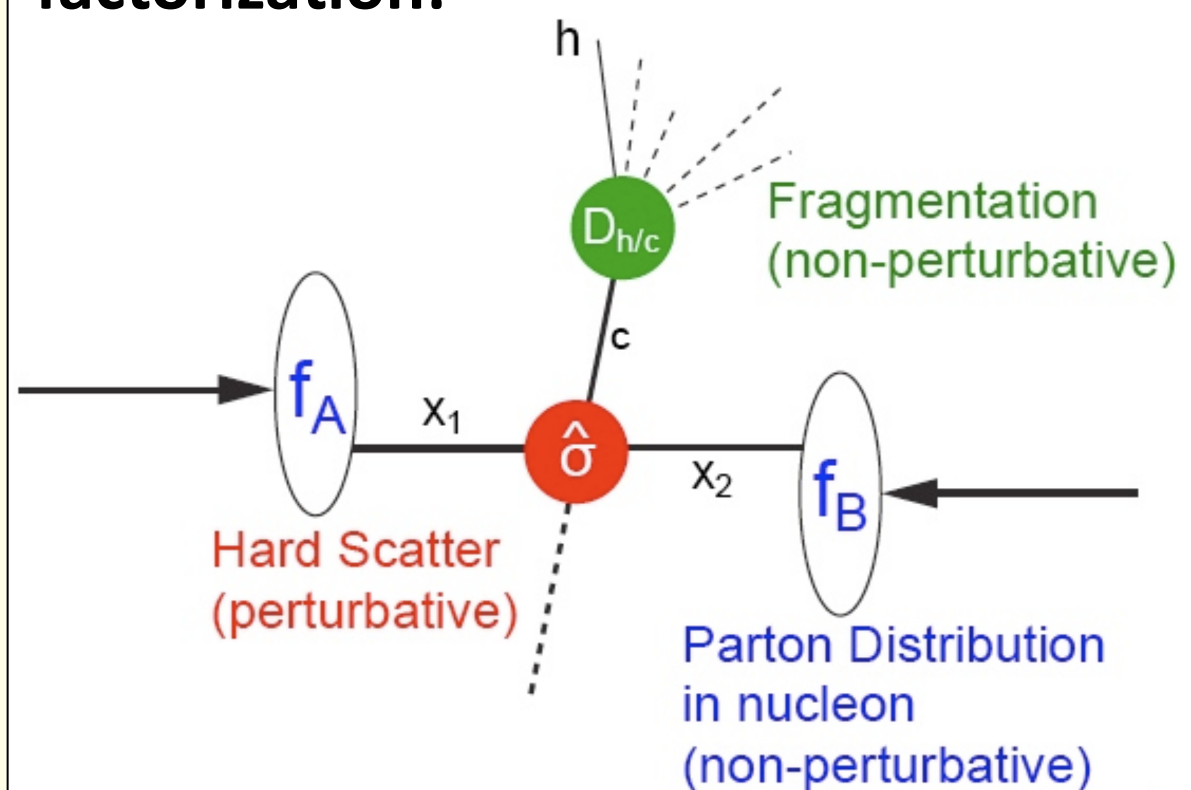
- **Scattering of pointlike partons described by QCD perturbation theory (pQCD)**
- **Soft processes described by universal, phenomenological functions**
  - **Parton distribution function from deep inelastic scattering**
  - **Fragmentation functions from e^+e^- collisions**



Leading hadron:

$$\langle z \rangle = \langle p_{\text{hadron}} / p_{\text{parton}} \rangle \approx 0.25$$

factorization:



$$d\sigma = \sum_{a,b,c} f_a \otimes f_b \otimes d\hat{\sigma}_{ab}^{c} \otimes D_c^{Hadron}$$

Hadron Production in Leading Order QCD

Parton distributions
(functions of x_{Bjorken}
and momentum transfer Q^2)

Inv. Cross section

Fragmentation function

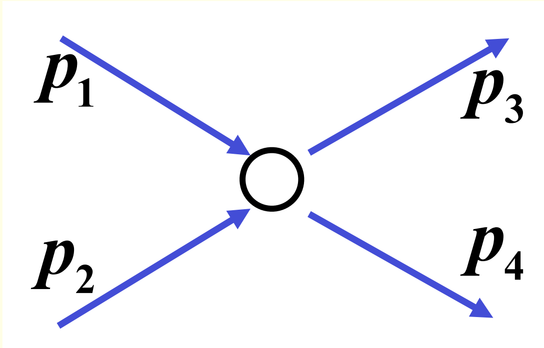
$$E\frac{d^3\sigma}{dp^3} = K \sum_{a,b,c,d=q,\bar{q},g} \int_{x_{a,\min}}^{1} dx_a \int_{x_{b,\min}}^{1} dx_b \, f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \frac{d\sigma^{ab\to cd}}{d\hat{t}} \frac{1}{\pi z_c} D_{h/c}(z_c, Q^2)$$

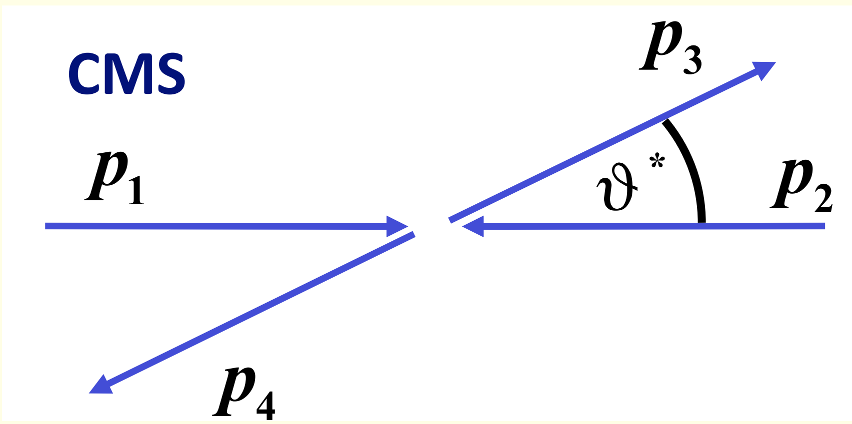
Phenomenological factor
which takes higher order
contributions into account

Elementary QCD
parton-parton
cross section

$z_c = p_{\mathrm{T,Hadron}} / p_{\mathrm{T,c}}$

Mandelstam Variables





Massless partons:

$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2 = -2\,p_1 p_3 = -\hat{s}\,\frac{1-\cos\vartheta^*}{2}$$

$$\hat{u} = (p_2 - p_3)^2 = -2\,p_2 p_3 = -\hat{s}\,\frac{1+\cos\vartheta^*}{2}$$

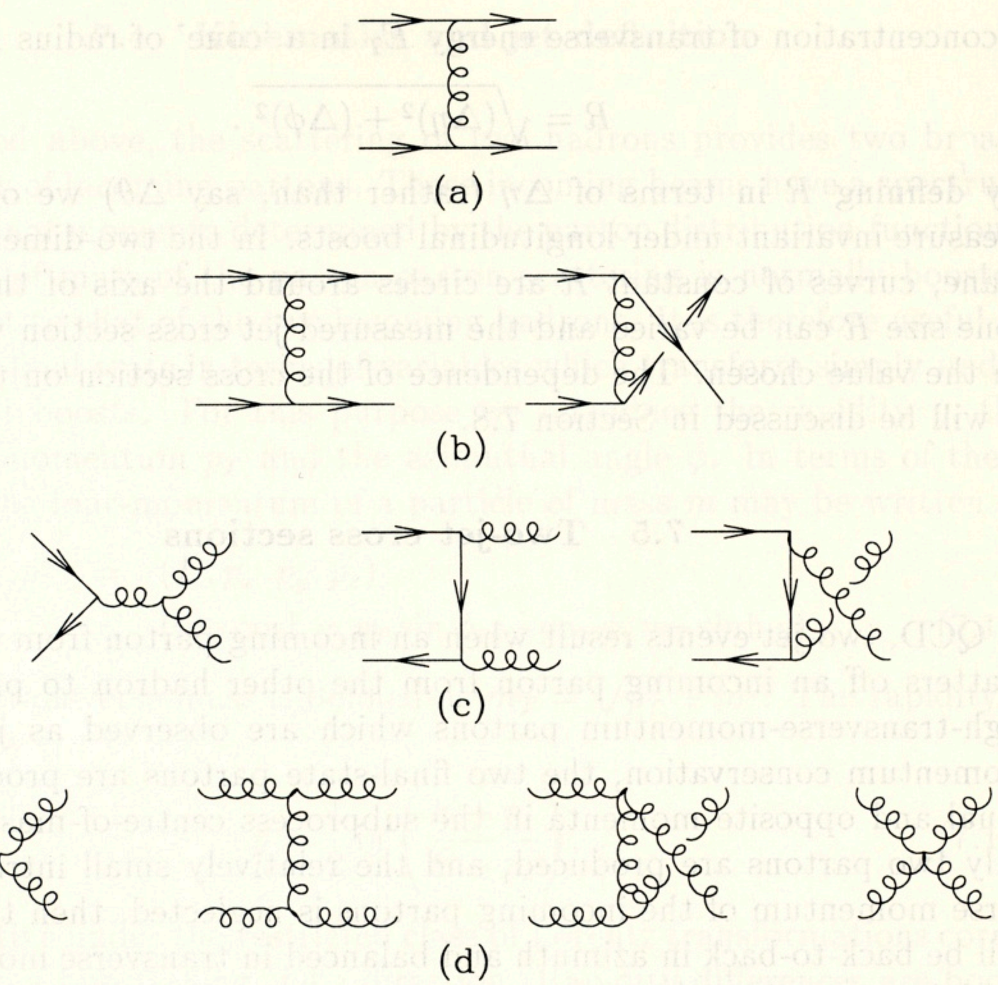
Example: Jet cross section (i.e., no fragmentation function):

$$E_{\text{jet}}\frac{d^3\sigma}{d^3 p_{\text{jet}}} = \frac{1}{16\pi s}\sum_{i,j,k,l=q,\bar{q},g}\int_0^1 \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, f_i(x_1,\mu^2) f_j(x_2,\mu^2)$$

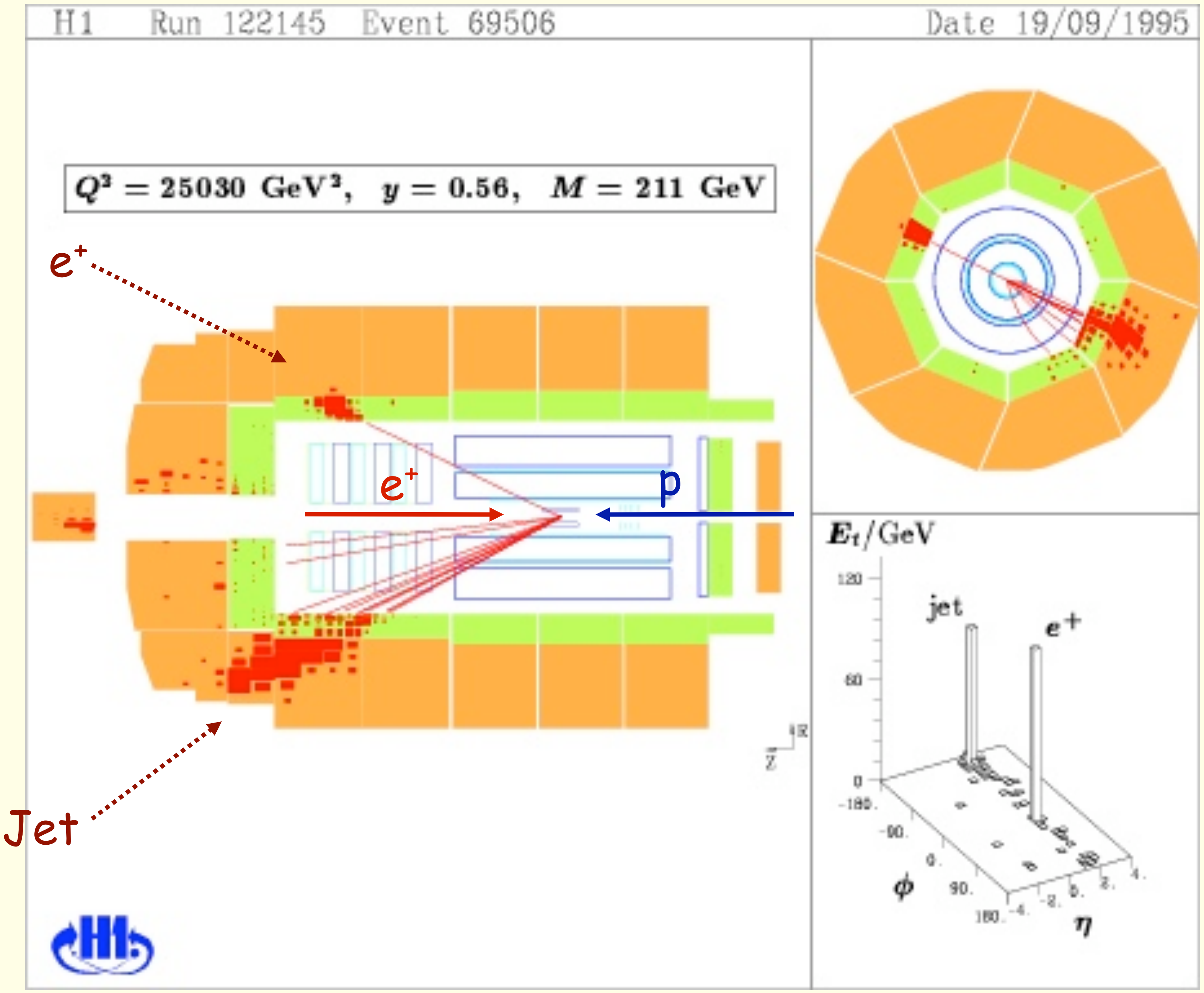
$$\times \sum \left|M(i\,j \to k\,l)\right|^2 \frac{1}{1+\delta_{kl}}\,\delta(\hat{s}+\hat{t}+\hat{u})$$

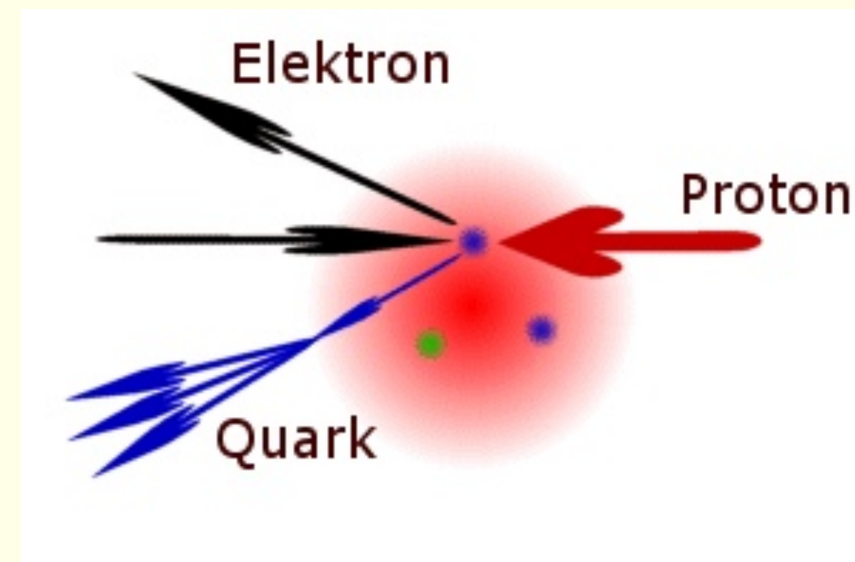
Point Cross Sections at Leading Order

Process	$\overline{\sum}\lvert\mathcal{M}\rvert^2/g^4$	$\theta^* = \pi/2$
$q\,q' \to q\,q'$	$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.22
$q\,\overline{q'} \to q\,\overline{q'}$	$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.22
$q\,q \to q\,q$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}$	3.26
$q\,\overline{q} \to q'\,\overline{q'}$	$\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.22
$q\,\overline{q} \to q\,\overline{q}$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)-\frac{8}{27}\frac{\hat{u}^2}{\hat{s}\hat{t}}$	2.59
$q\,\overline{q} \to g\,g$	$\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	1.04
$g\,g \to q\,\overline{q}$	$\frac{1}{6}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{3}{8}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.15
$g\,q \to g\,q$	$-\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}+\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}$	6.11
$g\,g \to g\,g$	$\frac{9}{2}\left(3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$	30.4

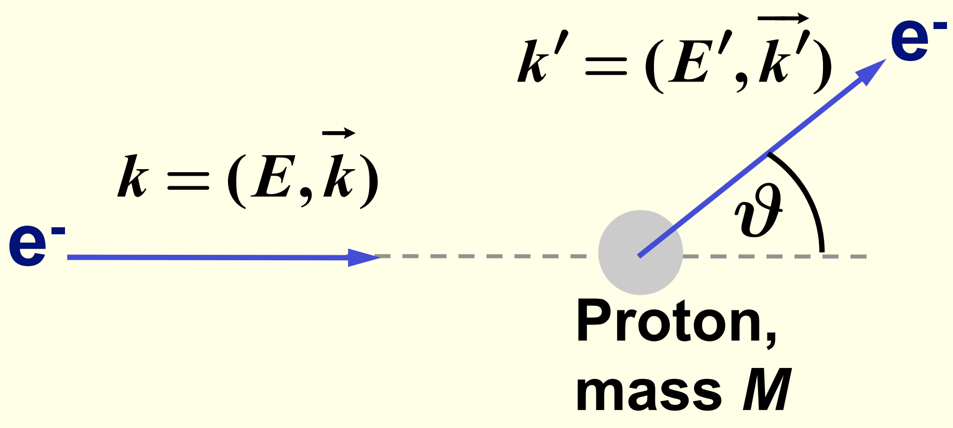


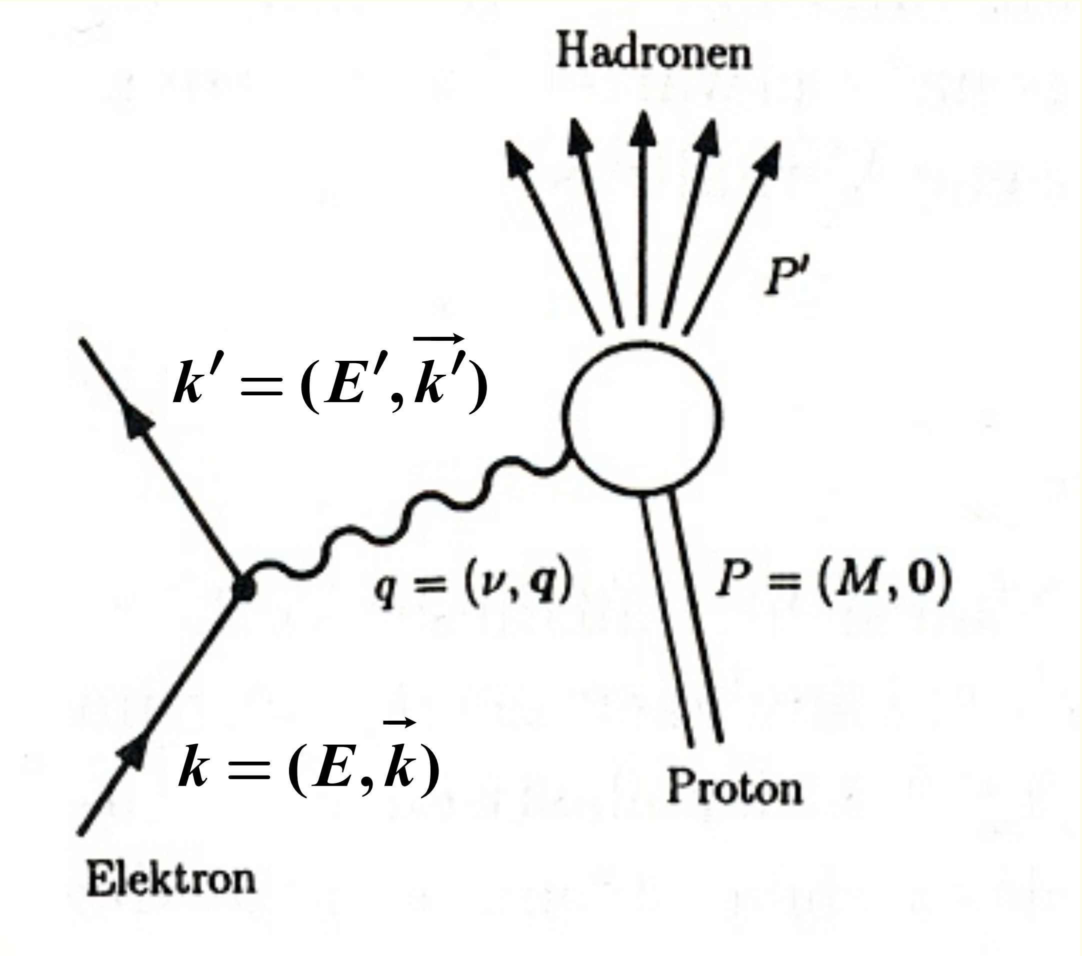
e^++p Scattering: H1 event display





Determination of Parton Distribution Functions: Deep-Inelastic Lepton-Nucleon Scattering: Kinematic Variables





$$\nu = E - E' \quad \text{(in lab. frame)}$$

$$Q^2 = -q^2 :$$

Structure functions to be determined in the experiment

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{(Q^2)^2} \cdot \frac{E}{E'} \left(W_2(Q^2,\nu) \cos^2\frac{\vartheta}{2} + 2W_1(Q^2,\nu) \sin^2\frac{\vartheta}{2} \right)$$

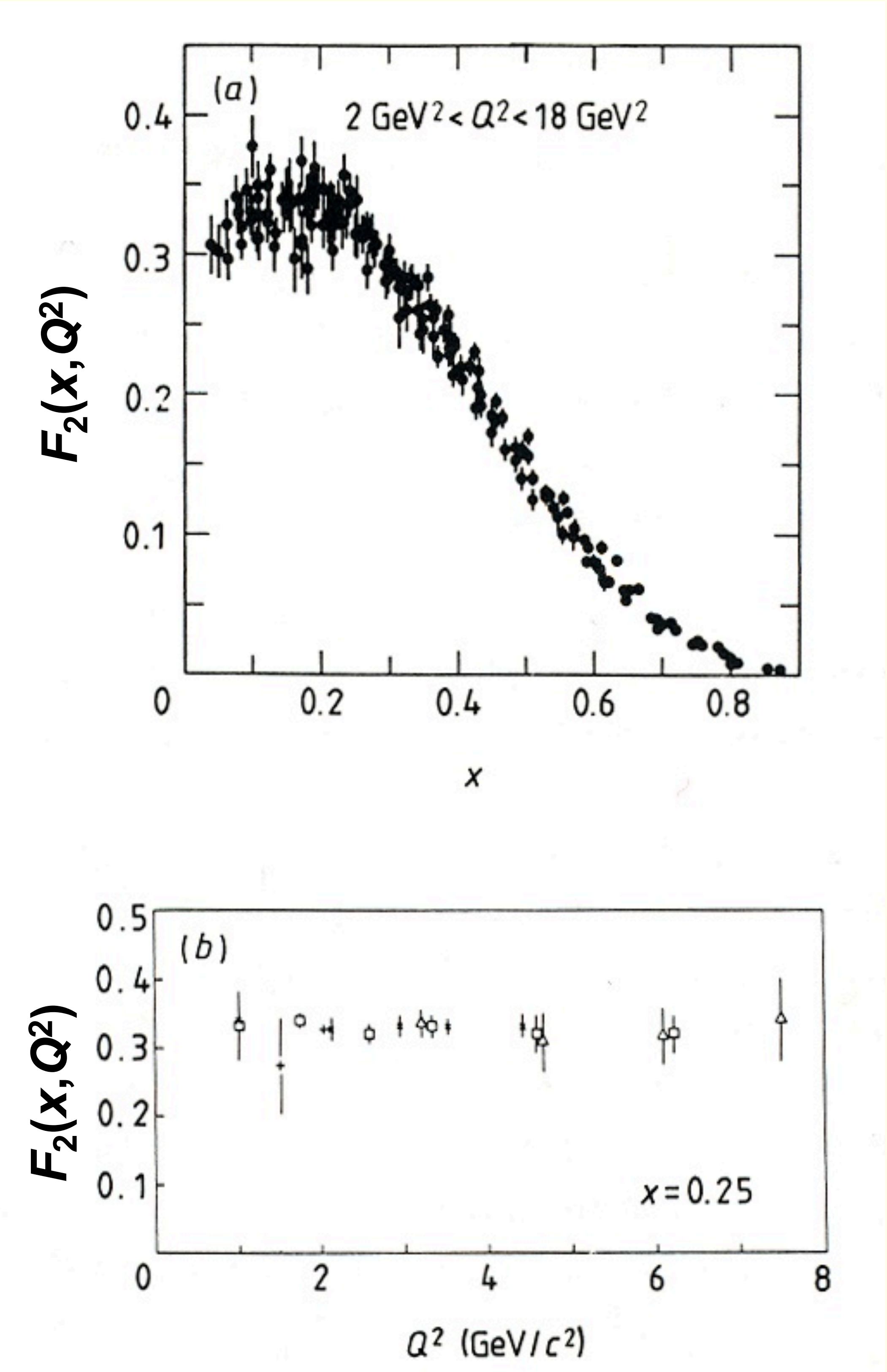
Discovery of Scaling at SLAC

Bjorken-x: $$x := \frac{Q^2}{2M\nu}$$

In the limit $Q^2, \nu \to \infty$
the structure functions only depend on *x*. This is called *scaling*:

$$W_1(Q^2,\nu) \qquad \to F_1(x),$$
$$\frac{\nu}{M} \cdot W_2(Q^2,\nu) \to F_2(x)$$

Interpretation:
Inelastic lepton-nucleon scattering can be regarded as incoherent elastic scattering of the lepton off pointlike constituents of the nucleon, called partons.



Quark-Parton Model

Quark-Parton Model:

- View nucleon in infinite momentum frame so that transverse momenta of the partons can be neglected
- The Bjorken-*x* can the be interpreted as the momentum fraction *x* ($0 \leq x \leq 1$) of the nucleon that is carried by the parton that participated in the scattering
- Identify partons with quarks and gluons

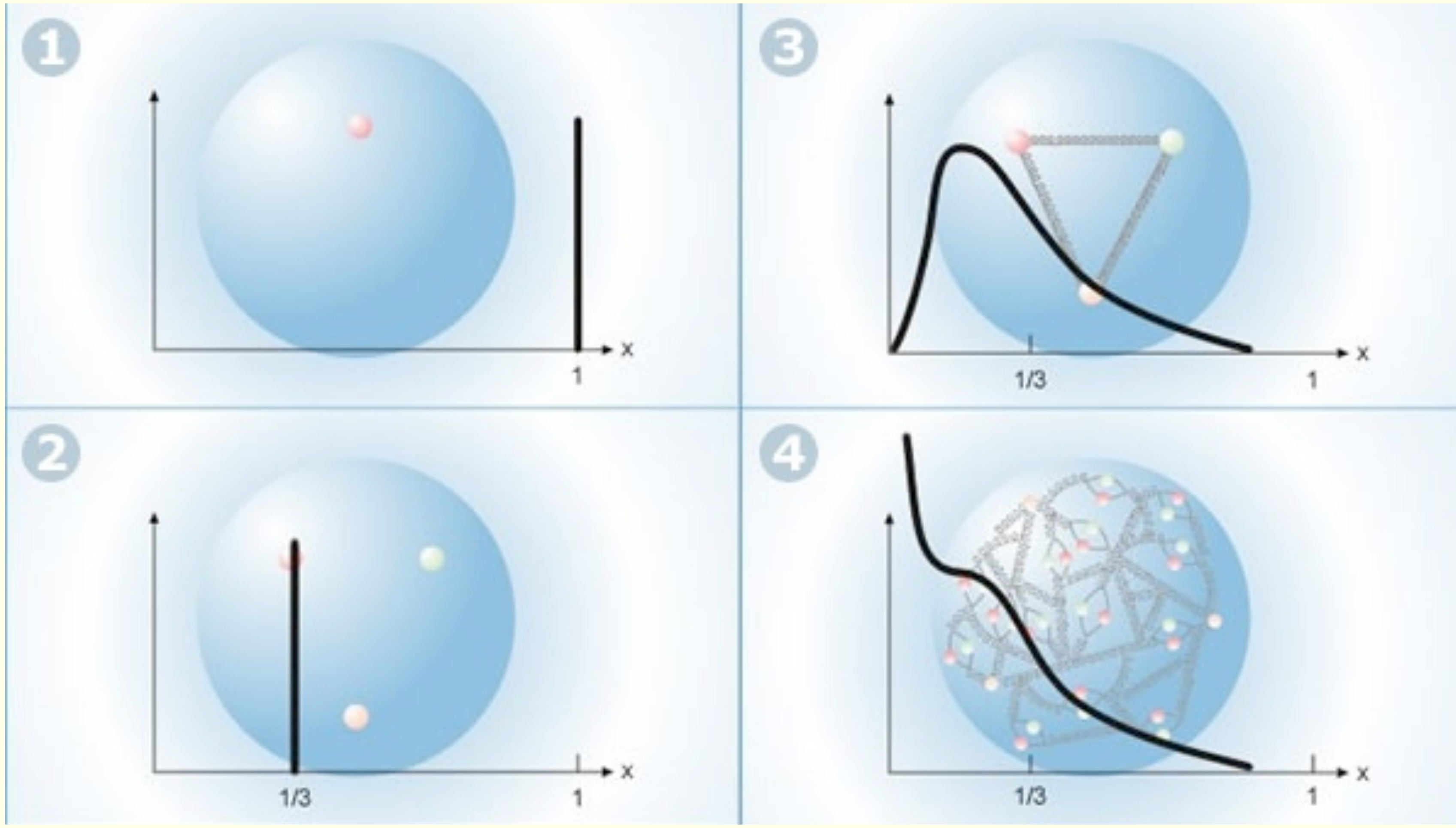
The structure function F_2(x) is then given by

$$F_2(x) = x \sum_f z_f^2 (q_f(x) + \bar{q}_f(x))$$

where

$q_f(x)\,\mathrm{d}x\left[\bar{q}_f(x)\mathrm{d}x\right]$:	number of quarks [antiquarks] of flavor f with fractional momenta between x...x+dx
z_f :	quark charge (e.g. 2/3 for u-quark)

Parton Distributions (I)



