QGP and High-p_T Physics

Lecture 1: The Physics of the QGP

Helmholtz Graduate School of Fundamental Physics

Winter School Obergurgl 2010

January 29 - February 2, 2010

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Books/Paper

Introduction to High-Energy Heavy-Ion Collisions
Cheuk-Yin Wong
World Scientific

Quark-Gluon Plasma
K. Yagi, T. Hatsuda, and Y. Miake,
(Cambridge Monographs, ed. T. Ericson, P.V. Landshoff)
ISBN 0-521-56108-6

Ultrarelativistic Heavy-Ion Collisions (Elsevier)
R. Vogt
ISBN 978-0-444-52196-5

Quark Gluon Plasma 3
(World Scientific Publishing, ed. R.C. Hwa and X.-N. Wang)
ISBN 981-238-077-9

The Large Hadron Collider, **Nature 448 (2007) 269**

Jet Quenching in Heavy-Ion collisions, **U. Wiedemann, arXiv 0908.2306**

Introduction

Strong Interaction

Nobel prize in physics (2004)
(work done in 1973 = Birth of QCD)

David J. Gross

H. David Politzer

Frank Wilczek

- Confinement:
  Isolated quarks and gluons cannot be observed, only color-neutral hadrons
- Asymptotic freedom:
  Coupling α_s between color charges gets weaker for high momentum transfers, i.e., for small distances r
  (Perturbative methods applicable for $r < 1/10$ fm)
- Limit of low particle densities and weak coupling experimentally well tested ($\rightarrow$ QCD perturbation theory)
- Nucleus-Nucleus collisions: QCD at high temperatures and density („QCD thermodynamics“)

Ultrarelativistic Heavy-Ion Physics

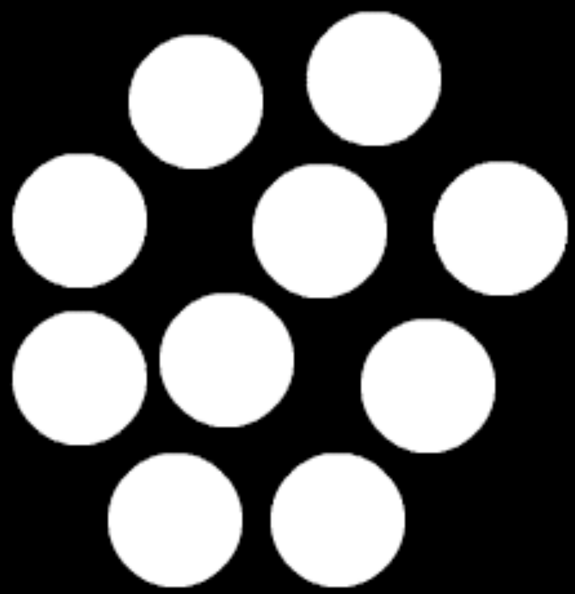
Basic, childlike questions addressed in ultrarelativistic heavy-ion physics:

What happens to matter if you make it

- hotter and hotter?
- denser and denser?

With increasing temperature *T*:
solid $\rightarrow$ liquid $\rightarrow$ gas $\rightarrow$ plasma $\rightarrow$ QGP

Quark-Gluon-Plasma



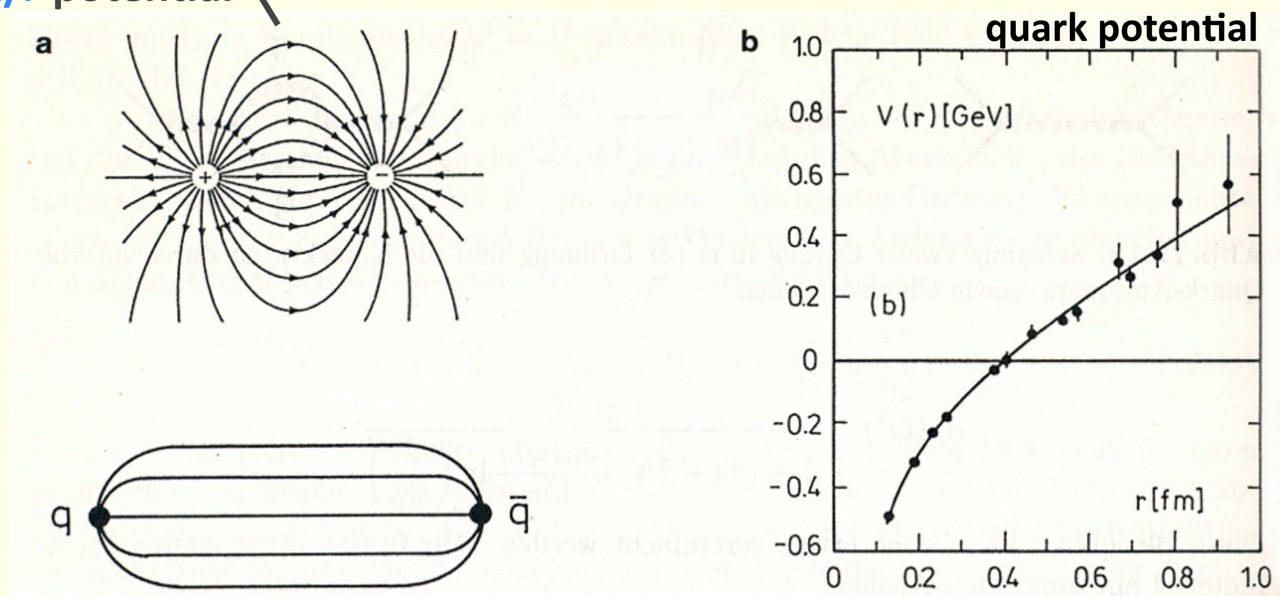
Confinement

$$\text{Quark potential: } V(r) = -\frac{4}{3}\frac{\alpha_s(r)\hbar c}{r} + k \cdot r$$

Dominant at small distances (1-gluon exchange)

Dominant at large distances (related to confinement)

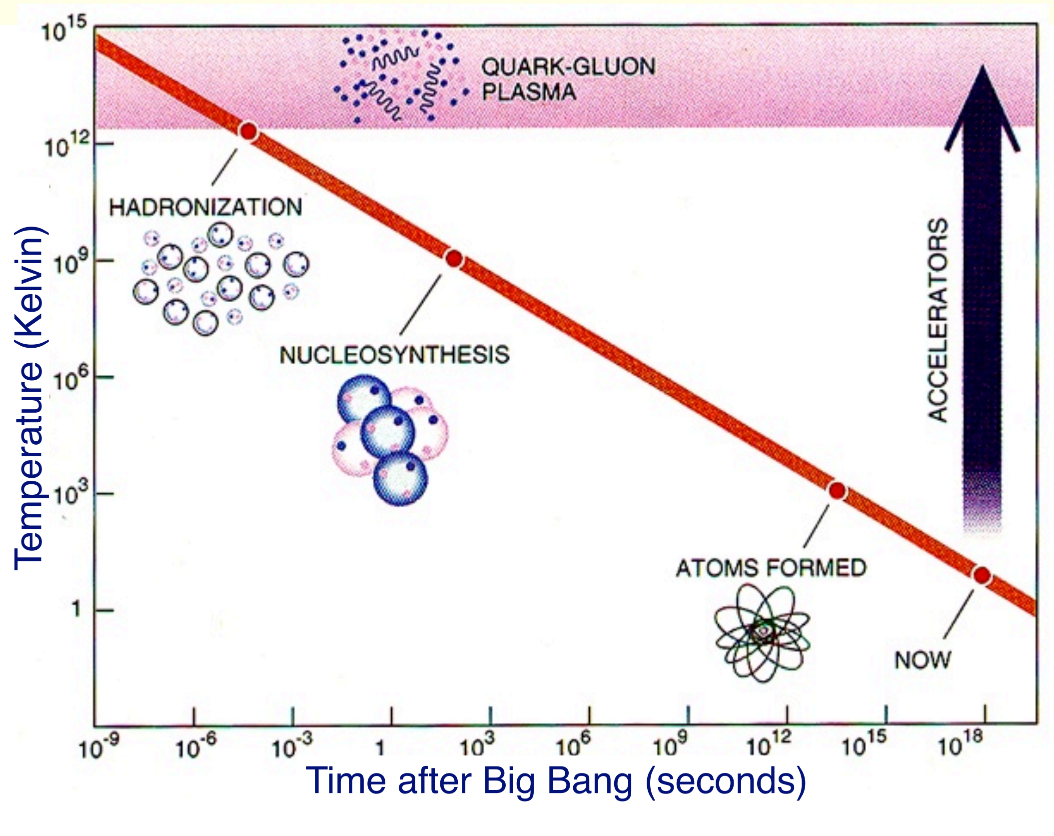
QED: flux is not confined:
1/*r* potential



The long distance term $k \cdot r$ is expected to disappear in the QGP

QCD: flux is confined (flux tubes form): potential ~ *r*

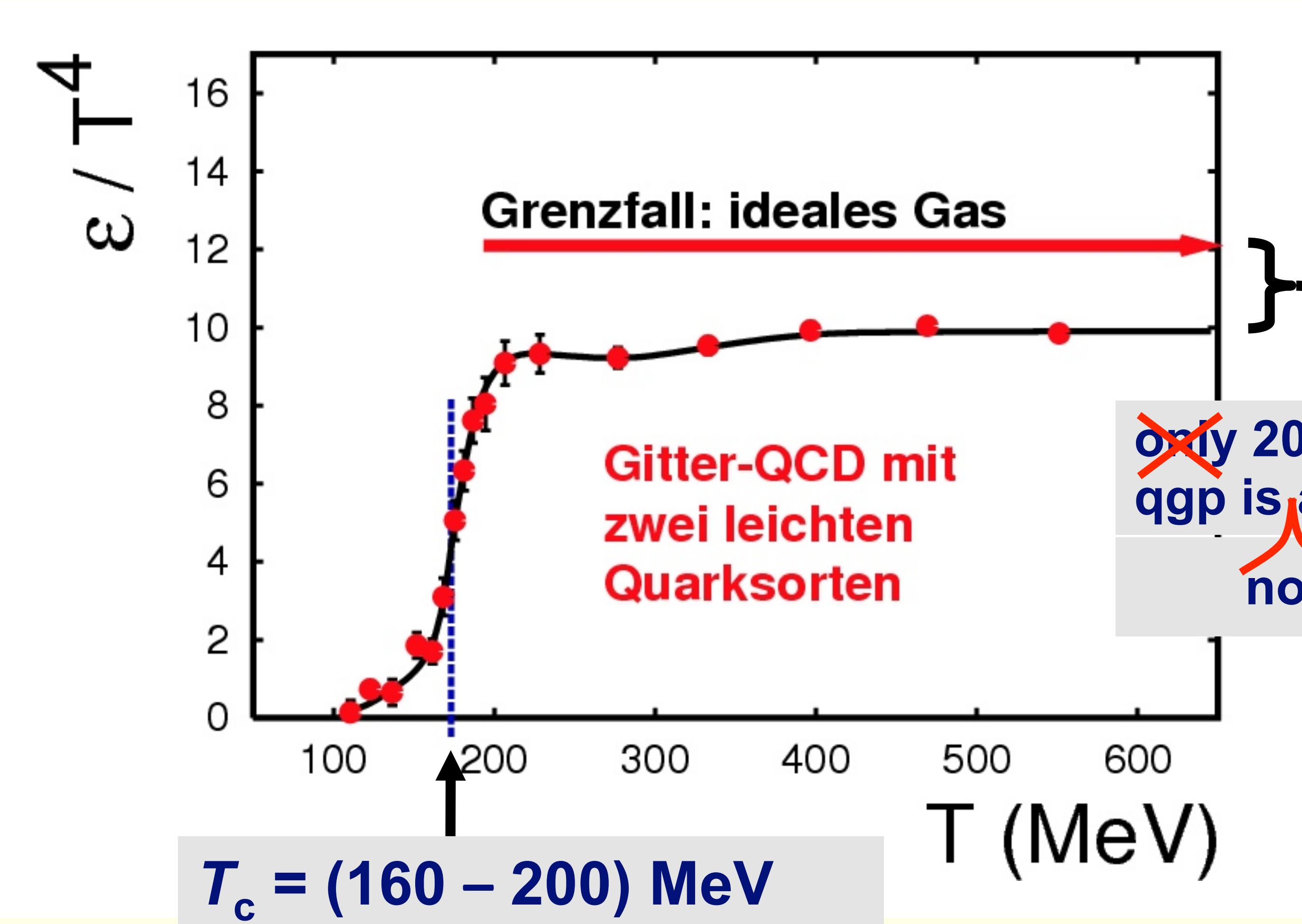
Nucleus-Nucleus Collisions: „Mini Big Bang in the Laboratory"



