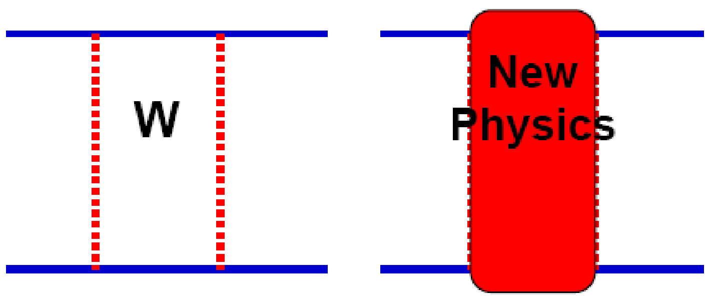
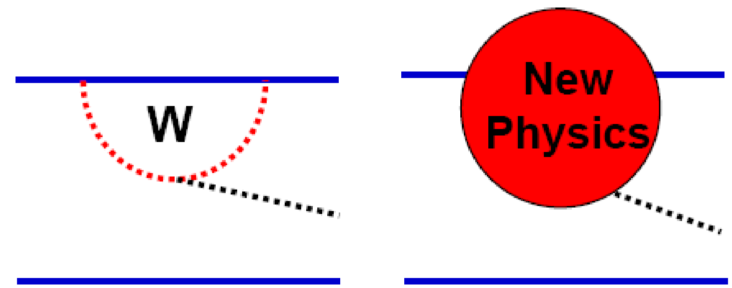
Overview of flavour physics

Probes for New Physics

Precision B Meson Physics as Probe for New Physics in Loop-Processes:



Box Diagrams (Oscillation)



Penguin Decays

Popular New Physics Scenarios: SUSY, Little Higgs Models

→ Deviations from Standard Model predictions

Complementary to direct New Physics searches by ATLAS and CMS

Example from the past:

GIM Mechanism

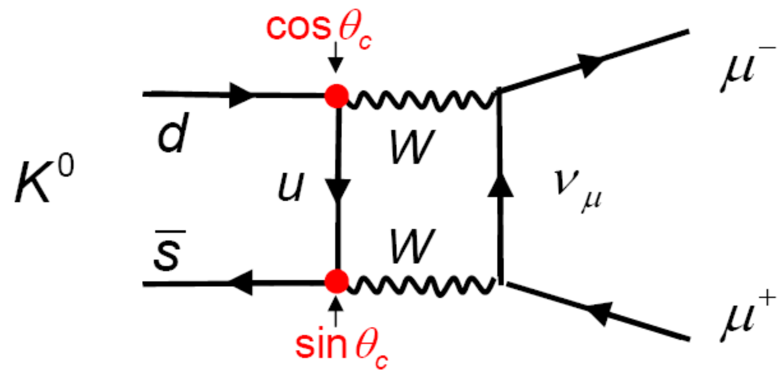
Observed branching ratio $K^0 \rightarrow \mu\mu$

$$\frac{BR(K_L \rightarrow \mu^+\mu^-)}{BR(K_L \rightarrow all)} = (7.2 \pm 0.5) \cdot 10^{-9}$$

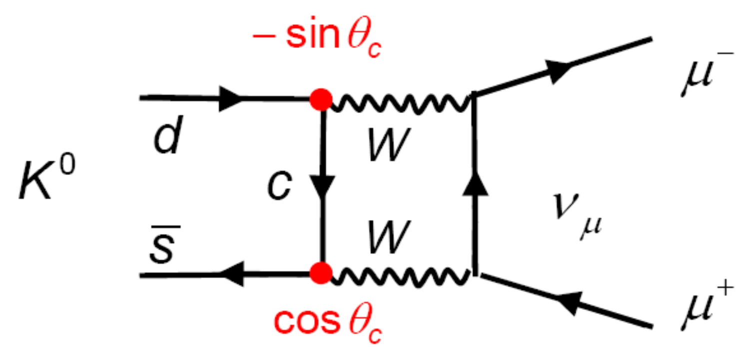
In contradiction with theoretical expectation in the 3-Quark Model

Glashow, Iliopolus, Maiani (1970):

Prediction of a 2nd up-type quark, additional Feynman graph cancels the “u box graph”.



$$M \sim \sin\theta_c \cos\theta_c$$



$$M \sim -\sin\theta_c \cos\theta_c$$

Probes for New Physics searches

The aim of heavy flavour physics is to study *B* and *D* decays and to look for anomalous effects beyond the Standard Model.

Requirements to look for New Physics effects:
- Should not be ruled out by existing measurements.
- Prediction from SM should be well known.

These requirements are fulfilled for these processes:
- CP violation
- Rare decays

→ CP violation and rare decays of *B* and *D* hadrons are the main focus of LHCb.

Today: CP violation

Symmetries

The (probably) most important concept in physics: concept of symmetry

T.D.Lee:
"The root to all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; the non-observables"

$\Rightarrow$ If a quantity is fundamentally non-observable it is related to an exact symmetry

$\Rightarrow$ If a quantity could in principle be observed by an improved measurement; the symmetry is said to be broken

Noether Theorem: symmetry $\Longleftrightarrow$ conservation law

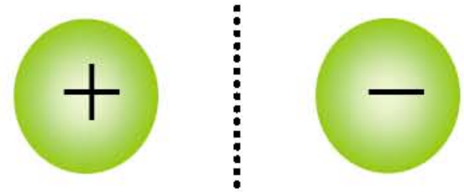
Few examples:

Non-observables	Symmetry Transformations	Conservation Laws
Absolute spatial position	Space translation $\vec{r} \rightarrow \vec{r} + \vec{\Delta}$	Momentum
Absolute time	Time translation $t \rightarrow t + \tau$	Energy
Absolute spatial direction	Rotation $\hat{r} \rightarrow \hat{r}'$	angular momentum

Three discrete symmetries

Charge conjugation C

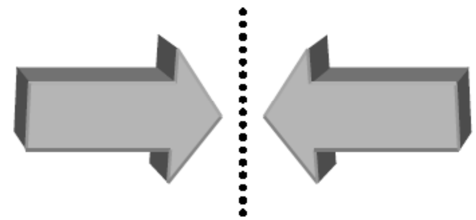
Particle ⇔ Anti-particle



$e^- \to e^+$

$\gamma \to \gamma$

Parity P

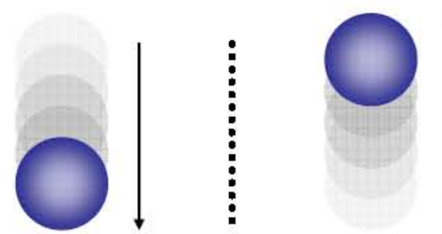


$\vec{r} \to -\vec{r}$

$\vec{p} \to -\vec{p}$

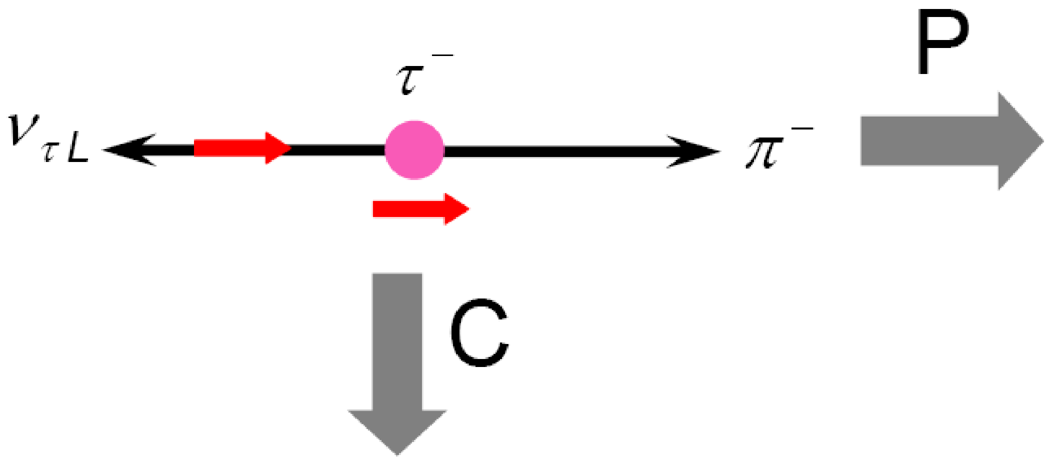
$\vec{L} \to \vec{L}$

Time inversion T



$t \to -t$

C, P and CP in weak interactions

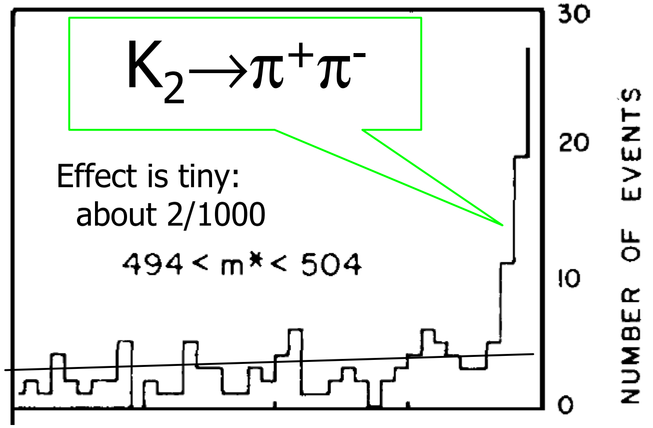


CP violation in Kaon system

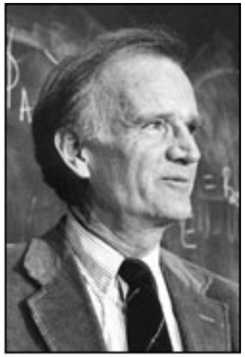
Under CP symmetry:
K_S (CP=+1): can only decay to ππ (CP=+1)
K_L (CP=-1): can only decay to πππ (CP=-1)

Why does the K_L live so much longer than the K_S ?

