Advanced Topics in Particle Physics: LHC Physics

Part III: Heavy-Ion Physics

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Contents

Books

Introduction to High-Energy Heavy-Ion Collisions
Cheuk-Yin Wong
World Scientific, 1994

Quark-Gluon Plasma
K. Yagi, T. Hatsuda, and Y. Miake,
Cambridge Monographs, ed. T. Ericson, P.V. Landshoff, 2005, ISBN 0-521-56108-6

Phenomenology of Ultra-Relativistic Heavy-Ion Collisions
W. Florkowski
World Scientific, 2010

Ultrarelativistic Heavy-Ion Collisions
R. Vogt, Elsevier, 2007, ISBN 978-0-444-52196-5

Quark Gluon Plasma 3
World Scientific Publishing, ed. R.C. Hwa and X.-N. Wang, ISBN 981-238-077-9

The Large Hadron Collider, Nature 448 (2007) 269

1. Introduction

Strong Interaction

Nobel prize in physics (2004)


David J. Gross

H. David Politzer

Frank Wilczek

- Confinement:
  Isolated quarks and gluons cannot be observed, only color-neutral hadrons
- Asymptotic freedom:
  Coupling α_s between color charges gets weaker for high momentum transfers, i.e., for small distances ($\alpha_s(q^2) \rightarrow 0$ for $q^2 \rightarrow \infty$)
  (Perturbative methods applicable for $r < 1/10$ fm)
- Limit of low particle densities and weak coupling experimentally well tested ($\longrightarrow$ QCD perturbation theory)
- Nucleus-Nucleus collisions: QCD at high temperatures and density („QCD thermodynamics“)

Ultrarelativstic Heavy-Ion Physics

Basic, childlike questions addressed in ultra-relativistic heavy-ion physics:

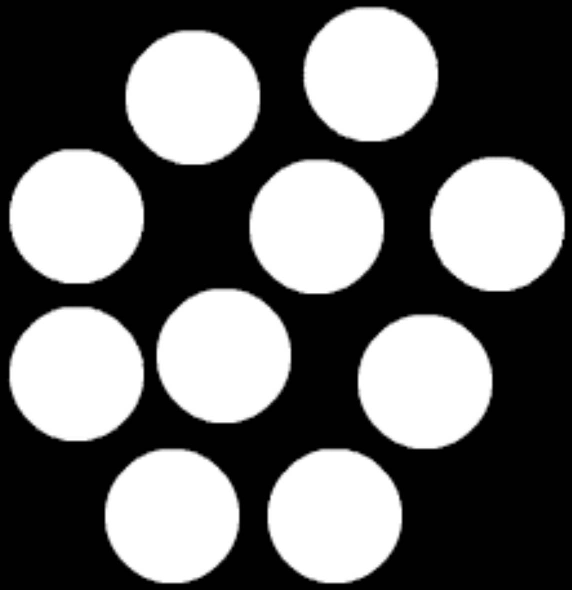
What happens to matter if you make it

- hotter and hotter?
- denser and denser?

With increasing temperature *T*:

solid → liquid → gas → plasma → QGP

Quark-Gluon-Plasma

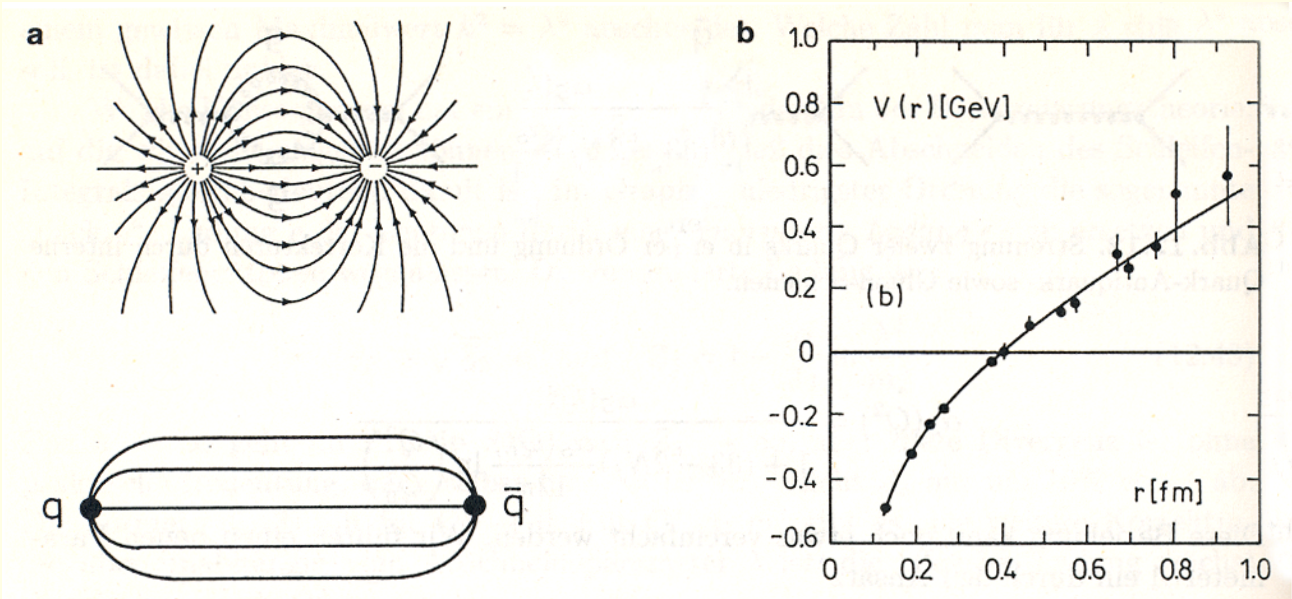


Confinement

$$\text{Quark potential: } V(r) = -\frac{4}{3}\frac{\alpha_s(r)\hbar c}{r} + k\cdot r$$

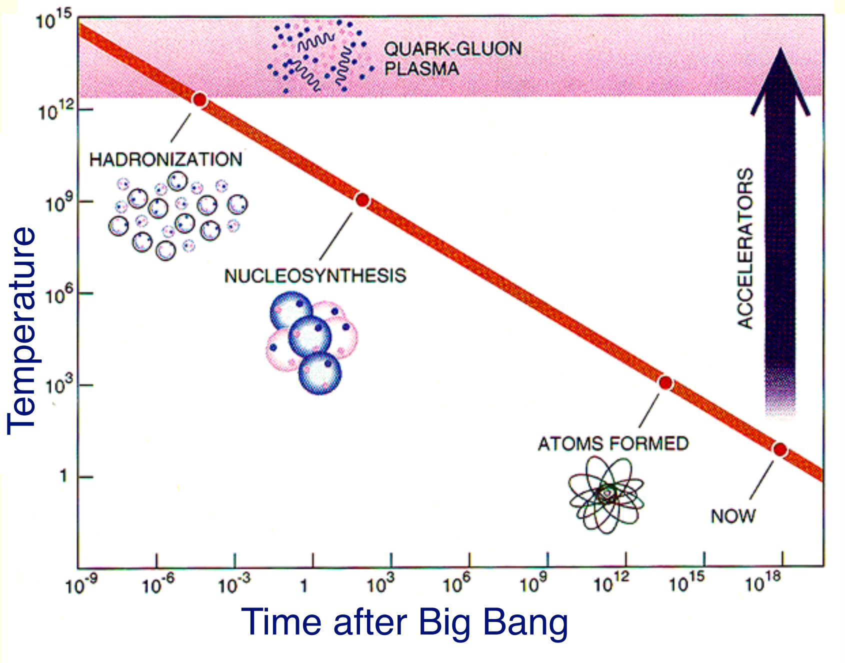
Dominant at small distances (1-gluon exchange)