- Transition from the Quark-Gluon Plasma to a gas of hadrons at ~ 10^{12} °C
- 100 000 hotter than the core of the sun
- Early universe: QGP $\rightarrow$ hadron gas a few microseconds after the Big Bang

Predictions from First Principles: Lattice QCD

F. Karsch, E. Laermann, hep-lat/0305025



2 quark flavors:

$$\varepsilon_{SB} = g \cdot \frac{\pi^2}{30} \cdot T^4$$

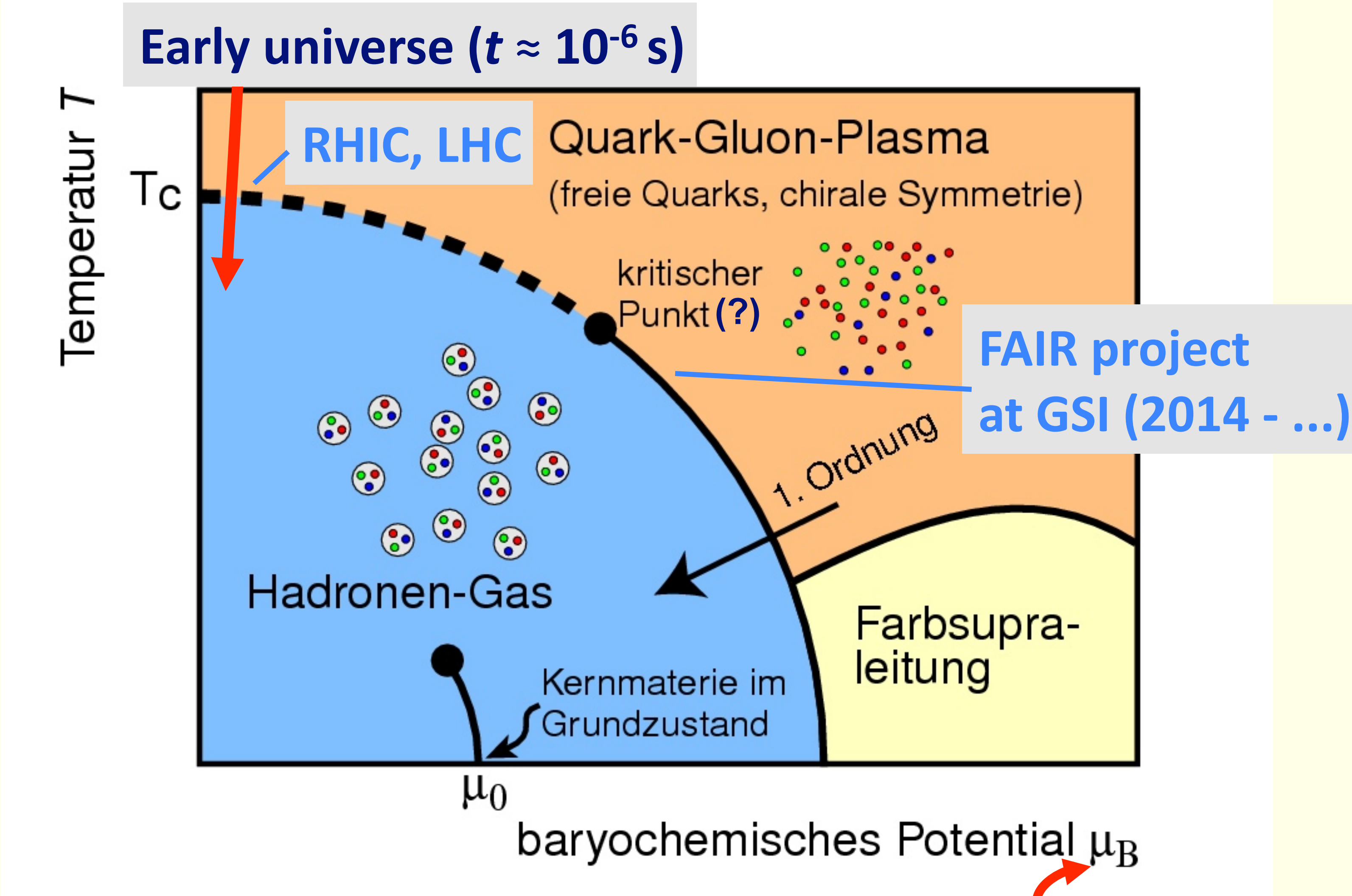
with $g = 37$

~~only~~ 20% deviation: qgp is not an ideal gas

T_c = (160 – 200) MeV

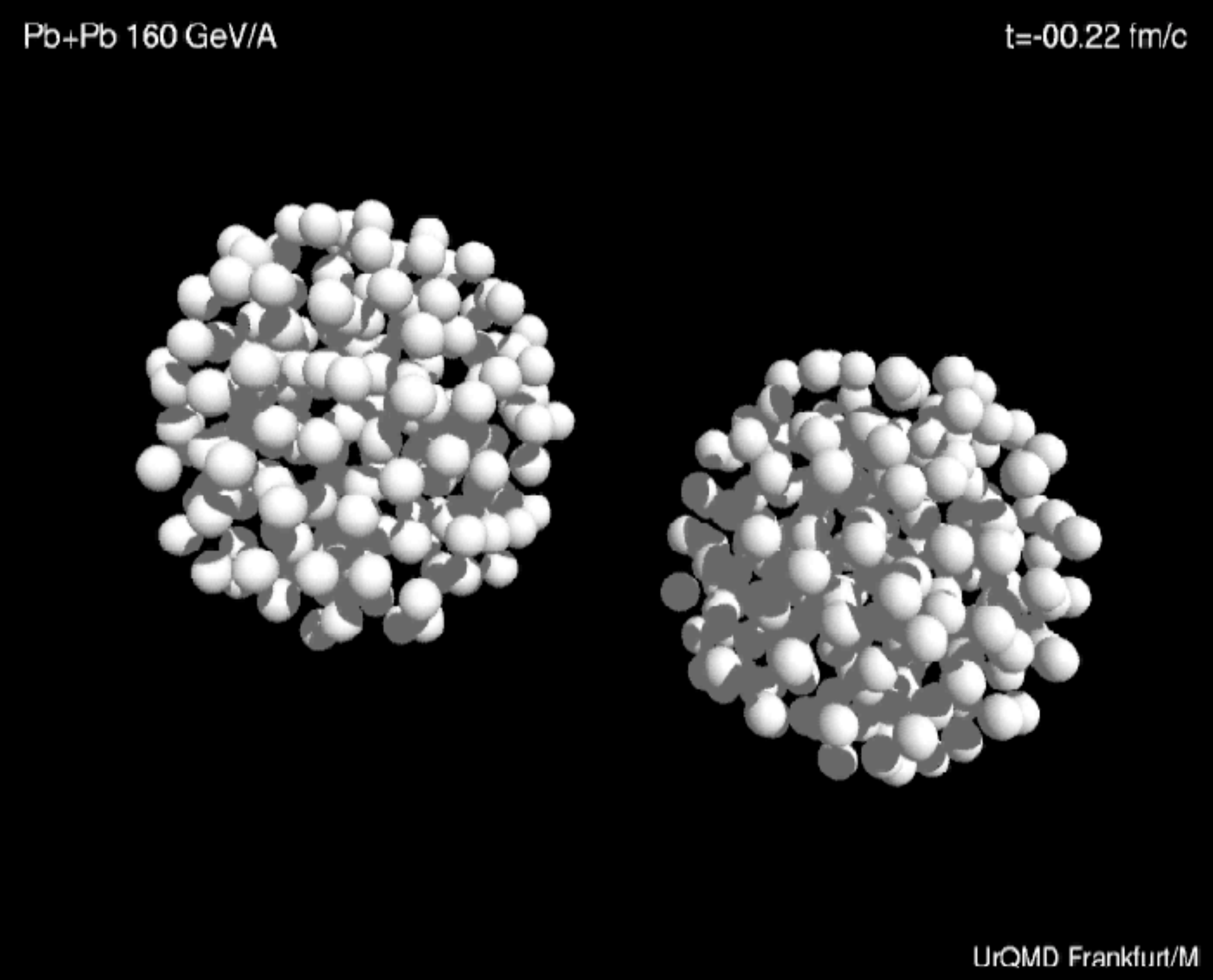
$\varepsilon_c \approx$ 0.7 – 1.0 GeV/fm³