Proton =

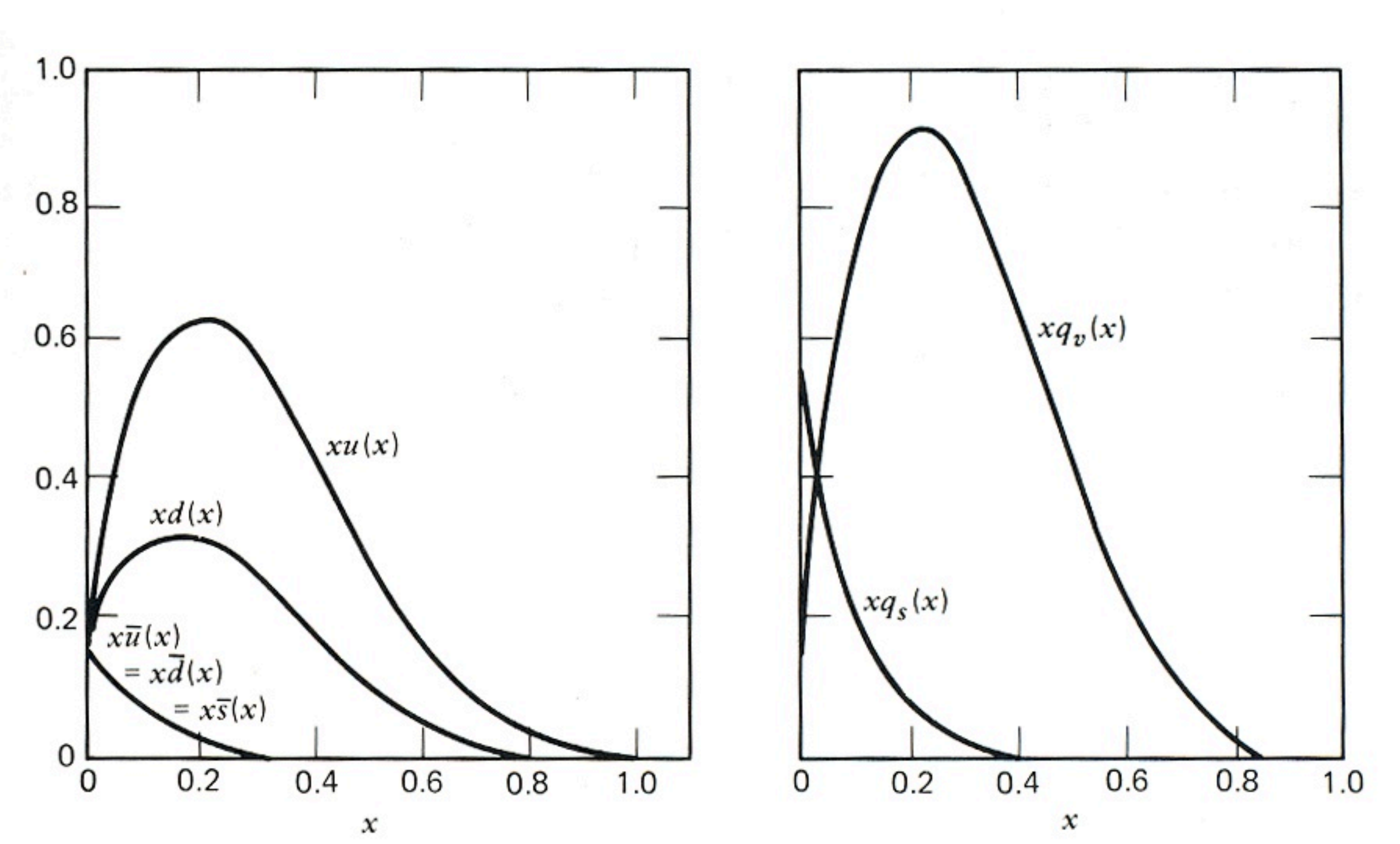
1. 1 valence quark
2. 3 valence quarks
3. 3 valence quarks
   + gluons
4. 3 valence quarks
   + gluons
   + sea quarks

Interpretation of x_{Bjorken} in the parton model:

In a reference frame in which the parton transverse momentum can be neglected (infinite momentum frame) x_{Bjorken} represents the fraction of the 4-momentum of the nucleon carried by the parton

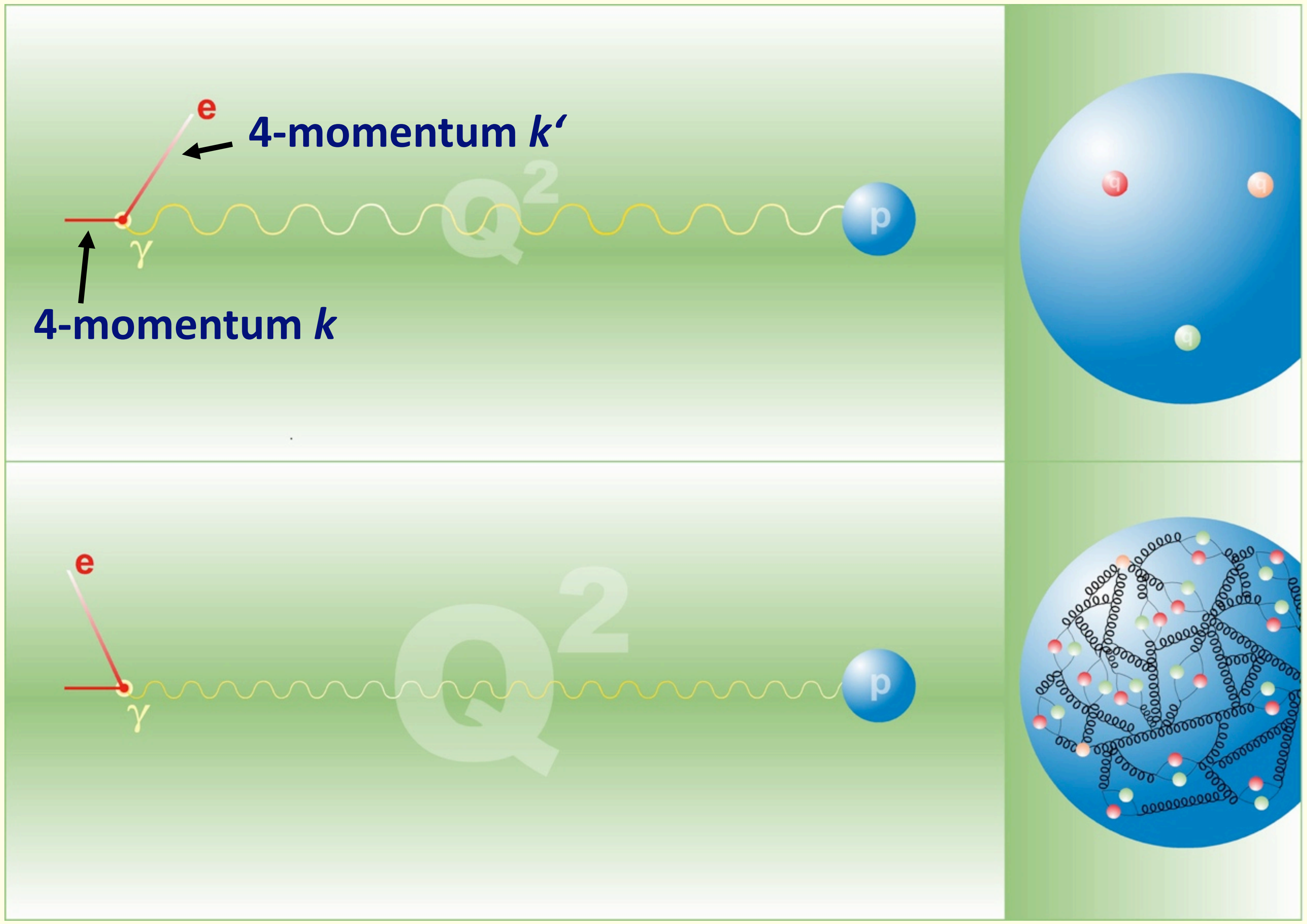
Parton Distributions (II)

parton distributions in the proton:



$q(x)\,\mathrm{d}x$: **parton distribution = number of partons in [*x*, *x*+d*x*]**

Resolution of the Virtual Photon



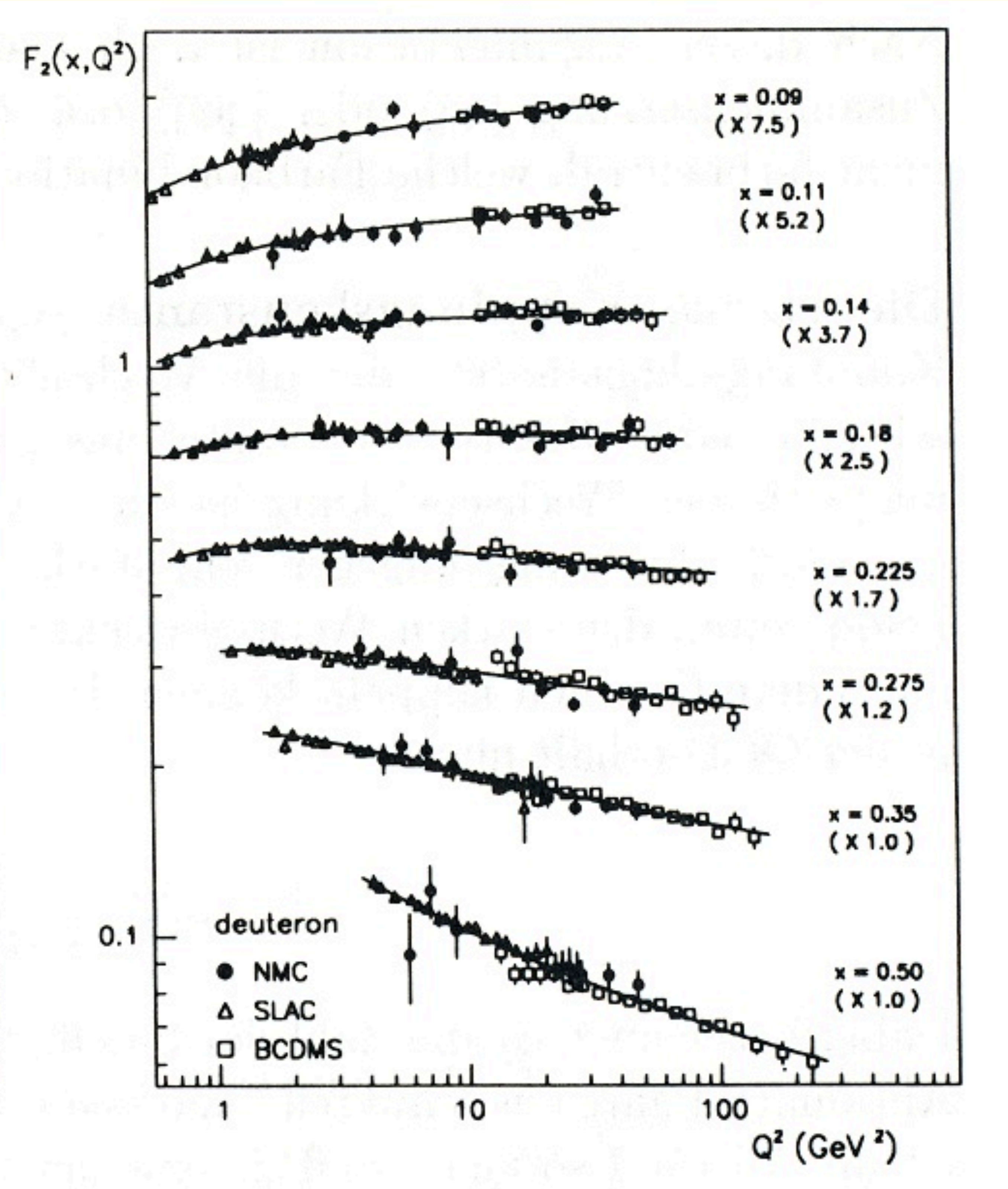
Valence quarks

Valence quarks + sea quarks

4-momentum transfer: $q = (k - k'), \quad Q^2 := -q^2$

wave length of the virtual photon: $\lambda = \dfrac{\hbar}{\sqrt{Q^2}}$

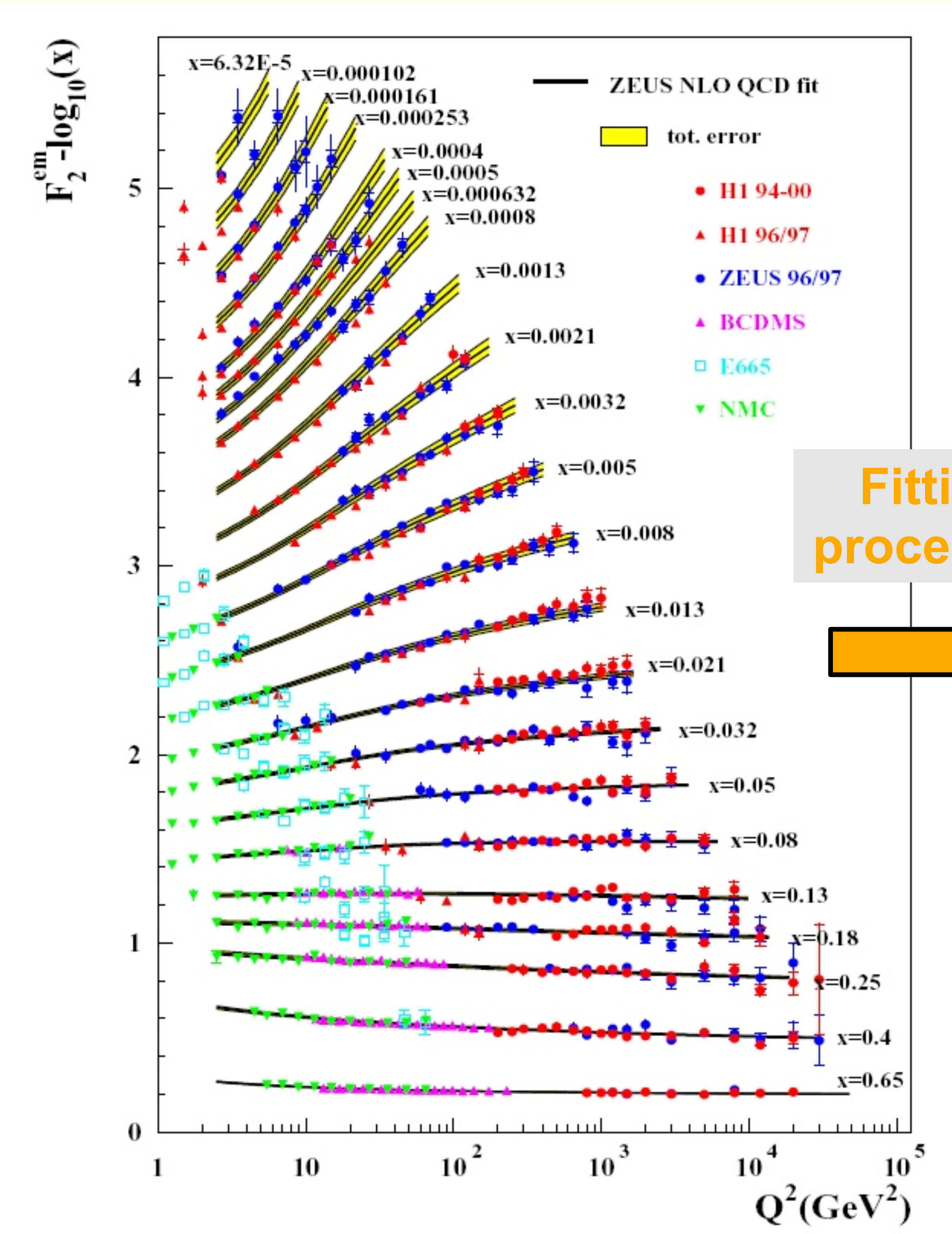
Scaling Violations



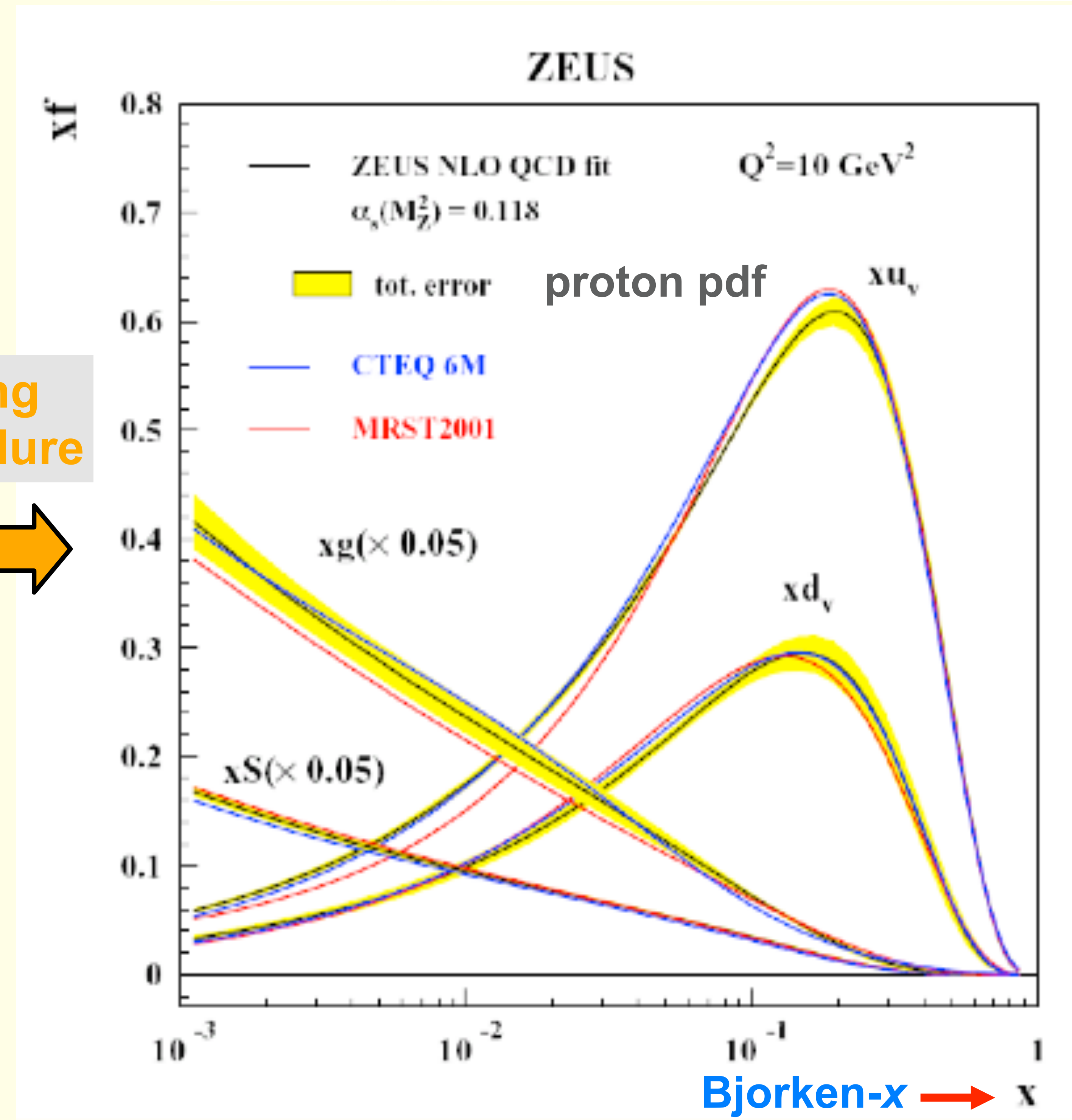
- **Scaling violation**
  - **Small x (< 0.1):** **$F_2(x,Q^2)$ increases with Q^2**
  - **Large x (> 0.3):** **$F_2(x,Q^2)$ decreases with Q^2**

Parton Distributions: High Precession Data from HERA

$$d\sigma = \sum_{a,b,c} f_a \otimes f_b \otimes d\hat{\sigma}^c_{ab} \otimes D^{Hadron}_c$$

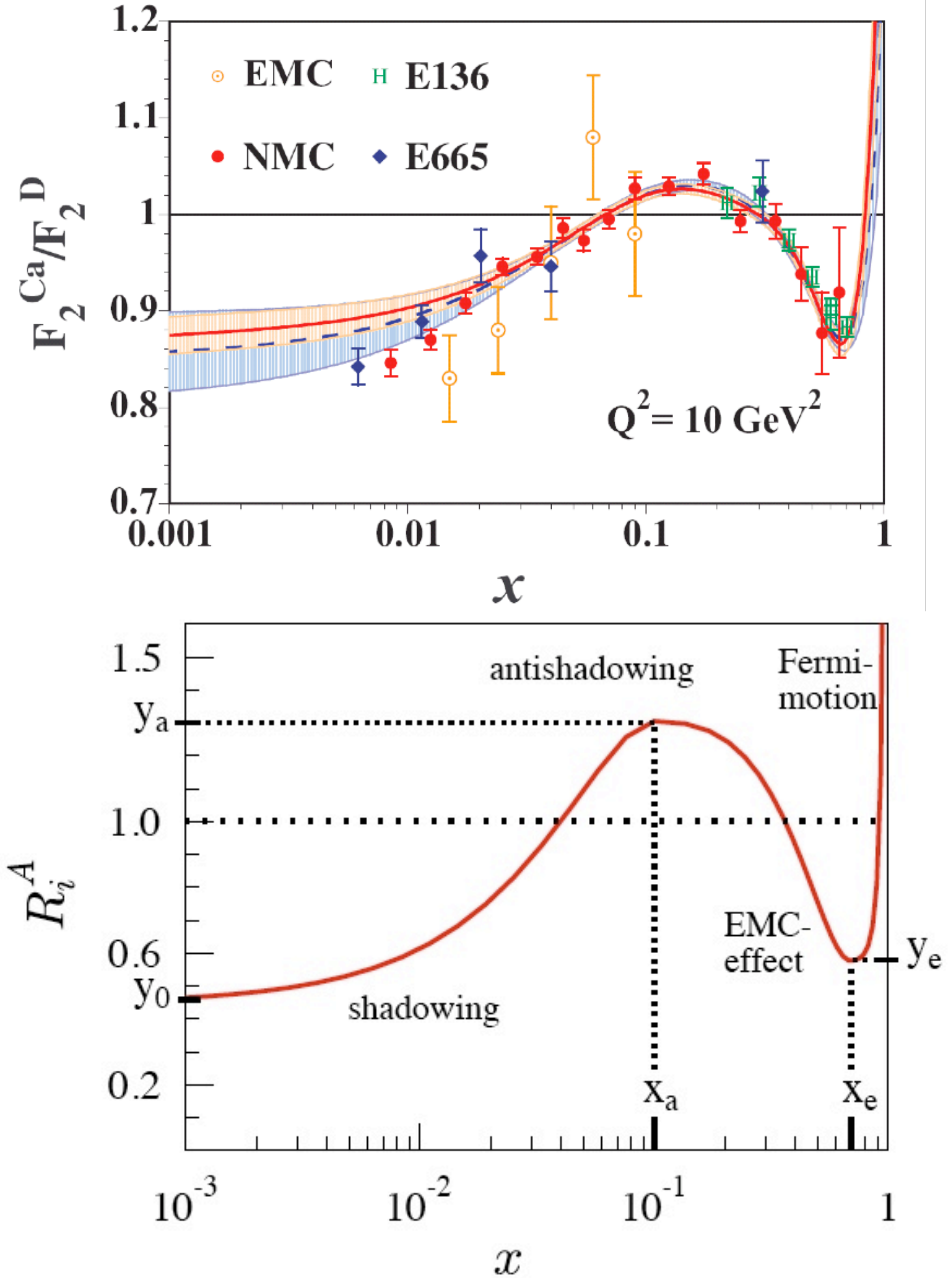


Fitting procedure



Bjorken-x → x

Modification of the Structure Functions in Nuclei



$x < 0.1$: "shadowing region"

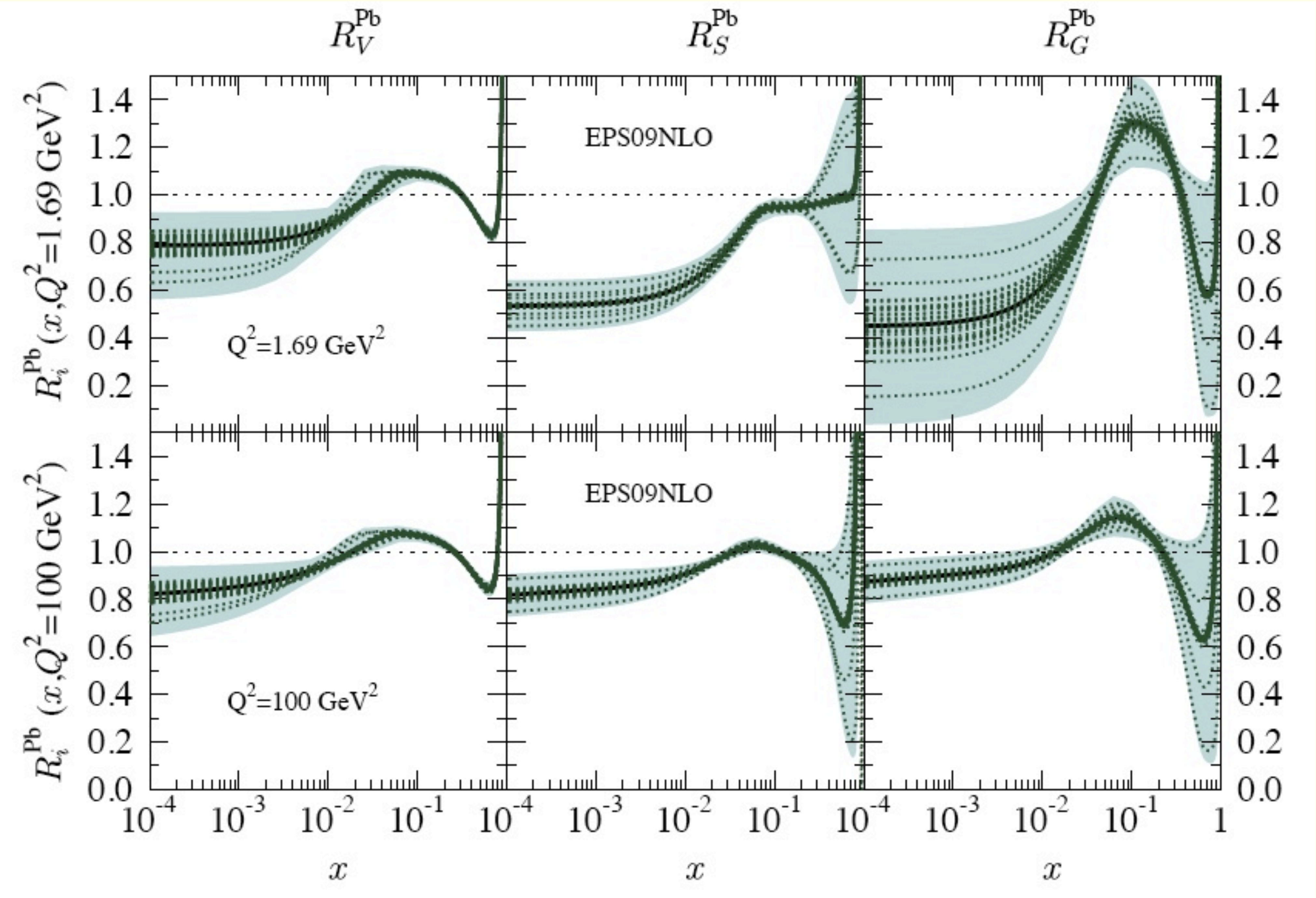
$0.1 < x < 0.3$: "anti-shadowing"

$0.3 < x < 0.7$: "EMC effect"

$0.7 < x < 1.0$: Fermi-motion of nucleons in nuclei

An Example: Nuclear PDF's in Pb (EPS09NLO)

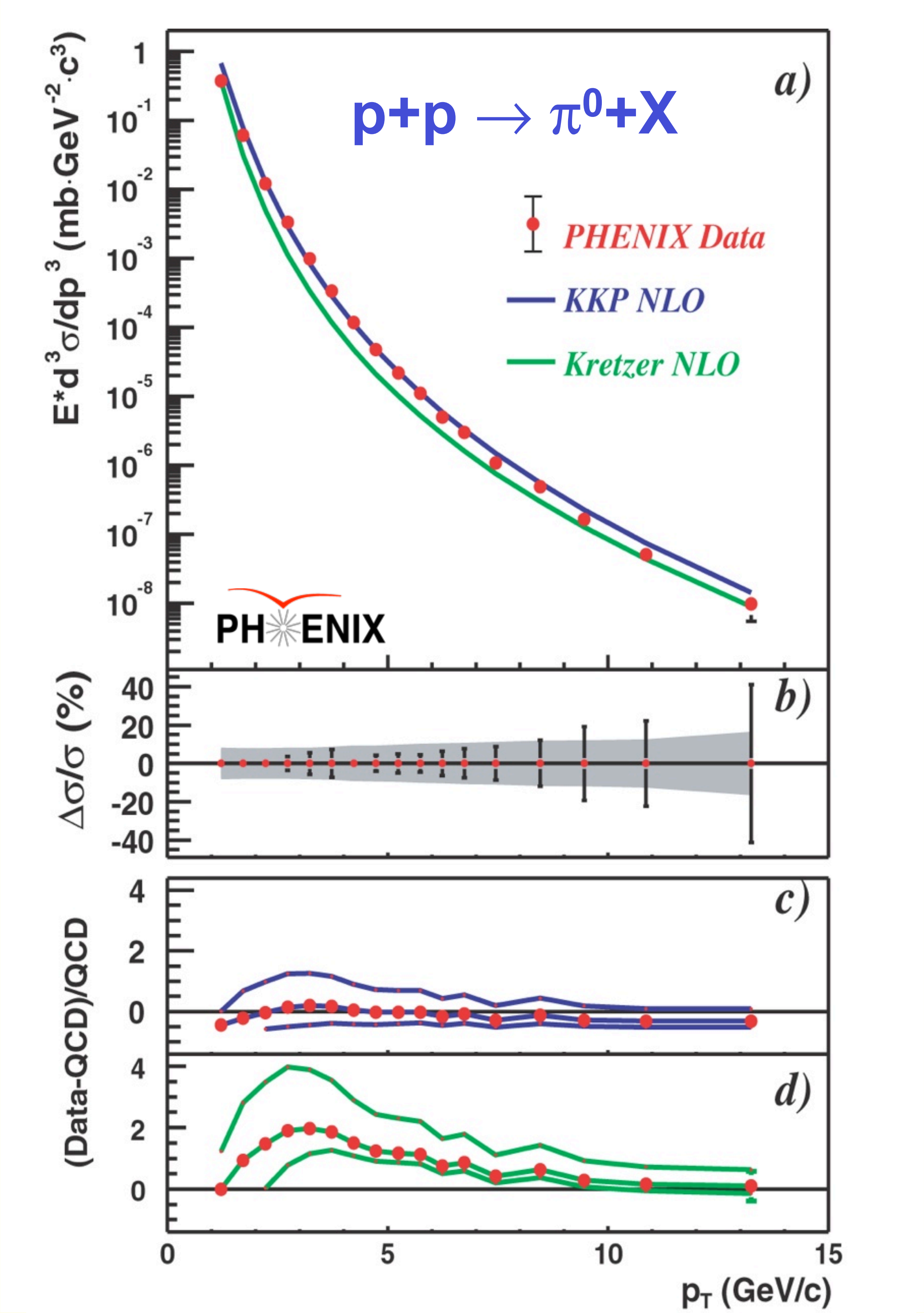
Valence quarks Sea quarks gluons



Large uncertainties for gluon PDF's at small *x*

Eskola et al.,
arXiv:0902.4154v2 [hep-ph]