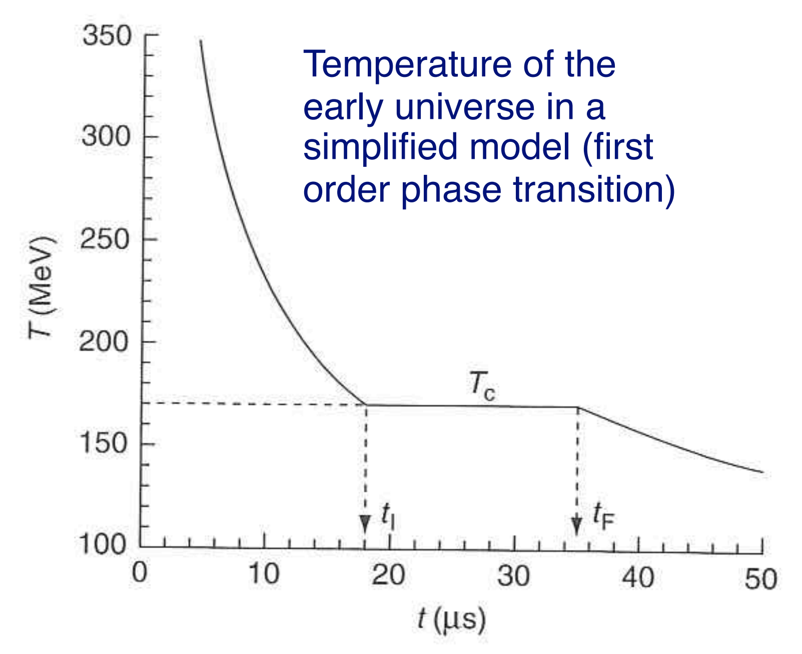
Dominant at large distances (related to confinement)



The long distance term $k\cdot r$ is expected to disappear in the QGP

Nucleus-Nucleus Collisions:
„Mini Big Bang in the Laboratory“



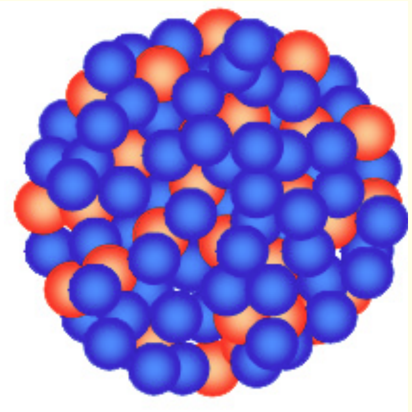


Temperature of the early universe in a simplified model (first order phase transition)

- Transition from the Quark-Gluon Plasma to a gas of hadrons at ~ 10^{12} °C
- 100 000 hotter than the core of the sun
- Early universe:
  QGP → hadron gas a few microseconds after the Big Bang