Expected QCD Phase Diagram

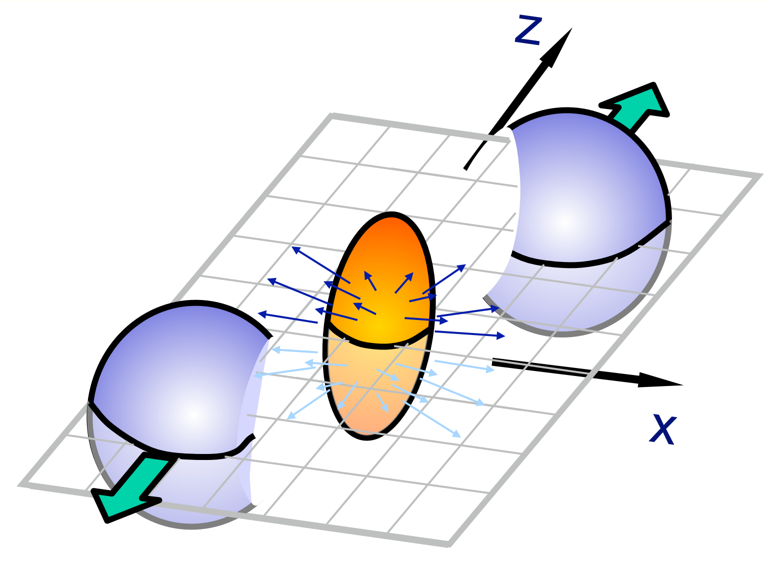


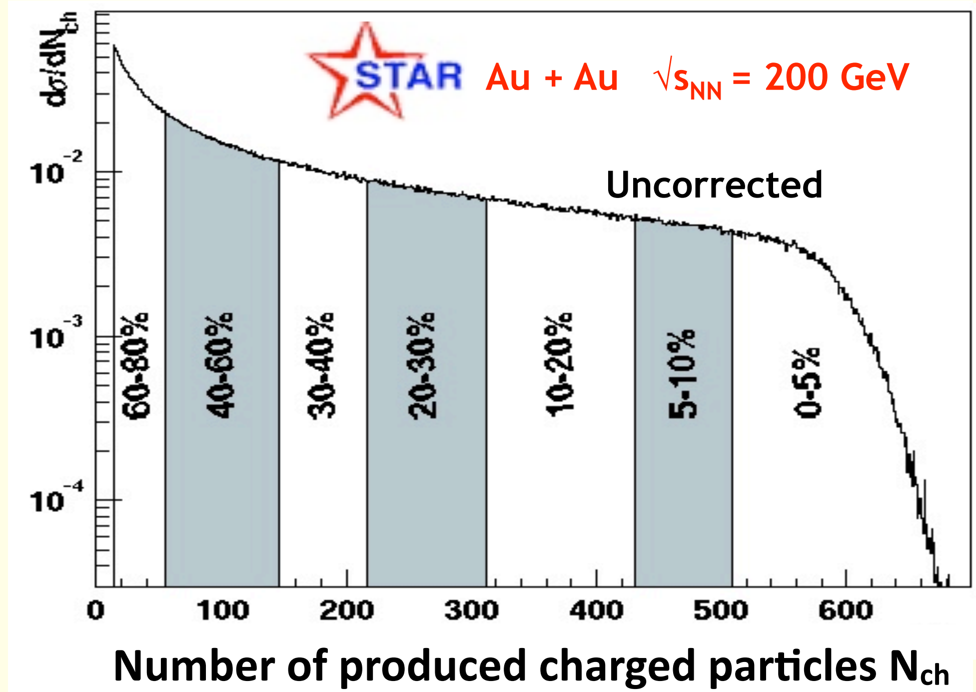
Measure of the net baryon density ρ

Ultra-Relativistische Schwerionenkollision



Geometry Plays a Key Role in Ultra-Relativistic Heavy-Ion Physics





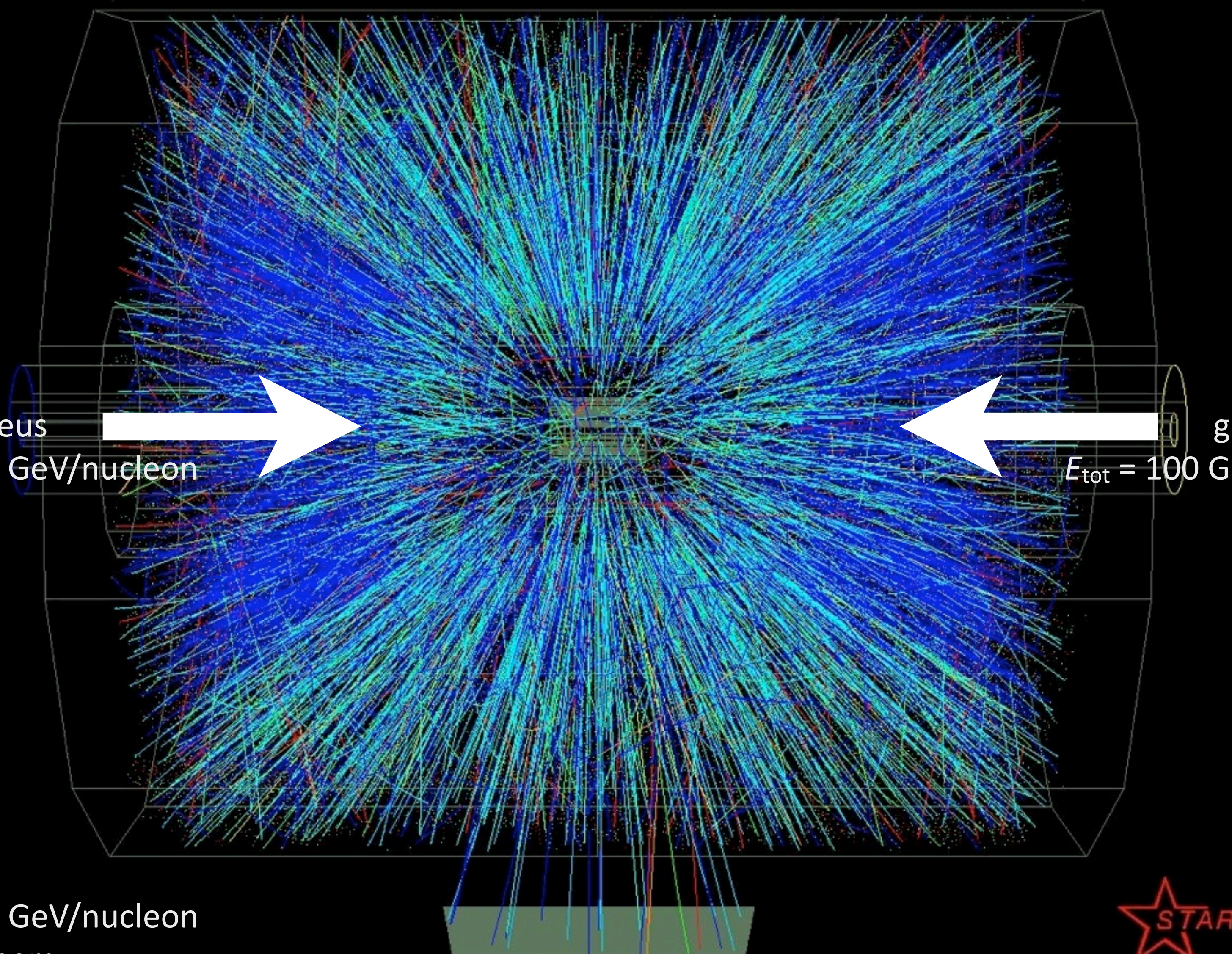
Number of participants: number of nucleons in the overlap region