Testing CP conservation:
Create a pure K_L (CP=-1) beam: (Cronin & Fitch in 1964)
Easy: just "wait" until the K_s component has decayed…
If CP conserved, should *not* see the decay $K_L \rightarrow$ 2 pions



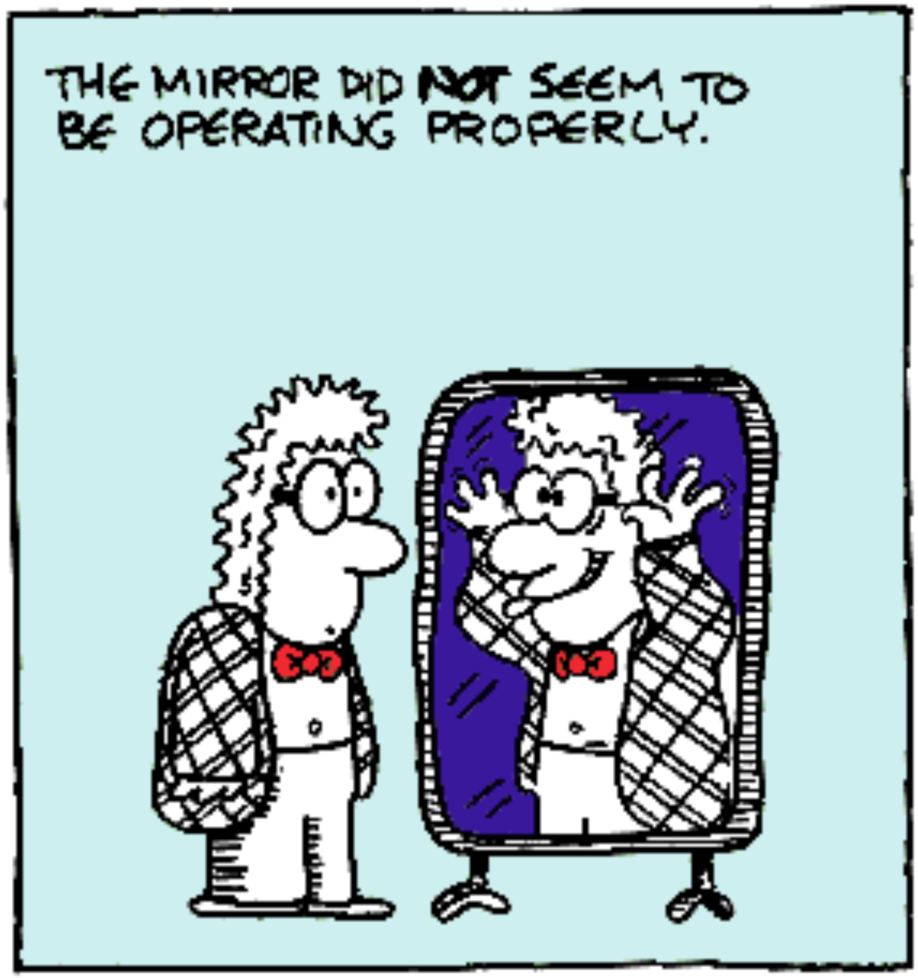

James Cronin


Val Fitch

The discovery emphasizes, once again, that even almost self evident principles in science cannot be regarded fully valid until they have been critically examined in precise experiments.

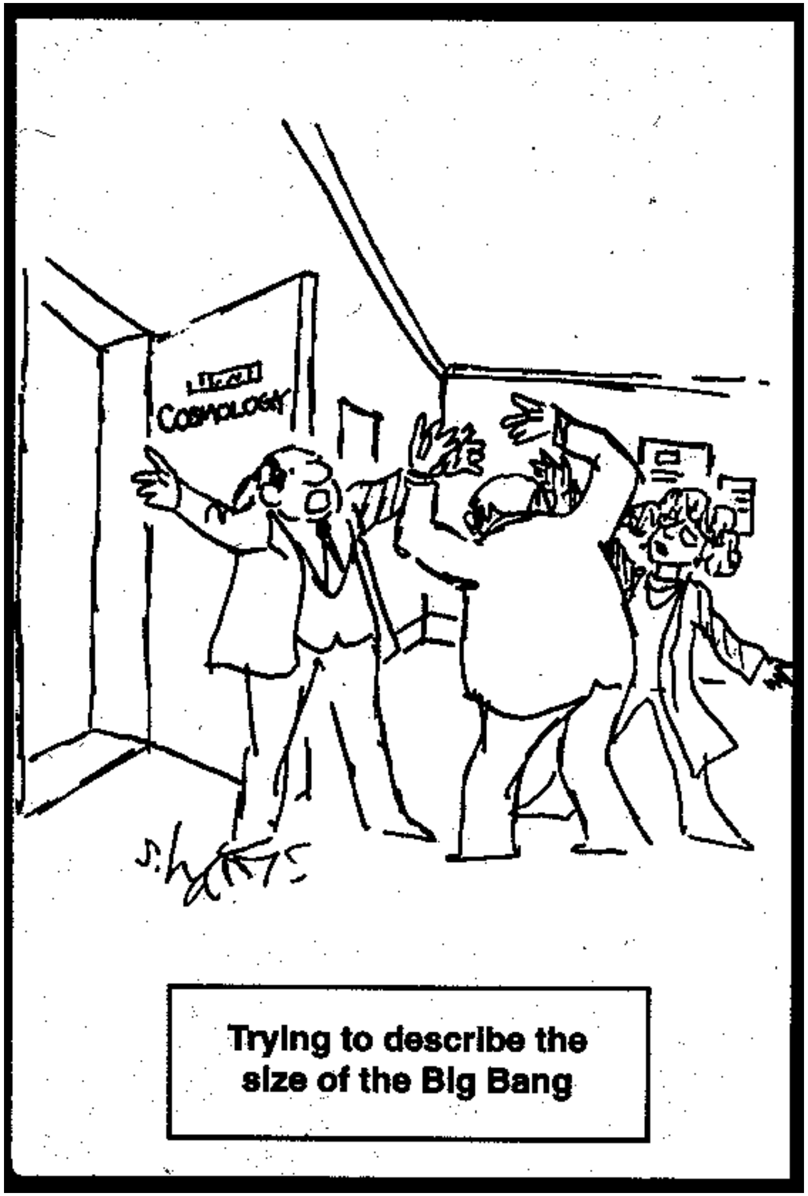
... and for this experiment they got the Nobel price in 1980...

CP symmetry is broken



There is an absolute difference between matter and anti-matter.
Actually we could have known this already…

… because of the Big Bang

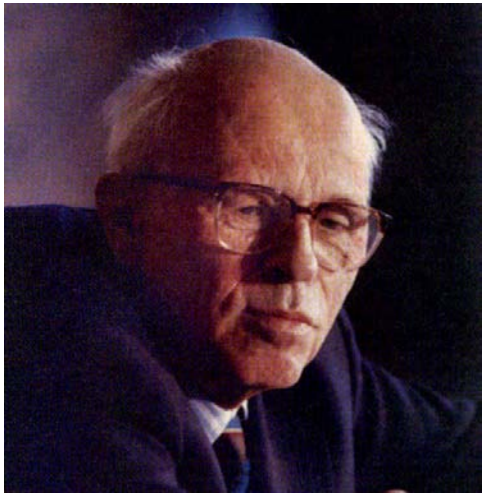


Baryon asymmetry in Universe

We know that the matter – anti-matter asymmetry in the Universe is broken: the Universe consists of matter.

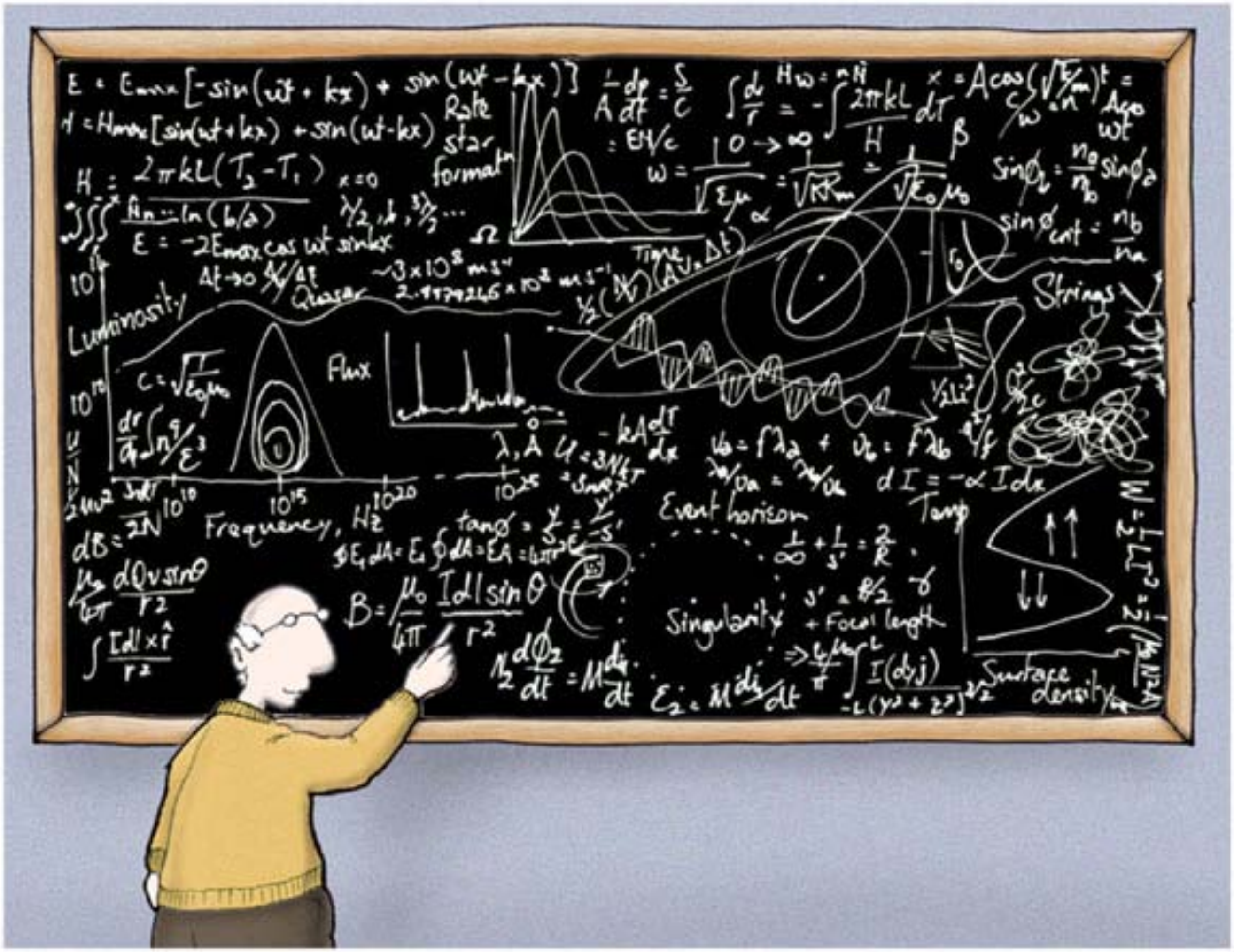
But, shortly after the Big Bang, there should have been equal amounts of matter and anti-matter
→ how did the Universe develop a preference of matter?