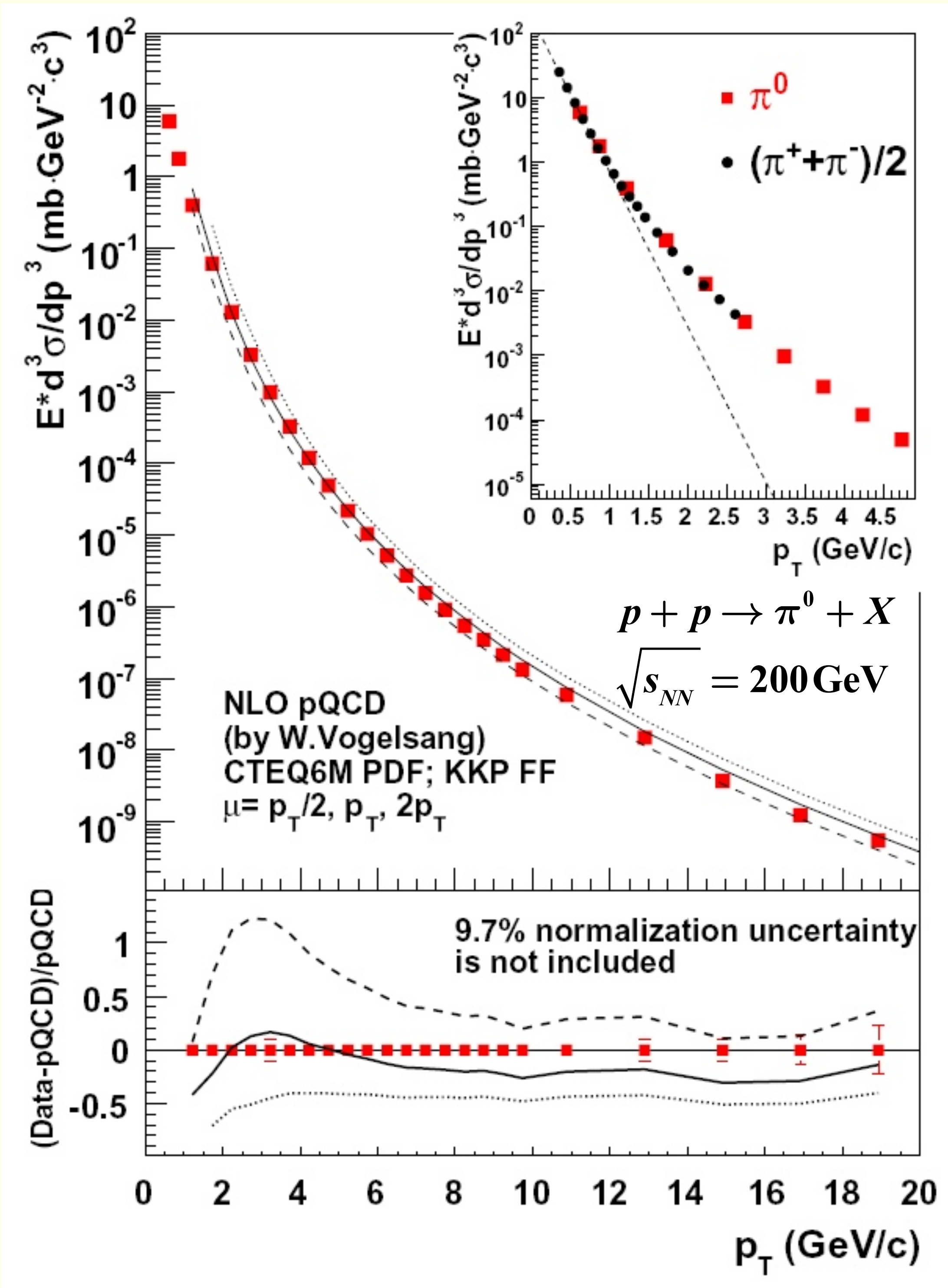
pQCD works in p+p at √s = 200 GeV

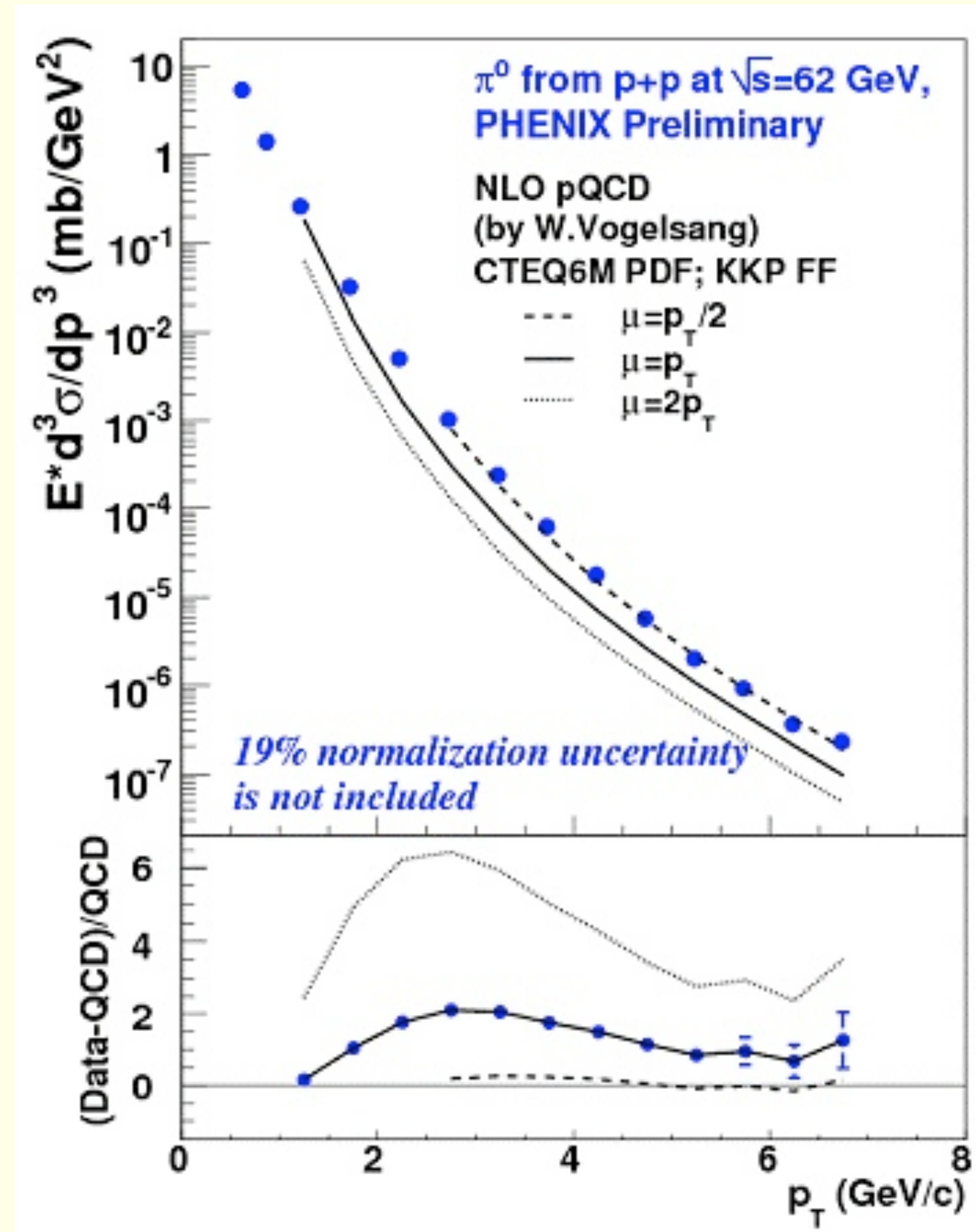


- π^0 production in p+p well described by perturbative QCD
- Data sensitive to differences in gluon fragmentation function (KKP vs. Kretzer)
- Reference for Au+Au
- Description of hard processes under control at RHIC energy

PRL 91, 241802 (2003)

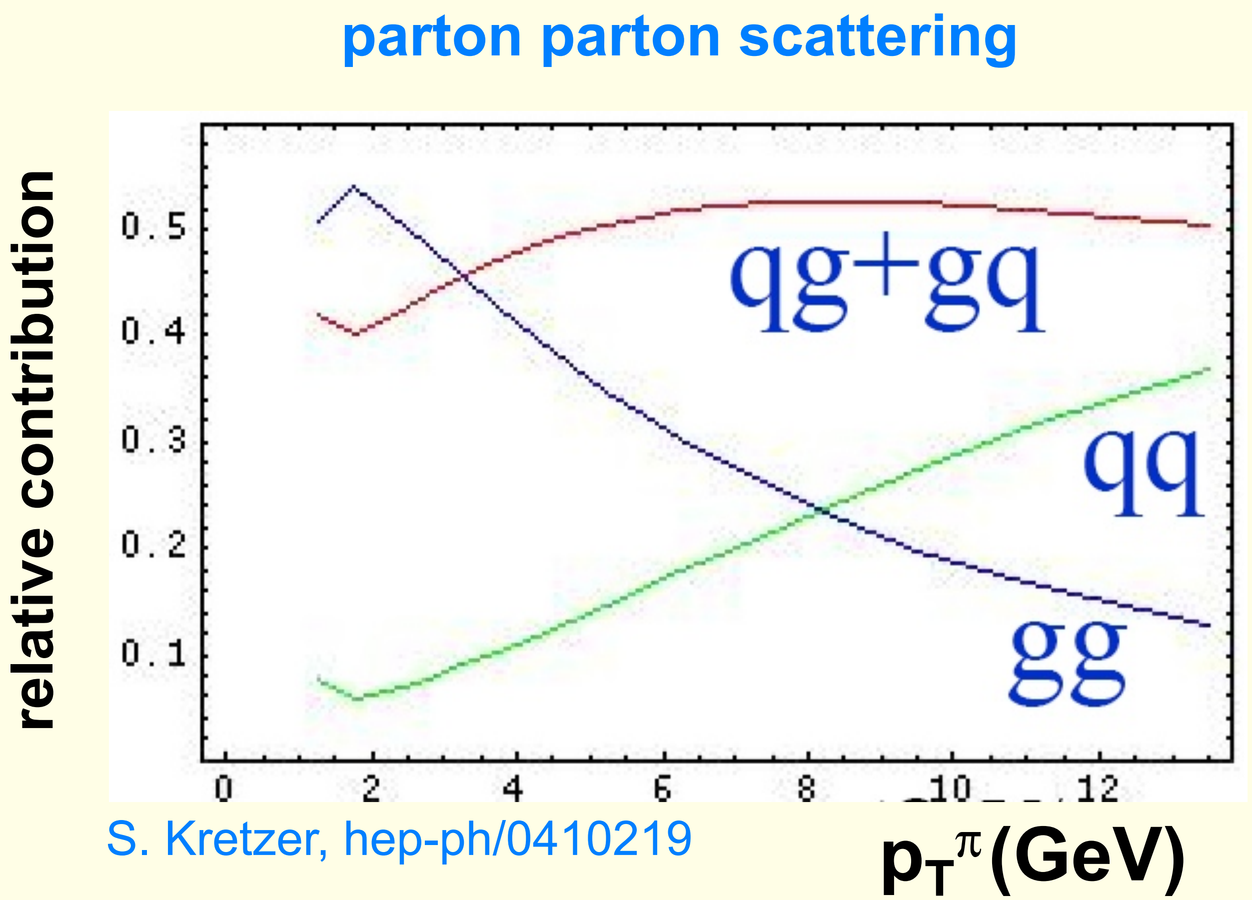
More Recent Data with Higher Statistics: Same Conclusion: pQCD Works at RHIC Energies



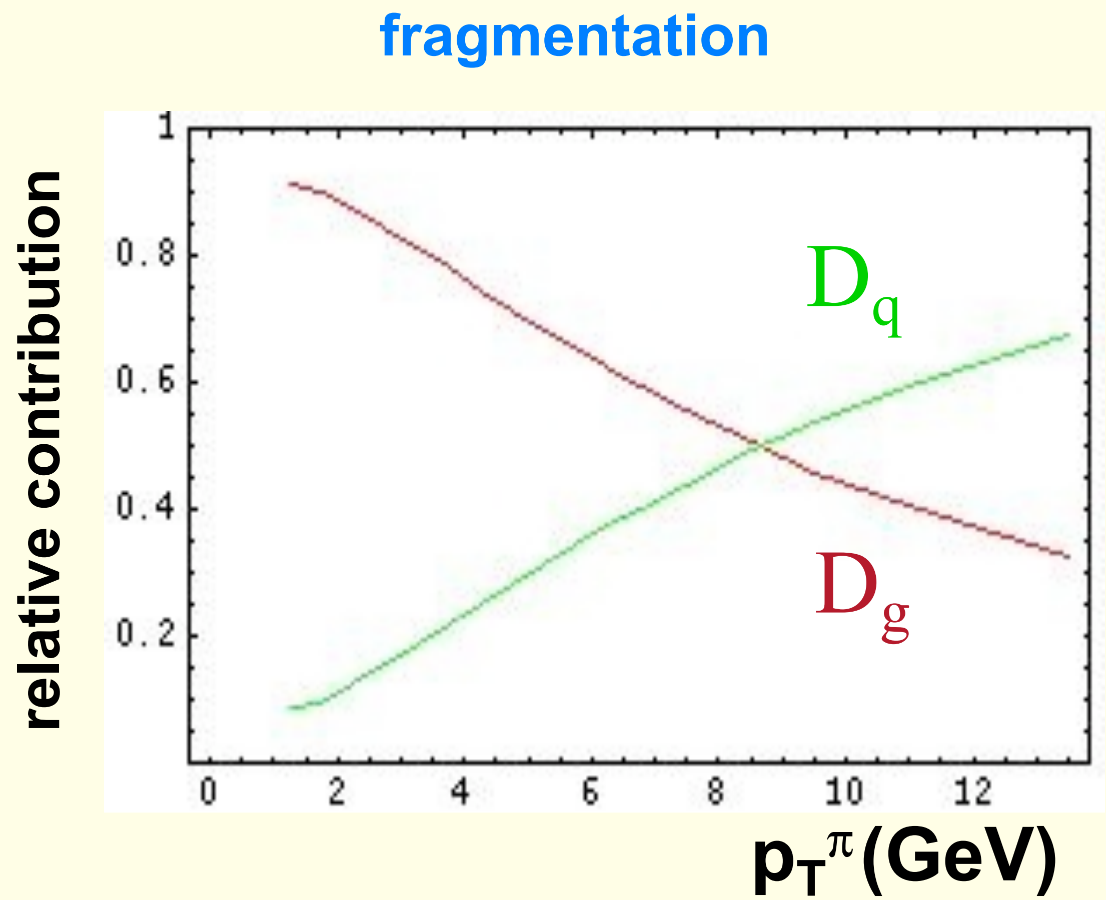


Agreement with pQCD in p+p is a prerequisite for parton energy loss calculations

Different Contributions to the Pion Spectrum in p+p at $\sqrt{s}$ = 200 GeV



S. Kretzer, hep-ph/0410219

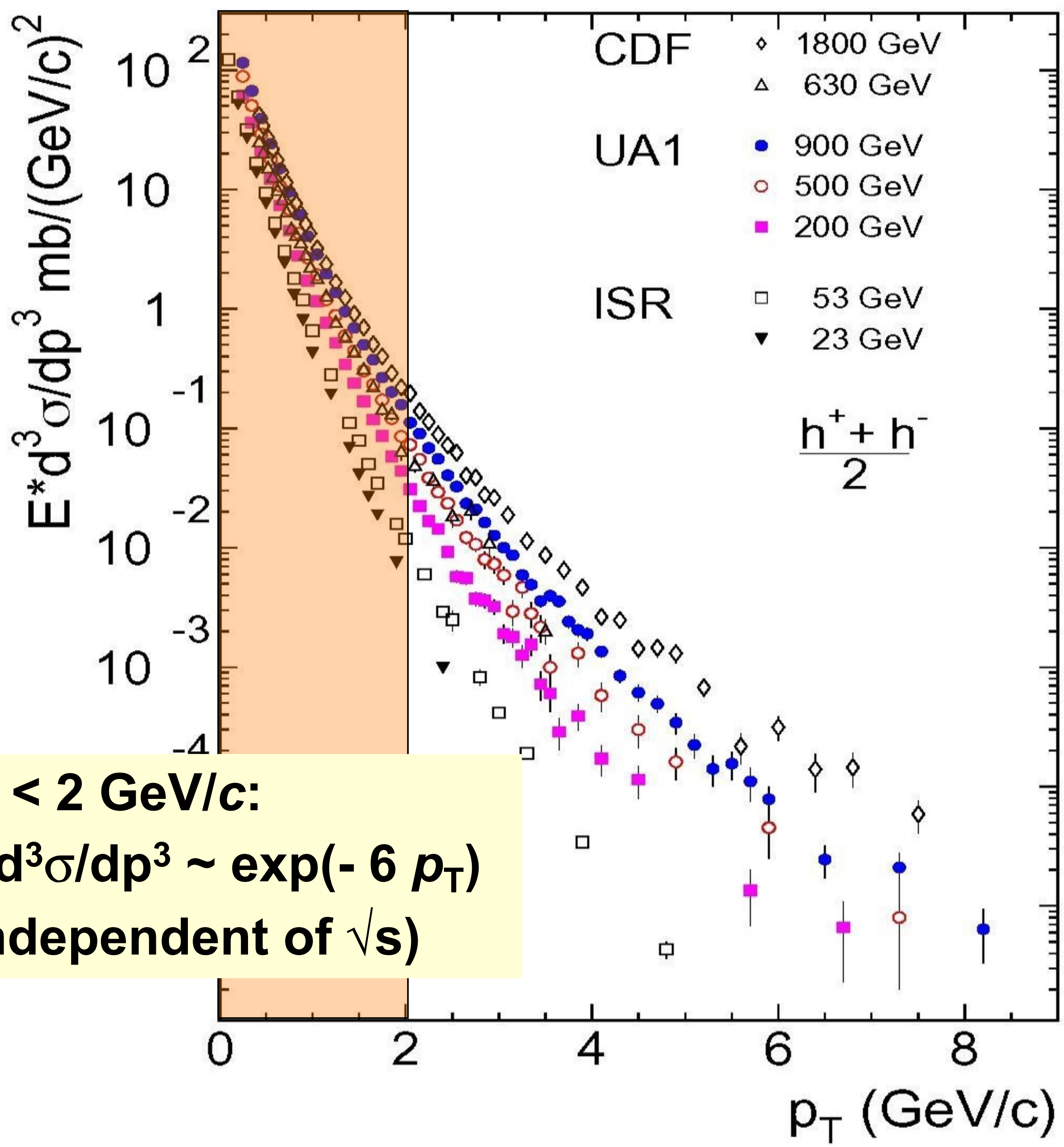


p_T < 9 GeV/c: gluon fragmentation dominates

p_T > 9 GeV/c: quark fragmentation dominates

Particle Production at High p_T (I)

p_T spectra of charged particles for various √s



p_T < 2 GeV/c:
$E \cdot d^3\sigma/dp^3 \sim \exp(-6\, p_T)$
(independent of √s)

High p_T part of the spectrum flattens with increasing √s

Low p_T (< 2 GeV/c):

$$\frac{1}{p_T}\frac{dN_x}{dp_T} = A(\sqrt{s}) \cdot e^{-6\,p_T}$$

High p_T:

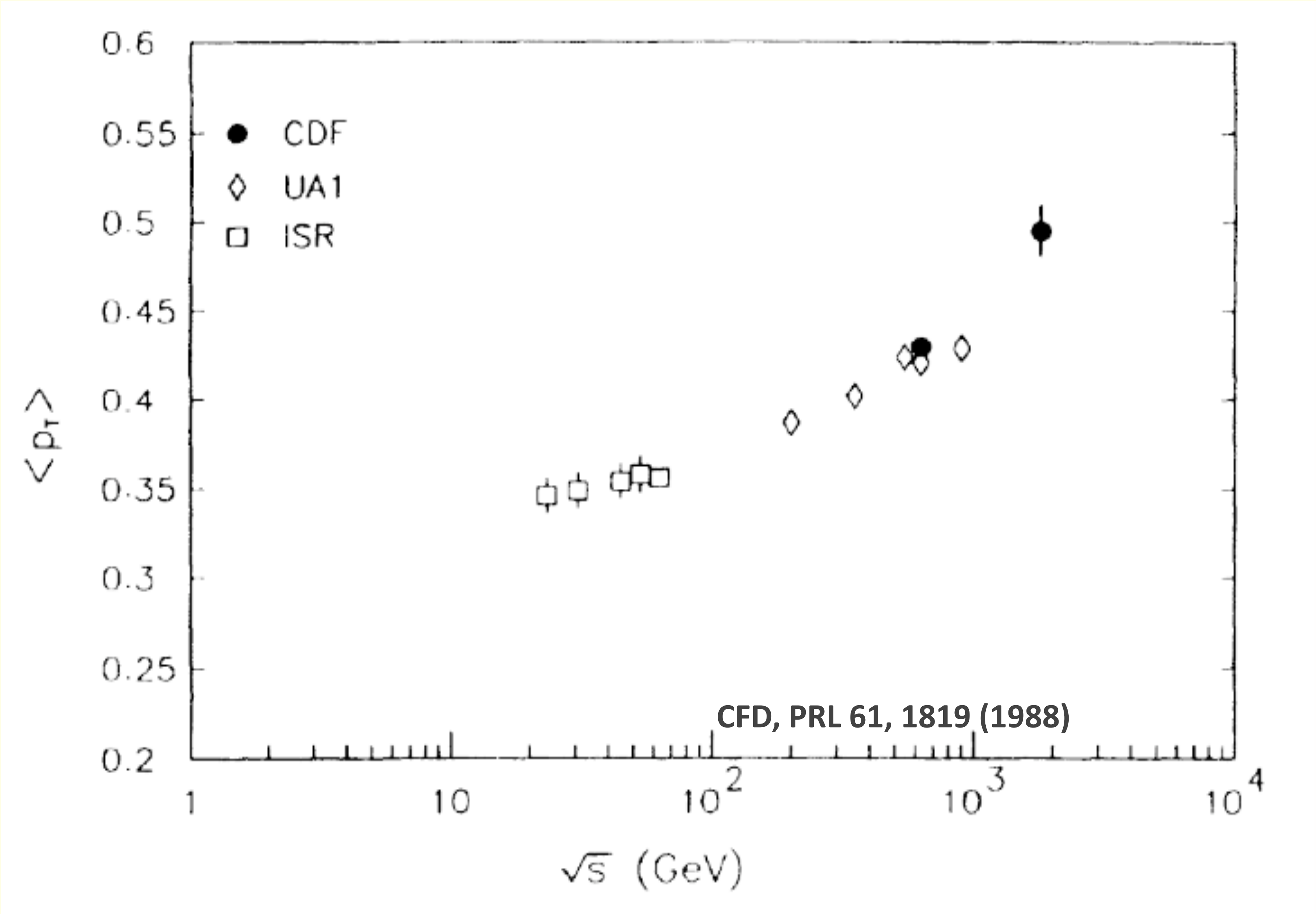
$$\frac{1}{p_T}\frac{dN_x}{dp_T} = A(\sqrt{s}) \cdot \frac{1}{p_T^n}$$

Average transverse momentum:

$$\langle p_T \rangle = \frac{\int_0^\infty p_T \frac{dN_x}{dp_T} dp_T}{\int_0^\infty \frac{dN_x}{dp_T} dp_T} \approx 300 - 400 \text{ MeV}/c$$

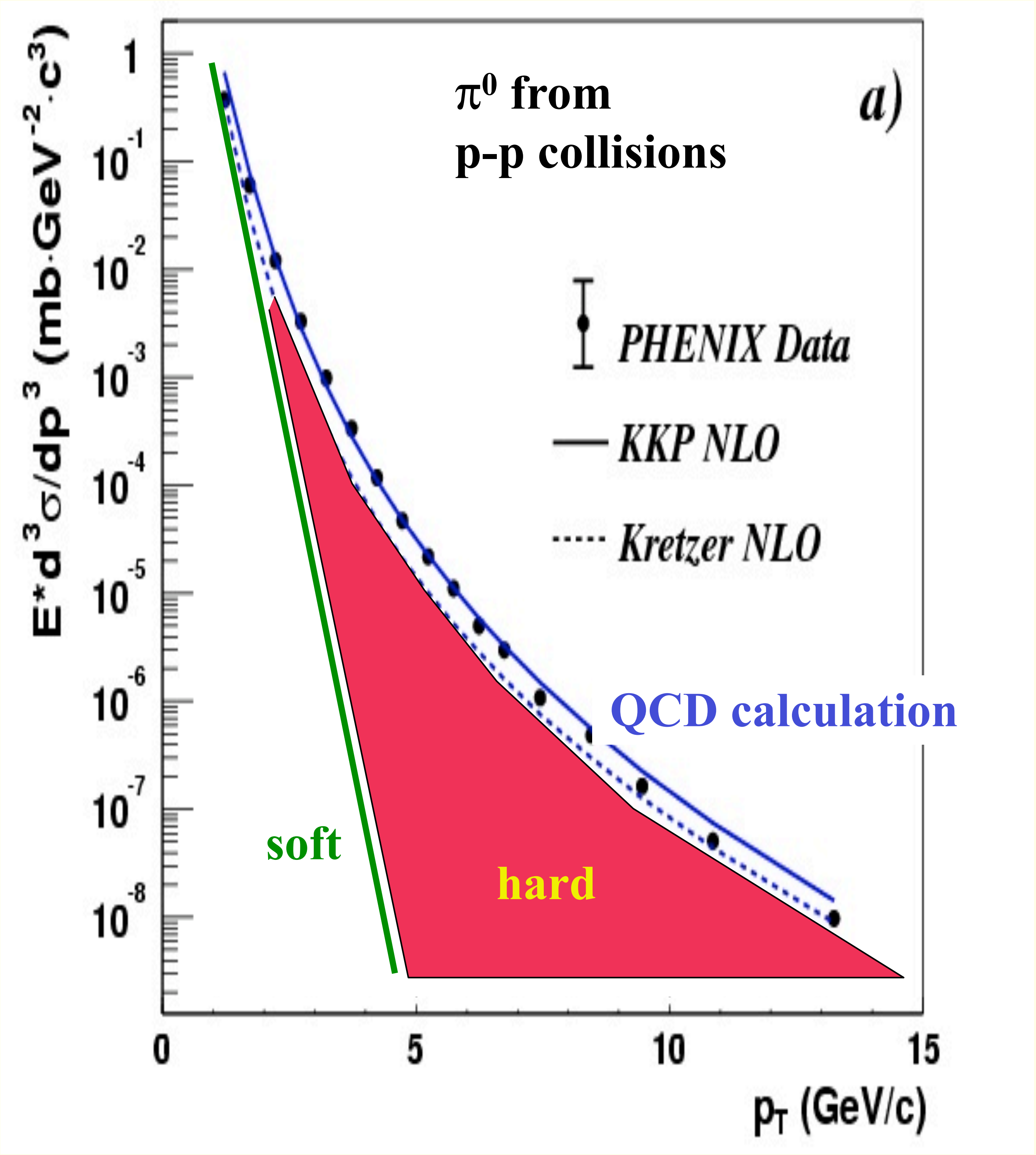
Largely independent of √s (for √s < 200 GeV)

$\langle p_T \rangle$ vs. $\sqrt{s}$ in p+p(bar p)



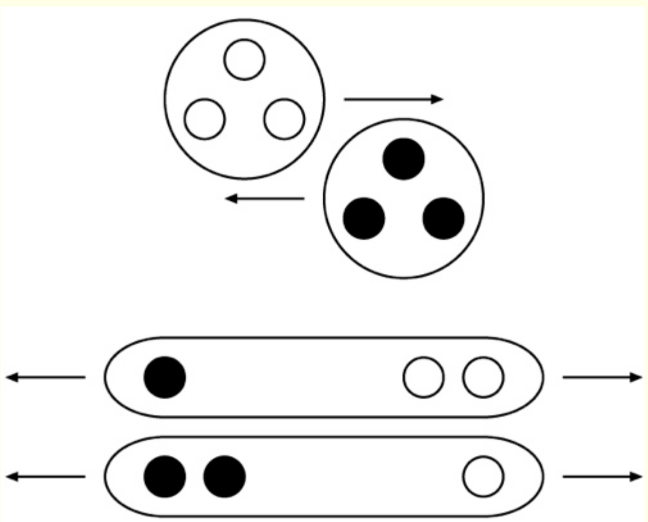
Increase of $\langle p_T \rangle$ reflects increase in hard scattering