Number of binary collisions: number of inelastic nucleon-nucleon collisions

Small impact parameter *b* corresponds to large particle multiplicity

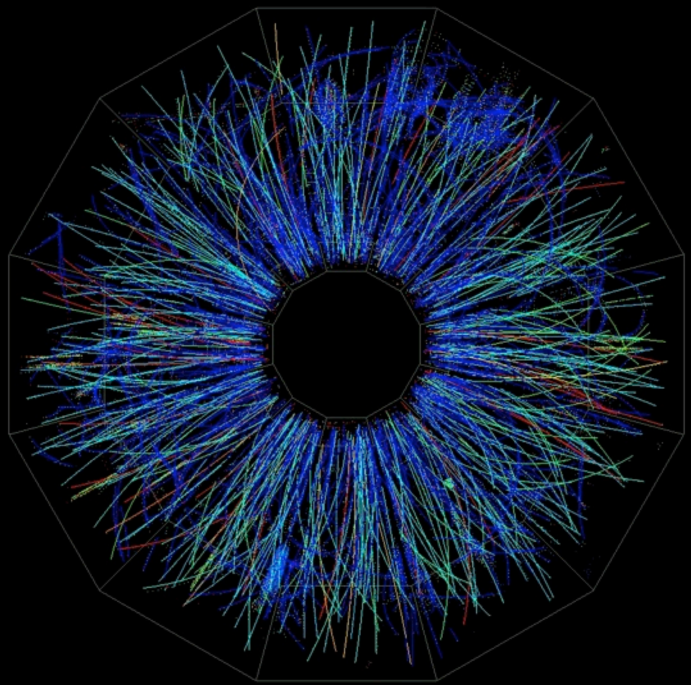
Reaction plane: *x*-*z* plane

Au+Au Collision at the Relativistic Heavy Ion Collider (RHIC) in the USA

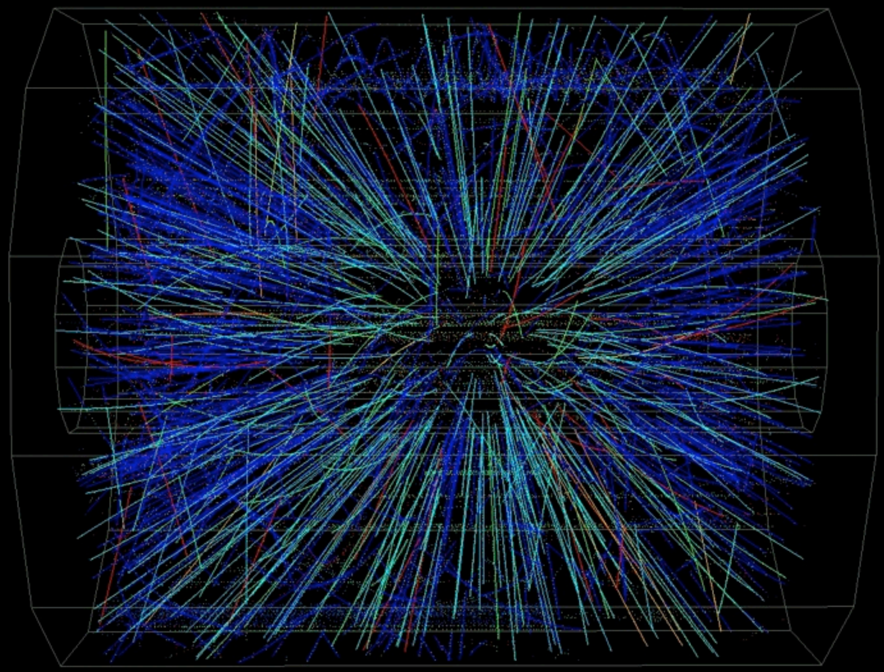


LHC:
E_{tot} = 2750 GeV/nucleon
for each beam

Au + Au Collisions at RHIC

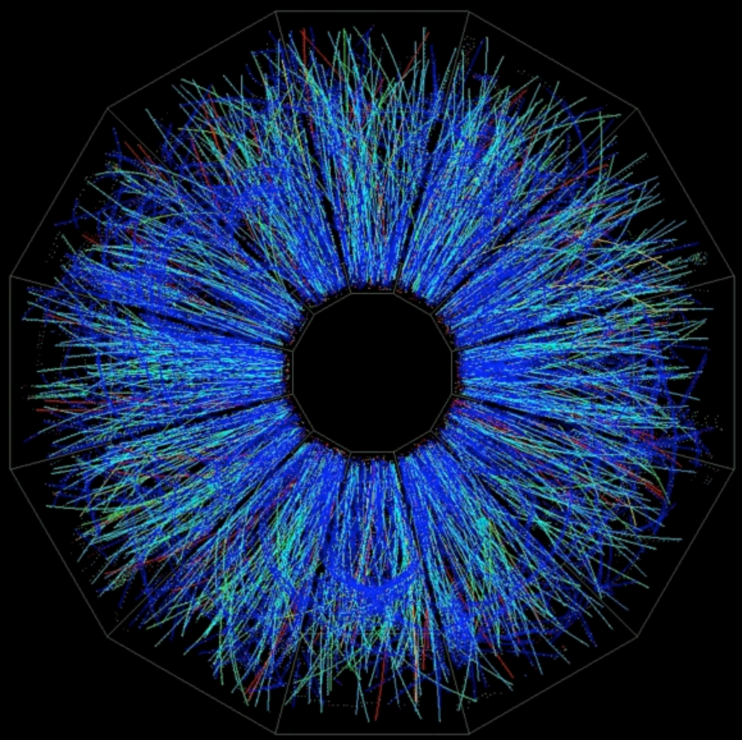


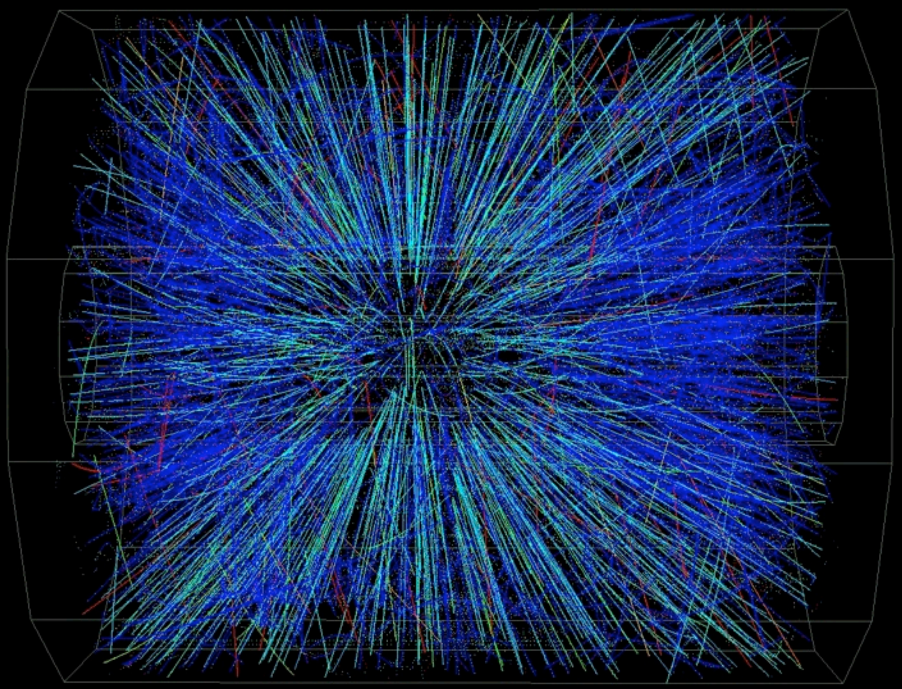
Peripheral Event



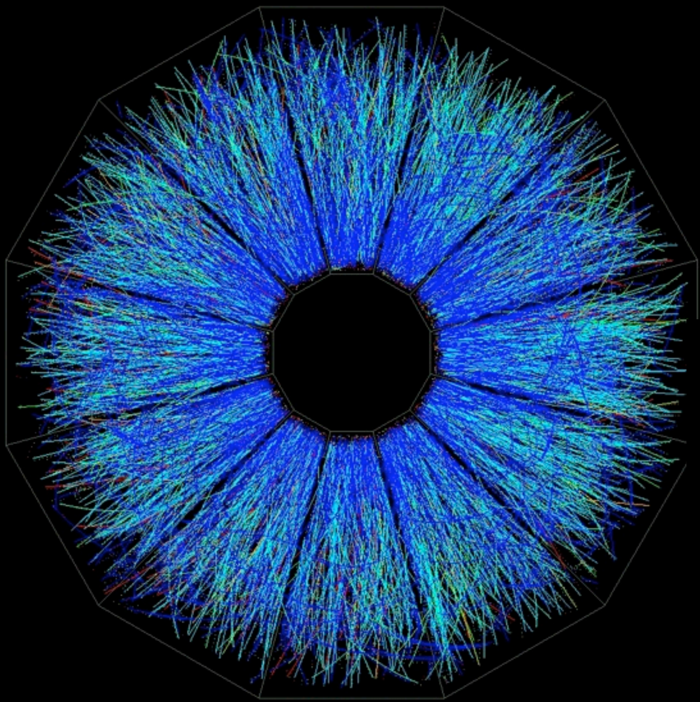
Au + Au Collisions at RHIC

Mid-Central Event

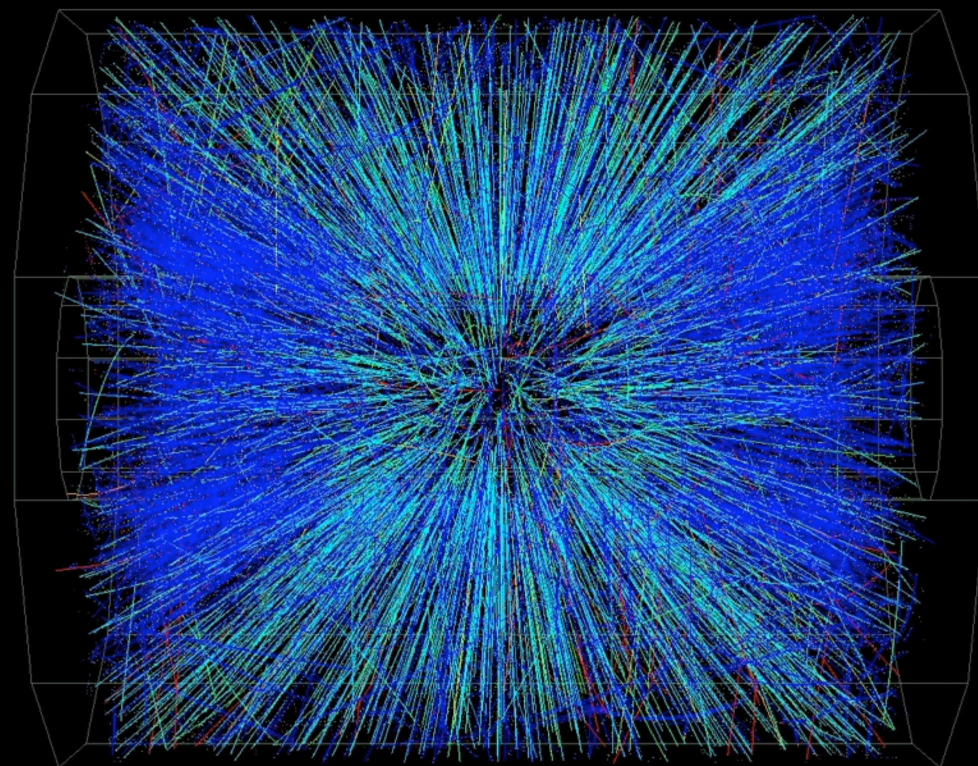




Au + Au Collisions at RHIC



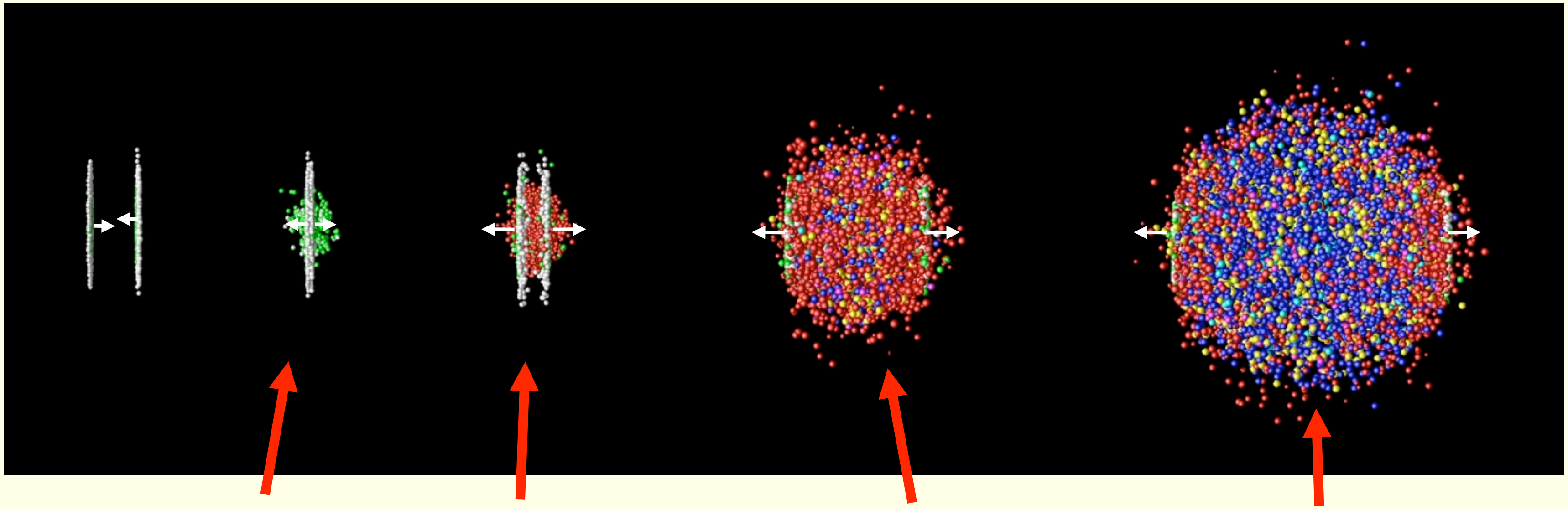
Central Event



about 5000 charged
particles per
central collisons

Ultra-Relativistic Nucleus-Nucleus Collisions

time →



Early hard parton-parton scatterings ($Q^2 \gg \Lambda^2_{QCD}$)

Thermalized medium (QGP!?) ($T_0 > T_c$, $T_c \approx$ 160–190 MeV)

Transition QGP → hadron gas

Freeze-out

- Time scales (RHIC, $\sqrt{s_{NN}}$ = 200 GeV):
  - Thermalization: $\tau_0 < \sim 1$ fm/c
  - QGP lifetime (center of a central Au+Au coll.): ~ 5 fm/c

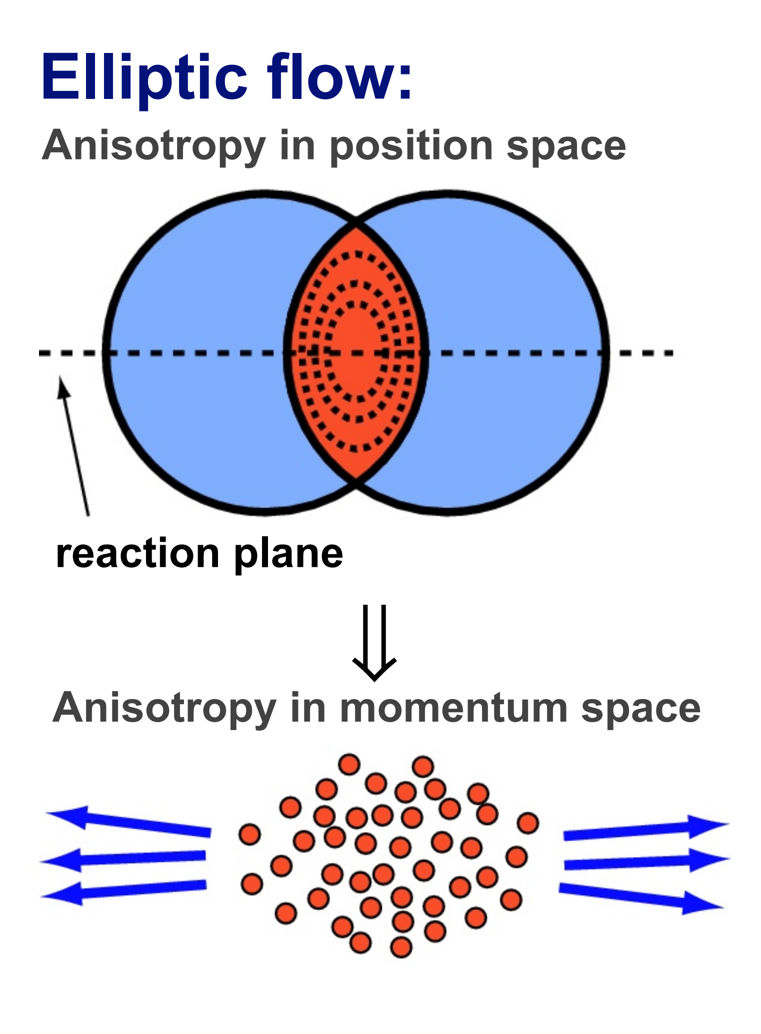
Note:
$1\ \mathrm{fm}/c = 0.33 \cdot 10^{-23}\ \mathrm{s}$