- In 1966, Andrei Sakharov showed that the generation of a net baryon number requires:
  1. Baryon number violating processes (*e.g.* proton decay)
  2. Non-equilibrium state during the expansion of the universe
  3. Violation of *C* and *CP* symmetry



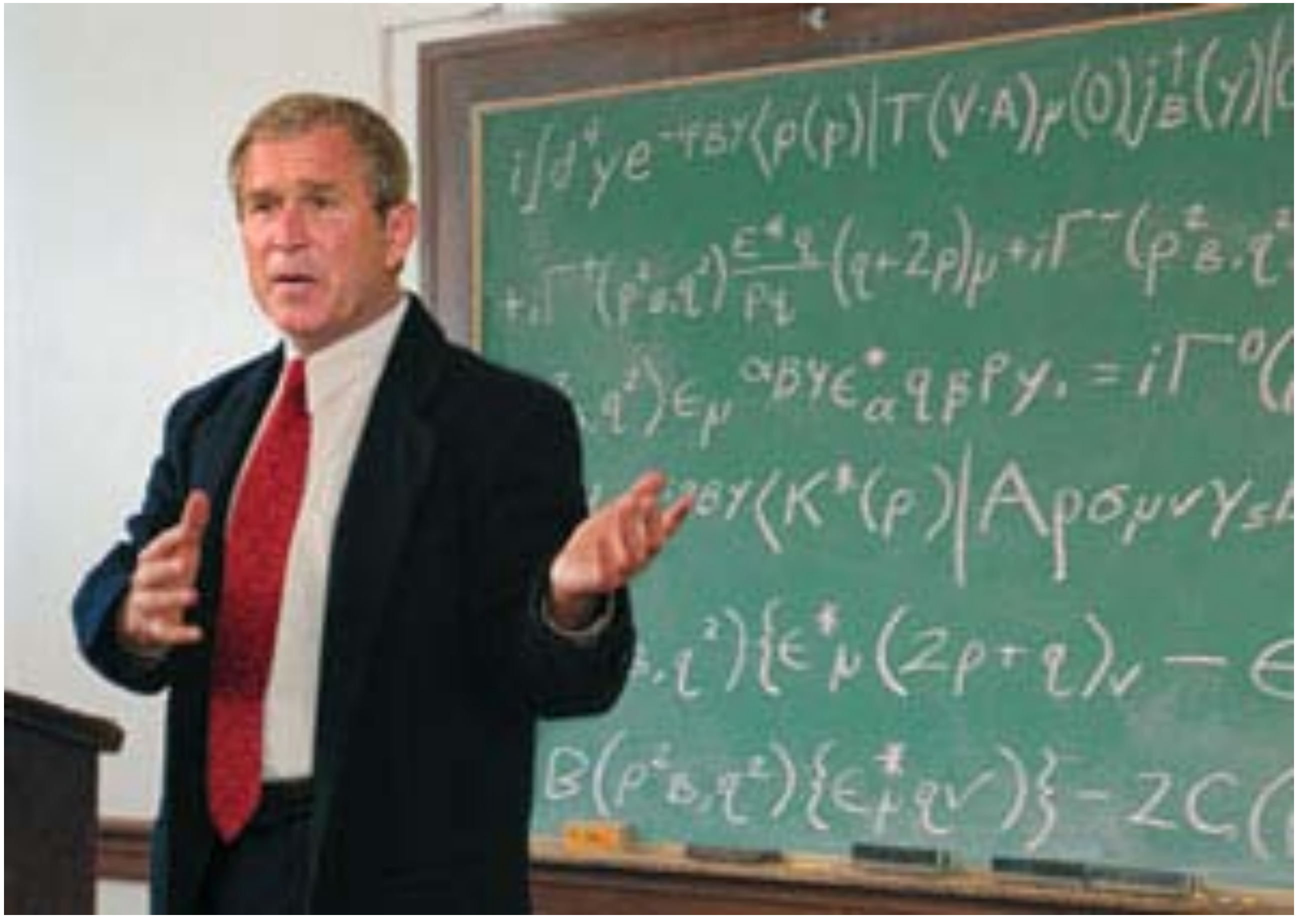
- Standard Model *CP* violation is very unlikely to be sufficient to explain matter asymmetry in the universe
  - It means there is something *beyond* the SM in *CP* violation somewhere, so a good place for further investigation

In more details…



Astrophysics made simple

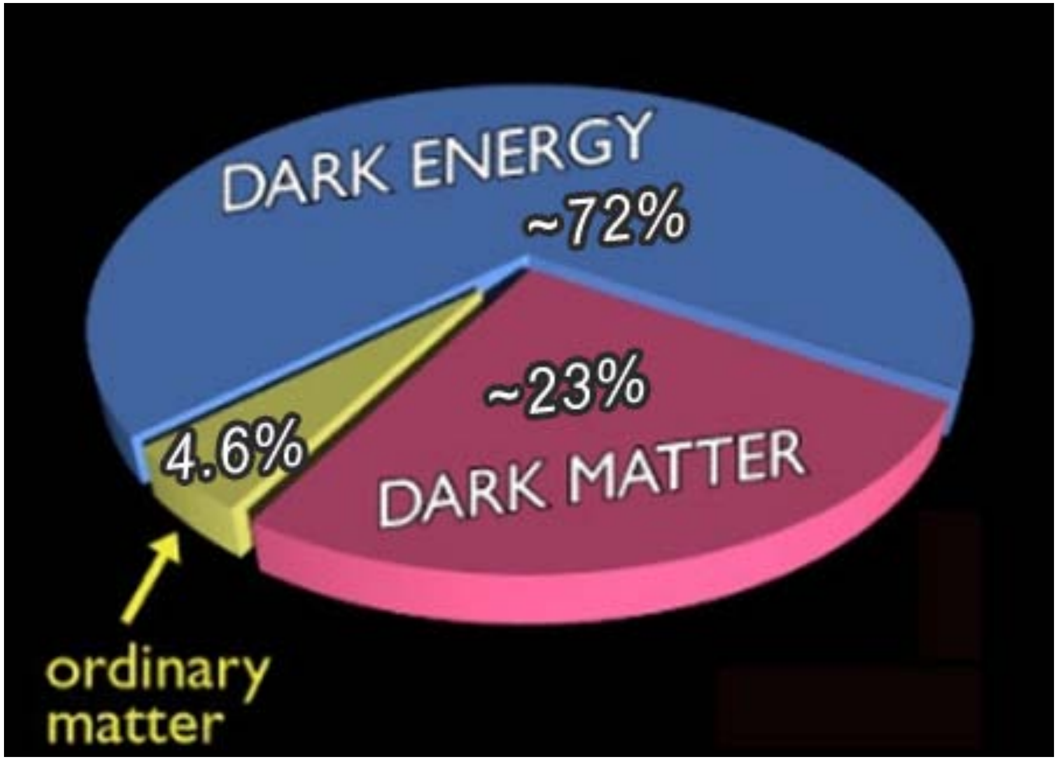
Even more details…



Particle physics made simple

Now to some simpler questions ☺ …

What is the origin of mass in the Universe?



Answers:

- Actually, We don’t know (dark matter, dark energy)
- Ordinary matter: mainly QCD (mass proton=1 GeV, mass u,d quarks few MeV)

Flavour in Standard Model

Higgs field was introduced to give masses to W^+, W^- and Z^0 bosons (after SBB).

Since we have a Higgs field we can add (ad-hoc) interactions between the Higgs field ϕ and the fermions in a gauge invariant way (Yukawa couplings):

doublets   singlet

$$-L_{Yukawa} = Y_{ij}\left(\overline{\psi}_{Li}\,\phi\right)\psi_{Rj} + h.c.$$

The fermions are in the weak interaction basis. We can diagonalize the Y_{ij} matrices, such that we arrive in the "mass basis". However, then the Lagrangian of the charged weak current should also be rewritten:

$$-L_{W^+} = \frac{g}{\sqrt{2}}\left(\overline{u},\overline{c},\overline{t}\right)_L \left(V_{CKM}\right)\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

CKM matrix

Bottom line: V_{CKM} originates from the diagonalization of the Yukawa couplings.

Weak interactions in the SM

After SSB, the charged current of a W⁻ exchange can be written as

$$J^{\mu -} = (\overline{u}_L, \overline{c}_L, \overline{t}_L)\gamma^\mu V_{\mathrm{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Exchange of W⁺ obtained from Hermitian conjugate.

Weak interaction only couples to left-handed field: Left-handed quarks or right-handed anti-quarks. Manifestly violates parity.