Getting a Feel for the Relevant Energy Densities

1. Inside a nucleus

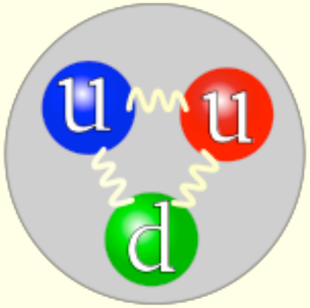


nucleon density: $\rho_0 = 0.16 \text{ nucleons/fm}^3$

nucleon mass: $m_n \approx 0.931 \text{ GeV}$

energy density: $\varepsilon = \rho_0 \cdot m_n \approx 0.15 \text{ GeV/fm}^3$

2. Inside the nucleon



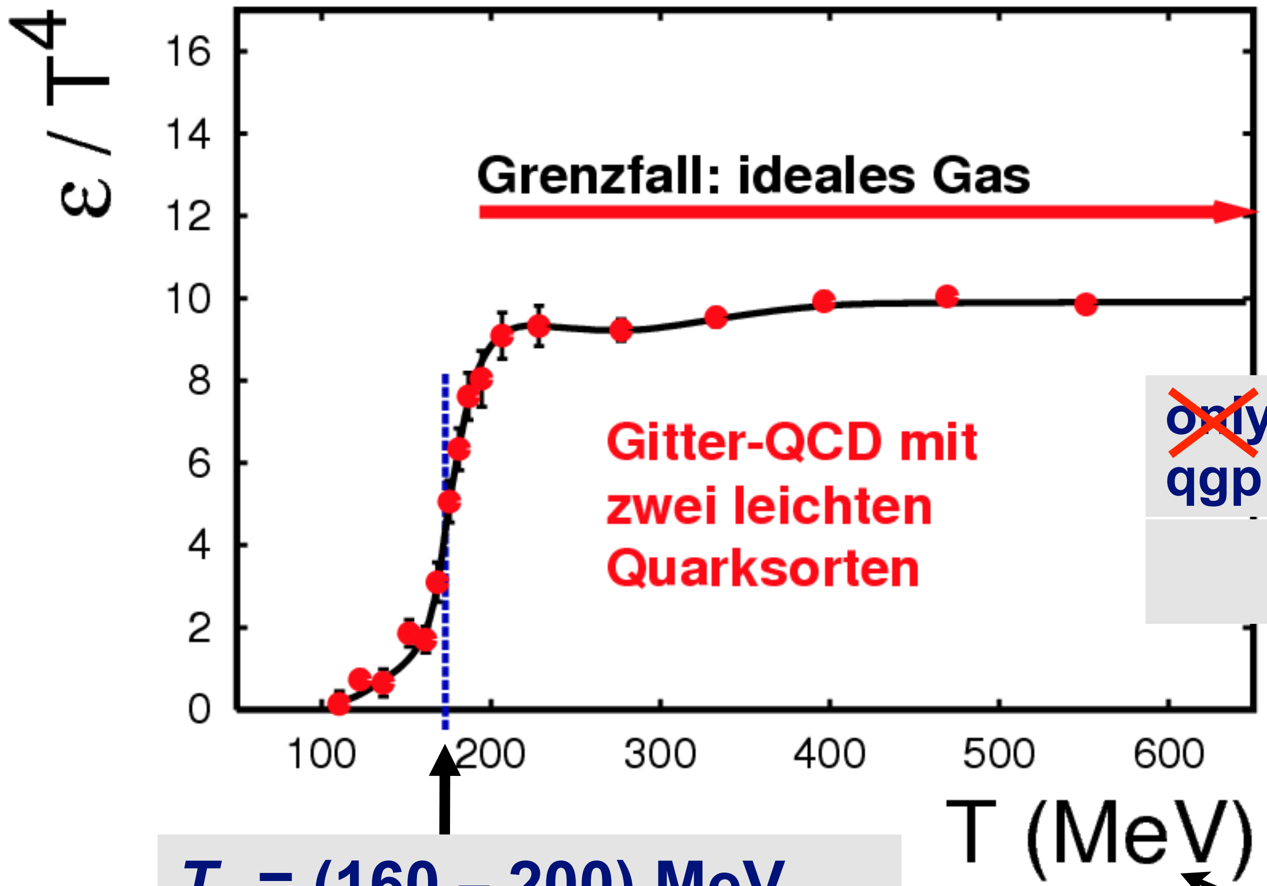
radius: $r_N \approx 0.8 \text{ fm}$

mass (free nucleon): $m_N \approx 0.94 \text{ GeV}$

energy density: $\varepsilon = \dfrac{0.94 \text{ GeV}}{4/3\, \pi r_N^3} \approx 0.44 \text{ GeV/fm}^3$