Brief History of Heavy-Ion Physics

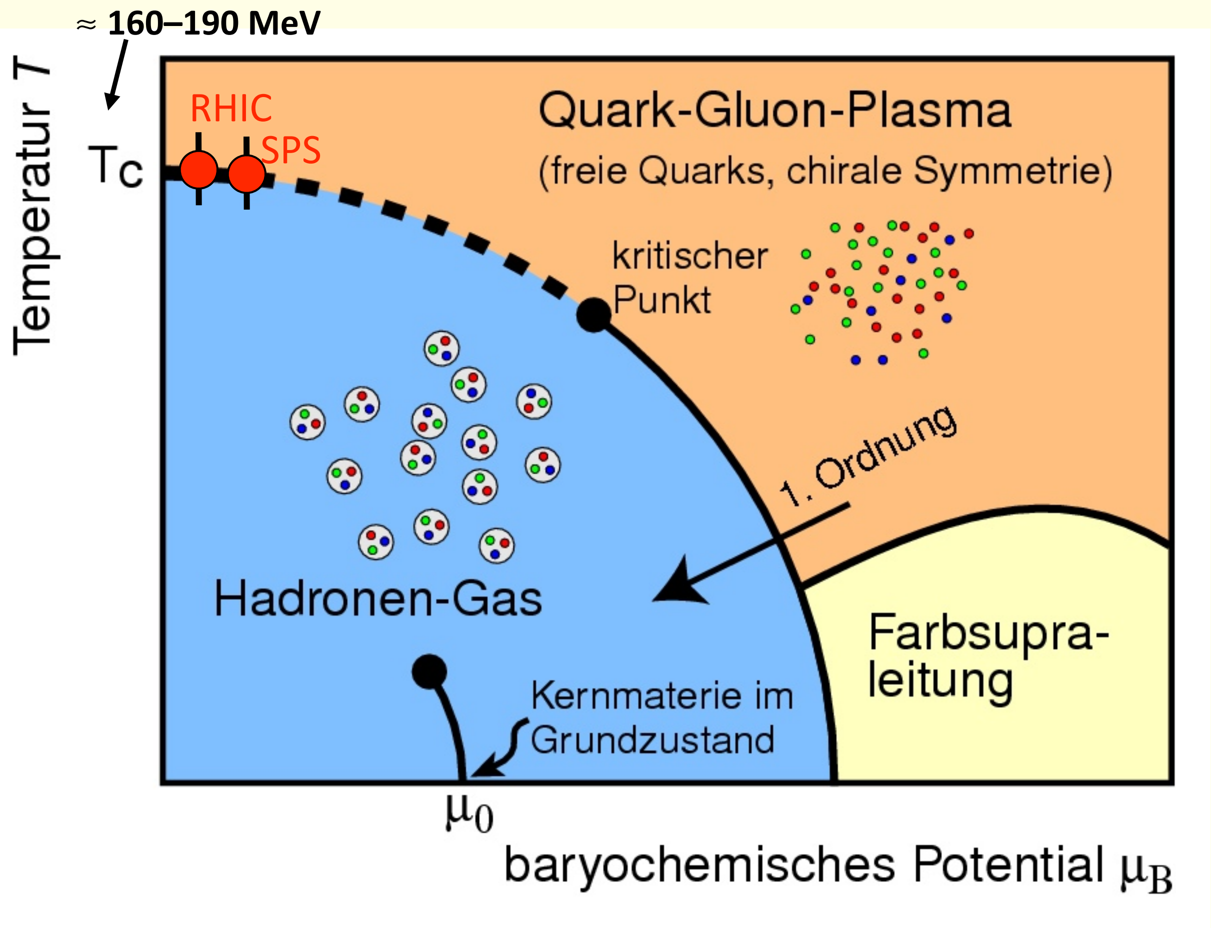
Start	Accelerator	Projectile	Energy ($\sqrt{s}$) per NN pair
~1985	AGS (BNL)	Si	~5 GeV
~1985	SPS (CERN)	O, S	~20 GeV
1994	SPS (CERN)	Pb	17 GeV
2000	RHIC (BNL)	Au	200 GeV
2010	LHC (CERN)	Pb	5500 GeV

Important Results of the RHIC Heavy-Ion Program

- Hadron suppression at high p_T
  - Medium is to large extent opaque for jets ("jet quenching")
- Elliptic Flow at low p_T
  - Ideal hydro close to data
    $\Rightarrow$ Small viscosity: "perfect liquid"
  - Evidence for early thermalization ($\tau \lesssim 1$ fm/c)
- All hadron species in chemical equillibrium
  ($T \approx 160$ MeV, $\mu_B \approx 20$ MeV)



Nucleus-Nucleus Collisions: Freeze-out Parameters



Freeze-out parameters *T* and μ_B approximately at expected phase boundary

Points to Take Home

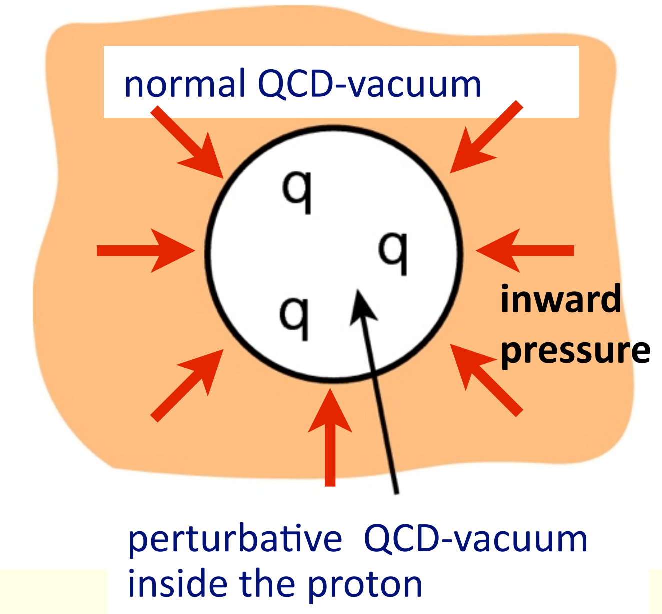
- Ultrarelativistic Heavy-Ion Collisions:
  Study of QCD in the regime of extreme temperatures and densities
- Goal: Characterization of the Quark-Gluon Plasma
- Transition QGP $\rightarrow$ hadrons about 10^{-6} s after the Big Bang
- QCD phase diagram: QGP reached
  - at high temperature (about 160-200 MeV [$\sim 2 \cdot 10^{12}$ K])
  - and/or add high baryochemical potential μ_B
- RHIC/LHC: $\mu_B \approx 0$
- Experiments at FAIR:
  $\mu_B > 0$ search for critical point

Thermodynamics of the QGP

How to Estimate the Transition Temperature for the QGP ↔ Hadron Gas Transition?

- Compute the pressure p in each phase
- The phase with the higher pressure wins

Bag Model



- Hadron = „bag“ filled with quarks
- Two kinds of vacuum
  - Normal QCD-Vacuum outside of the bag
  - Perturbative QCD-Vacuum within the bag

Energy density: $\varepsilon = E / V$

Energy density in the bag is higher than in the vacuum: $\varepsilon_{\mathrm{BAG}} - \varepsilon_{vacuum} =: B > 0$

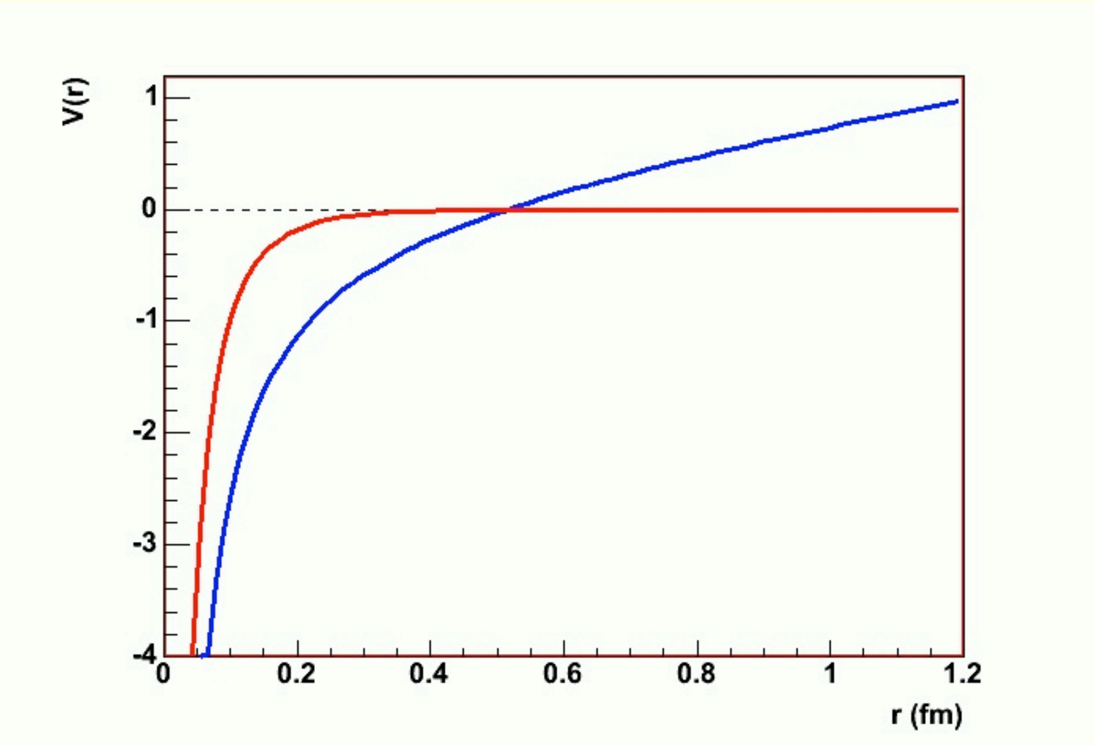
kinetic energy of *N* particles in a spherical box of radius *R*

Energy of *N* quarks in a bag of radius *R*: $E = \frac{2.04 N}{R} + \frac{4\pi}{3} \cdot R^3 \cdot B$

Condition for stability: d*E*/d*R* = 0 (minimum):

$$B^{1/4} = \left(\frac{2.04 N}{4\pi}\right)^{1/4} \cdot \frac{1}{R} \quad \overset{N=3,\ R=0,8\ \mathrm{fm}}{\Longrightarrow} \quad B^{1/4} = 206\ \mathrm{MeV} \quad (\hbar = c = 1)$$

Quark-Antiquark-Potential in the QGP



- Within a QGP, the long-range part of the potential of a quark-antiquark-pair vanishes
- Consequently, at high temperatures the J/Ψ cannot exist anymore
- Asymptotic freedom ($\alpha_s(q^2) \rightarrow 0$ for $q^2 \rightarrow \infty$): QGP is an ideal gas of quarks and gluons at very high temperatures