The weak eigenstates are related to the mass eigenstates by the CKM matrix:

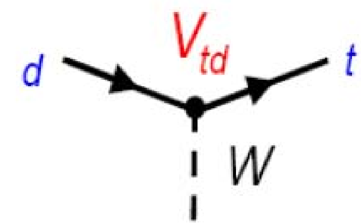
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak eigenstates — Mass eigenstates

CP transformation & the weak interaction

Quarks

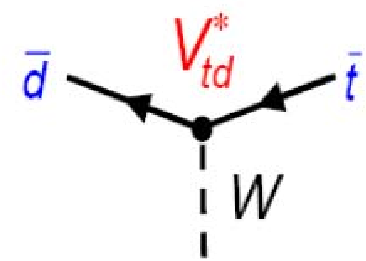
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



----- CP -----

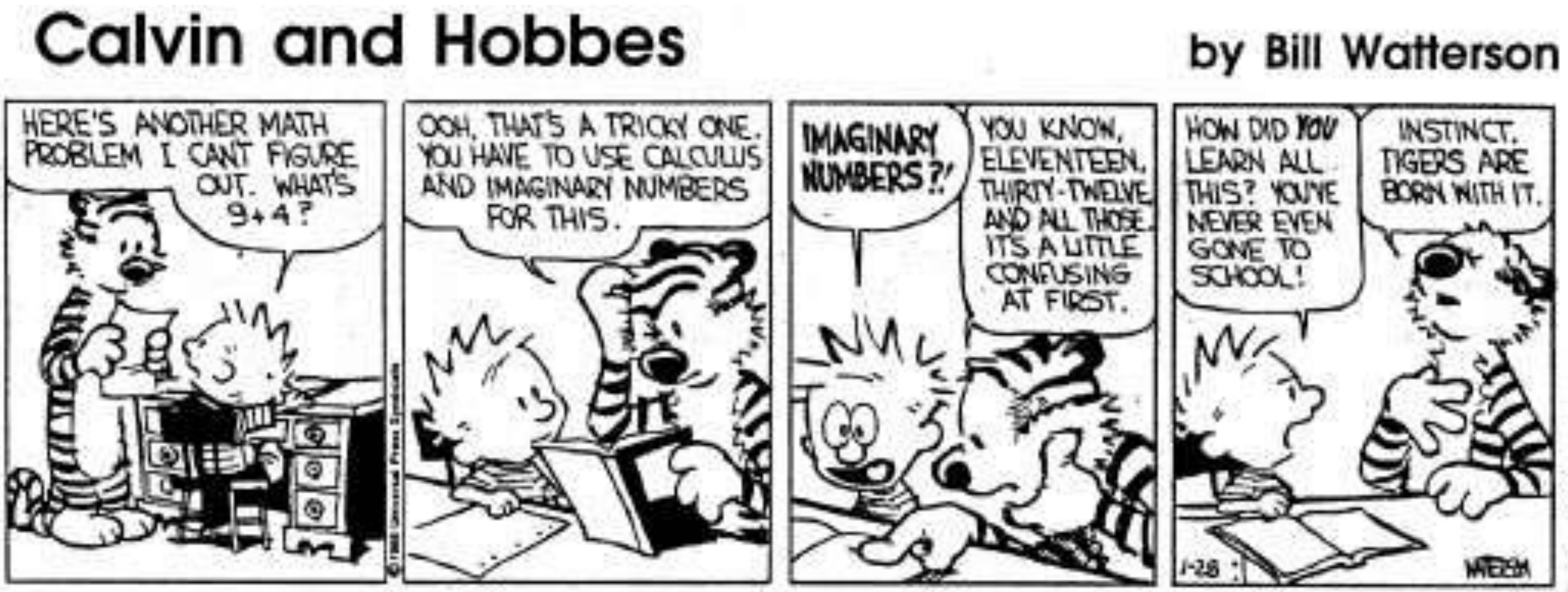
Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V^*_{ud} & V^*_{us} & V^*_{ub} \\ V^*_{cd} & V^*_{cs} & V^*_{cb} \\ V^*_{td} & V^*_{ts} & V^*_{tb} \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$



CP violation requires complex matrix elements.

It's all about imaginary numbers



CKM matrix

Q: *How many parameters does the CKM matrix have?*

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Remember:
- V_{CKM} is unitary
- Not all phases are observable.

18 parameters (9 complex numbers)
− 9 unitary conditions: $V_{\text{CKM}} V_{\text{CKM}}^{\dagger} = 1$
− 5 relative phases of the quark fields

4 parameters: 3 (real) Euler angles and 1 phase.

This phase is the single source of CP violation in the SM.

With 2 generations there is only one real (Euler) angle: the Cabbibo angle.
CP violation requires 3 generations.

That is why Kobayashi and Maskawa proposed a third generation in 1973
(CP violation in K decay was just observed).
At the time only u,d,s were known!

Relative phases

When I do a phase transformation of the (left-handed) quark fields:

$$u_{Li} \rightarrow e^{i\phi_{u\,i}} u_{Li} \qquad\qquad d_{Li} \rightarrow e^{i\phi_{d\,i}} d_{Li}$$

And a simultaneous transformation of the CKM matrix:

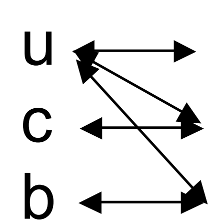
$$V \rightarrow \begin{pmatrix} e^{-\phi_u} & & \\ & e^{-\phi_c} & \\ & & e^{-\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-\phi_d} & & \\ & e^{-\phi_s} & \\ & & e^{-\phi_b} \end{pmatrix}$$

or $V_{\alpha j} \rightarrow \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$

The charged current remains invariant

$$J^{\mu}_{CC} = \overline{u_{Li}} \gamma^{\mu} V_{ij} d_{Lj}$$

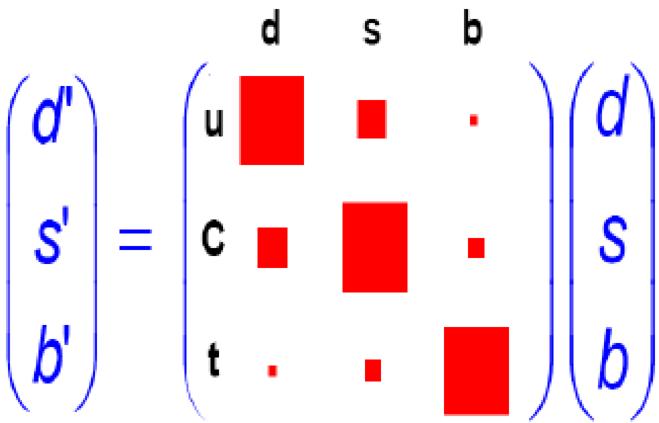
There are only 5 relative phases
(+ one overall phase)

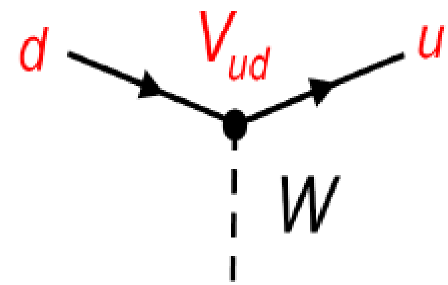


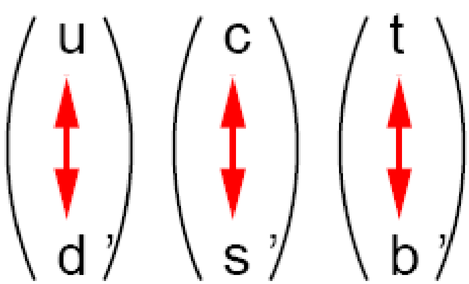
In other words, I can always absorb the 5 relative phases by redefining the quark fields

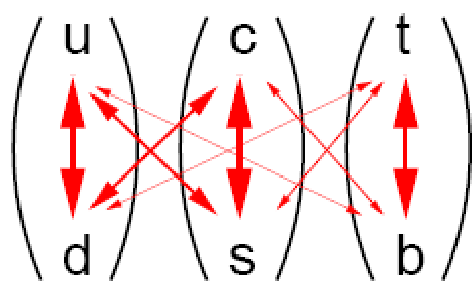
→ These 5 phases are unobservable.

Size of elements









Diagonal elements of CKM matrix are close to one.

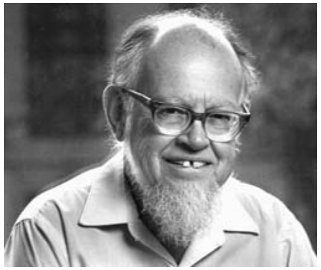
Only small of diagonal contributions.

Mixing between quark families is “CKM suppressed”.

Wolfenstein Parametrization

Makes use of the fact that the off-diagonal elements are small compared to the diagonal elements.
→ Expansion in $\lambda = V_{us}$, $A = V_{cb}/\lambda^2$ and ρ, η.

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$



$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

λ ~ 0.22 (sinus of Cabibbo angle)
A ~ 1 (actually 0.80)
ρ ~ 0.14
η ~ 0.34

CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \sim \lambda^3 e^{-i\gamma} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \sim \lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

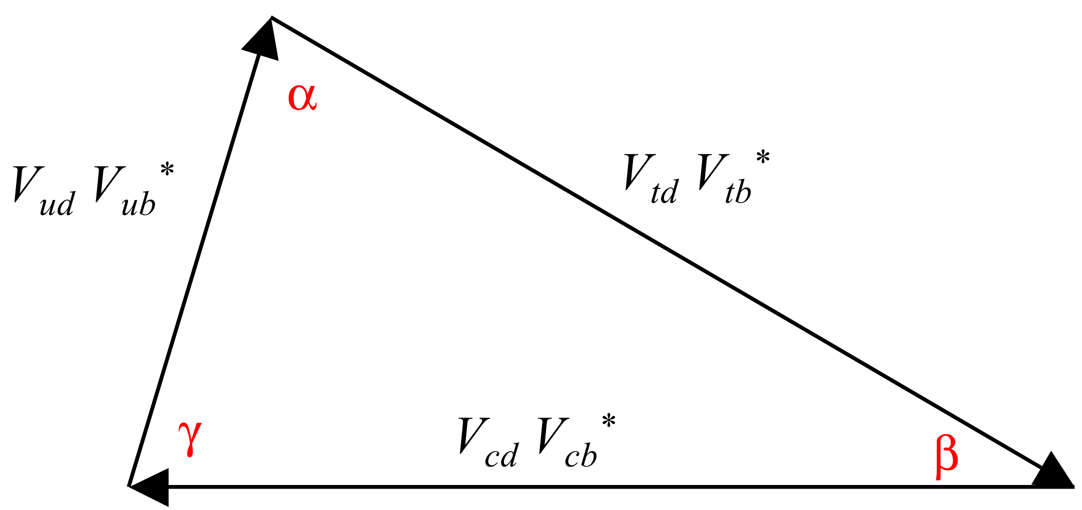
$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{tb}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using one of the 9 unitarity relations: $V_{\mathrm{CKM}}^\dagger V_{\mathrm{CKM}} = 1$
Multiply first “d” column with last “b” column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \sim \lambda^3 e^{-i\gamma} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \sim \lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

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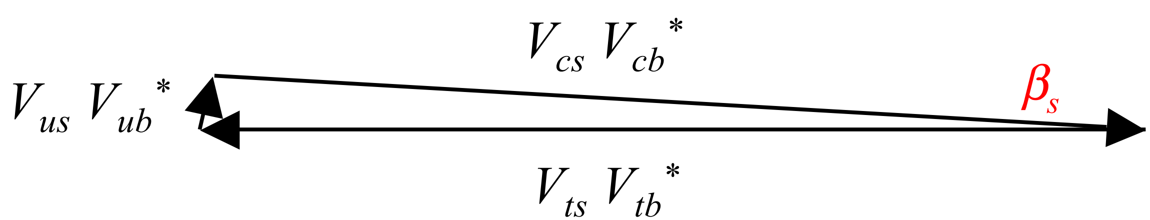
$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{tb}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using another unitarity relations: $V_{\rm CKM}^\dagger V_{\rm CKM} = 1$
Multiply second “s” column with last “b” column.