Predictions from First Principles: Lattice QCD

F. Karsch, E. Laermann, hep-lat/0305025



2 quark flavors:

$$\varepsilon_{\mathrm{SB}} = g \cdot \frac{\pi^2}{30} \cdot T^4$$

with $g = 37$

~~only~~ 20% deviation:
qgp is not an ideal gas

T_c **= (160 – 200) MeV**

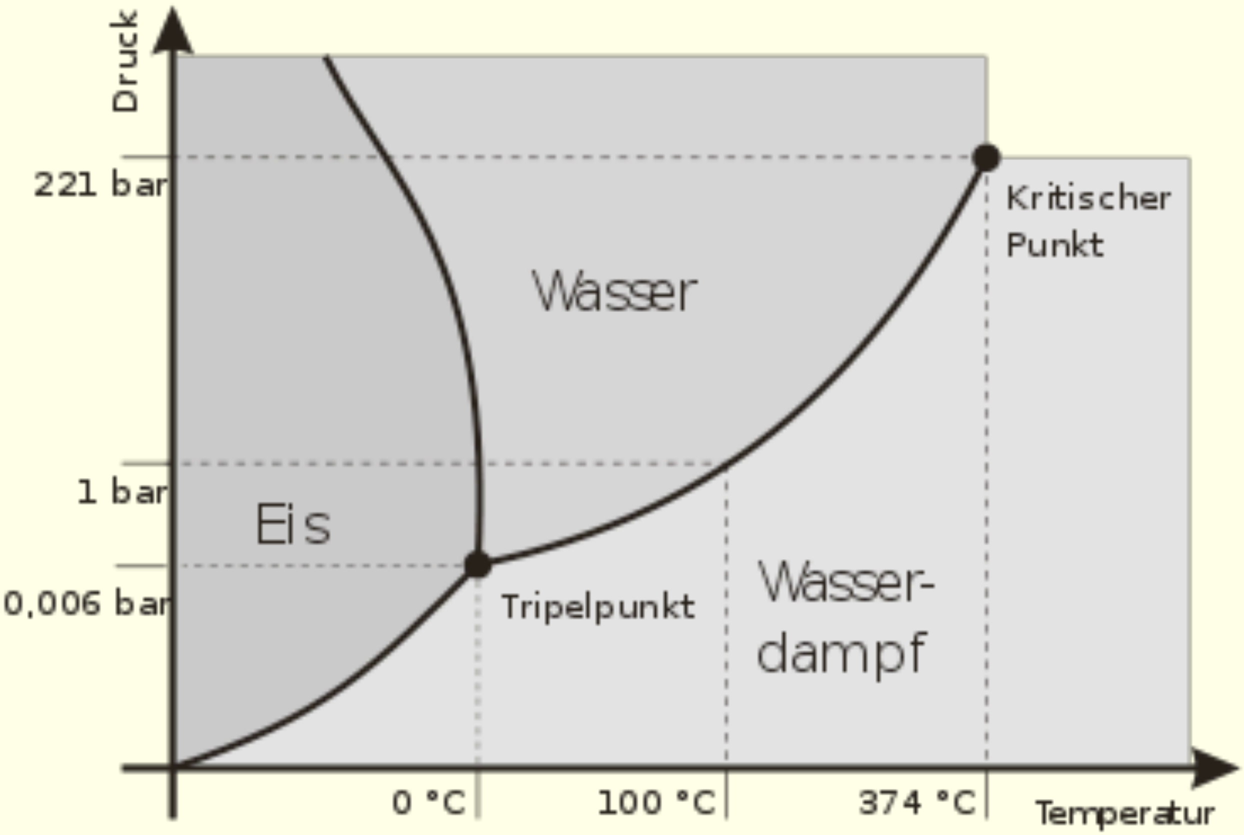
$\varepsilon_c \approx$ **0.7 – 1.0 GeV/fm³**

temperatures in eV:

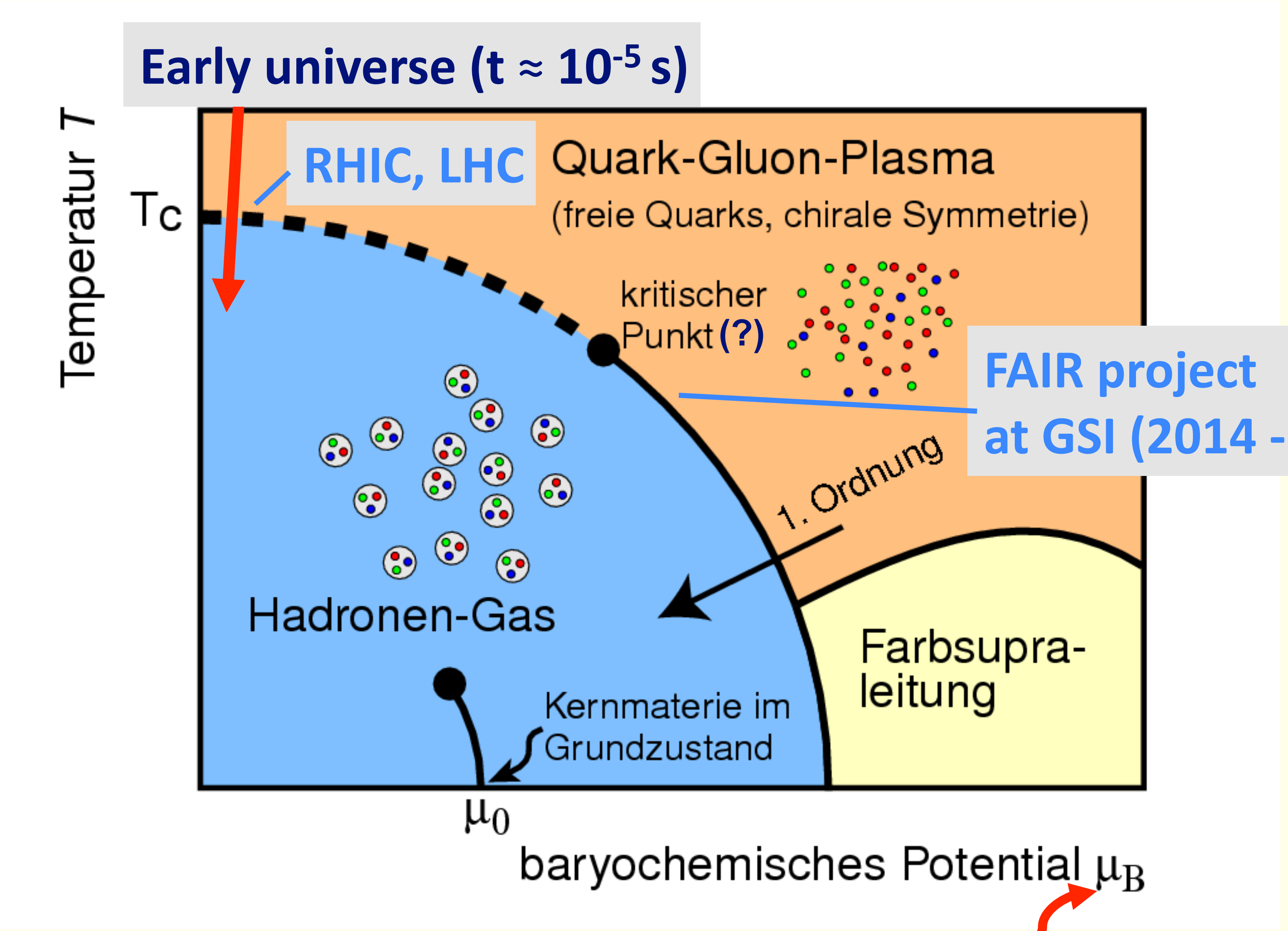
Example: room temp.