QCD-vacuum: $V(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + kr$

QGP: $V(r) = -\frac{4}{3}\frac{\alpha(r)}{r} \cdot \exp\left(-\frac{r}{\lambda_d}\right)$

Debye-screening-length, approx. 0,1 fm

Number of States

Number of states between momentum p and p+dp
(each state occupies a volume h^3 in phase space):

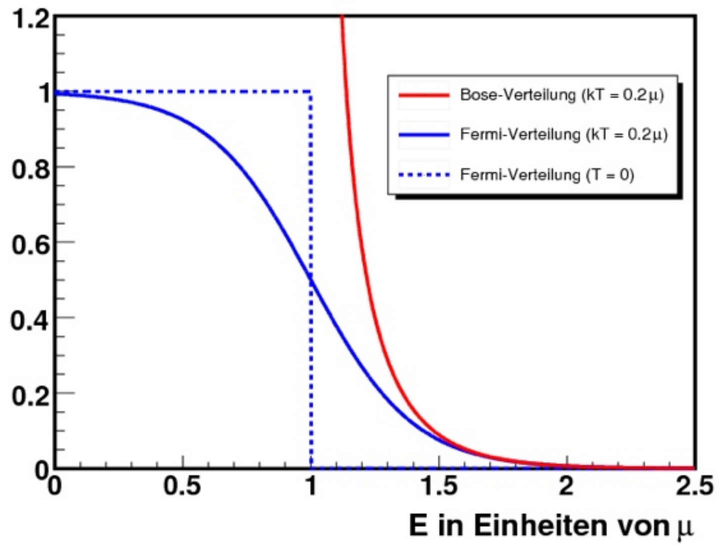
$$\mathrm{d}N = \frac{V}{h^3} 4\pi p^2 \mathrm{d}p$$

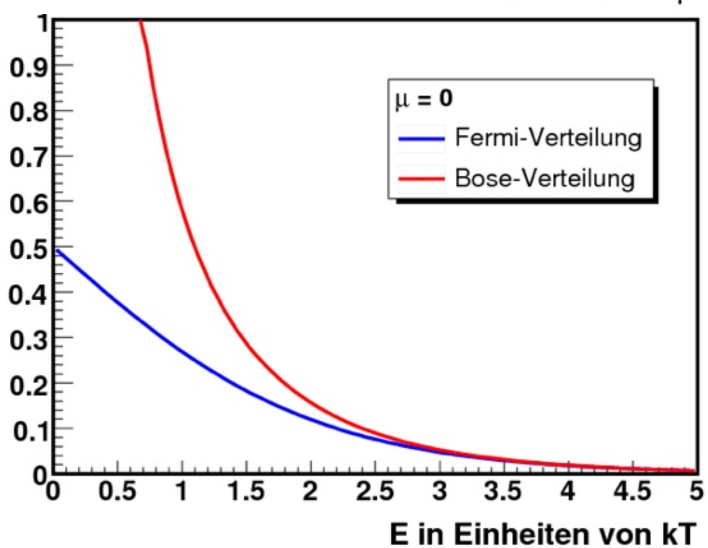
number of states

physical volume

volume of a spherical shell with radius p
and thickness dp in momentum space

Fermi-Dirac and Bose-Einstein Distribution





Number of particles with energy *E*

... for fermions (half-integer spin):

$$n(E) = \frac{g}{1 + e^{(E-\mu)/kT}}$$

(Fermi-Dirac distribution)

... for bosons (integer spin):

$$n(E) = \frac{g}{e^{(E-\mu)/kT} - 1}$$

(Bose-Einstein distribution)

g : # degrees of freedom (degeneracy)

μ : Chemical potential

T : Temperature

Degeneracy

QGP:

$$g_{\text{Bosons}} = 8_{\text{Color}} \times 2_{\text{Polarisation}} = 16$$

$$g_{\text{Fermions}} = g_{\text{Quarks}} + g_{\text{Antiquarks}} = 2 \times g_{\text{Quarks}}$$

$$= 2 \times 3_{\text{Color}} \times 2_{\text{Flavour}} \times 2_{\text{Spin}} = 24$$

assume only *u* and *d* quarks can be produced in the QGP, the rest too heavy

Pion-Gas:

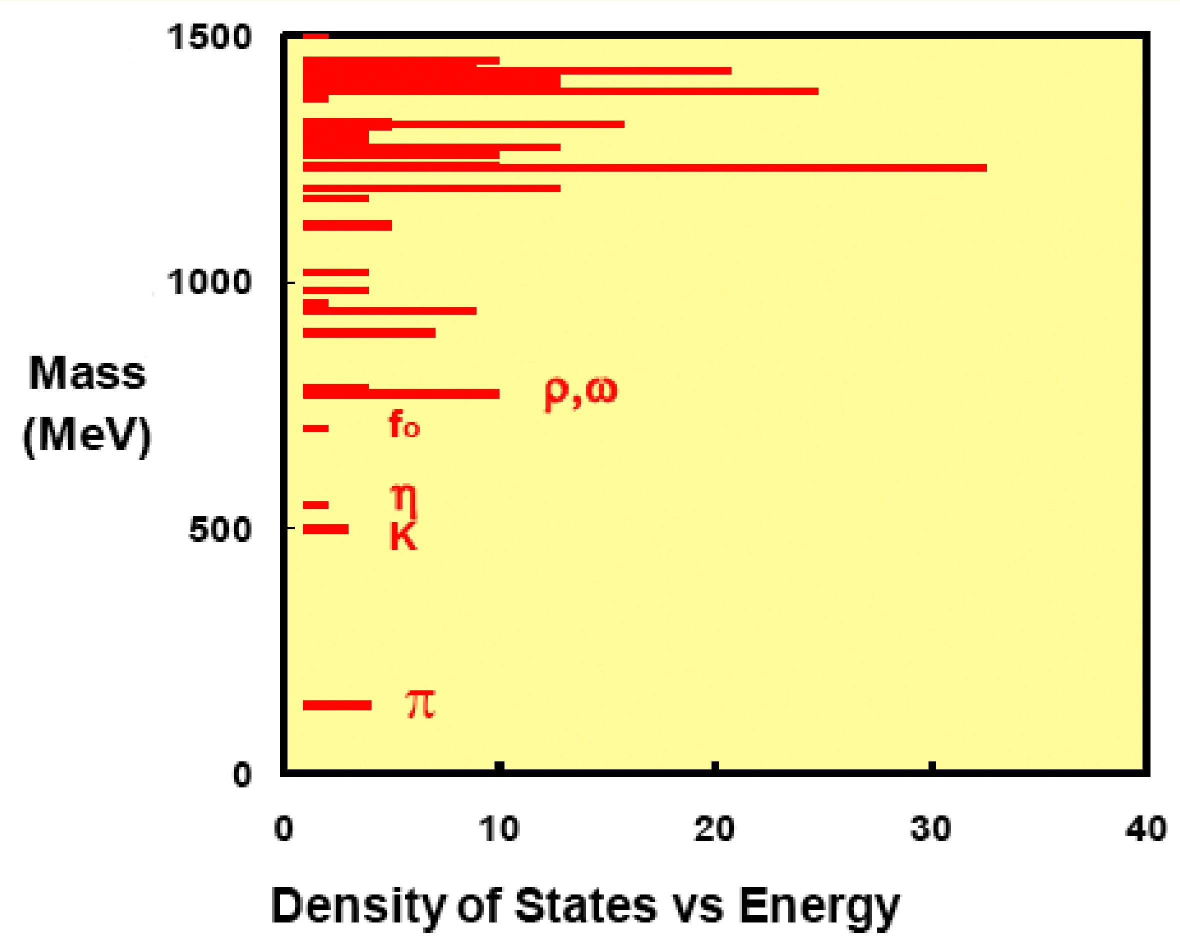
$$g_{\text{Bosons}} = 3_{\text{Type}} \qquad g_{\text{Fermions}} = 0$$

$$(\pi^+, \pi^-, \pi^0)$$

bottom line:

$$\text{ndf}_{\text{QGP}} \sim 10 \times \text{ndf}_{\text{Hadrons}}$$

Why Do We Consider Only Pions in the Hadronic Phase?



Assume T << 500 MeV for the hadronic phase

Then only pions should be relevant

Integration Yields Total Quark Density in the Ideal (= non interacting) QGP at Temperature T

Massless quarks, Fermi-Dirac distribution:

degrees of freedom — occupation factor

$$\mathrm{d}N_\mathrm{q} = g_\mathrm{q} \cdot \frac{V}{h^3} \cdot 4\pi p^2 \left(\frac{1}{1+e^{(E-\mu_\mathrm{q})/kT}} \right) \mathrm{d}p$$

$$\overset{\hbar=k=c=1}{=} g_\mathrm{q} \frac{p^2 V}{2\pi^2} \left(\frac{1}{1+e^{(p-\mu_\mathrm{q})/T}} \right) \mathrm{d}p$$

Quark density:

$$n_\mathrm{q}(\mu_\mathrm{q}) = \frac{N_\mathrm{q}}{V} = g_\mathrm{q} \frac{4\pi}{(2\pi)^3} \int_0^\infty \left(\frac{p^2}{1+e^{(p-\mu_\mathrm{q})/T}} \right) \mathrm{d}p$$

holds for massless quarks

Antiquarks ($\mu_{\bar{\mathrm{q}}} = -\mu_\mathrm{q}$):

$$n_{\bar{\mathrm{q}}}(\mu_{\bar{\mathrm{q}}}) = \frac{N_{\bar{\mathrm{q}}}}{V} = g_\mathrm{q} \frac{4\pi}{(2\pi)^3} \int_0^\infty \left(\frac{p^2}{1+e^{(p+\mu_\mathrm{q})/T}} \right) \mathrm{d}p$$

Quark-Gluon Plasma with μ = 0: Quarks

Quark density ($\mu_q = 0$):

$$n_q = n_{\bar{q}} = \frac{N_q}{V} = \frac{3}{2}\zeta(3)\frac{g_q}{2\pi^2}\frac{\pi^2}{30}T^3$$

1.20205

Total energy of the quarks:

$$E_q = \int_0^\infty p\,\mathrm{d}N_q$$

Calculating the integral for vanishing energy density and pressure ($\mu_q = 0$) yields:

$$\varepsilon_q = \frac{E_q}{V} = \frac{7}{8}g_q\frac{\pi^2}{30}T^4, \qquad p_q = \frac{1}{3}\varepsilon_q$$

(identical result for antiquarks ($\mu_q = 0$))

Example: $T = 200\ \mathrm{MeV},\ g_q = 12 \quad \Rightarrow \quad n_q = n_{\bar{q}} = 1{,}71\ /\ \mathrm{fm}^3$

Quark-Gluon Plasma with $\mu = 0$: Gluons

Gluons, Bose-Einstein distribution:

$$dN_g = \frac{V g_g}{2\pi^2} \cdot \frac{p^2}{e^{p/T} - 1} dp, \quad n_g = \frac{N_g}{V} = \frac{1}{V}\int_0^\infty dN_g, \quad E_g = \int_0^\infty p\, dN_g$$

Solution:

Energy density:	Pressure:	Particle density:
$\varepsilon_g = \frac{E_g}{V} = g_g \frac{\pi^2}{30} T^4,$	$p_g = \frac{1}{3}\varepsilon_g,$	$n_g = \frac{g_g}{\pi^2}\zeta(3) T^3$

$\zeta(3)$: 1.20205

Example: $T = 200\ \text{MeV}, g_g = 16 \quad \Rightarrow \quad n_g = 2{,}03\ \text{Gluons}/\text{fm}^3$