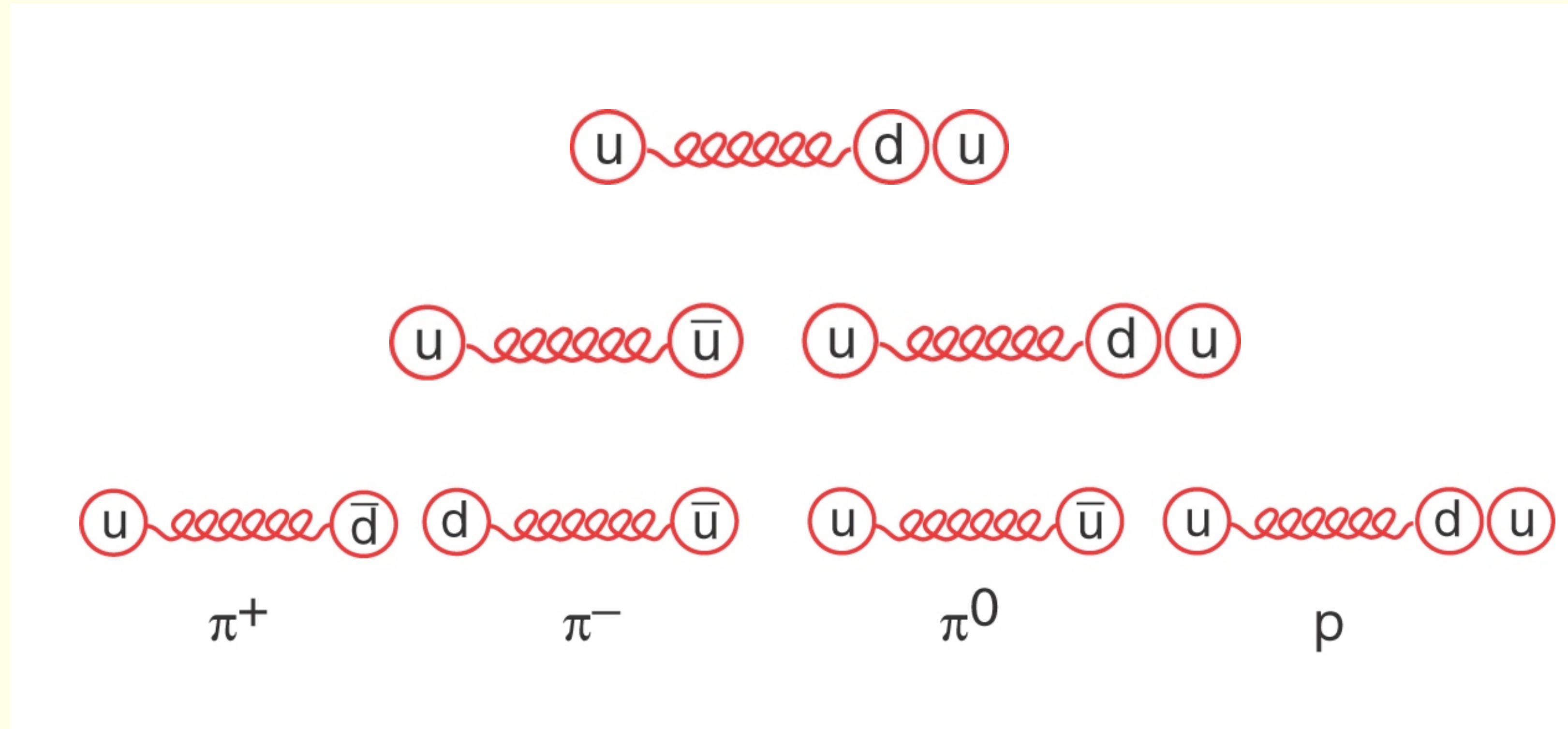
Particle Production at High p_T (II)



Hard vs. soft particle production

A Model for Particle Production at Low p_T in Nucleon-Nucleon Collisions: String Fragmentation

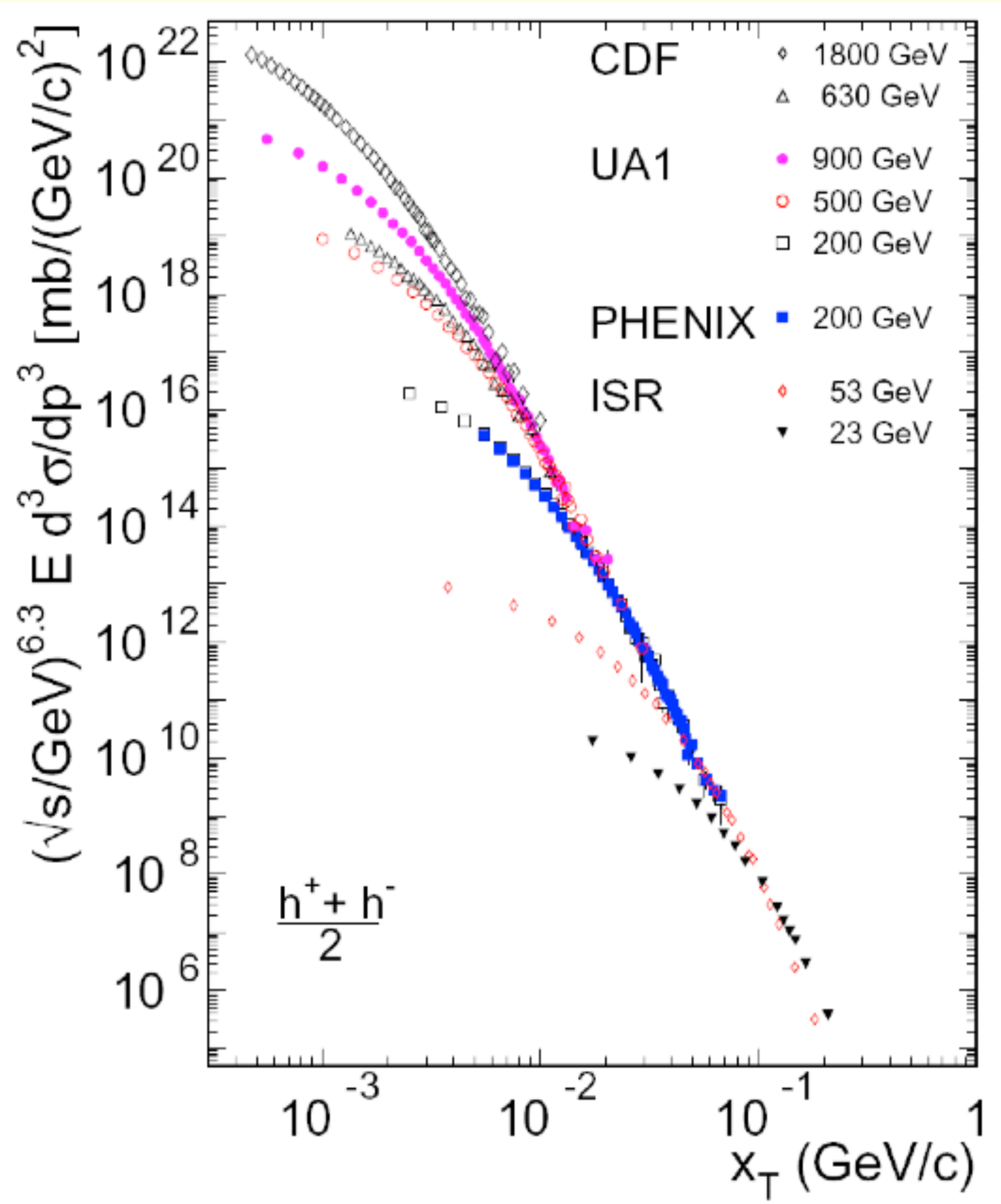




String fragmentation models explains:

- **$\sqrt{s}$ independence of the p_T of produced particles (p_T ~ 350 MeV/c) („string breaking is a local process“)**
- **Shape of the rapidity distribution of produced particles, in particular the plateau at mid-rapidity**

Where is the Transition from Soft to Hard Particle Production?



Scaling expected for parton-parton scattering with high momentum transfer (hard scattering):

$$E\frac{d^3\sigma}{dp^3} = \frac{1}{\sqrt{s}^{\,n(x_T,\sqrt{s})}}\, G(x_T)$$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

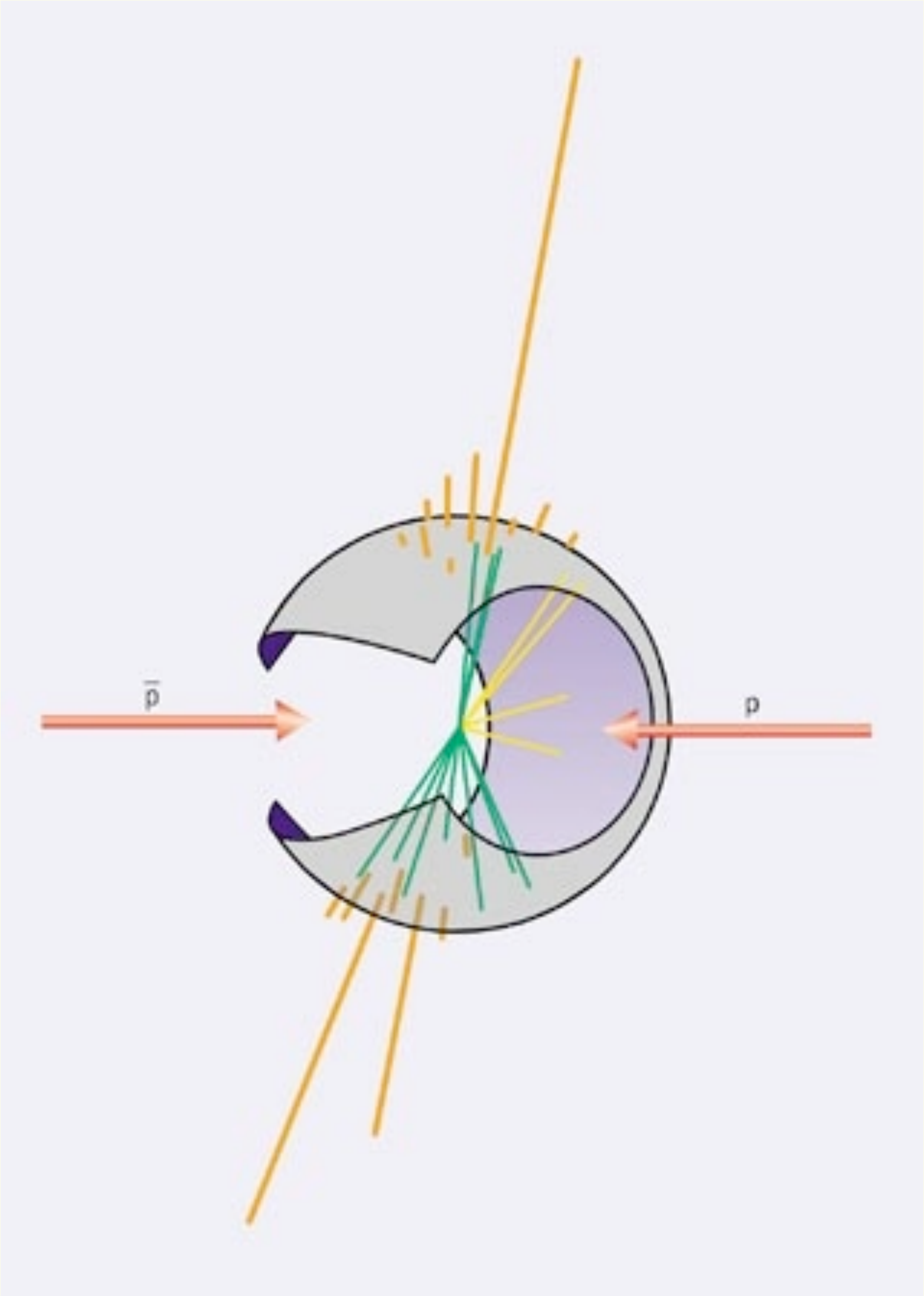
x_T at which scaling behavior is observed decreases with increasing $\sqrt{s}$.

Upshot: Particle production dominated by hard processes for

$$p_T > \sim 2\ \mathrm{GeV}/c$$

3.3 Jets in Nucleon-Nucleon Collisions

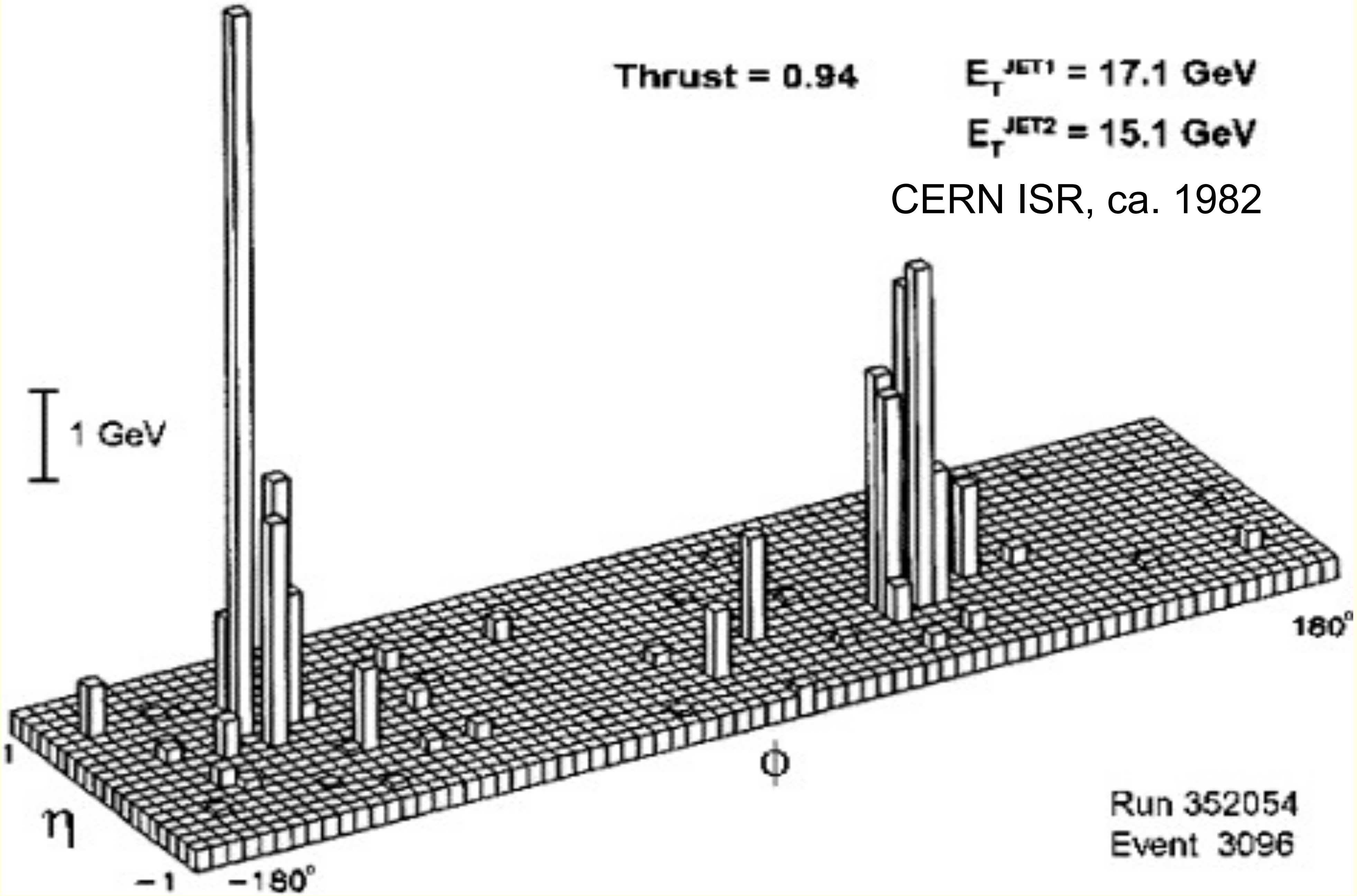
Jets and Hard Scattering in p+p



UA2 two-jet event, ca 1982

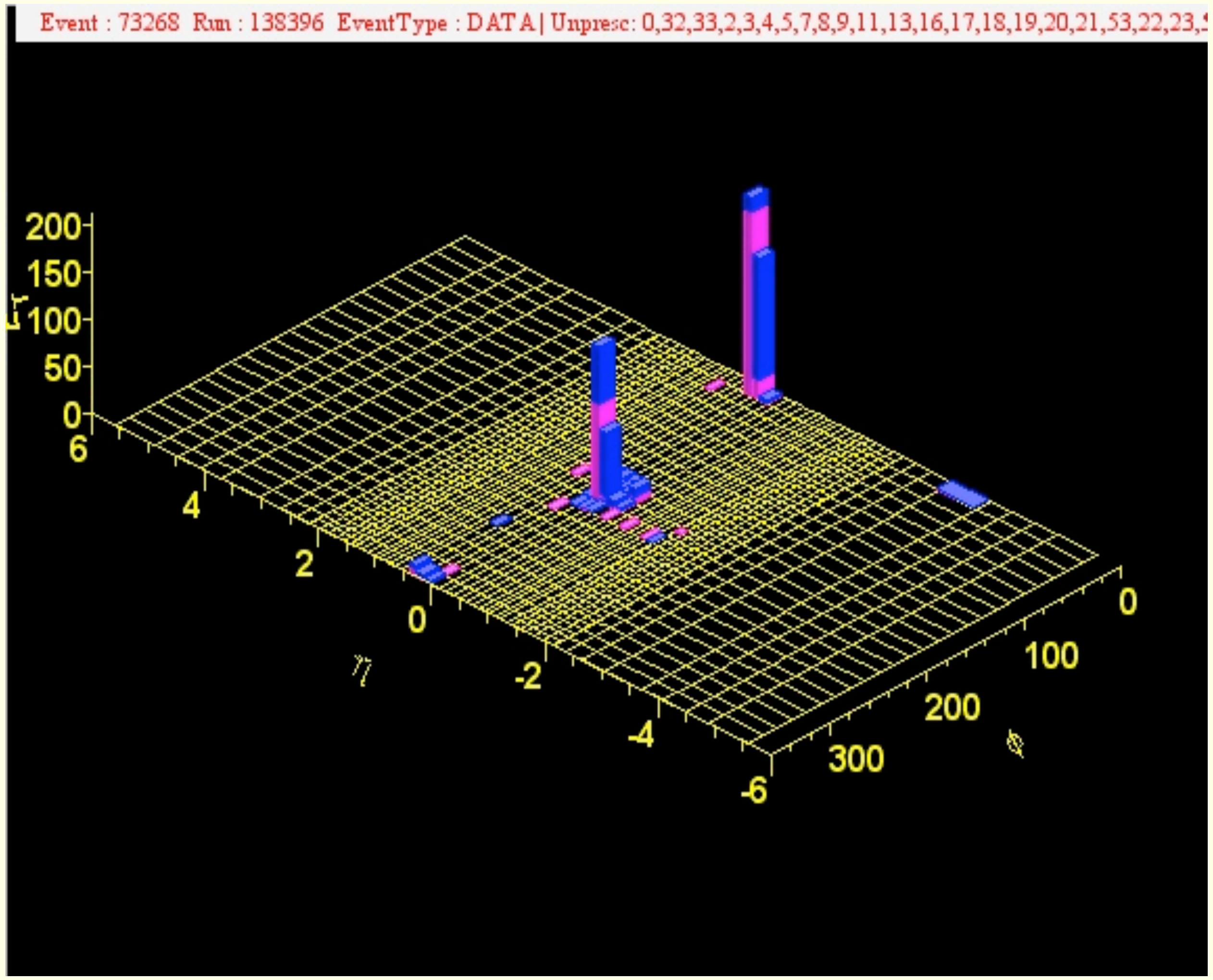
- Jets in p+p were discovered in the late seventies, early eighties
- Confirmation of QCD
- Note: jet cross section small (a typical „minimum bias“ event looks different)

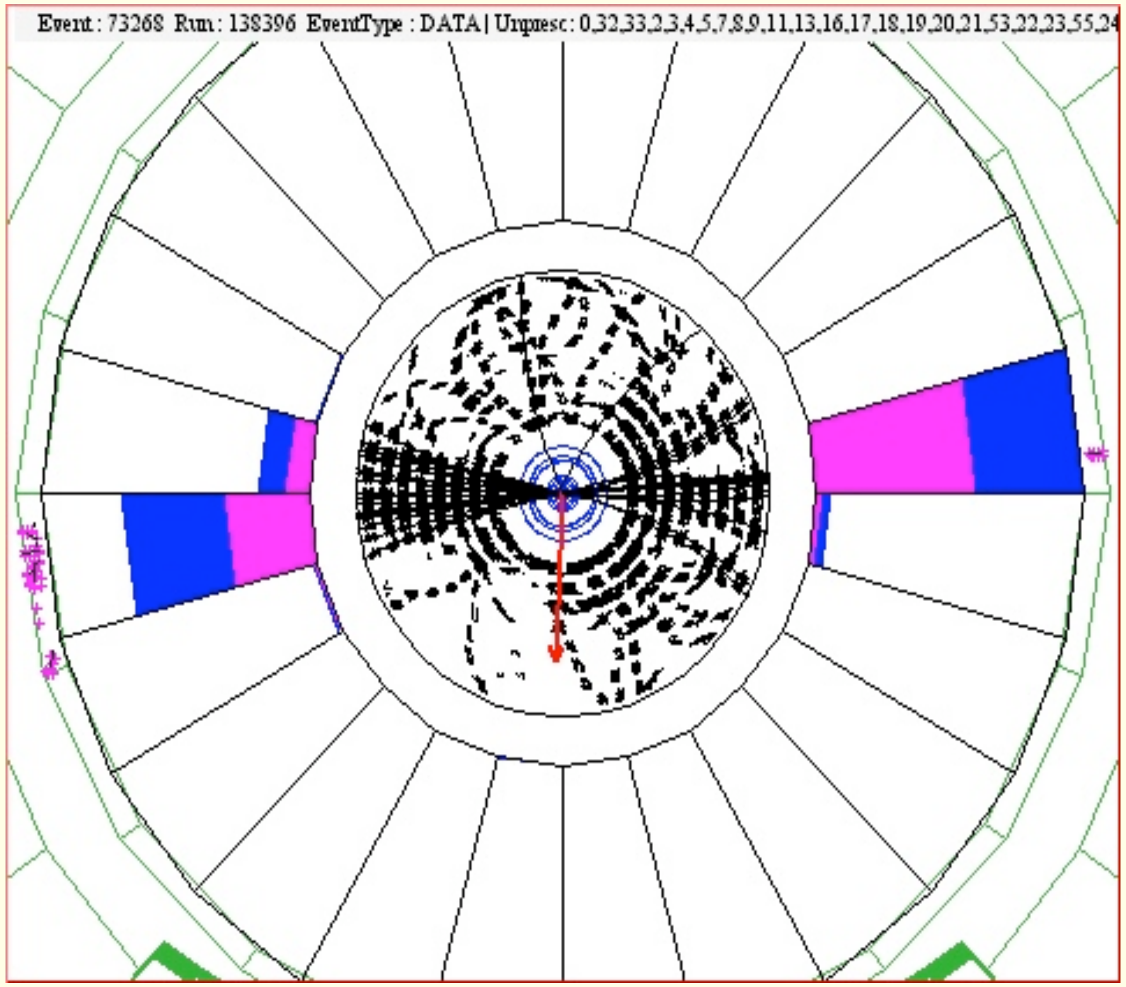
Jet Event in a p+p Collision at $\sqrt{s}$ = 63 GeV



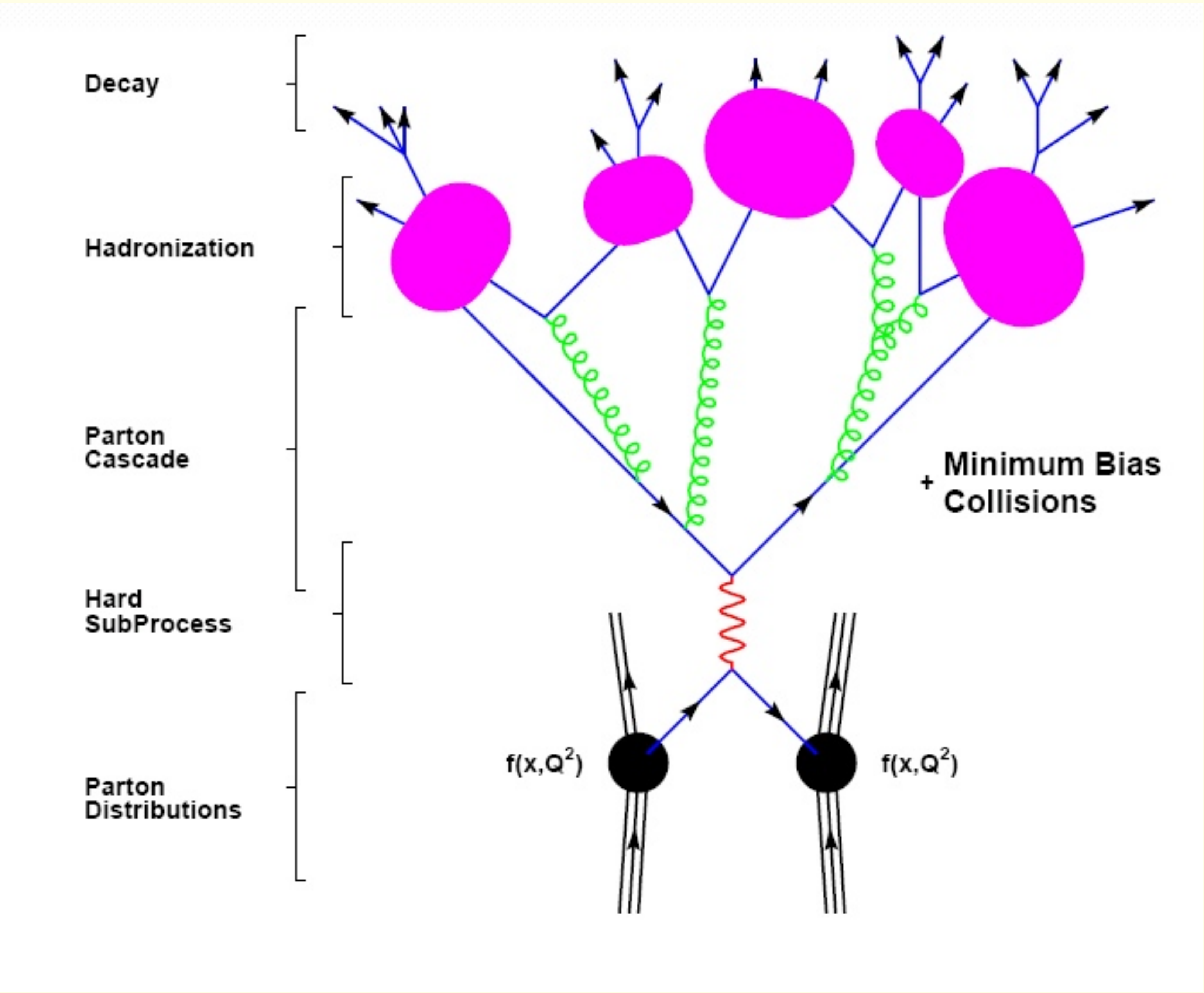
Lego plot shows energy vs. pseudorapidity η and azimuthal angle φ

Jet Event at the Tevatron





Evolution of a Jet Event



$$\underbrace{\text{Hard Process} \rightarrow \text{Parton Cascade}}_{\text{describable with pQCD}} \rightarrow \underbrace{\text{Hadronization}}_{\text{not describable with pQCD}}$$

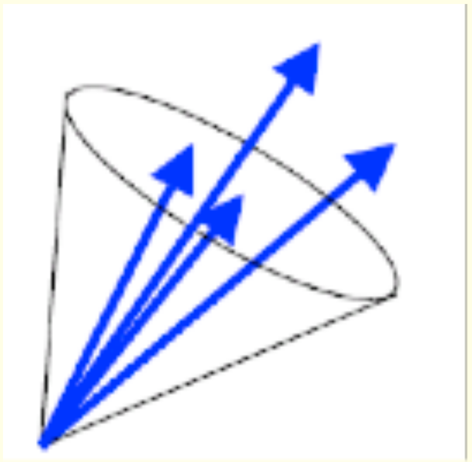
describable with pQCD

not describable with pQCD
(only phenomenological models)

Jet-Finding Algorithms

- Objective: reconstruct energy and direction of initial parton
- Must be unambiguously applicable at level of experimental data (tracks/towers) and in perturbative QCD calculation (parton level)
- Starting point: list of calorimeter towers and/or charged hadron tracks
- Two classes of algorithms:
  - Cone algorithm: traditional choice in hadron-hadron collisions
  - k_T algorithm: traditional choice in e+e- collisions

Cone algorithm:

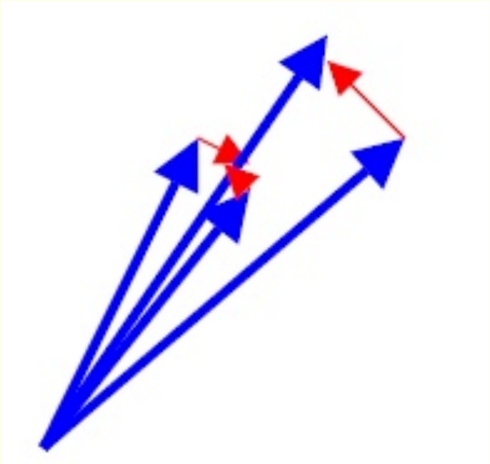


Sum content in cone with radius

$$R = \sqrt{(\Delta\eta)^2 + (\Delta\varphi)^2}$$

Typical choice:
$R = 0.7$

k_T algorithm:



Successively merge "particles" in order of relative transverse momentum.