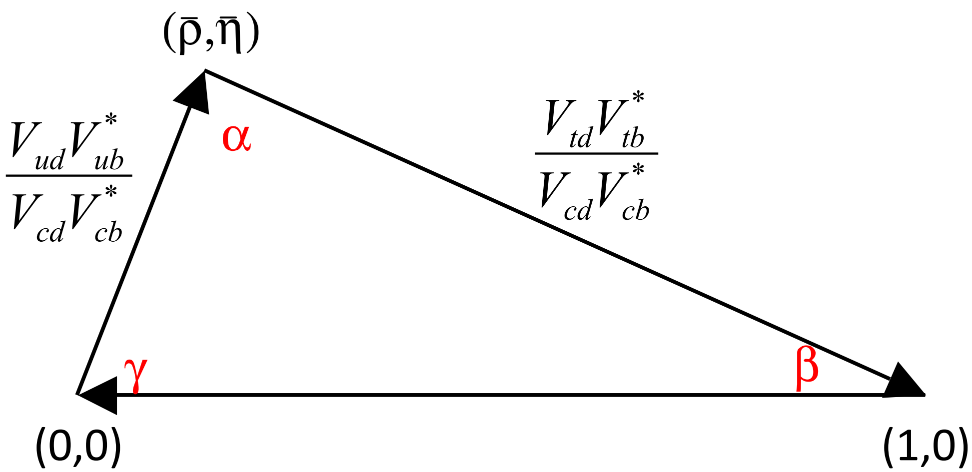
$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$



“Squashed unitarity triangle”

Back to *The* Unitarity Triangle

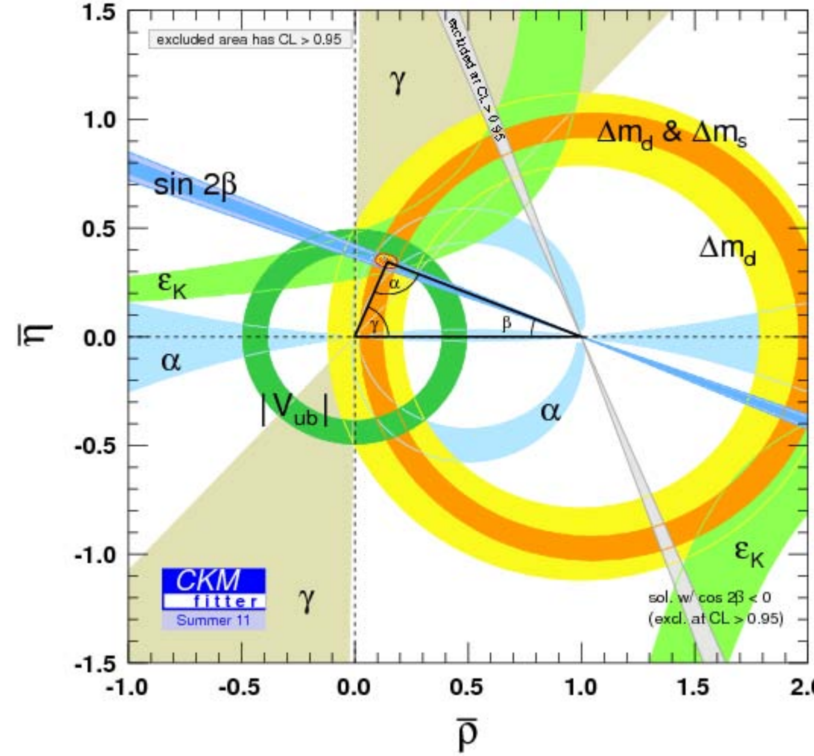
Normalized CKM triangle:
→ Divide each side by $V_{cd}\, V_{cb}{}^*$



The "apex" of this triangle is then:

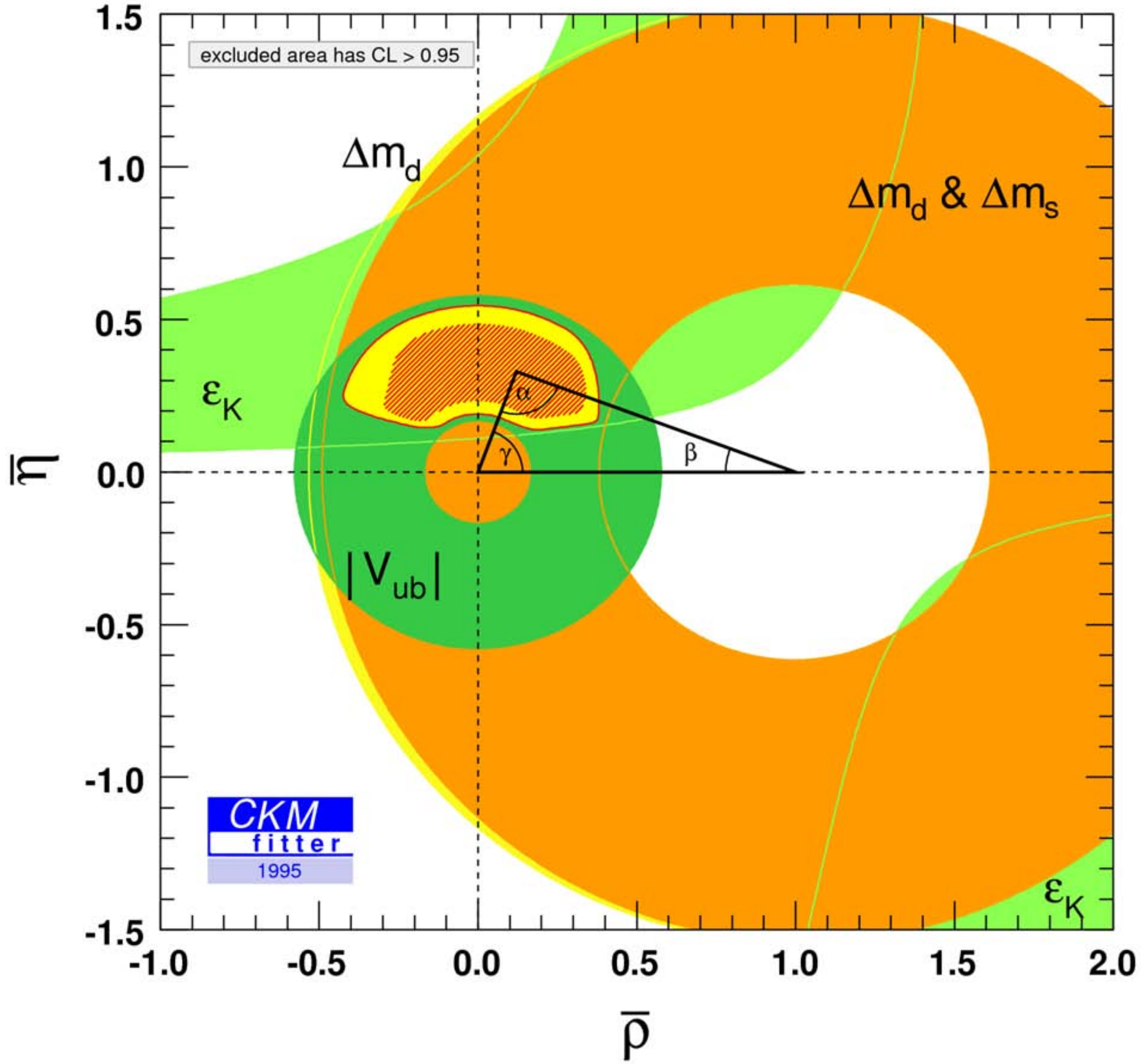
$$\bar{\rho} = \rho(1-\lambda^2/2)$$
$$\bar{\eta} = \eta(1-\lambda^2/2)$$

Current knowledge of UT:
(from CKMFitter)