Quark-Gluon Plasma with μ = 0: Pressure and Energy Density

Pressure and energy density in a Quark-Gluon-Plasma at μ = 0 without particle interactions:

$$p_{\mathrm{QGP}} = \left(g_{\mathrm{g}} + \frac{7}{8}(g_{\mathrm{q}} + g_{\bar{\mathrm{q}}}) \right) \frac{\pi^2}{90} T^4 = 37 \frac{\pi^2}{90} T^4$$

$$\varepsilon_{\mathrm{QGP}} = 3 p_{\mathrm{QGP}} = 37 \frac{\pi^2}{30} T^4$$

Example: $T = 200\ \mathrm{MeV} \quad \Rightarrow \quad \varepsilon_{\mathrm{QGP}}^{\mathrm{id.\ Gas}} = 2{,}54\,\mathrm{GeV/fm}^3$

Quark-Gluon Plasma with μ = 0: Critical Temperature (I)

Accounting for the QCD-vacuum:

$$\varepsilon_{\text{QGP}}^{\text{QCD-Vac.}} = \varepsilon_{\text{QGP}} + B$$

$$p_{\text{QGP}}^{\text{QCD-Vac.}} = p_{\text{QGP}} - B$$

$$E = T\,S - pV \qquad (\mu = 0)$$

$$\Rightarrow \; p = Ts - \varepsilon, \qquad s := \frac{S}{V}$$

So we have:

$$p_{\text{HG}} = 3aT^4 \qquad \varepsilon_{\text{HG}} = 9aT^4$$

$$p_{\text{QGP}}^{\text{QCD-Vac.}} = 37aT^4 - B \qquad \varepsilon_{\text{QGP}}^{\text{QCD-Vac.}} = 111aT^4 + B \qquad a := \frac{\pi^2}{90}$$

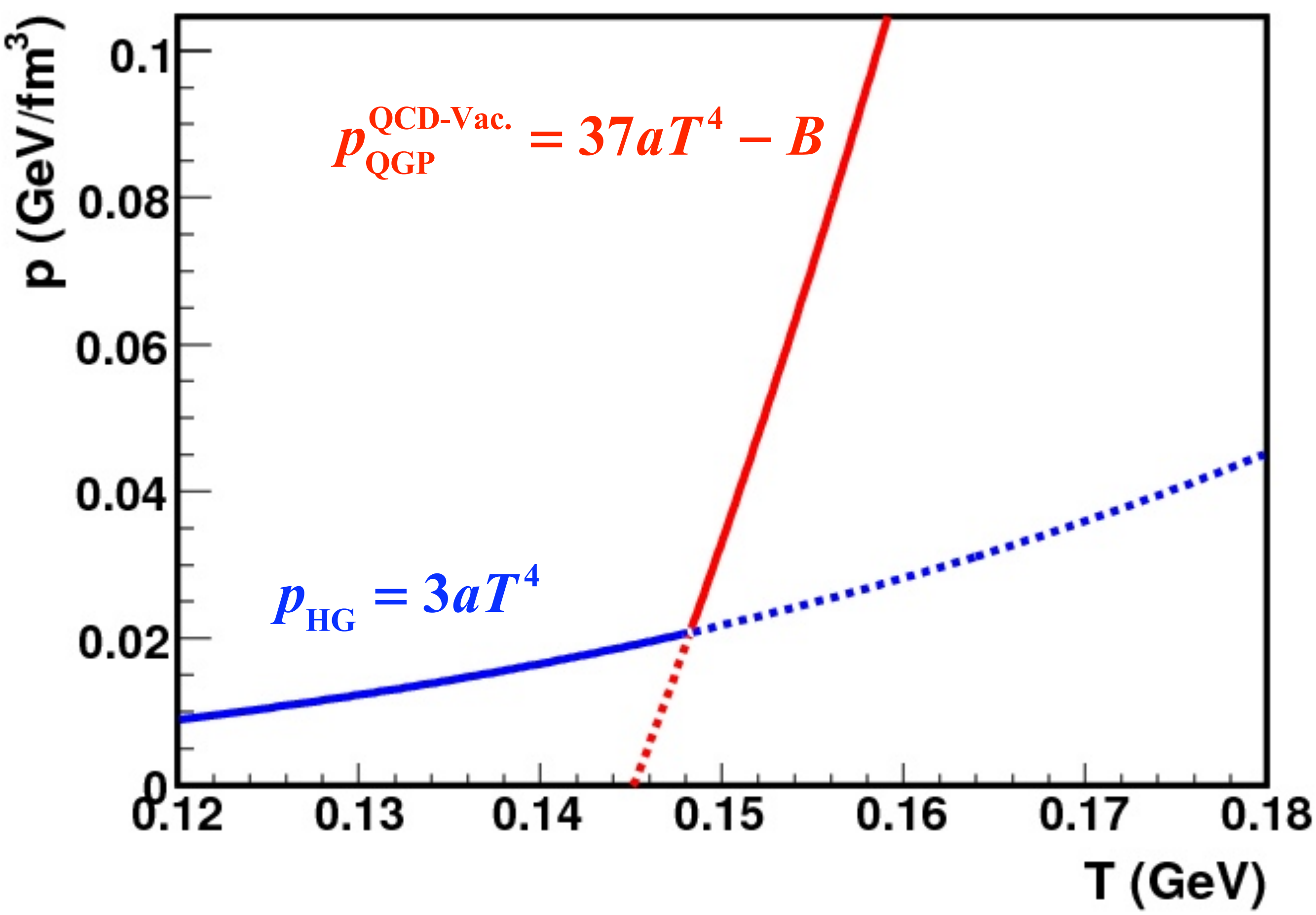
Gibbs criterion for the phase transition:

$$p_{\text{HG}} = p_{\text{QGP}}^{\text{QCD-Vac.}}, \; T_{\text{HG}} = T_{\text{QGP}} = T_{\text{c}} \quad \Rightarrow \quad T_{\text{c}} = \left(\frac{B}{34a}\right)^{1/4} \approx 150\,\text{MeV}$$

Phase transition in the bag model is of first order. Latent heat:

$$\varepsilon_{\text{QGP}}^{\text{QCD-Vac.}}(T_c) - \varepsilon_{\text{HG}}(T_c) = 102aT_c^4 + B = 4B$$

Quark-Gluon Plasma with μ = 0: Critical Temperature (II)



Quark-Gluon Plasma mit μ = 0: Entropy

Entropy density for constant temperature and pressure:

$$E = TS - pV \quad (\mu = 0) \quad \Rightarrow \quad \frac{S}{V} = s = \frac{\varepsilon + p}{T} = 4\frac{p}{T}$$

Ratio of entropy density (QGP / pion gas):

$$s_{\mathrm{QGP}} = 148\, a T^3, \quad s_{\mathrm{HG}} = 12\, a T^3 \quad \Rightarrow \quad \frac{s_{\mathrm{QGP}}}{s_{\mathrm{HG}}} \approx 12,3$$

Entropy per particle:

Pion gas:

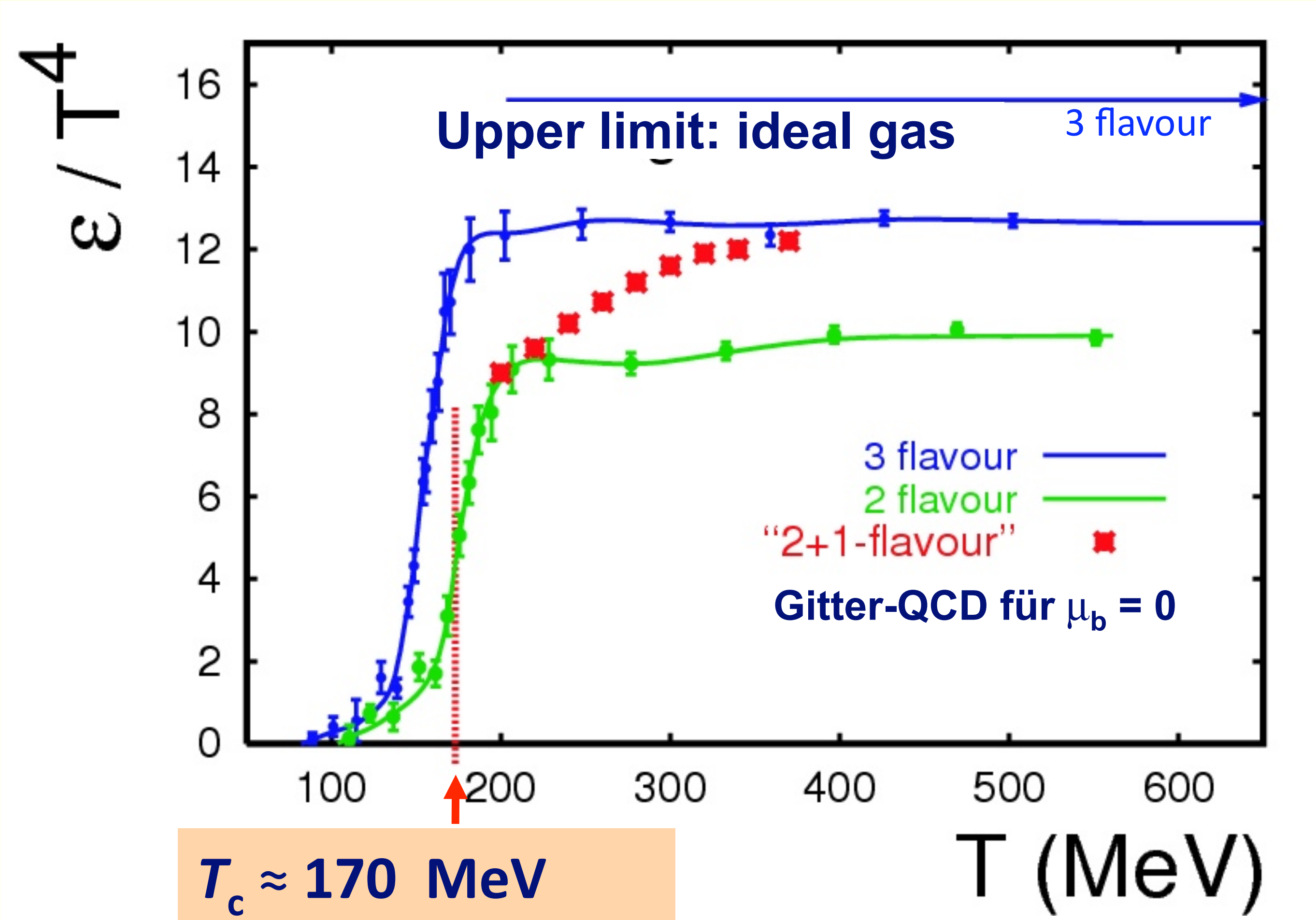
$$\frac{s_{\mathrm{HG}}}{n_\pi} = \frac{12\pi^2 / 90 \cdot T^3}{g_\pi \cdot 1,202 / \pi^2 \cdot T^3} = 3.6$$

QGP:

$$\frac{s_{\mathrm{q}}}{n_{\mathrm{q}}} = 1,4 \qquad \frac{s_{\mathrm{g}}}{n_{\mathrm{g}}} = 1,2$$

Quark-Gluon Plasma with $\mu = 0$: QGP with particle interactions (I)