Termination of merging controlled by a parameter D

Cone Algorithm

Fermilab Run II jet physics:
hep-ex/0005012

- (Normally) start with seed (e.g. calorimeter module with high energy)
- Consider all particles in cone around seed and calculate

$$\eta^C = \frac{\sum_{i \subset C} E_T^i \eta^i}{E_T^C}, \quad \phi^C = \frac{\sum_{i \subset C} E_T^i \phi^i}{E_T^C}$$

$$(E_T = E \sin\vartheta)$$

- Repeat this procedure with new cone center (η^C, ϕ^C)
- Terminate when "flow" of cone center stops
- Calculate jet energy as

$$E_T^C = \sum_{i \subset C} E_T^i$$

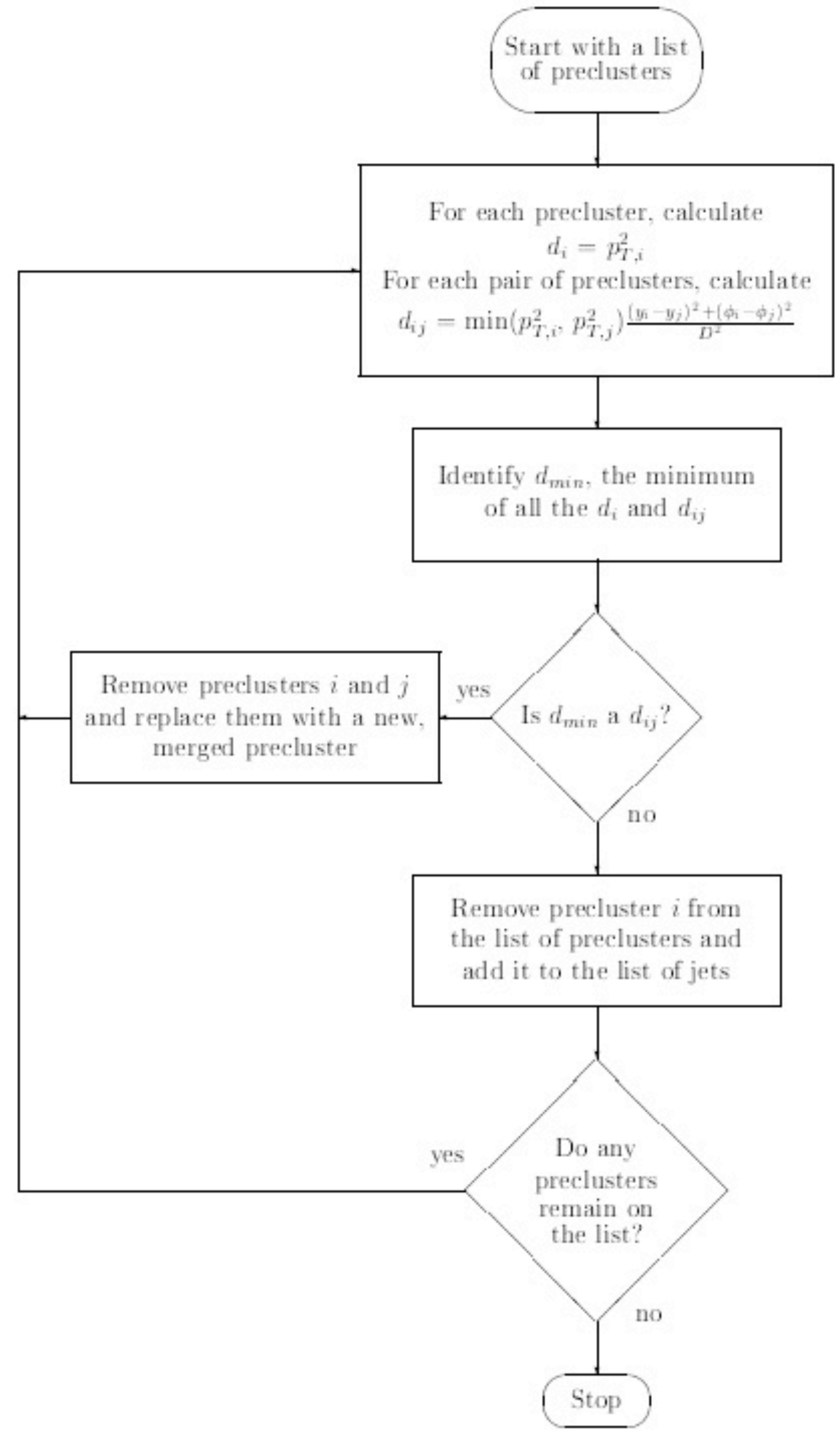
- A particle may belong to two cones:
  split energy among jets
- Subtract background energy from underlying event

k_T Algorithm (I)

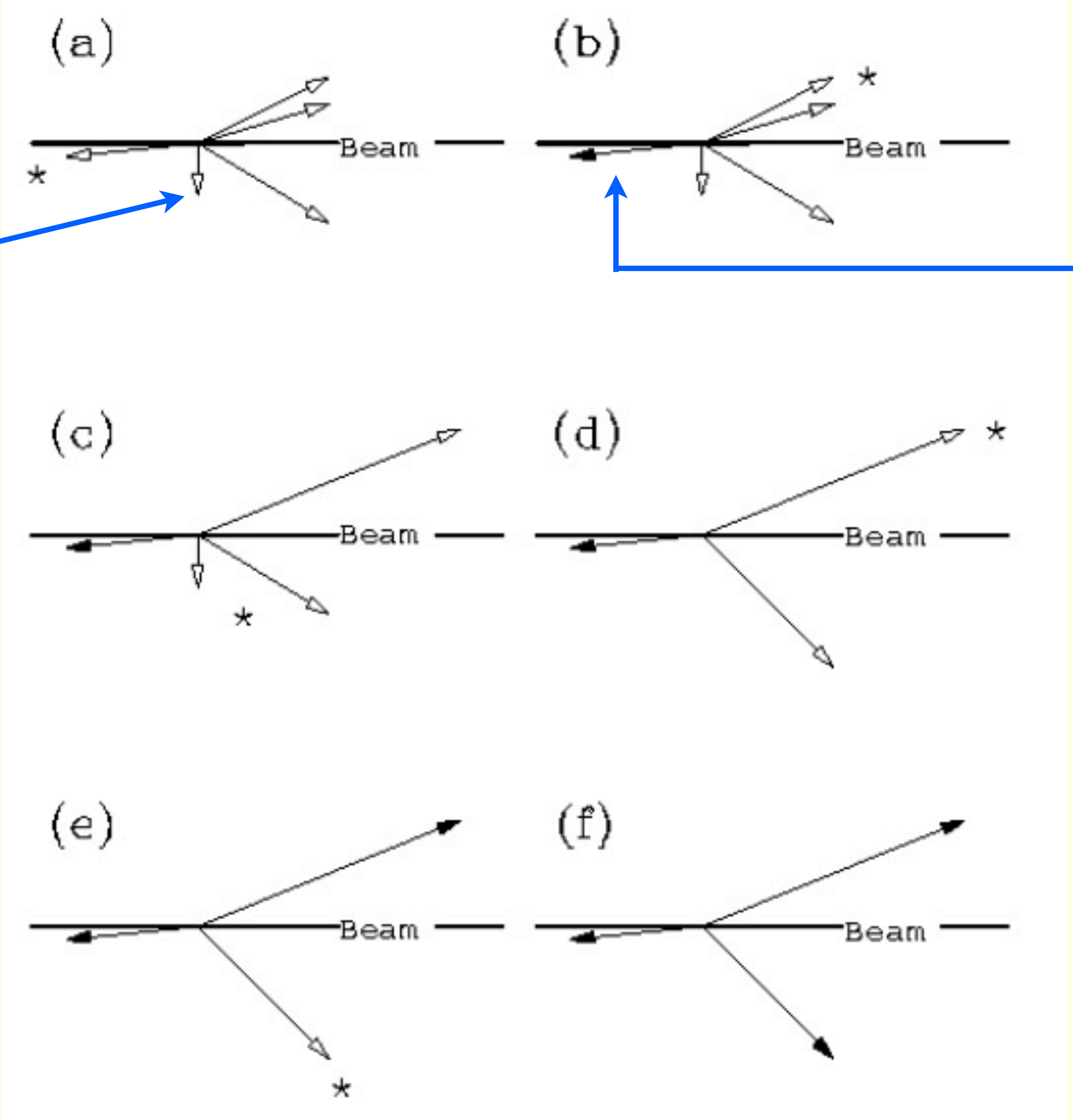
- **Alorithms starts with a list of preclusters (calorimeter cells, particles, or partons)**
- **Calculate p_T and rapidity y for each precluster**
- **For each precluster define** $d_i = p_{T,i}^2$
- **For each pair (i,j) of preclusters define**

$$d_{ij} = \min\left(p_{T,i}^2, p_{T,j}^2\right)\frac{\Delta\mathcal{R}_{ij}^2}{D^2}$$
$$= \min\left(p_{T,i}^2, p_{T,j}^2\right)\frac{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}{D^2}$$

- **For D = 1 and $\Delta R_{ij}^2 \ll 1$, d_{ij} is the minimal transverse momentum k_T (squared) of one vector with respect to the other**
- **Find minimum d_{min} of all d_i and d_{ij}**
- **Merge preclusters i and j if d_{min} is a d_{ij}**
- **Else: Remove precluster i with $d_{min} = d_i$ from list of preclusters and add it to the list of jets**
- **Repeat until list of preclusters is empty**



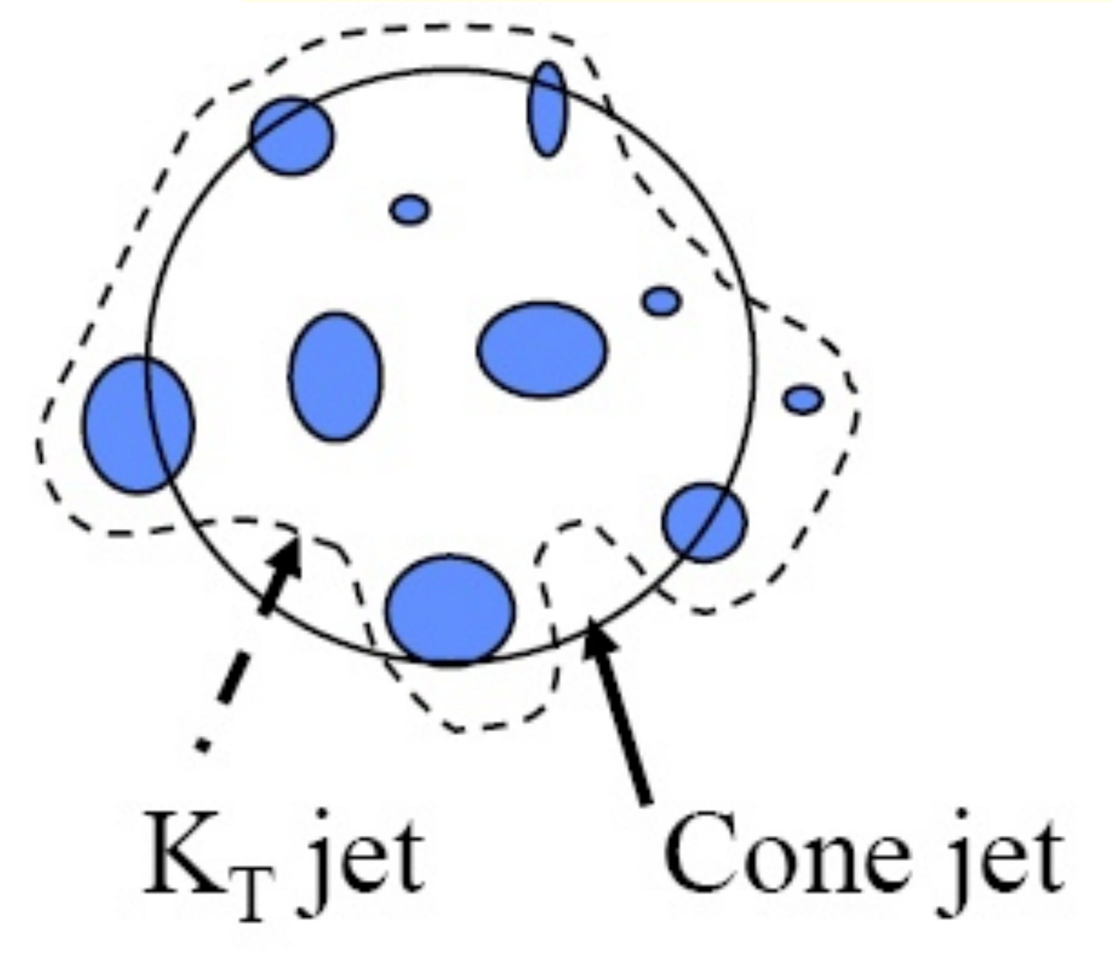
k_T Algorithm (II): A Simple Example



open arrows:
preclusters

closed arrows:
jets

Why the k_T Algorithm is Typically only used in e^+e^- Collisions?

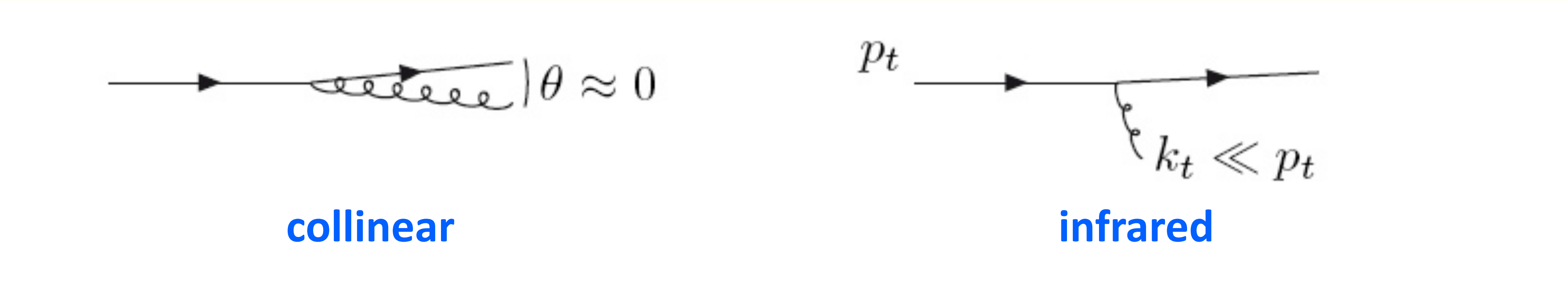


k_T jet has no fixed shape/area

$\rightarrow$ Difficult to subtract background from underlying event

Jet Finding Algorithms: Typical Requirements (I)

Two types of divergences:



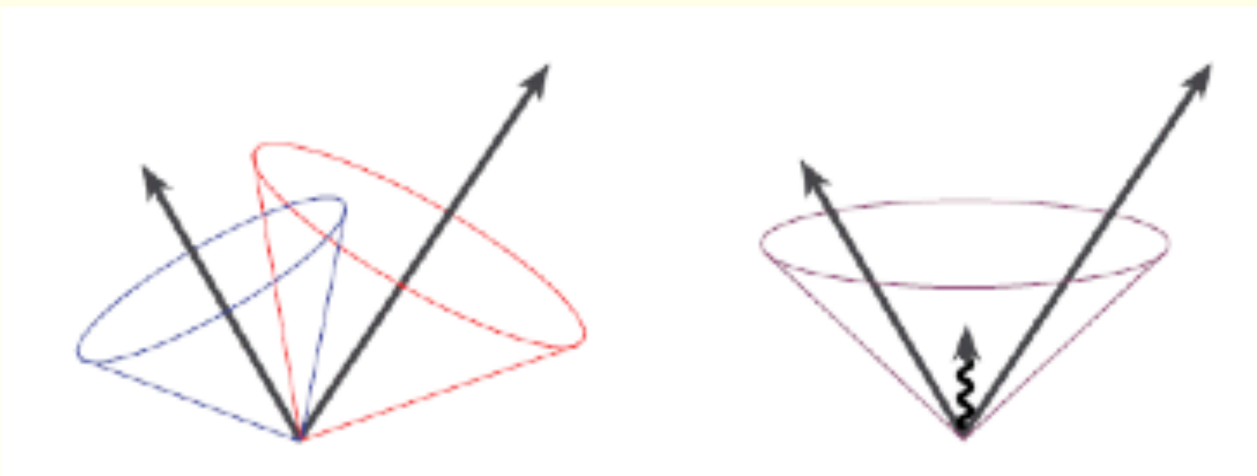
The reconstructed jets should not change in case of

- **collinear splitting, i.e., if one parton is replaced by two partons at the same place**
- **soft emission, i.e., if a low energy parton is added**

Jet Finding Algorithms: Typical Requirements (II)

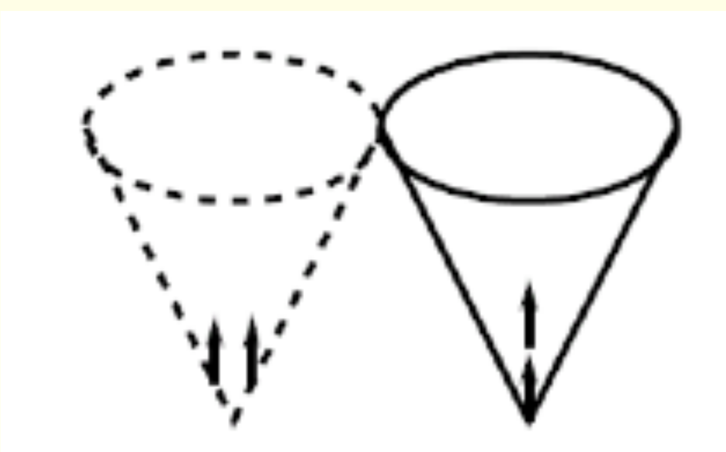
- **Infrared safety**

Fermilab Run II jet physics: hep-ex/0005012

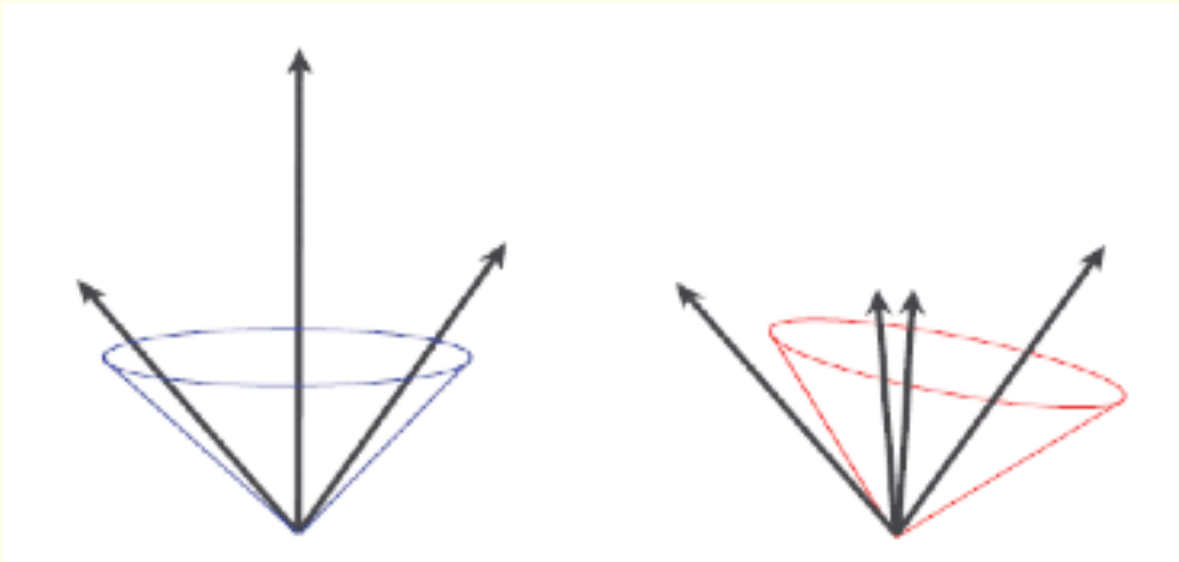


Example of infrared sensitivity: Soft radiation (right plot) causes merging of jets which would have been separated otherwise

- **Collinear safety**

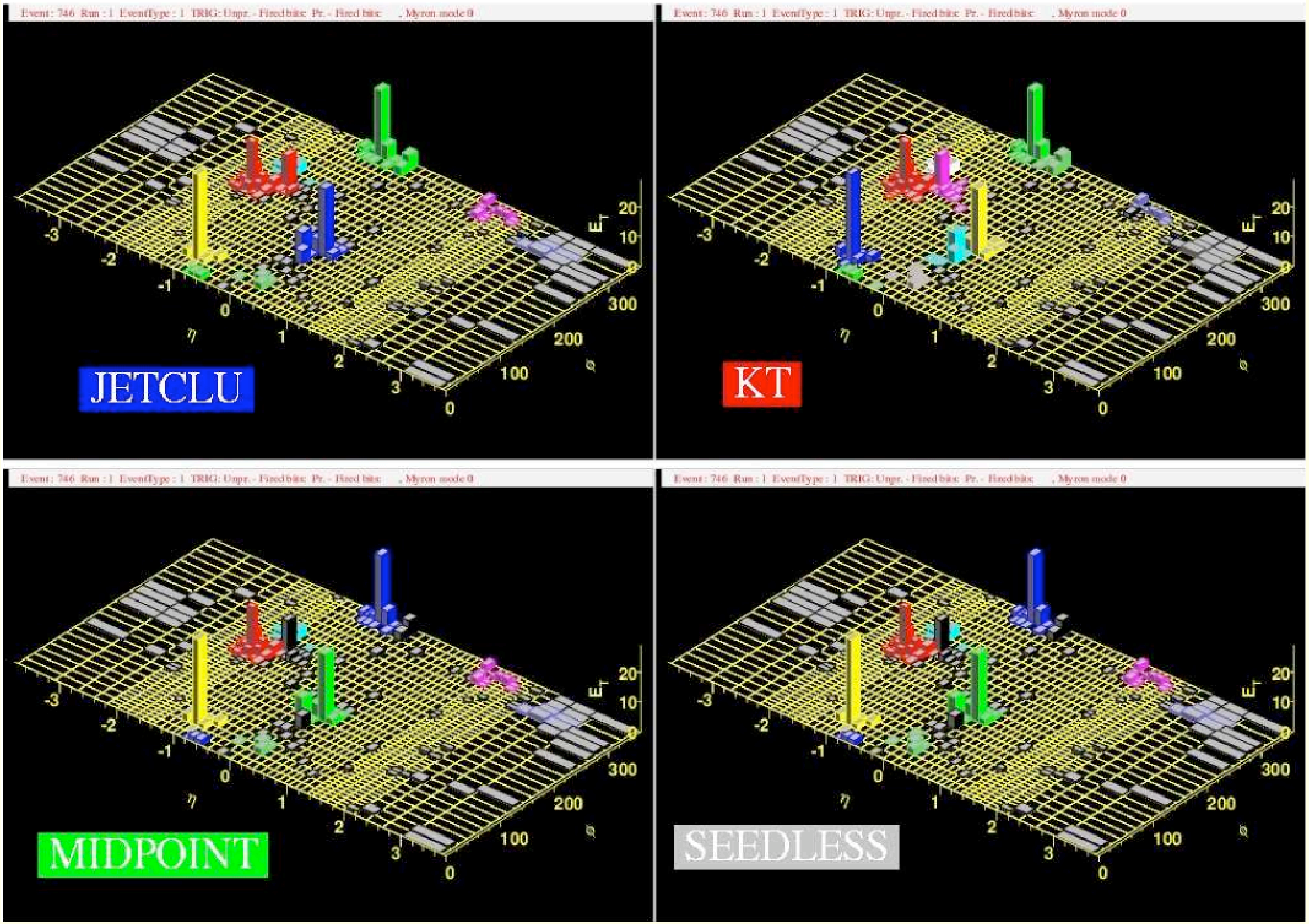


Example of collinear sensitivity: Seed and therefore jet not found in left picture



Example of collinear sensitivity: Reconstructed jet depends on seed

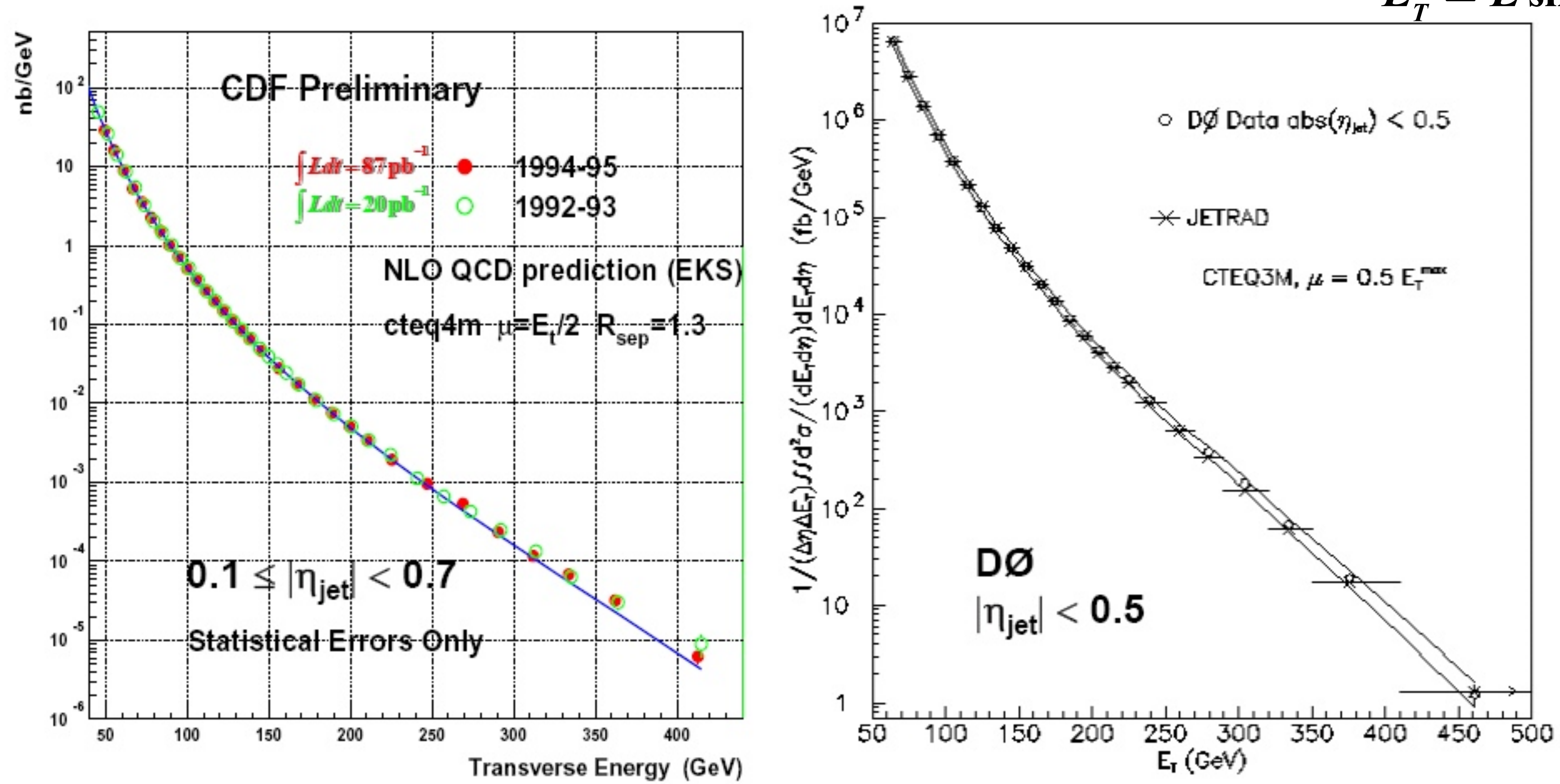
Jet Ambiguities: Different Algorithms Find Different Jets



Jet Cross Sections at the Tevatron

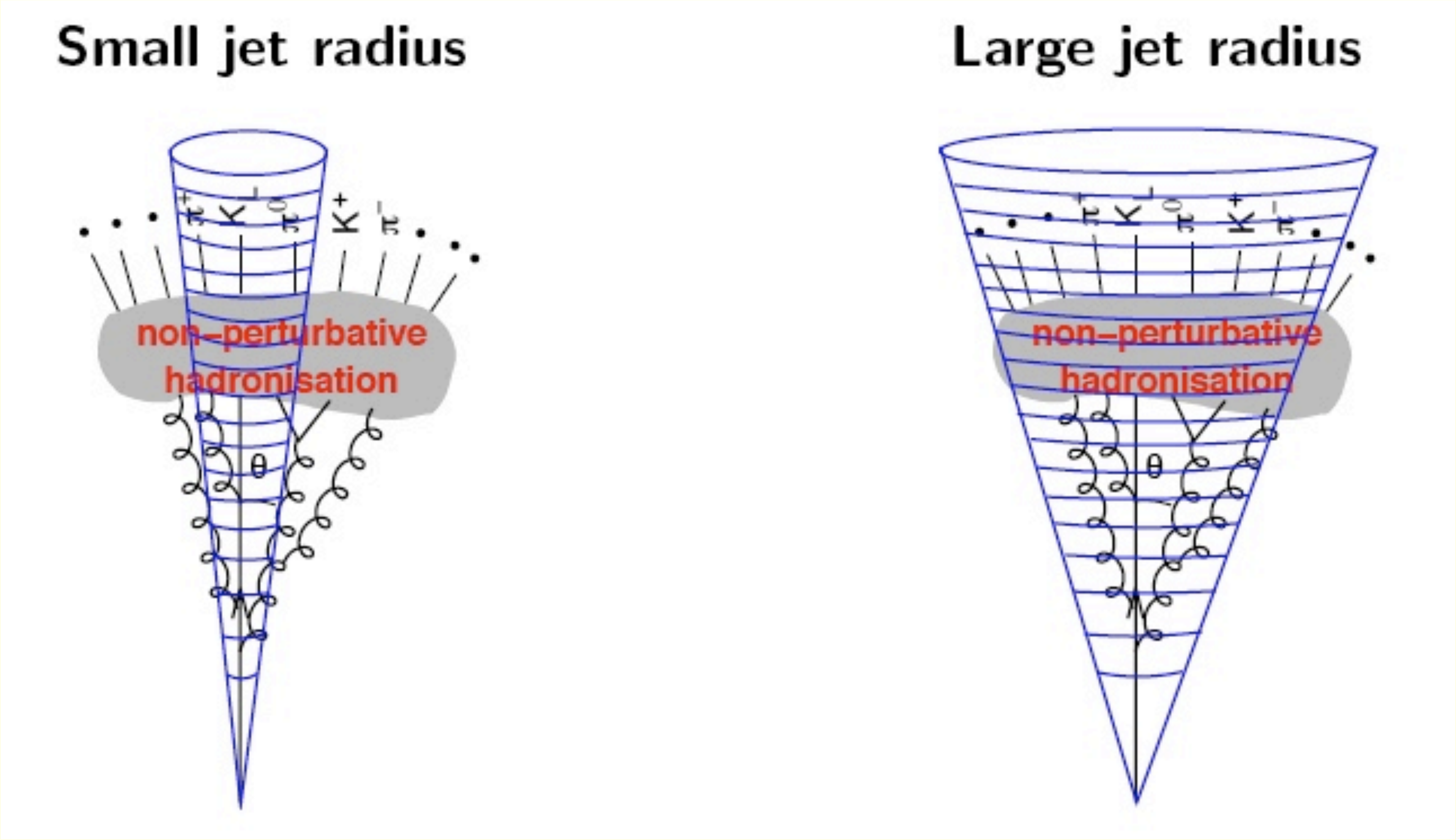
http://www-d0.fnal.gov/Run2Physics/displays/presentations/klima_lecture_jan2003/Lecture_1.pdf

$$E_T = E \sin\vartheta$$



Pretty good agreement between data and NLO pQCD predictions

Choice of the Optimal Cone Radius

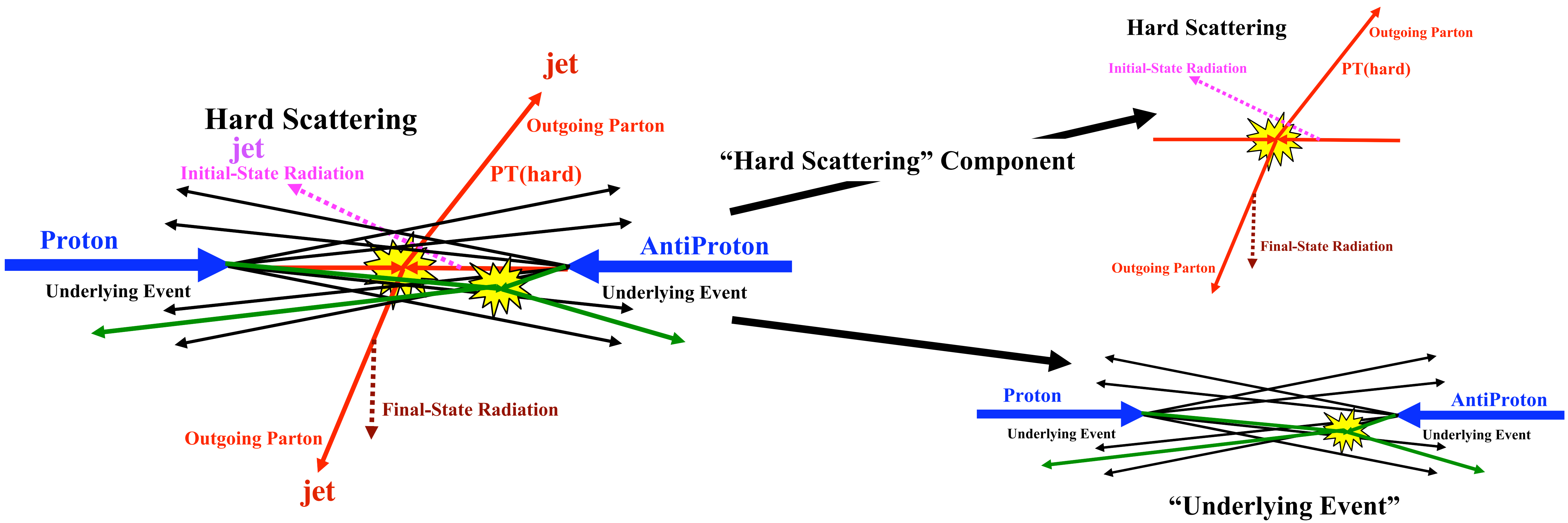


Small cone radius:
Small background from the underlying event at the expense of some loss of jet particles