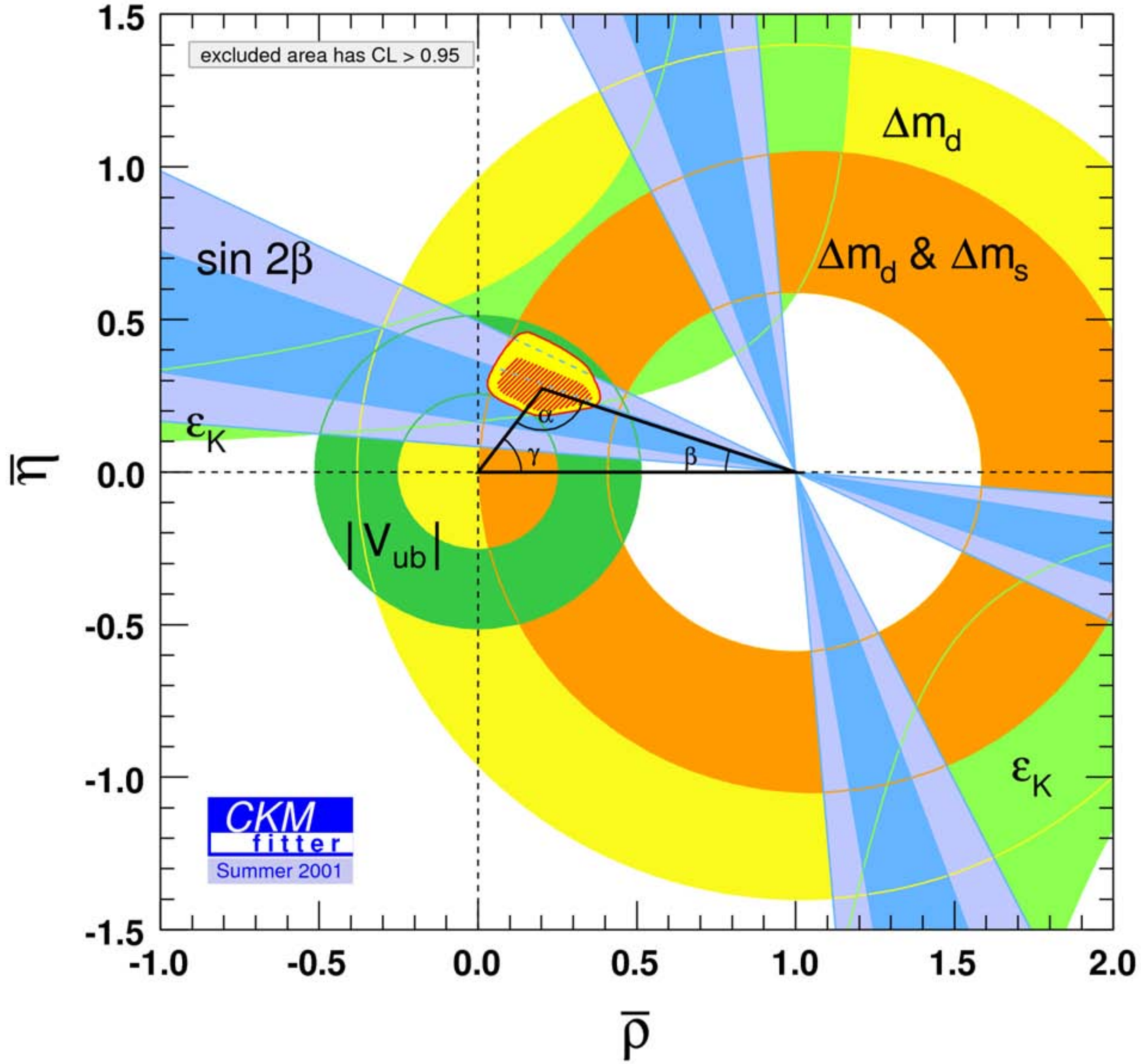
Progress in UT

1995



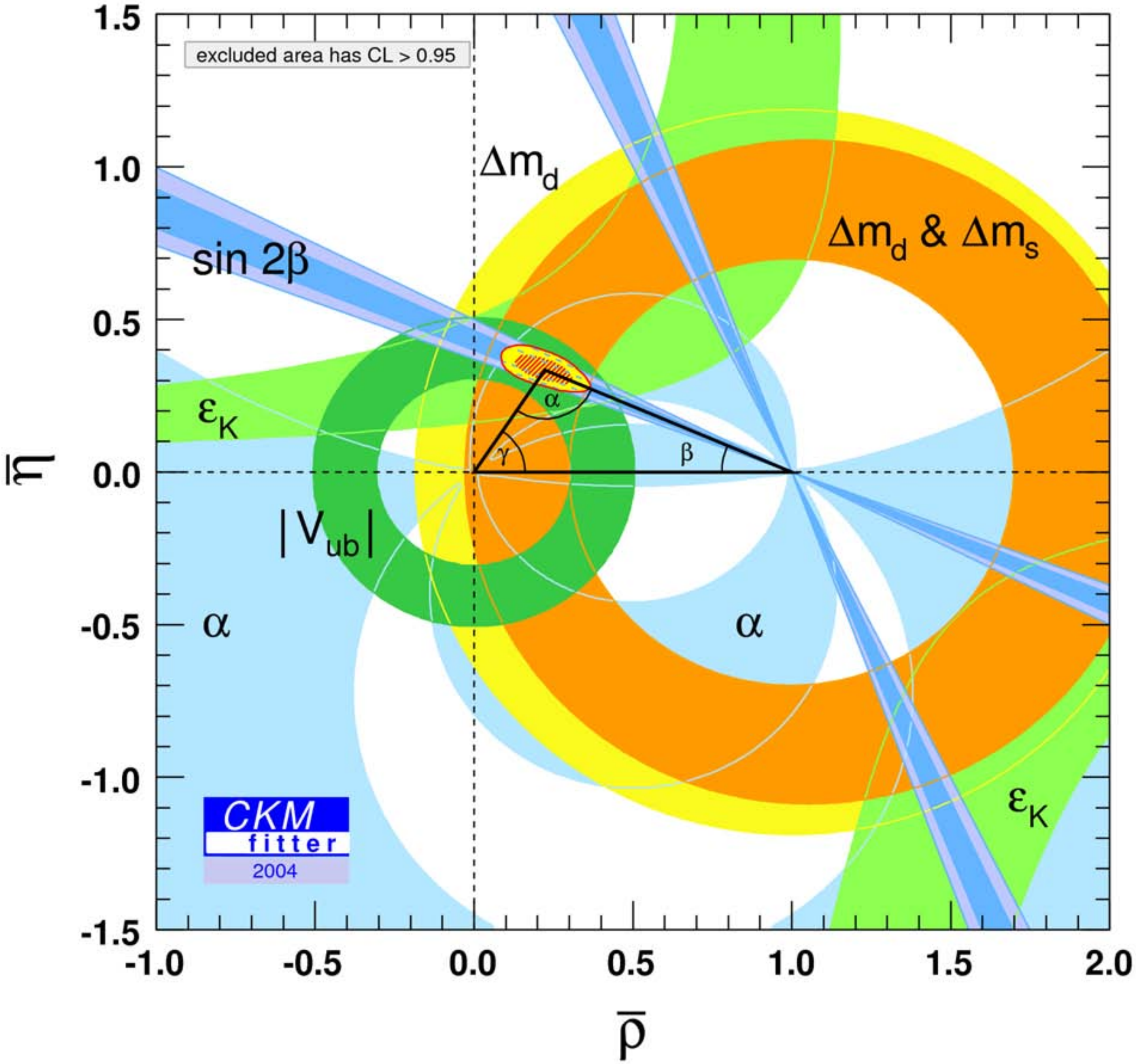
Progress in UT

2001



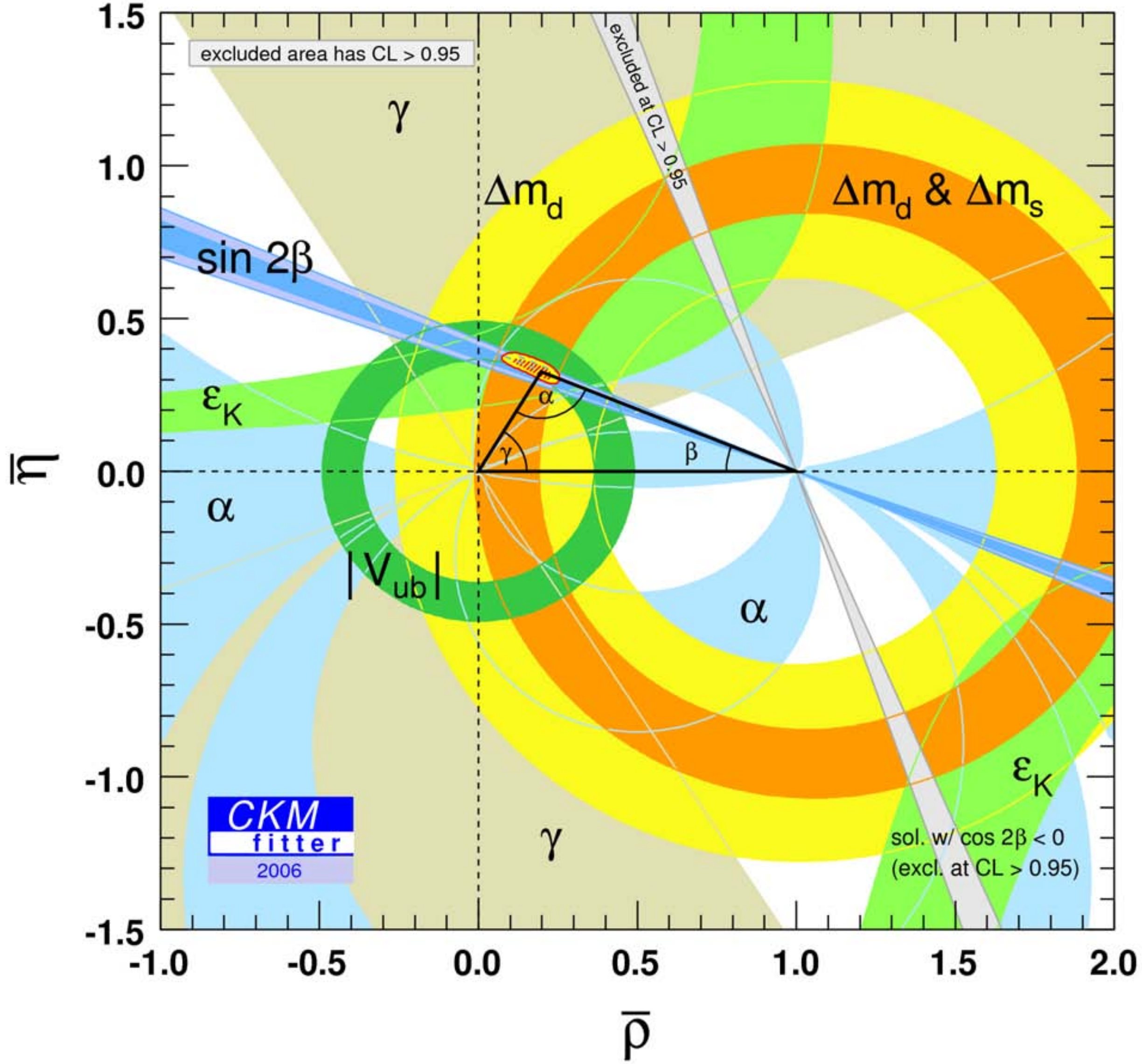
Progress in UT

2004



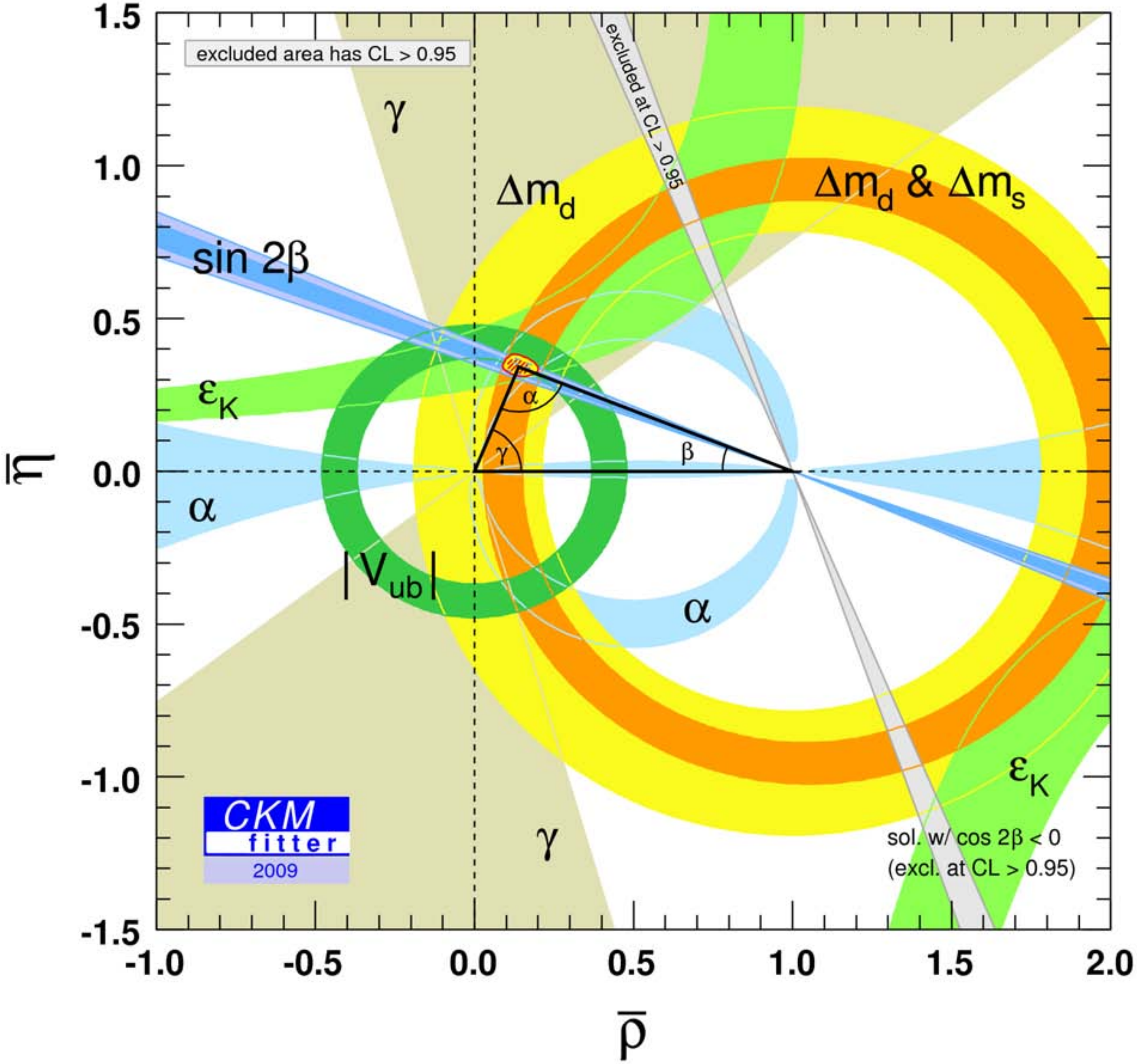
Progress in UT

2006



Progress in UT

2009



Neutral meson mixing

What are the possible neutral meson systems?

Possible neutral meson systems:

K^0-$\bar{K}^0$ system: Mass eigenstates: K_S and K_L

D^0-$\bar{D}^0$ system: Mass eigenstates: D_+ and D_-

B_d-$\bar{B}_d$ system: Mass eigenstates: $B_{H,d}$ and $B_{L,d}$

B_s-$\bar{B}_s$ system: Mass eigenstates: $B_{H,s}$ and $B_{L,s}$

Math in the following slides for Bd system

Applies to all systems, nevertheless phenomenology very different.

B_d and B_s systems:

$$
\begin{aligned}
|B^0\rangle &= |\bar{b}d\rangle \quad , \quad |\overline{B}^0\rangle = |b\bar{d}\rangle \\
|B_s^0\rangle &= |\bar{b}s\rangle \quad , \quad |\overline{B}_s^0\rangle = |b\bar{s}\rangle
\end{aligned}
$$

Beautiful example of quantum mechanics at work!

Neutral meson mixing

Time evolution of B^0 or $\overline{B^0}$ can be described by an *effective* Hamiltonian:

$$i\frac{\partial}{\partial t}\Psi = H\Psi \qquad \Psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

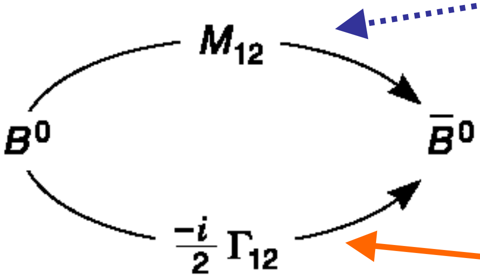
Mass term: "dispersive"

Decay term: "absorptive"

Note that H is not Hermitian!
(due to decay term; this is not the full Hamiltonian; all final state terms are missing)

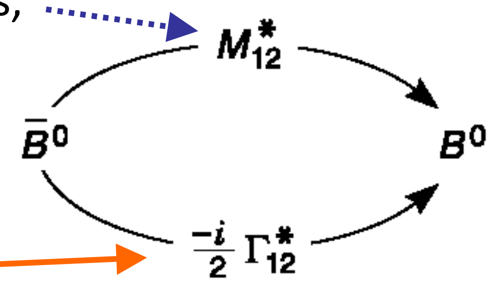
CPT symmetry: $M_{11} = M_{22} = M_B$
$\Gamma_{11} = \Gamma_{22} = 1/\tau_B$

The off-diagonal elements describe mixing – but what is the difference between M_{12} and Γ_{12}?



M_{12} describes $B^0 \leftrightarrow \overline{B^0}$ via *off-shell* states, e.g. the weak box diagram