F. Karsch, E. Laermann, hep-lat/0305025



$T_c \approx 170$ MeV

$\varepsilon_c \approx 0{,}7$ GeV/fm^3

Different calculation yield results in the range T_c = 160 - 190 MeV

Quark-Gluon Plasma with $\mu \neq 0$
Energy and Particle Number Density of the Quarks

For $\mu_q \neq 0$ a solution in closed form can be found for $\varepsilon_q + \varepsilon_{\bar{q}}$ but not for ε_q and $\varepsilon_{\bar{q}}$ separately:

$$\varepsilon_q + \varepsilon_{\bar{q}} = g_q \left(\frac{7\pi^2}{120} T^4 + \frac{1}{4} \mu_q^2 T^2 + \frac{1}{8\pi^2} \mu_q^4 \right)$$

Accordingly one finds for the quark density

$$n_q - n_{\bar{q}} = g_q \left(\frac{1}{6} \mu_q T^2 + \frac{1}{6\pi^2} \mu_q^3 \right), \quad g_q = 12$$

From this the net baryon density can be determined as:

$$n_B = \frac{n_q - n_{\bar{q}}}{3} = \frac{2}{3} \mu_q T^2 + \frac{2}{3\pi^2} \mu_q^3 = \frac{2}{9} \mu_B T^2 + \frac{2}{81\pi^2} \mu_B^3 \qquad (\mu_B = 3\mu_q)$$

Quark-Gluon Plasma with μ ≠ 0: Critical Temperature and Critical Quark Potential

Energy density in a QGP with μ ≠ 0 (without particle interactions):

$$\varepsilon_{\mathrm{QGP}} = \frac{37}{30}\pi^2 T^4 + 3\mu_{\mathrm{q}}^2 T^2 + \frac{3}{2\pi^2}\mu_{\mathrm{q}}^4$$

Condition for QGP stability:

$$p_{\mathrm{QGP}} = \frac{1}{3}\varepsilon_{\mathrm{QGP}} \overset{!}{=} B \quad \Rightarrow \quad T_{\mathrm{c}}(\mu_{\mathrm{q}})$$

Condition for QGP:
QGP-pressure ≥ pressure of the QCD-vacuum
(similar, but not identical, to the previous condition $p_{\mathrm{HG}} = p_{\mathrm{QGP}}$)

Critical temperature / quark potential:

$$T_{\mathrm{c}}(\mu_{\mathrm{q}} = 0) = \left(\frac{90B}{37\pi^2}\right)^{1/4}$$

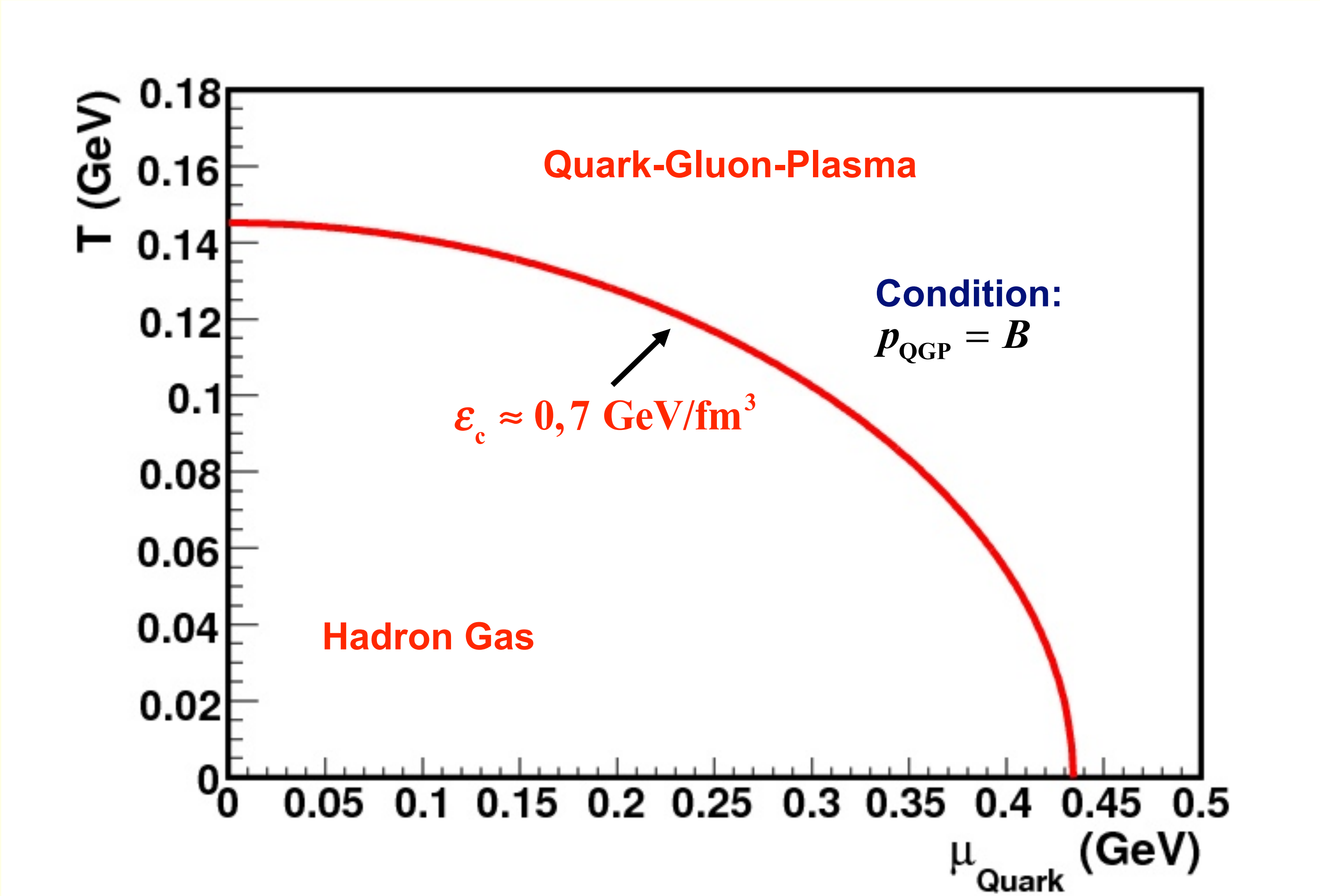
$$\mu_{\mathrm{q}}^{\mathrm{c}}(T = 0) = \left(2\pi^2 B\right)^{1/4} = 0{,}43\ \mathrm{GeV}$$

$$n_{\mathrm{B}}^{\mathrm{c}}(T = 0) = \frac{2}{3\pi^2}\left(2\pi^2 B\right)^{3/4}$$

$$= 0{,}72\ \mathrm{fm}^{-3} \approx 5 \times n_{\mathrm{nucleus}}$$

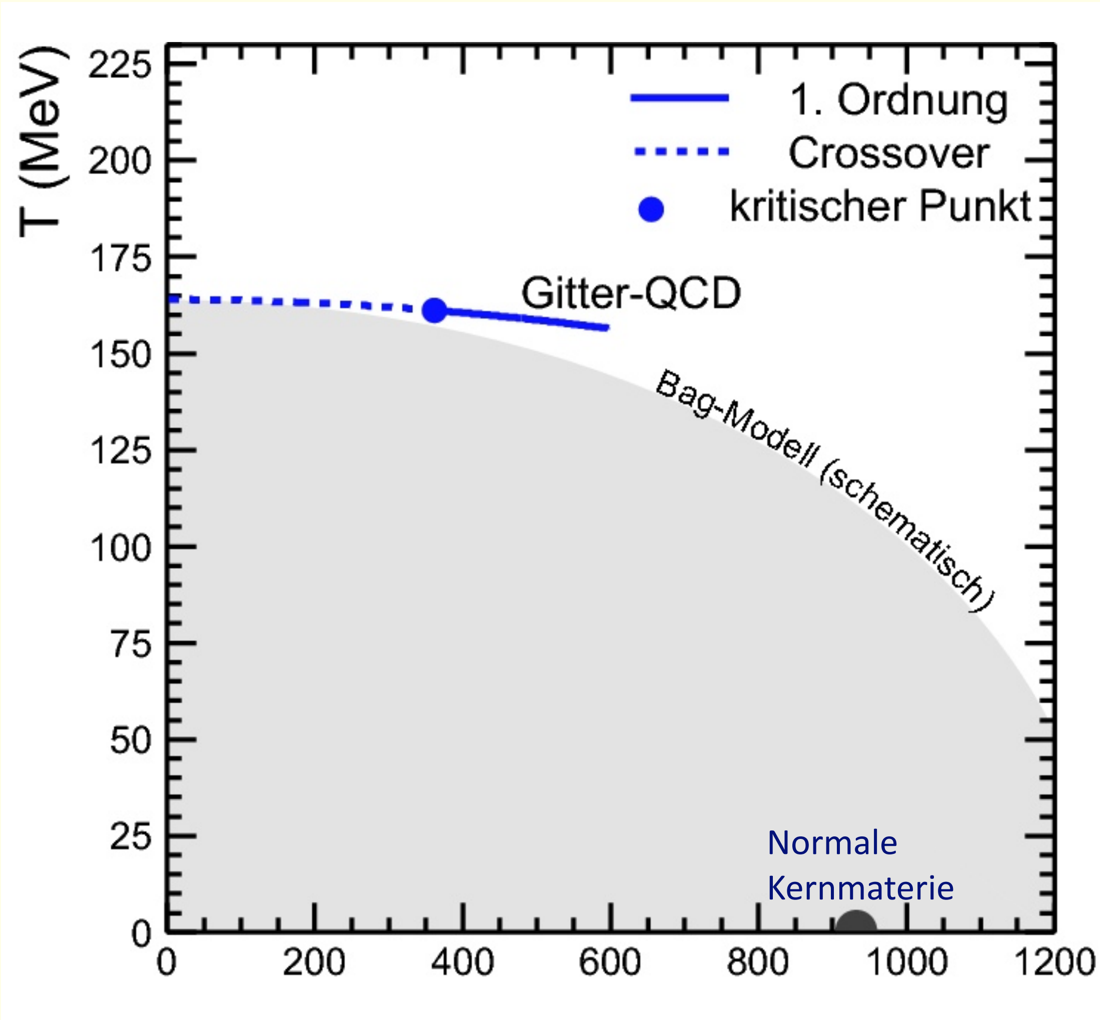
Possibly reached in neutron stars

Quark-Gluon Plasma with $\mu \neq 0$: Phase Diagram of the Non-Interacting QGP



$\mu_B = 3 \cdot \mu_{Quark}$

Quark-Gluon Plasma with $\mu \neq 0$: Phase Diagram from Lattice-QCD



Z. Fodor, S.D. Katz,
JHEP 404 (2004) 50 [hep-lat/0402006].

- Lattice-calculations for $\mu_b \neq 0$
- Transition temperature for $\mu_b = 0$: $T_c = 164$ MeV
- Critical point:
  - $T = 162$ MeV
  - $\mu_b = 340$ MeV

The existence and exact position of the critical point remains an open question

Points to Take Home

- When treated as a relativistic ideal gas, parameters for the transition Hadron Gas ⟷ QGP are:
  - T_c (μ_b=0) ≈ 150 MeV
  - $\mu_b(T=0) = 3\ \mu_{Quark}(T=0)$ ≈ 1,3 GeV (this corresponds to approximately five times the density of „normal“ nuclear matter)
- Lattice QCD calculations show that for temperatures up to several times T_c the assumption of an ideal gas is a poor approximtion
- Transition temperature from Lattice QCD:
  T_c (μ_b=0) = 160 - 190 MeV