Large cone radius:
Large background from the underlying event but nearly all jet particles included

Due to higher backgrounds the cone radius in heavy-ion collisions typically smaller than in p+p

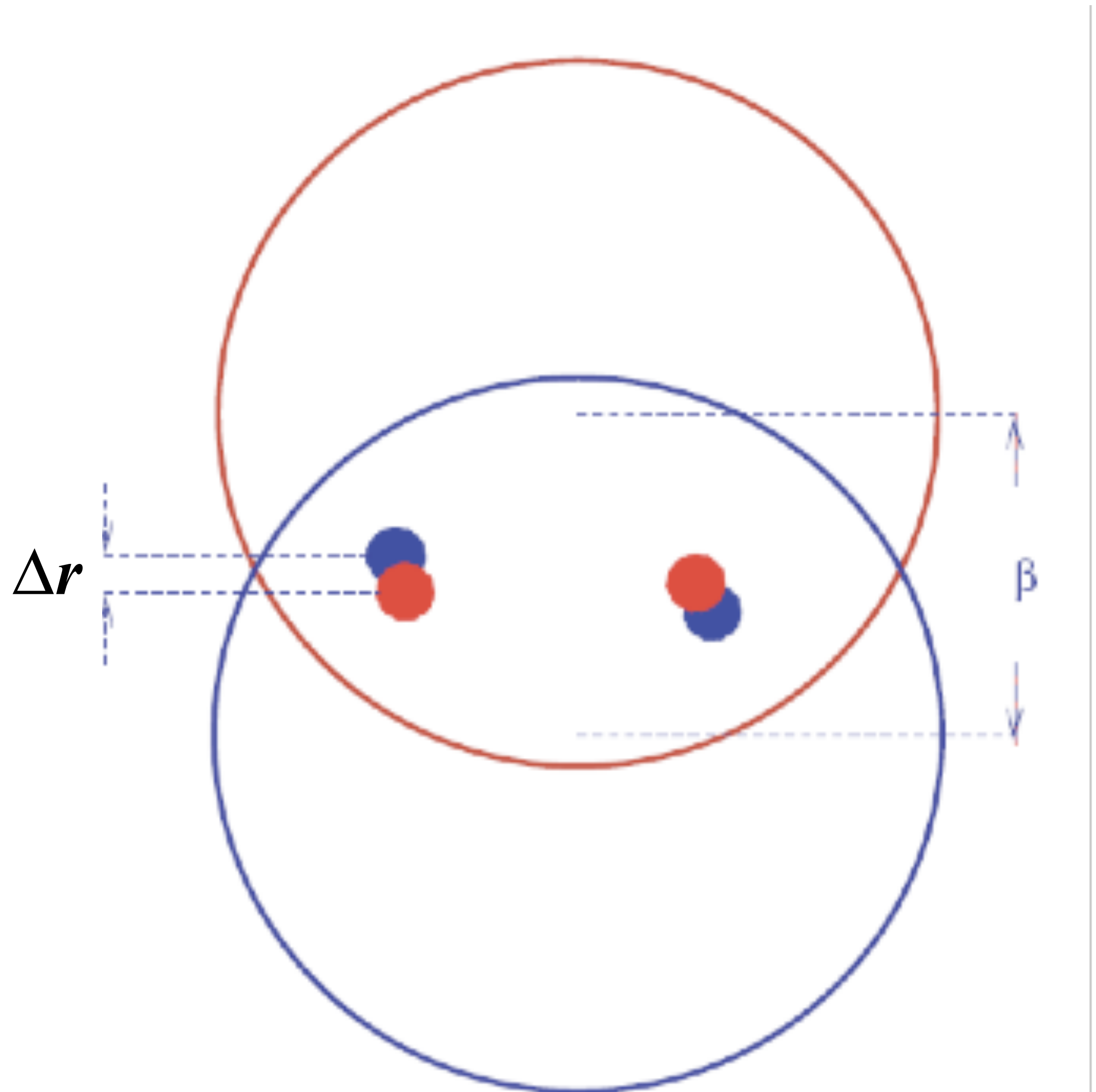
Underlying Event (UE)



Rick Field: https://agenda.infn.it/conferenceDisplay.py?confId=599

- The hard scattering component (the „signal"):
  - $2 \to 2$ (sometimes $2 \to 3$) parton-parton scattering
  - plus initial and final state radiation (however, sometimes also attributed to the UE)
- The "underlying event" consists of the "beam-beam remnants" and from particles arising from soft or semi-soft multiple parton interactions (MPI).

What are Multiple Partonic Interactions?



$$\Delta p_T^{\min} \cdot \Delta r \approx \hbar$$

$$\Delta r \approx \frac{\hbar c}{\Delta p_T^{\min} \cdot c} = \frac{0{,}2\ \mathrm{GeV\ fm}}{\Delta p_T^{\min}\ (\mathrm{GeV})}$$

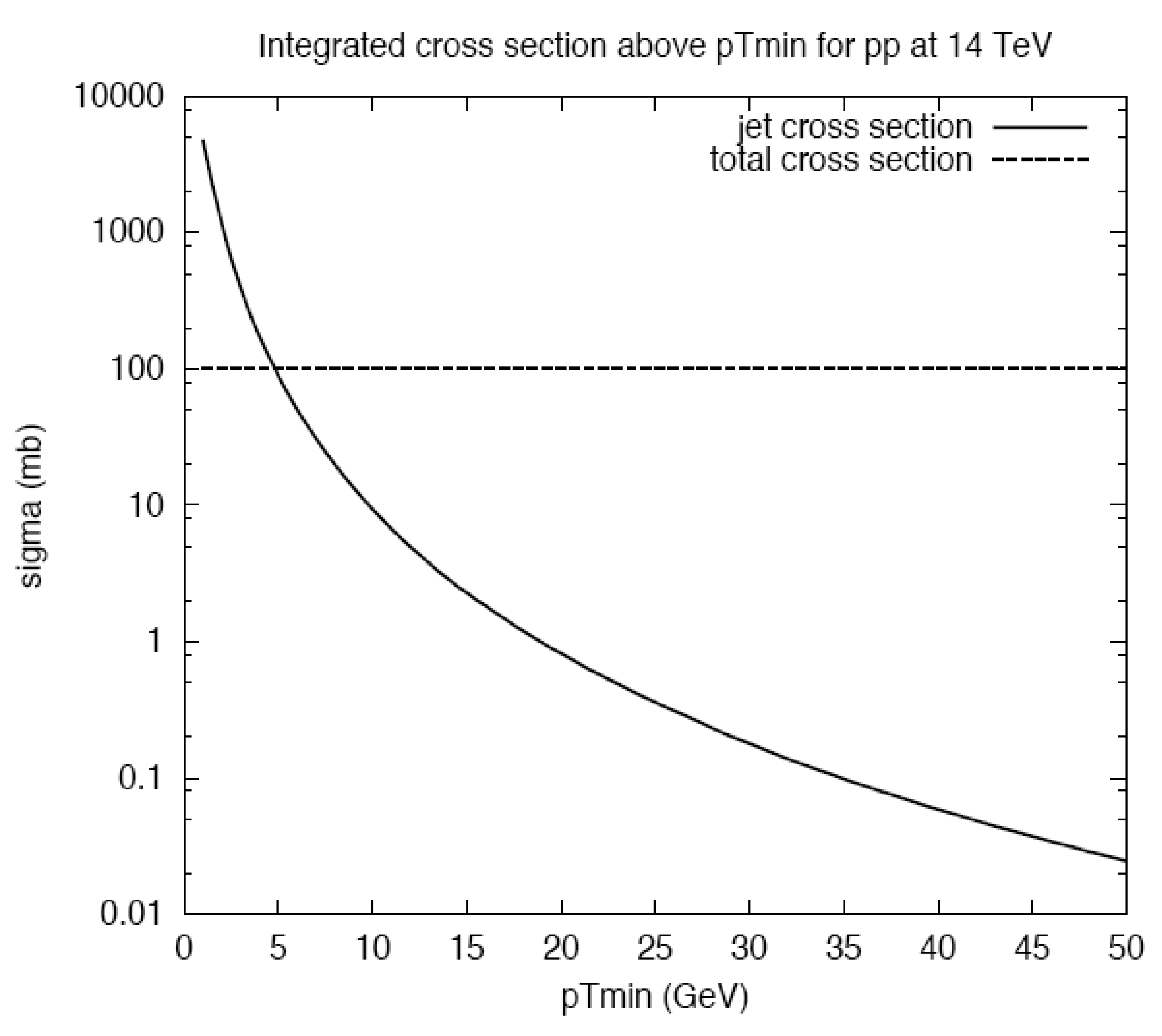
Multiple parton interaction:

- **Two or more pairs of partons interacting in the same inelastic p+p collision**
- **Momentum transfer larger than some lower cut-off $p_T^{\min}$ which establishes the hard scale**
- **$p_T^{\min}$ should correspond to a transverse size much smaller than the overlap area**
- **Thus, the two interaction region are well separated in space and should contribute incoherently to the cross section**

Multiple Partonic Interactions are Inevitable

Mini-jet cross section diverges for $p_{\perp\min} \to 0$:

$$\sigma_{\rm int}(p_{\perp\min}) = \int_{p_{\perp\min}}^{\sqrt{s}/2} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp} \mathrm{d}p_\perp \propto \frac{1}{p_{\perp\min}^2}$$

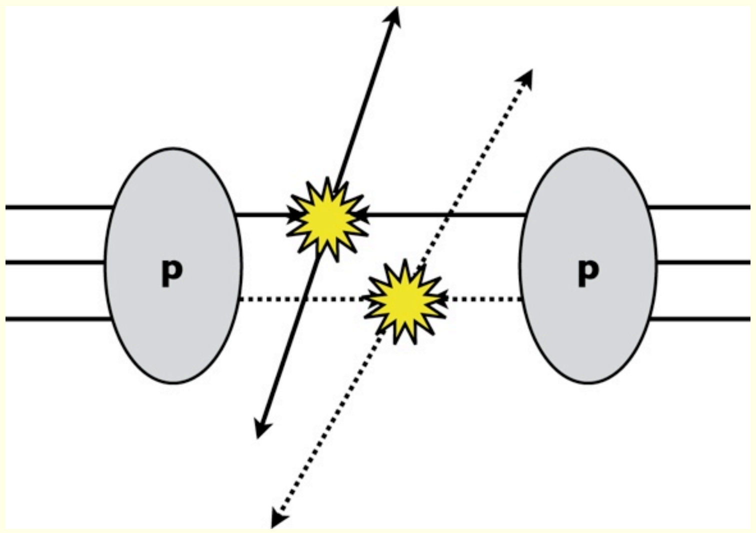


Even for scales $p_{\perp\min} \gg \Lambda_{QCD}$ one has $\sigma_{\rm int}(p_{\perp\min}) > \sigma_{\rm tot}$

The number of hard interactions per p+p collisions is thus given by

$$\langle n_{\rm hard} \rangle (p_{\perp\min}) = \frac{\sigma_{\rm int}(p_{\perp\min})}{\sigma_{\rm tot}}$$

Motivation for Studying Multiple Parton Interactions (I)



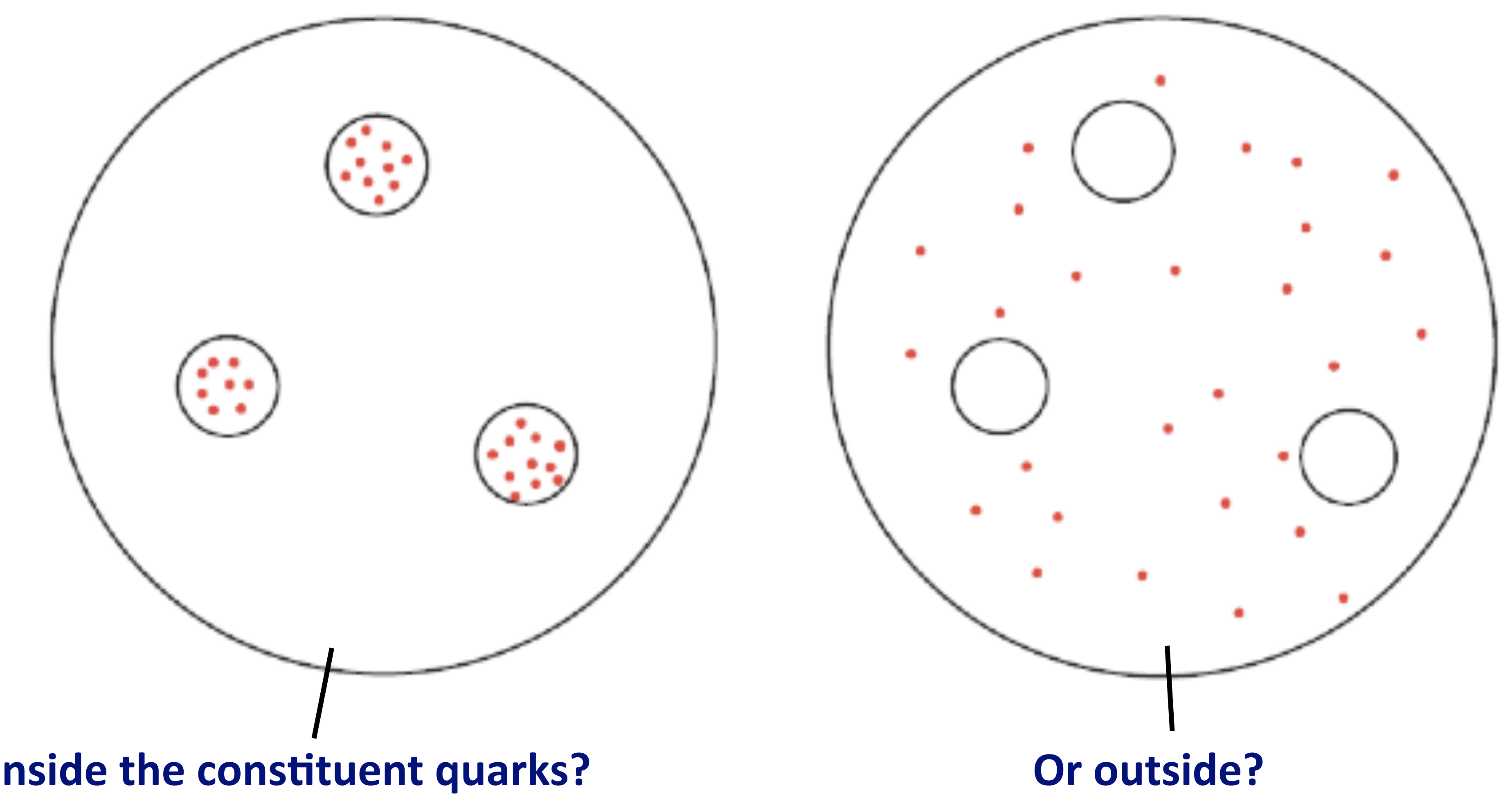
- Important for understanding of minimum bias p+p collisions at the Tevatron and the LHC
  - Tevatron: ~ 2 – 6 hard interactions per collision
  - LHC: ~ 4 – 10 hard interactions per collision

- Understanding of the "underlying event" important in specialized analyses, e.g., Higgs searches, jet production
  - Pedestal effect:
    Events with high-p_T jets have more underlying activity than minimum-bias events

- Study distribution of the partons and parton correlations in the plane transverse to the beam axis

Motivation for Studying Multiple Parton Interactions (II)
Transverse Profile of the Proton:

Where are the sea quarks and gluons inside the proton in the plane transverse to the beam axis?



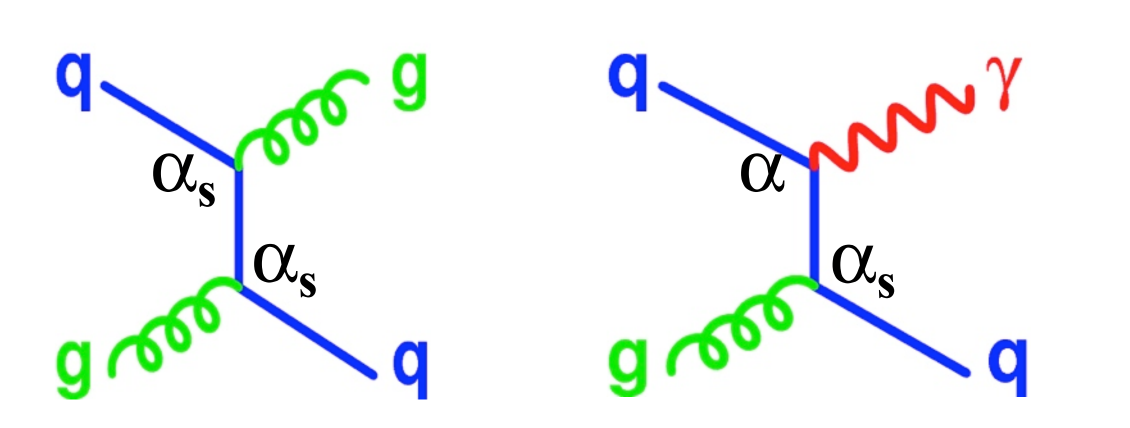
FELIX coll., J. Phys. G: Nucl. Part. Phys. 28 (2002) R117–R215

→ Study multiple parton interactions

3.4 Direct Photons

Direct Photons

- **Direct photons:**
  Photons emerging from parton-parton scattering
  (as opposed to hadron decay photons)
- **No complication due to parton-to-hadron fragmentation at the expense of smaller cross section**
- **Direct Photons allow to test QCD**
- **Sensitive to gluon distribution in the proton at leading order**
  (though this turns out to be difficult largely due to theoretical uncertainties and some possible inconsistencies in the data)



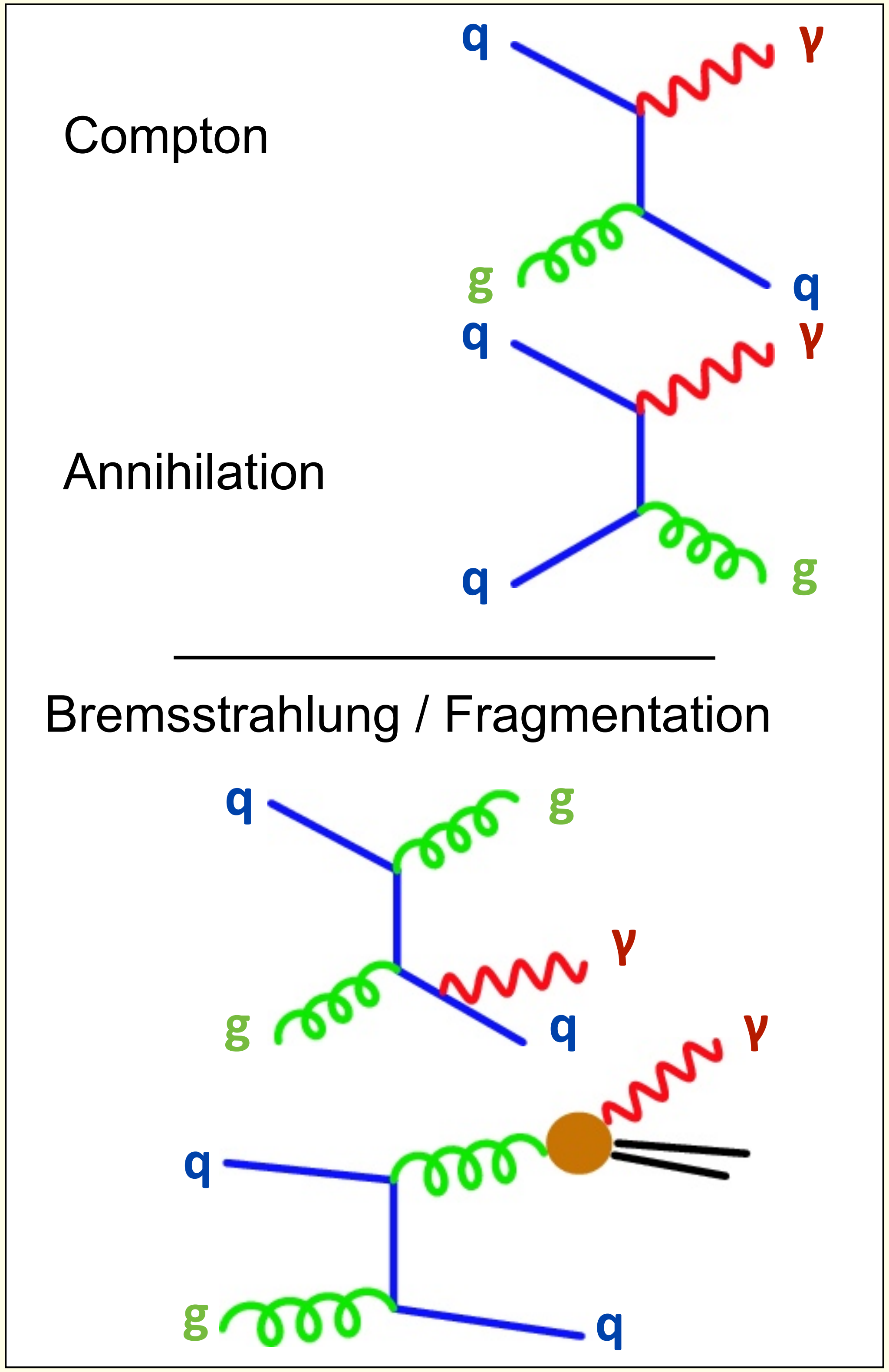
Direct Photon Production in p+p: Hard Scattering

- Processes in perturbative QCD
  - Compton: $q + g \rightarrow \gamma + q$
  - Annihilation: $q + \overline{q} \rightarrow \gamma + g$
  - Bremsstrahlung

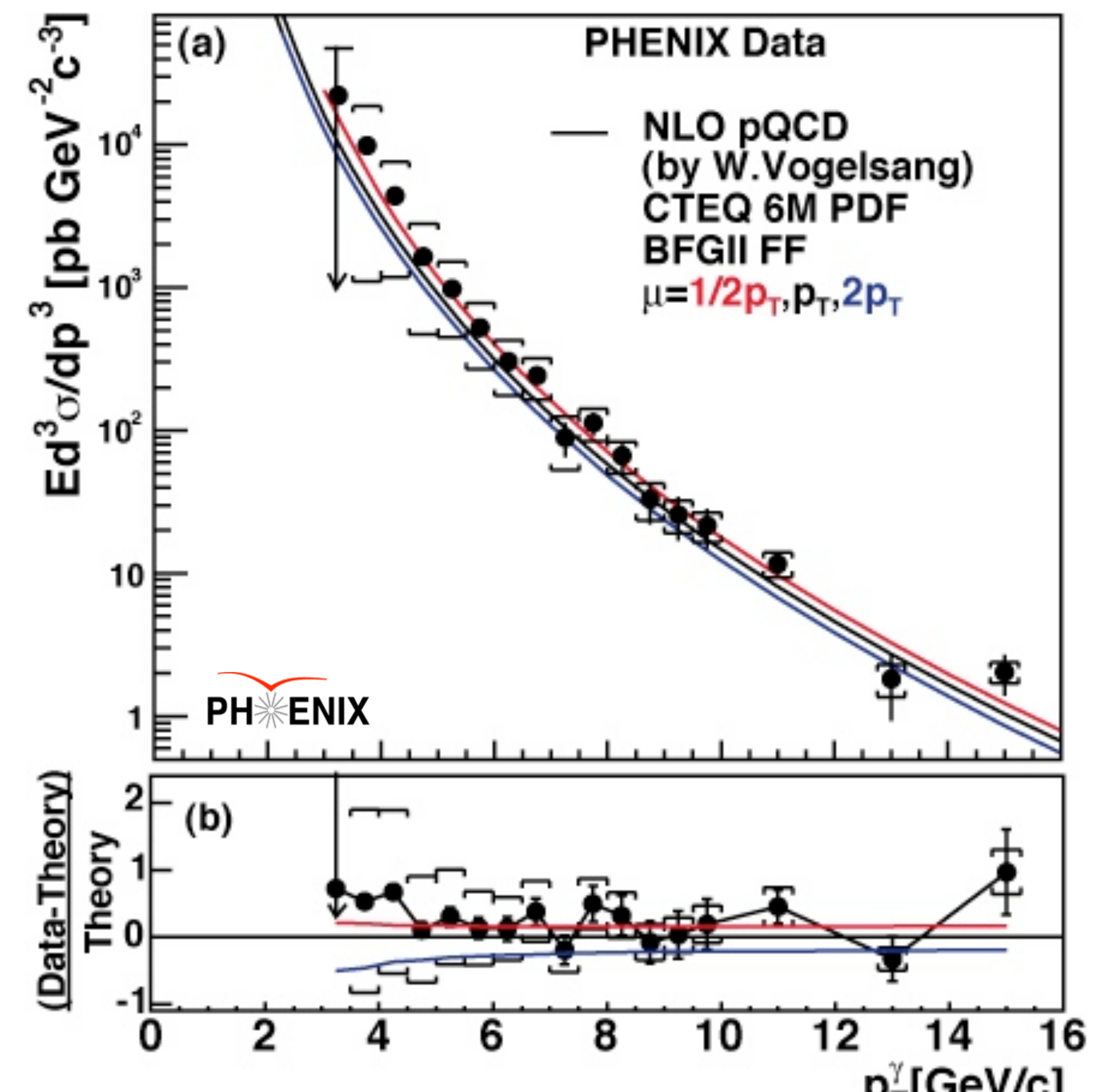
LO

NLO

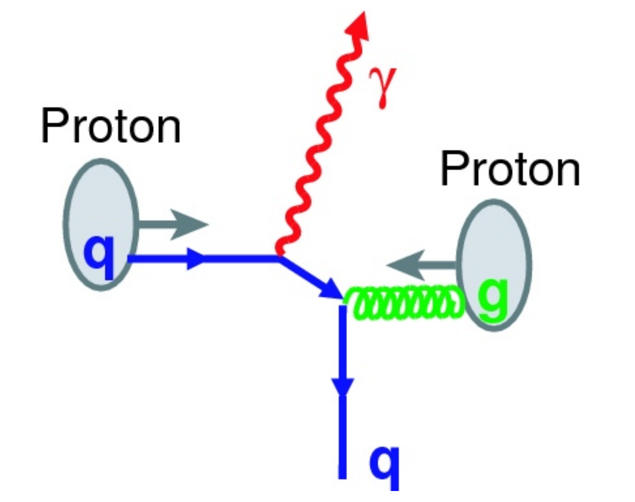
- Typically 20-30% uncertainty in pQCD calculations related to choice of unphysical scales



Direct Photons at RHIC: p+p at $\sqrt{s}$ = 200 GeV



PHENIX, Phys.Rev.Lett.98:012002,2007



- Direct photons measured on statistical basis

$$\gamma_{\mathrm{direct}} = \gamma_{\mathrm{total}} - \gamma_{\mathrm{decay}}$$

Calculated based on measured π^0, η spectra

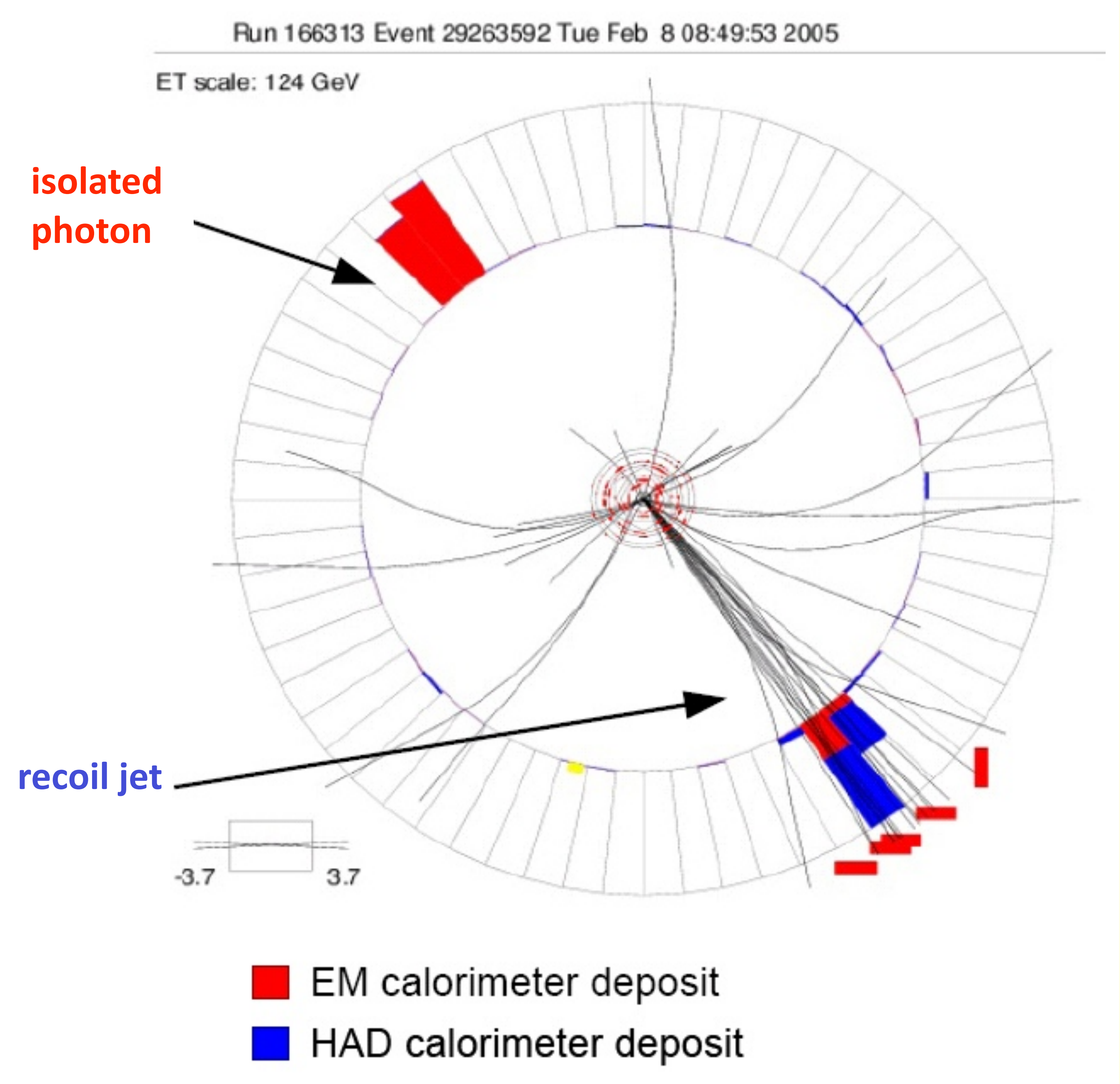
- Good agreement with NLO pQCD prediction
- Reference for Au+Au