Γ_{12} describes $B^0 \leftrightarrow f \leftrightarrow \overline{B^0}$ via *on-shell* states, e.g. $f=\pi^+\pi^-$



Solving the Schrödinger Equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = (M - \frac{i}{2}\Gamma)\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

Define the mass eigenstates:

$$|B_{H,L}\rangle = p|B^0\rangle \mp q|\overline{B}^0\rangle$$

The heavy and light B eigenstates have time dependence:

$$|B_{H,L}(t)\rangle = e^{-(im_{H,L}+\Gamma_{H,L}/2)t}|B_{H,L}(0)\rangle$$

The mass and decay width difference:

$$\begin{aligned} \Delta m &= m_H - m_L \\ \Delta\Gamma &= \Gamma_H - \Gamma_L \end{aligned}$$

Solving the Schrödinger equation gives:

$$\frac{q}{p} = -\sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$

$$\Delta m = 2\,\mathrm{Re}\sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)}$$

$$\Delta\Gamma = 2\,\mathrm{Im}\sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)}$$

Time evolution of neutral meson system

Remember that strong interaction produces quarks in their flavour eigenstate:
At time t=0 the *B* meson starts either as B^0 or $\overline{B}^0$ (not as superposition)

$$|B^0_{\rm phys}(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\overline{B}^0\rangle$$
$$|\overline{B}^0_{\rm phys}(t)\rangle = g_+(t)|\overline{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle$$

with

$$g_\pm(t) = \frac{1}{2}(e^{-(im_L+\Gamma_L/2)t} \pm e^{-(im_H+\Gamma_H/2)t})$$

So, the probability to observe a B^0 or $\overline{B}^0$ at after a given time t equals:

$$|\langle B^0|B^0_{\rm phys}(t)\rangle|^2 = |g_+(t)|^2 \quad ,$$
$$|\langle \overline{B}^0|B^0_{\rm phys}(t)\rangle|^2 = \left|\frac{q}{p}\right|^2 |g_-(t)|^2 \quad ,$$
$$|\langle B^0|\overline{B}^0_{\rm phys}(t)\rangle|^2 = \left|\frac{p}{q}\right|^2 |g_-(t)|^2 \quad ,$$
$$|\langle \overline{B}^0|\overline{B}^0_{\rm phys}(t)\rangle|^2 = |g_+(t)|^2 \quad ,$$