Direct Photons at the Tevatron (I): Isolated Photons

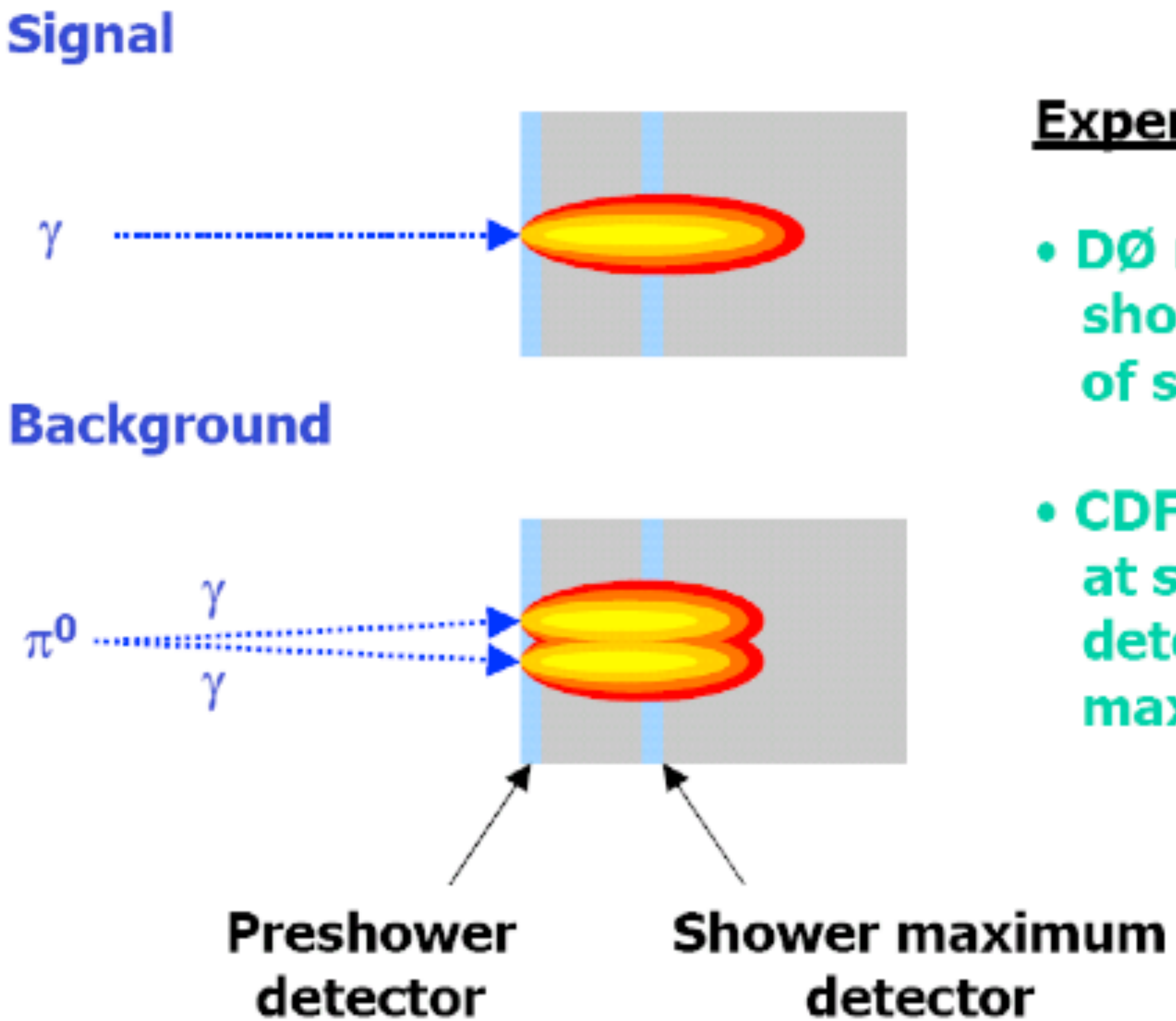
Tevatron experiments measure isolated direct photons (photons without associated charged track and small associated hadronic energy)

This rejects direct photons from fragmentation and bremsstrahlung

Rejection of background from $\pi^0 \to \gamma\gamma$ with measurement of shower profile



Direct Photons at the Tevatron (II): Rejection of $\pi^0 \to \gamma\gamma$ background



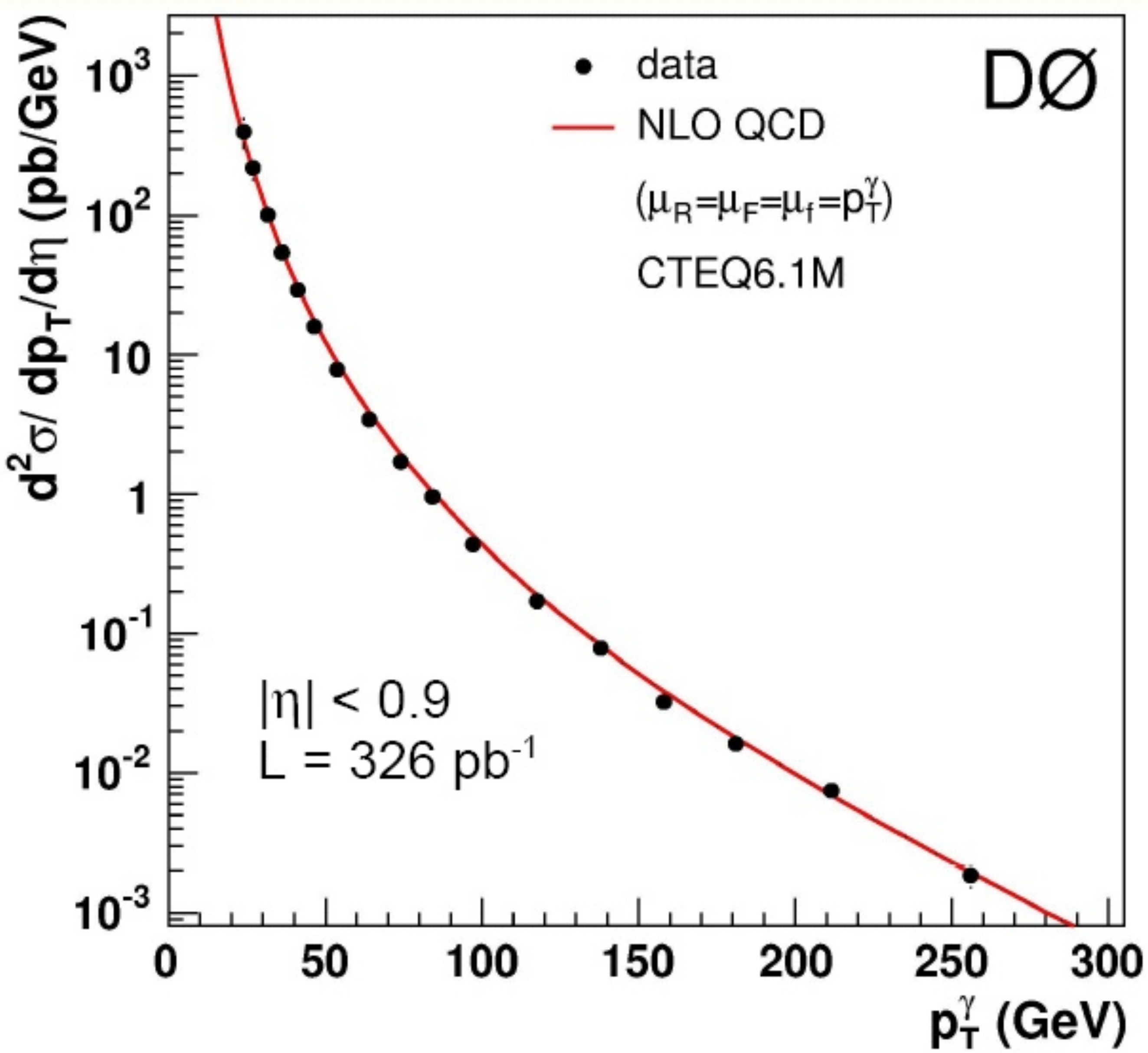
Experimental techniques

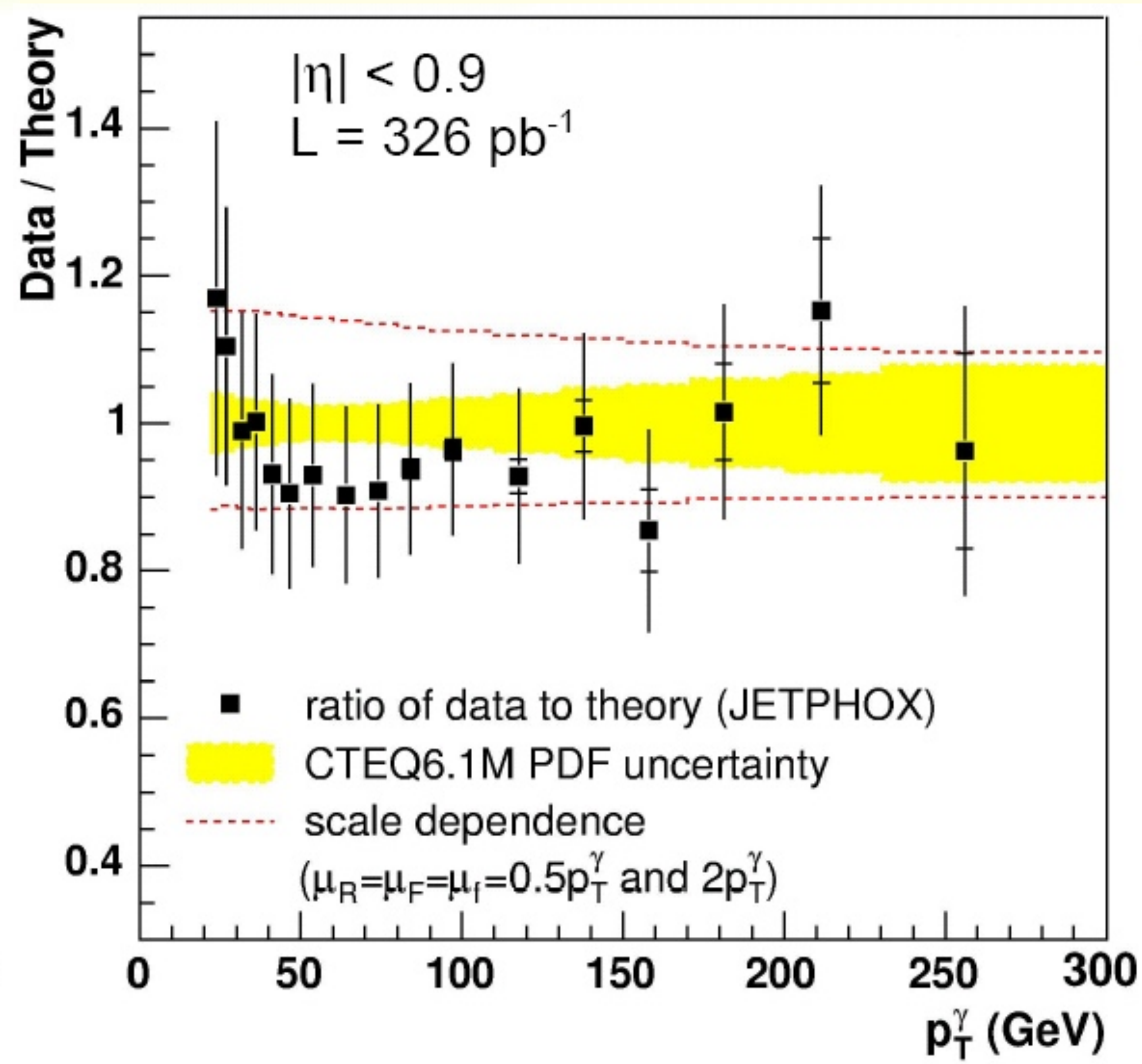
- DØ measures longitudinal shower development at start of shower
- CDF measures transverse profile at start of shower (preshower detector) and at shower maximum

Direct Photons at the Tevatron (III)

Isolated Photons (γ + jet):

PLB 639 (2006) 151





Good agreement with NLO QCD

Extra Slides

Mean *z* of the Fragmentation Products and the "Leading-Particle Effect" (I)

Consider exponential fragmentation function: $D(z) = B\, e^{-bz}$

Mean multiplicity of the fragments: $\langle m \rangle = \int_0^1 D(z)\, dz = B \int_0^1 e^{-bz}\, dz = \frac{B}{b}(1 - e^{-b})$

***B* and *b* are not independent since FF must satisfy:** $1 \equiv \int_0^1 z D(z)\, dz = \frac{B}{b^2}\left(1 - e^{-b}\left(1+b\right)\right)$

In particular: $B \approx b^2 \Rightarrow \langle m \rangle \approx b$

Mean *z* of the fragments: $\langle z \rangle = \int_0^1 z\, D(z)\, dz \Big/ \int_0^1 D(z)\, dz = \frac{1}{\langle m \rangle}$

RHIC: mean number of charged particles in a jet $\langle m \rangle \approx b \approx 8 - 10 \quad \Rightarrow \quad \langle z \rangle \approx 0.1 - 0.125$

Mean z of the Fragmentation Products and the “Leading-Particle Effect” (II)

Mean z for a fixed value of p_T:

$$\frac{1}{p_T}\frac{d^2 n_\pi}{dp_T dz} = f(\frac{p_T}{z}) \cdot D_{q/\pi}(z) \cdot z^{-2} \quad \Rightarrow \quad \langle z(p_T) \rangle = \frac{\int_{x_T}^{1} dz\; z\; f(\frac{p_T}{z}) \cdot D_{q/\pi}(z) \cdot z^{-2}}{\int_{x_T}^{1} dz\; f(\frac{p_T}{z}) \cdot D_{q/\pi}(z) \cdot z^{-2}}$$

$$= \ldots \approx \frac{n-1}{b}$$

$\langle z(p_T) \rangle$ **is n – 1 times larger than the unconditional** $\langle z \rangle$

This is called the leading-particle or “trigger bias” effect.

RHIC: $\langle z \rangle \approx 0.1 - 0.125 \quad \Rightarrow \quad \langle z(p_T) \rangle = 0.7 - 0.8$

A Simplified Analytic Model for the Calculation of the Inclusive Hadron p_T Spectrum (I)

How can we calculate the inclusive pion p_T distribution given the parton p_T distribution and the parton-to-pion fragmentation function?

Parton p_T distribution: $$\frac{1}{\hat{p}_T}\frac{dn}{d\hat{p}_T} = A \cdot \frac{1}{\hat{p}_T^n} =: f(\hat{p}_T)$$ ($n \approx 8$ for p+p at $\sqrt{s_{NN}}$ = 200 GeV)

Start with pion p_T spectrum as function of the parton $\hat{p}_T$ and $z = p_T / \hat{p}_T$:

pion transverse momentum

$$\frac{1}{\hat{p}_T}\frac{d^2 n_\pi}{d\hat{p}_T dz} = f(\hat{p}_T) \cdot D_{q/\pi}(z)$$

Change of variables:

$$\hat{p}_T = \frac{p_T}{z}, \quad \left.\frac{d\hat{p}_T}{dp_T}\right|_z = \frac{1}{z} \quad \Rightarrow \quad \frac{1}{p_T}\frac{d^2 n_\pi}{dp_T dz} = f\left(\frac{p_T}{z}\right) \cdot D_{q/\pi}(z) \cdot z^{-2}$$

A Simplified Analytic Model for the Calculation of the Inclusive Hadron p_T Spectrum (II)

Pion yield dN/dp_T at fixed p_T: Integration over z

$$\frac{1}{p_T}\frac{dn_\pi}{dp_T} = \int_{x_T}^{1} dz\, f\left(\frac{p_T}{z}\right) \cdot D_{q/\pi}(z) \cdot z^{-2}$$

Maximum parton energy: $\frac{\sqrt{s}}{2} \quad \Rightarrow z_{\min} = \frac{p_T}{\sqrt{s}/2} \equiv x_T$

Using the power law form of the parton spectrum:

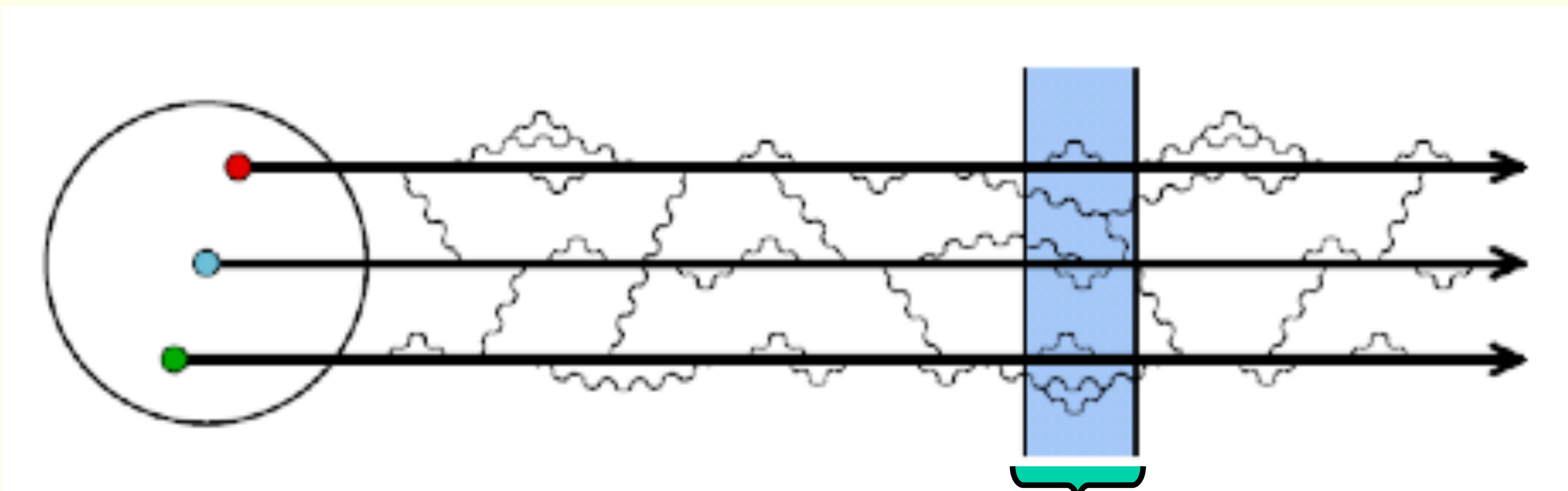
$$\frac{1}{p_T}\frac{dn_\pi}{dp_T} = A \cdot \frac{1}{p_T^n} \int_{x_T}^{1} dz\, D_{q/\pi}(z) \cdot z^{n-2}$$

Integral depends only weakly on p_T since $x_\mathrm{T} \approx 0$

The pion p_T spectrum is also a power-law with the same power n.
This is called the Bjorken parent-child relationship.

Interpretation of Scaling Violation (II)

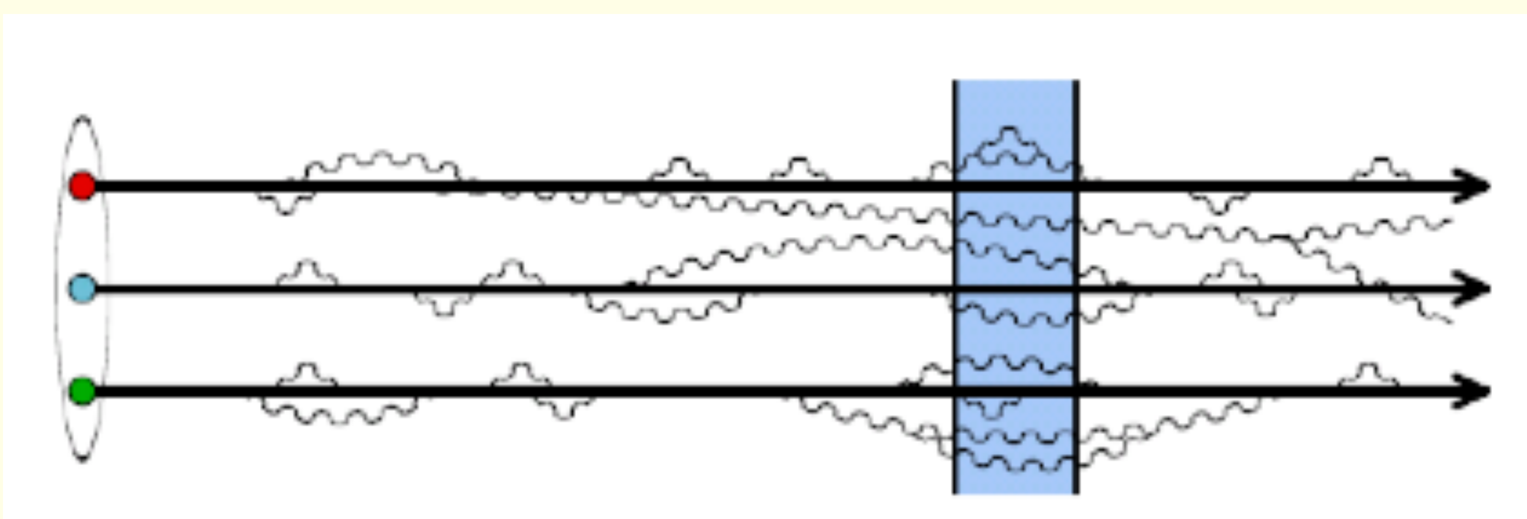
Low energy proton



Time resolution
of the probe

Fluctuations shorter than the resolution of the probe cannot be observed

High-energy proton

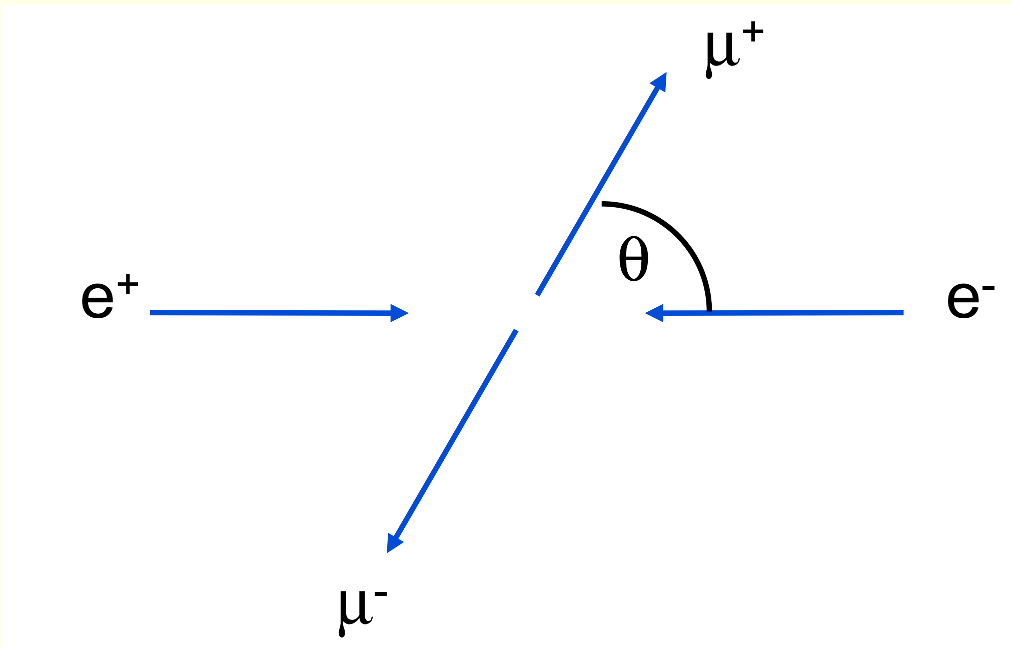


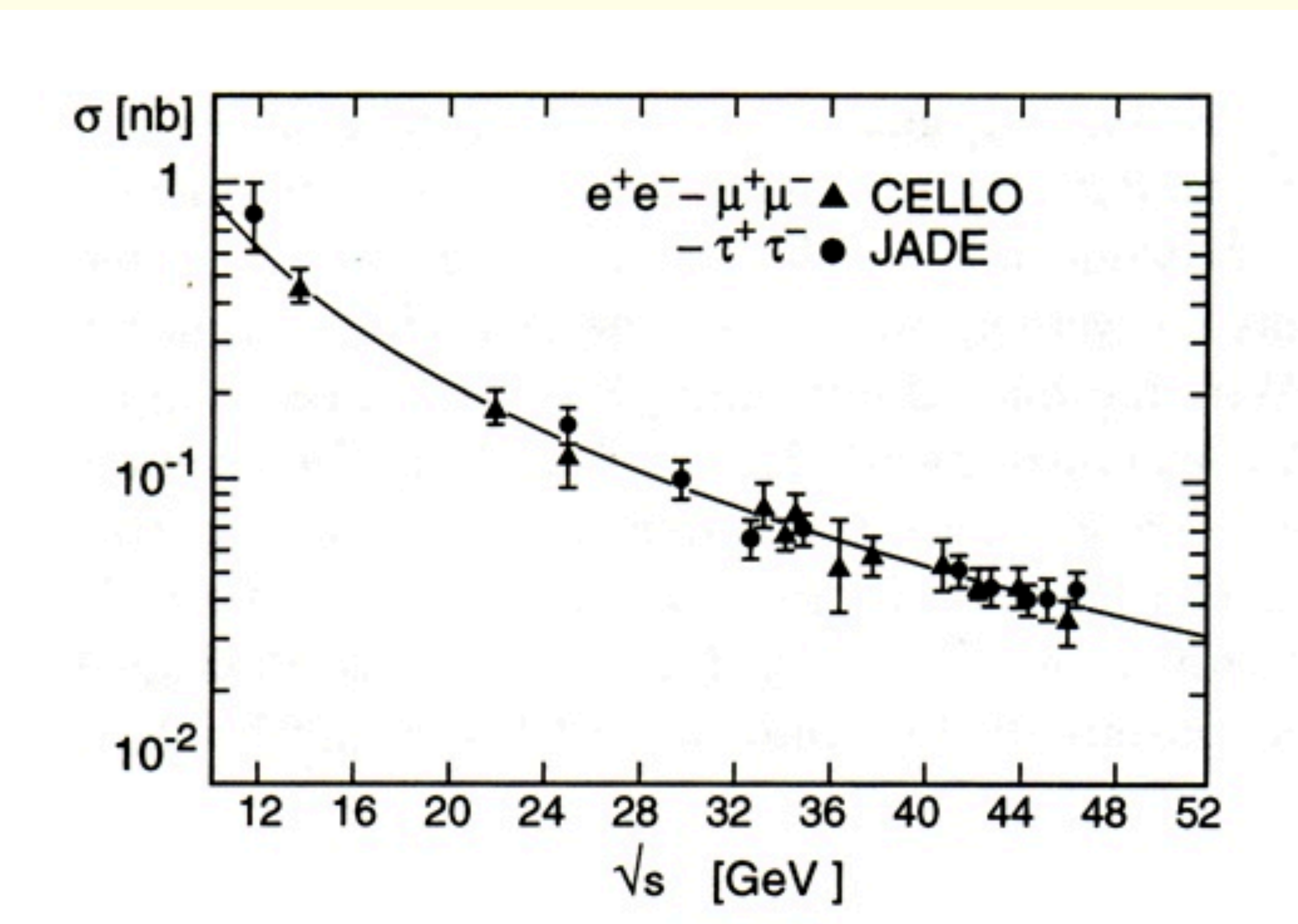
Gelis, Lappi, Venugopalan: arXiv:0708.0047

Time dilation allows more fluctuations to be resolved by the probe

High energy proton appears to contain more gluons than low energy proton

$e^{+}+e^{-} \rightarrow \mu^{+}+\mu^{-}$





Differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^{2}}{4s}(\hbar c)^{2}(1+\cos^{2}\theta)$$

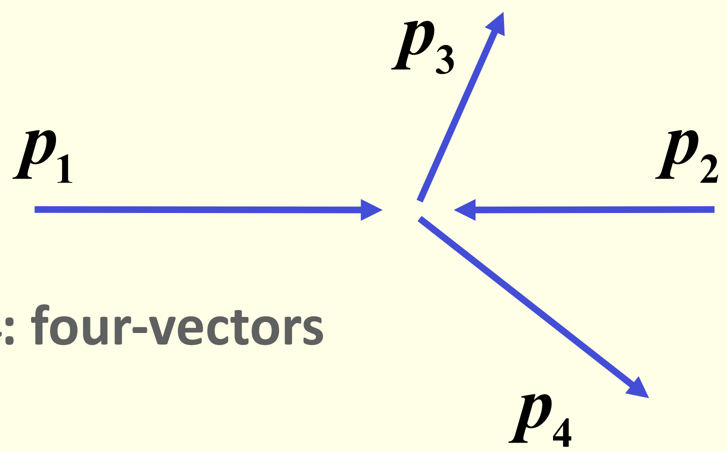
$\sqrt{s}$: center-of-mass energy

Total cross section:

$$\sigma = \frac{4\pi\alpha^{2}}{3s}(\hbar c)^{2}$$

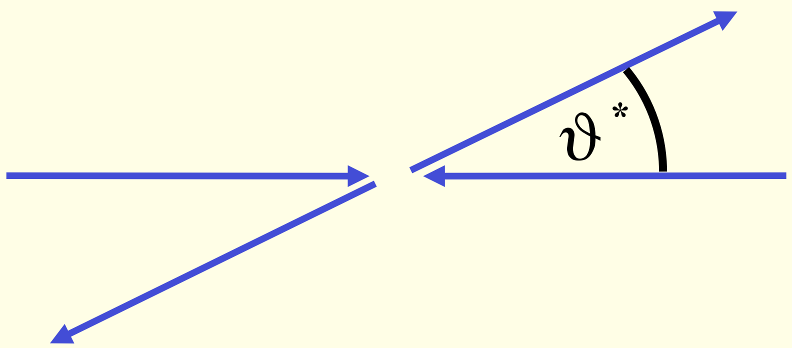
Parton Kinematics (I)

Laboratory reference frame



p_i: four-vectors

Center-of-mass system (CMS)



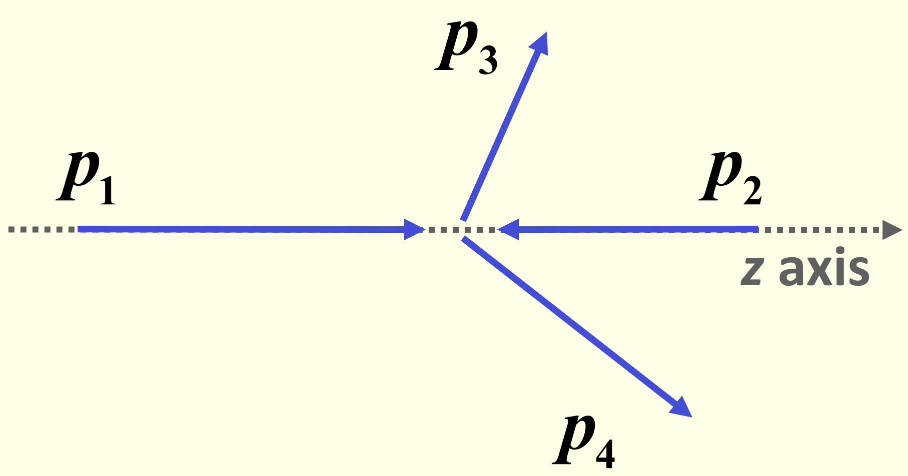
Parton 4-vectors conveniently represented as:

$$\begin{aligned} p^\mu &= (E, p_x, p_y, p_z) \\ &= (m_T \cosh y, p_T \cos\varphi, p_T \sin\varphi, m_T \sinh y) \end{aligned}$$

with transverse mass $m_T = \sqrt{p_T^2 + m^2}$ **and rapidity** $y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$

We will consider partons as massless so that $m_T = p_T$

Parton Kinematics (II)



four-vector of incoming proton 1

$$p_1 = x_1 P_1 = x_1 \left(\sqrt{s}/2, 0, 0, \sqrt{s}/2\right)$$

$$p_2 = x_2 P_2 = x_2 \left(\sqrt{s}/2, 0, 0, -\sqrt{s}/2\right)$$

$$p_3 = \left(p_{T,3} \cosh y_3, p_{T,3} \cos\varphi, p_{T,3} \sin\varphi, p_{T,3} \sinh y_3\right)$$

$$p_4 = \left(p_{T,4} \cosh y_4, p_{T,4} \cos\varphi, p_{T,4} \sin\varphi, p_{T,4} \sinh y_4\right)$$

$$p_{\text{beam}} \approx E_{\text{beam}} = \sqrt{s}/2$$

Energy conservation:

$$(x_1 + x_2)\frac{\sqrt{s}}{2} = p_T(\cosh y_3 + \cosh y_4)$$

Conservation of p_z:

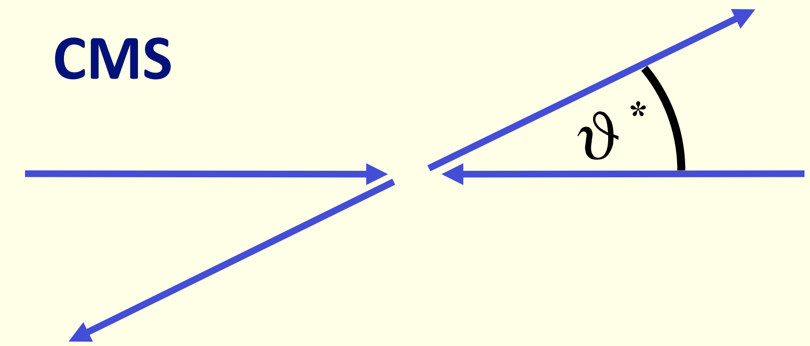
$$(x_1 - x_2)\frac{\sqrt{s}}{2} = p_T(\sinh y_3 + \sinh y_4)$$

This leads to:

$$x_1 = \frac{x_T}{2}(e^{y_3} + e^{y_4}) \qquad x_2 = \frac{x_T}{2}(e^{-y_3} + e^{-y_4}) \qquad \text{with} \qquad x_T = \frac{2 p_T}{\sqrt{s}}$$

Parton Kinematics (III)

√s of the parton system: $\hat{s} = (x_1 P_1 + x_2 P_2)^2 \approx x_1 x_2 s$



Jet transv. momentum: $p_T = p_T^* = \frac{\sqrt{\hat{s}}}{2} \sin\vartheta^*$

Energy-momentum vector
of the two-jet system: $x_1 P_1 + x_2 P_2 = \frac{\sqrt{s}}{2}(x_1 + x_2, 0, 0, x_1 - x_2)$

Rapidity of center-of-mass of the two-jet system: $y^{cm} = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} = \frac{1}{2} \ln \frac{x_1}{x_2}$

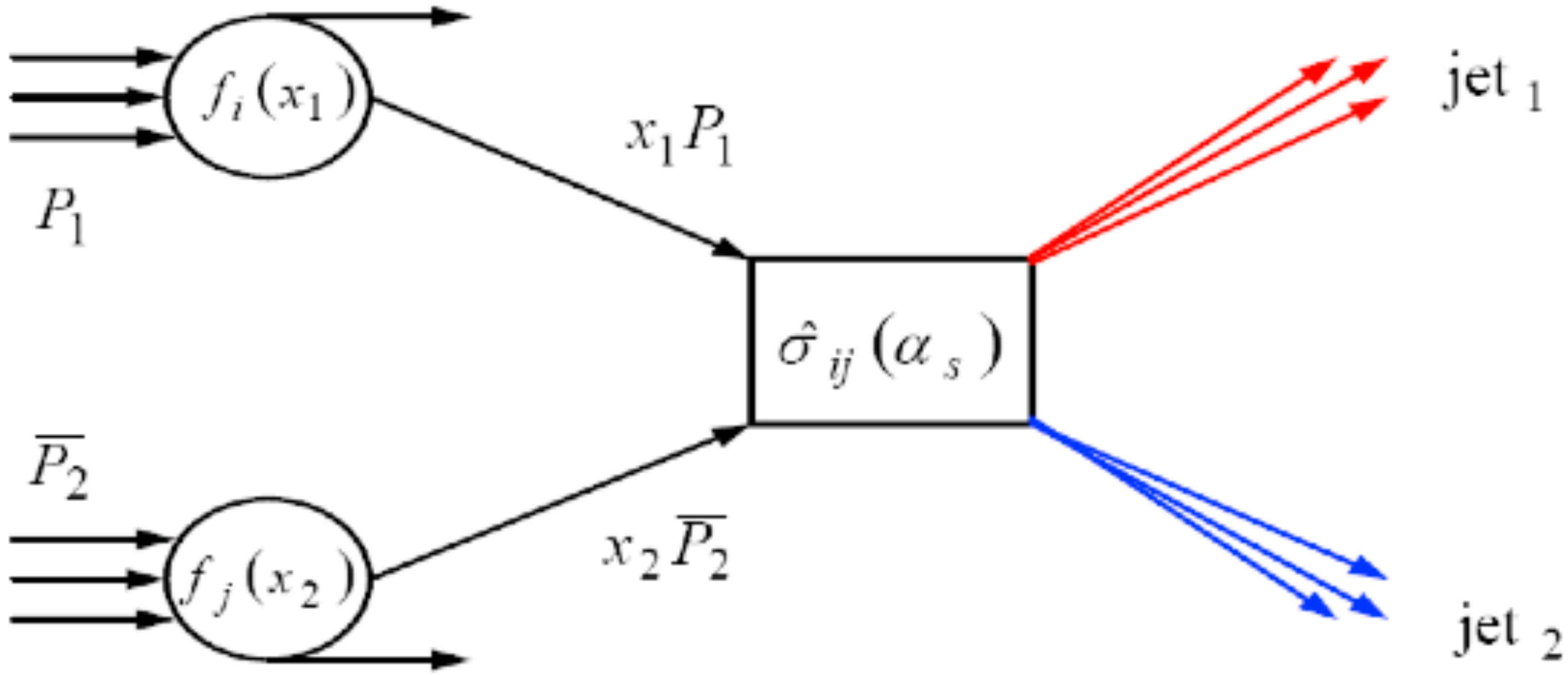
Rapidity y^* in the center-of-mass of the two-jet system:

$$\left.\begin{aligned} y_3 &= y^{cm} + y^* \\ y_4 &= y^{cm} - y^* \end{aligned}\right\} \quad y^{cm} = \frac{y_3 + y_4}{2}, \quad y^* = \frac{y_3 - y_4}{2}$$

Scattering angle: $\cos\vartheta^* = \frac{p_z^*}{E^*} = \frac{\sinh y^*}{\cosh y^*} = \tanh \frac{y_3 - y_4}{2}$

Upshot: Fractional momenta x_1, x_2, and scattering angle in the CMS measurable

Calculation of Jet Cross Sections: Factorization



$$\sigma = \sum_{ij} \int dx_1 dx_2 \, f_i\left(x_1, \mu_F^2\right) f_j\left(x_2, \mu_F^2\right) \hat{\sigma}_{ij}\left(\alpha_s^m\left(\mu_R^2\right), x_1 P_1, x_2 P_2, \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2}\right)$$

Sum over initial states

Parton Distributions

Factorization Scale

Point Cross Section

Order α_s^m

Renormalization Scale

Jet Fragmentation (II)

Energy conservation:

$$\sum_h \int_0^1 z\, D_q^h(z)\, dz = 1$$

Average number of hadrons arising from quark q:

$$\int_{z_{\min}}^1 D_q^h(z)\, dz = \left\langle n_q^h \right\rangle$$

$$z_{\min} = \frac{m_h}{E_q} = \frac{2m_h}{\sqrt{s}}$$

Threshold energy for producing hadron with mass m_h

Fragmentation function often parameterized as:

$$D_q^h(z) = N \frac{(1-z)^n}{z}$$

$$\left\langle n_q^h \right\rangle \approx \int_{z_{\min}}^1 N \frac{1}{z} dz = -N \ln z_{\min} = N \ln \frac{E_q}{m_h}$$

Particle multiplicity grows logarithmically with parton energy

Jet Fragmentation (III)

Differential cross section:

$$\frac{d\sigma}{dz}(e^- + e^- \to h + X) = \sum_q \sigma(e^- + e^- \to q + \bar{q})\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

Note that $\int \frac{d\sigma}{dz} dz = \langle n_h \rangle \cdot \sigma_{tot}$

Using

$$\sigma(e^+e^- \to q\,\bar{q}) = \frac{4\pi}{3}(\hbar c)^2 \cdot \frac{Q_q^2 \alpha^2}{s} \qquad \sigma_{tot} := \sigma(e^+e^- \to h + X) = \sum_q \sigma(e^+e^- \to q\,\bar{q})$$

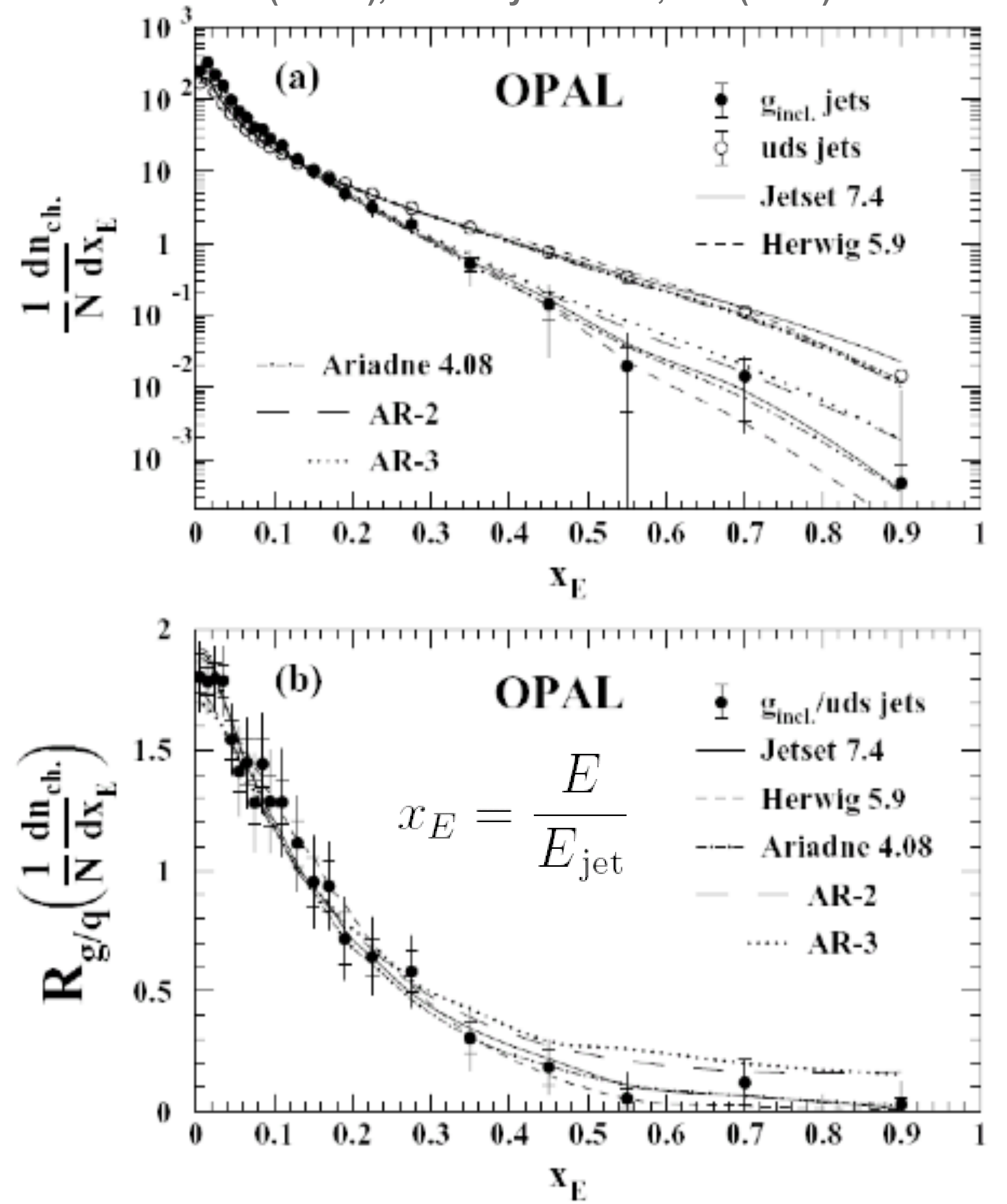
leads to

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{dz}(e^- + e^- \to h + X) = \frac{\sum_q Q_q^2 \left[D_q^h(z) + D_{\bar{q}}^h(z)\right]}{\sum_q Q_q^2} \equiv f(z)$$

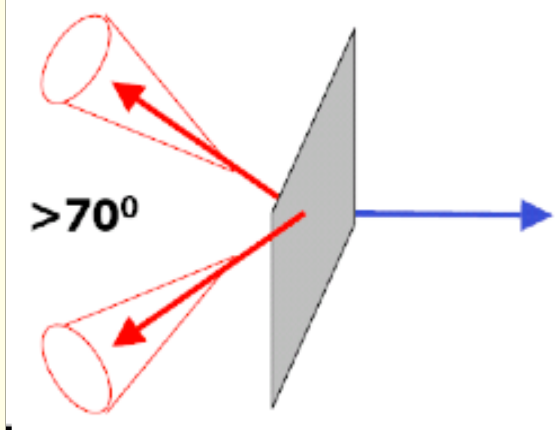
this cross section is expected to be a universal function, independent of √*s*

Quark vs. Gluon jet fragmentation (I)

G. Abbiendi et al (OPAL), Eur. Phys. J. C11, 217 (1999)



Gluon jet:



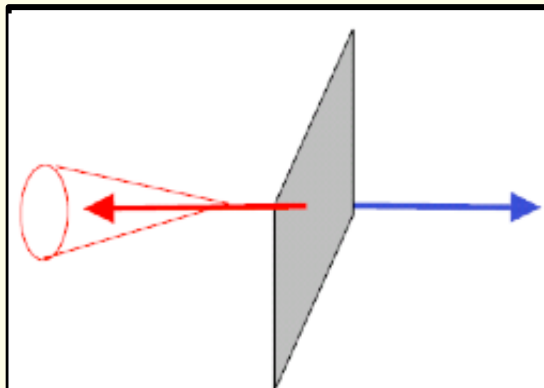
Treat hemisphere as a gluon jet

E ~ 40 GeV, purity ~ 82%

~ 400 events

Two b-tagged jets. B-tagging via displaced vertices.

u,d,s jet:



Treat hemisphere as a u,d,s jet

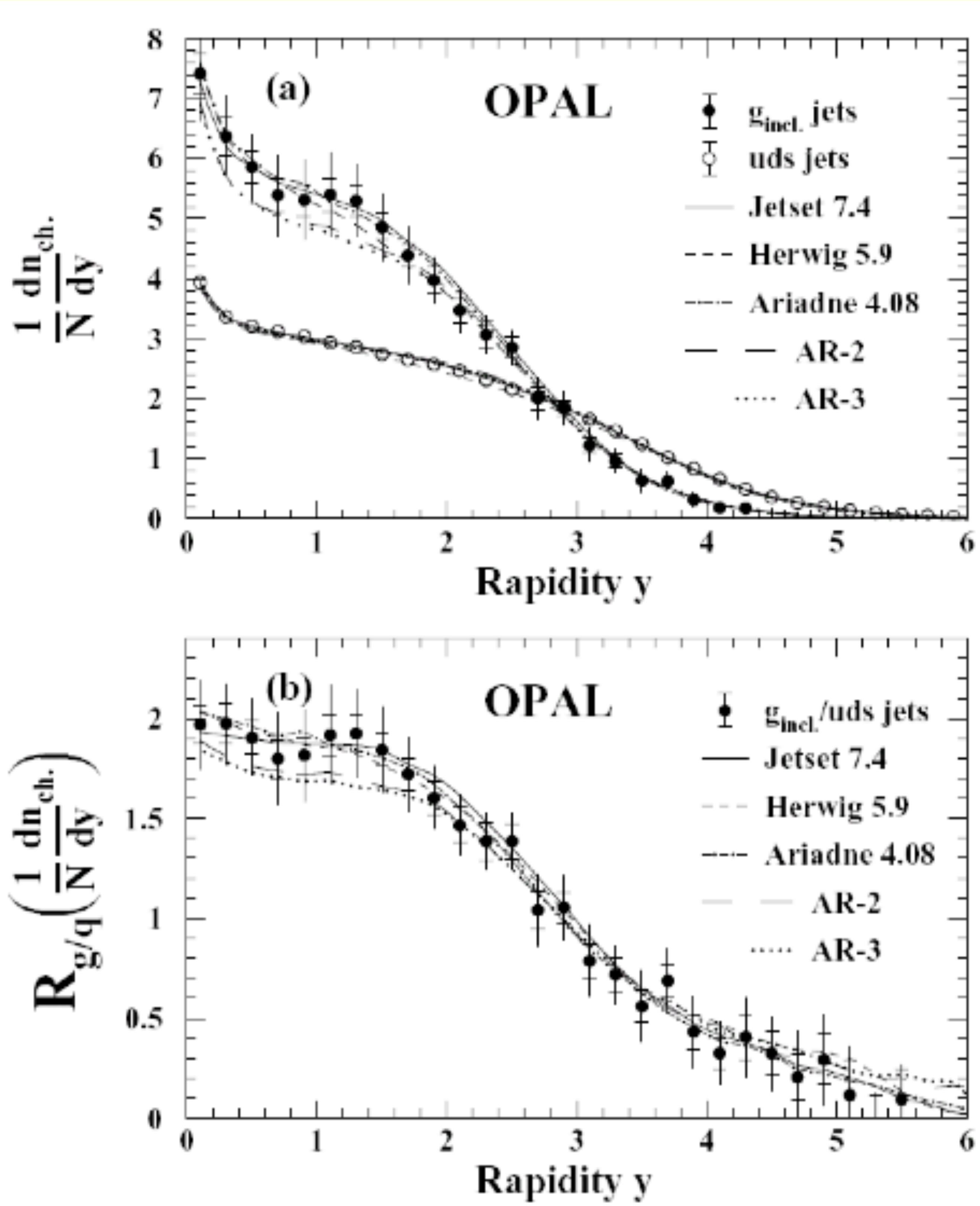
E = 45.6 GeV, purity ~ 86%

~ 200,000 events

Two anti-b-tagged jets.

Gluons fragment softer and broader than light quarks

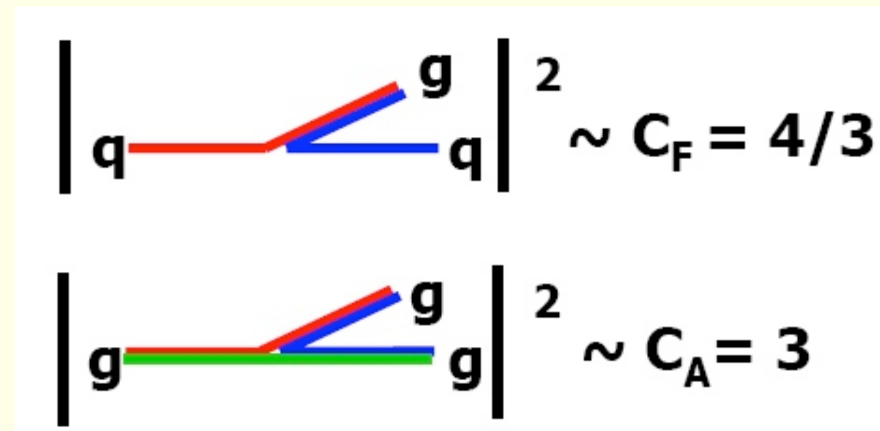
Quark vs. gluon jet fragmentation (II): Charged Particle Multiplicity



Rapidity w.r.t. thrust axis $\hat{r}$:

$$y = \frac{1}{2}\ln\left(\frac{E + \vec{p}\cdot\hat{r}}{E - \vec{p}\cdot\hat{r}}\right)$$

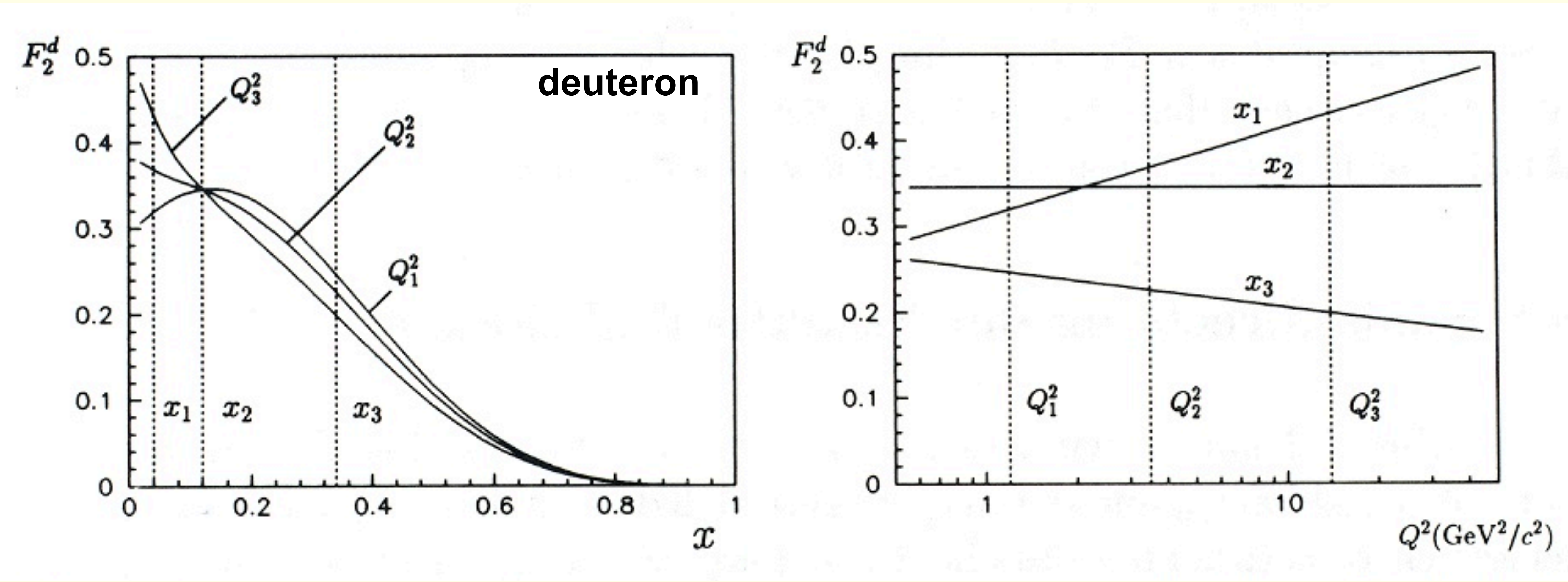
Naively expect:



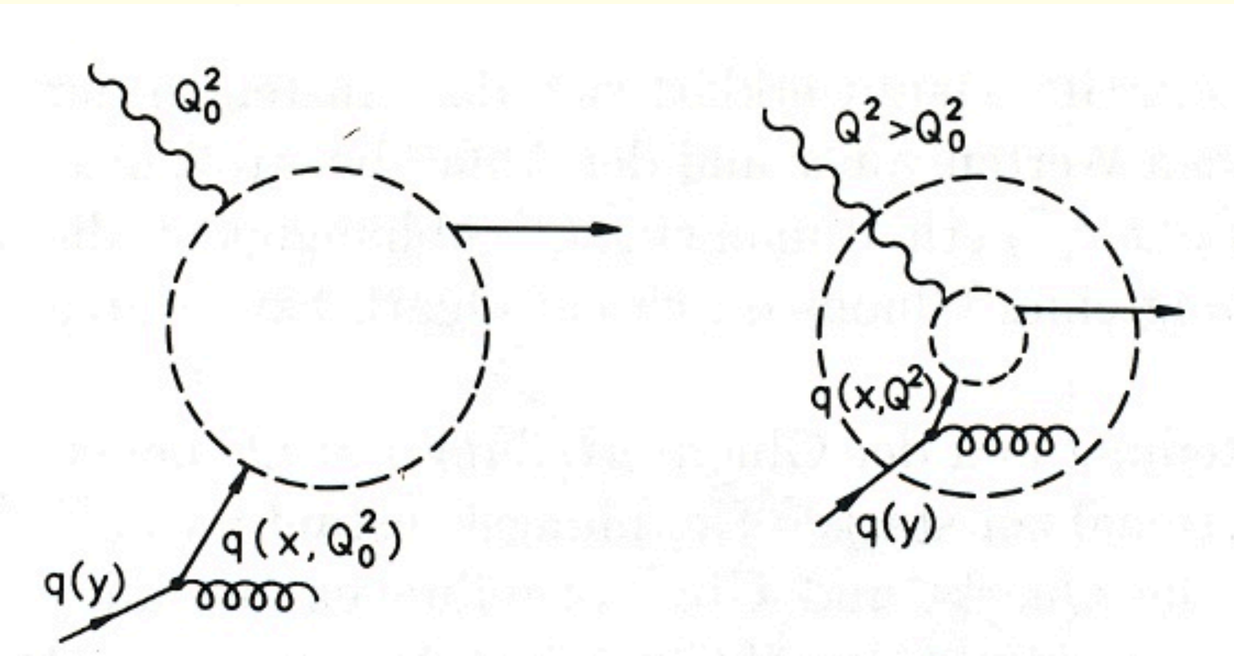
$$r \equiv \frac{<n_g>}{<n_q>} \equiv \frac{<\text{gluon jet multiplicity}>}{<\text{quark jet multiplicity}>} \sim \frac{C_A}{C_F} = \frac{9}{4}$$

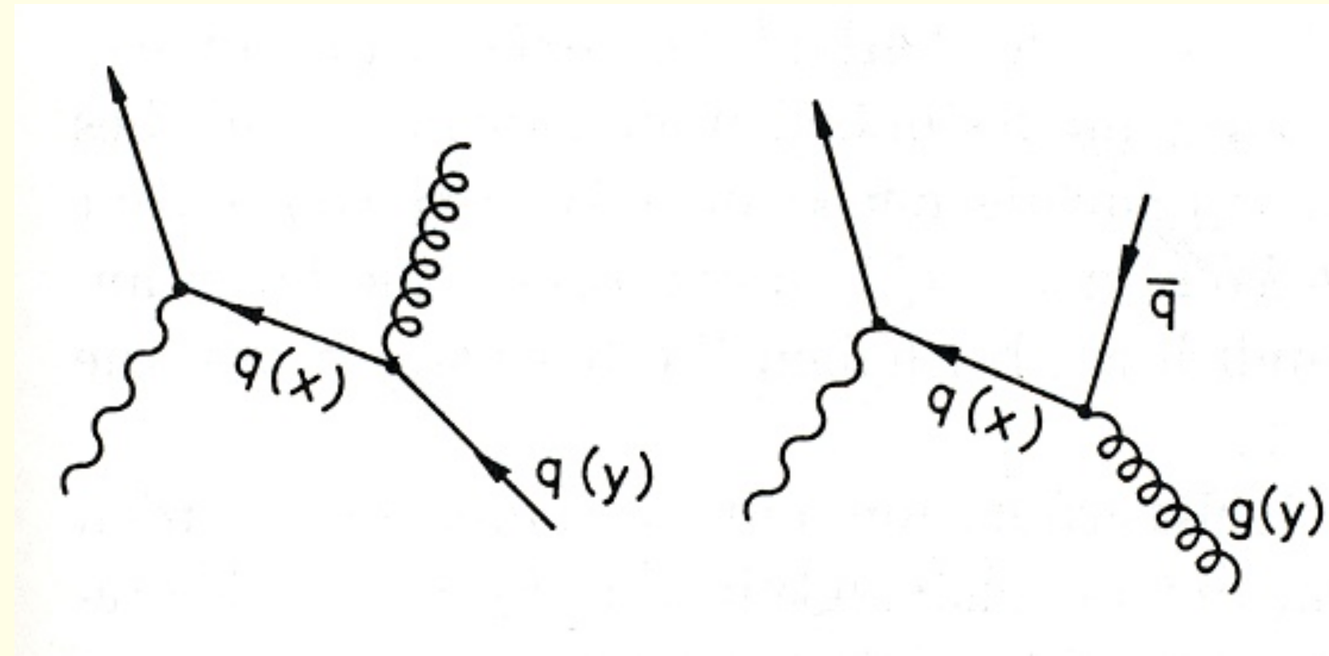
Indeed approximately observed at small rapidity

Scaling violations (II)



Interpretation of Scaling Violation (I)





Resolution of the virtual photon: $\lambda \approx \hbar / \sqrt{Q^2}$

- **Momentum continuously redistributed among partons**
- **Increasing resolution of the virtual photon with rising Q^2 explains scaling violations**
- **This is quantitatively explained in QCD**

Quark and Gluon distribution in the Proton as Function of x and Q^2