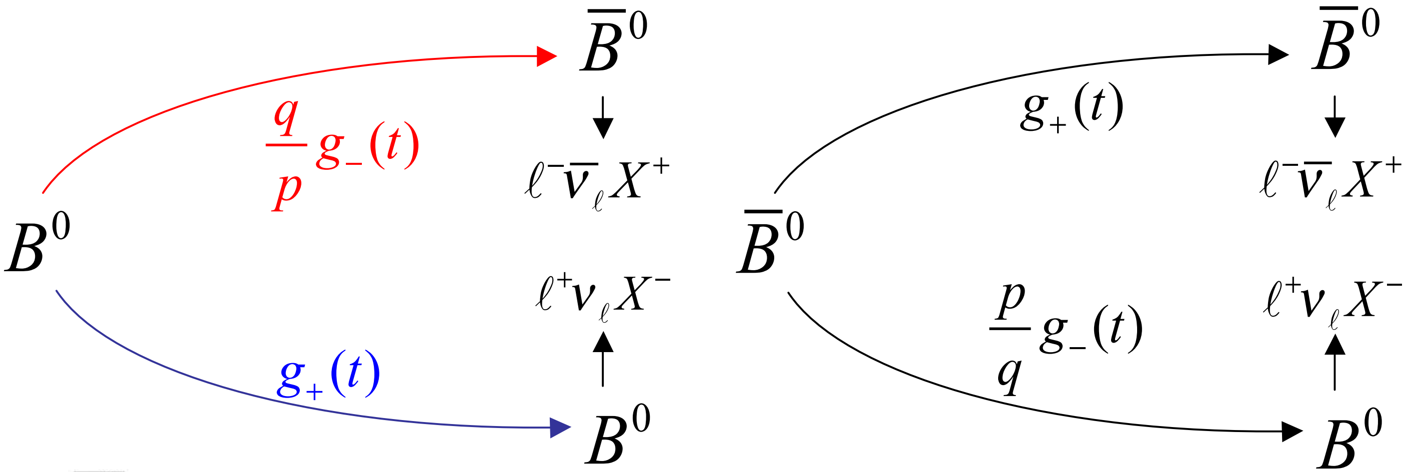
where: $|g_\pm(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_H t} + e^{-\Gamma_L t} \pm 2e^{-\Gamma t}\cos\Delta m t\right)$

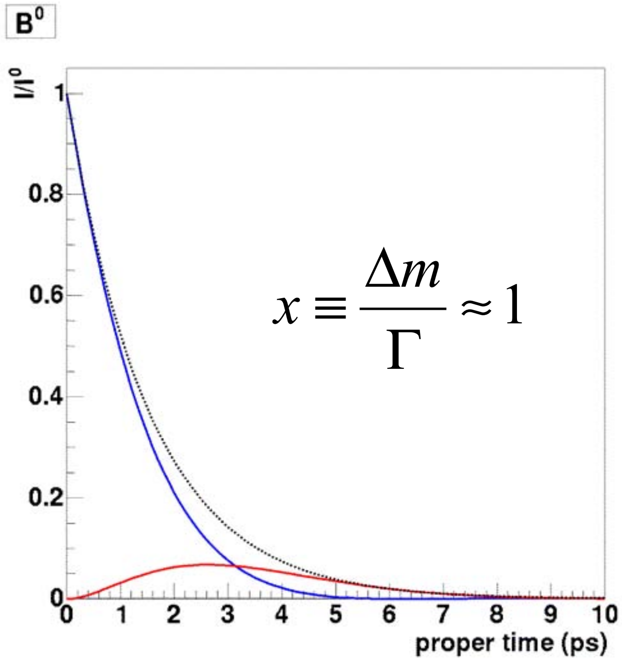
$\Delta\Gamma$ damps the oscillation (oscillation is gone when only B_L or B_H is left)

Δm describes the oscillation

Time evolution of neutral meson system

Example: *B* decay to flavour specific final state (semileptonic decay):





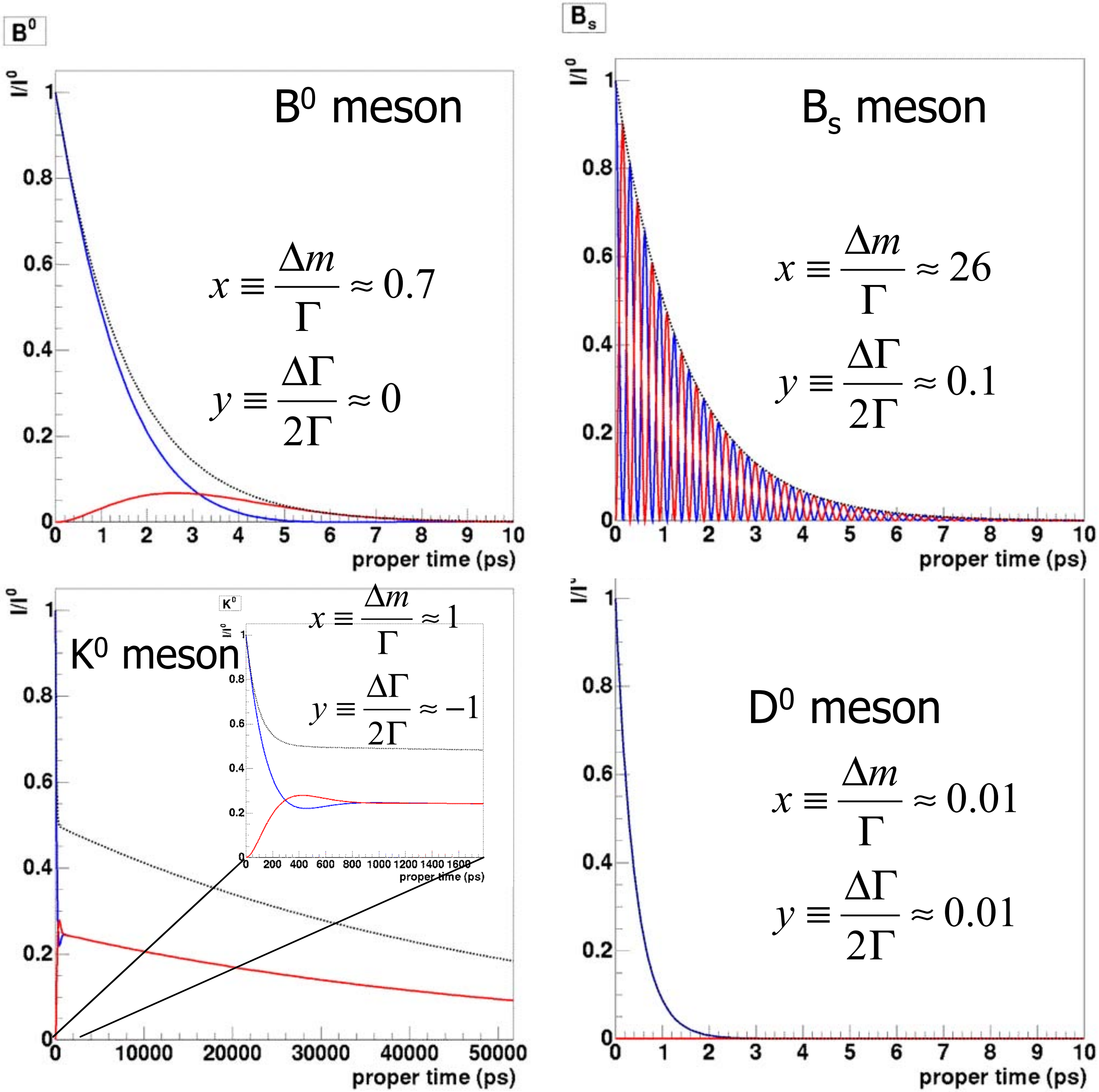
Black: Double exponential decay Γ_H and Γ_L

Blue: Probability of finding a B^0 at t for an initial B^0.

Red: Probability of finding a $\overline{B}^0$ at t for an initial B^0

x: the average number of oscillations before decay

Mixing of neutral mesons



The 4 different neutral meson systems have very different mixing properties.

B_s system: very fast mixing

Kaon system: large decay time difference.

Charm system: very slow mixing

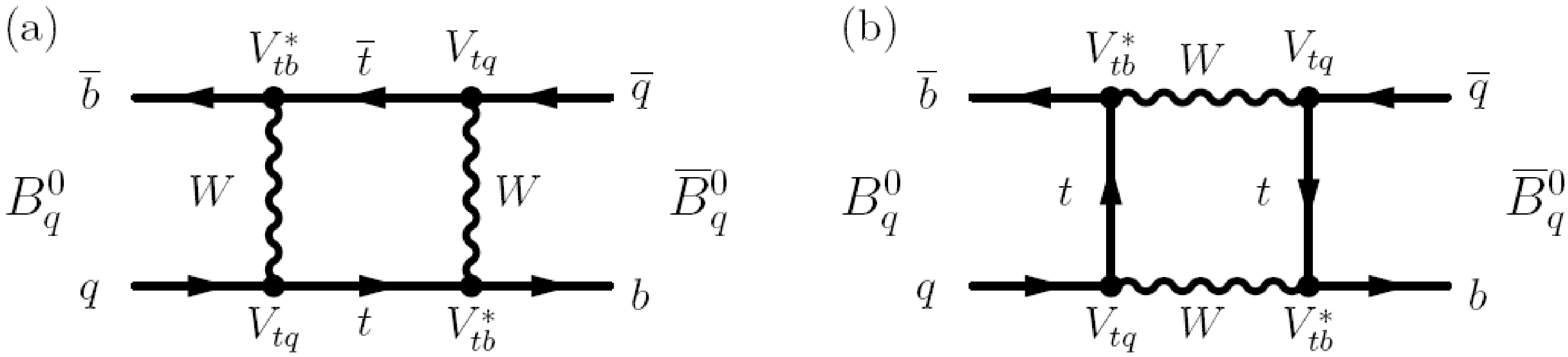
Mixing parameters

	$K^0/\bar{K}^0$	$D^0/\bar{D}^0$	$B^0/\bar{B}^0$	$B_s/\bar{B}_s$
τ [ps]*	89 51700	0.4	1.6	1.5
Γ [ps^{-1}]	5.6$\times$ 10^{-3}	2.4	0.64	0.62
y=$\frac{\Delta\Gamma}{2\Gamma}$	-0.997	0.01	|y|$<$0.01	0.03$\pm$0.03
Δm [ps^{-1}]	5.3 $\times$ 10^{-3}	0.02	0.5	17.8
x=$\frac{\Delta m}{\Gamma}$	0.95	0.01	0.8	26

Just for completeness

The weak box diagram

These two diagrams contribute to mixing in $B_{d,s}$ system:



The (heavy) top quark dominates the internal loop.
No GIM cancellation (if u,c,t would have the same mass these diagrams would cancel)

Why is are the oscillations in the B_s system so much faster than in B_d?
Why is the mixing in the D system so small?

Oscillations in B_d versus B_s system: V_{td} versus V_{ts}

Order λ^3 Order λ^2

→ Much faster oscillation in B_s system (less Cabbibo suppression).

In the D system, the d,s,b quarks in internal loop (no top): small mixing.