$$k \cdot T = k \cdot 300\ \mathrm{K} = 1/40\ \mathrm{eV}$$

Phase Diagrams: Water



Phase Diagrams: QCD Matter



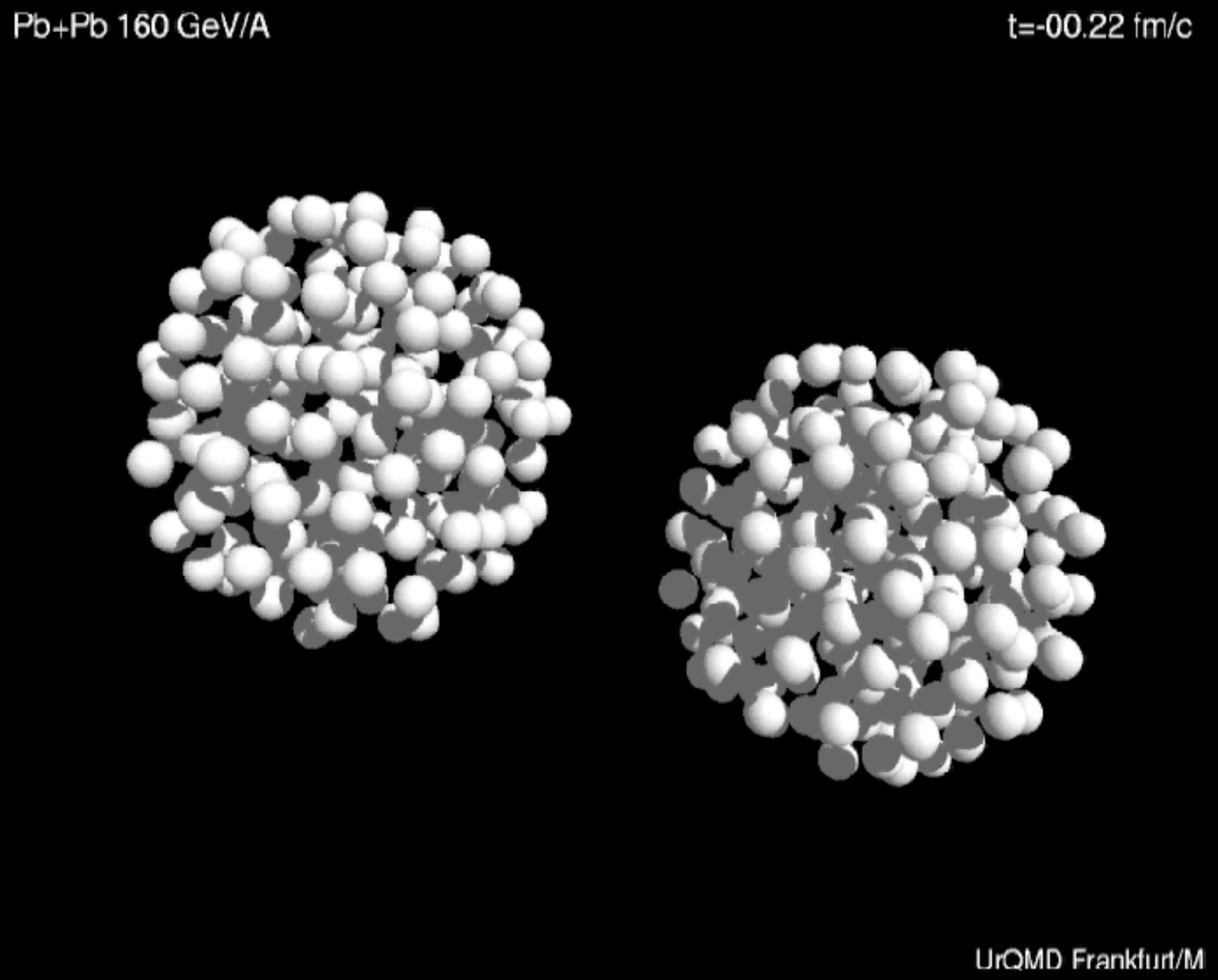
Early universe (t ≈ 10^{-5} s)

RHIC, LHC

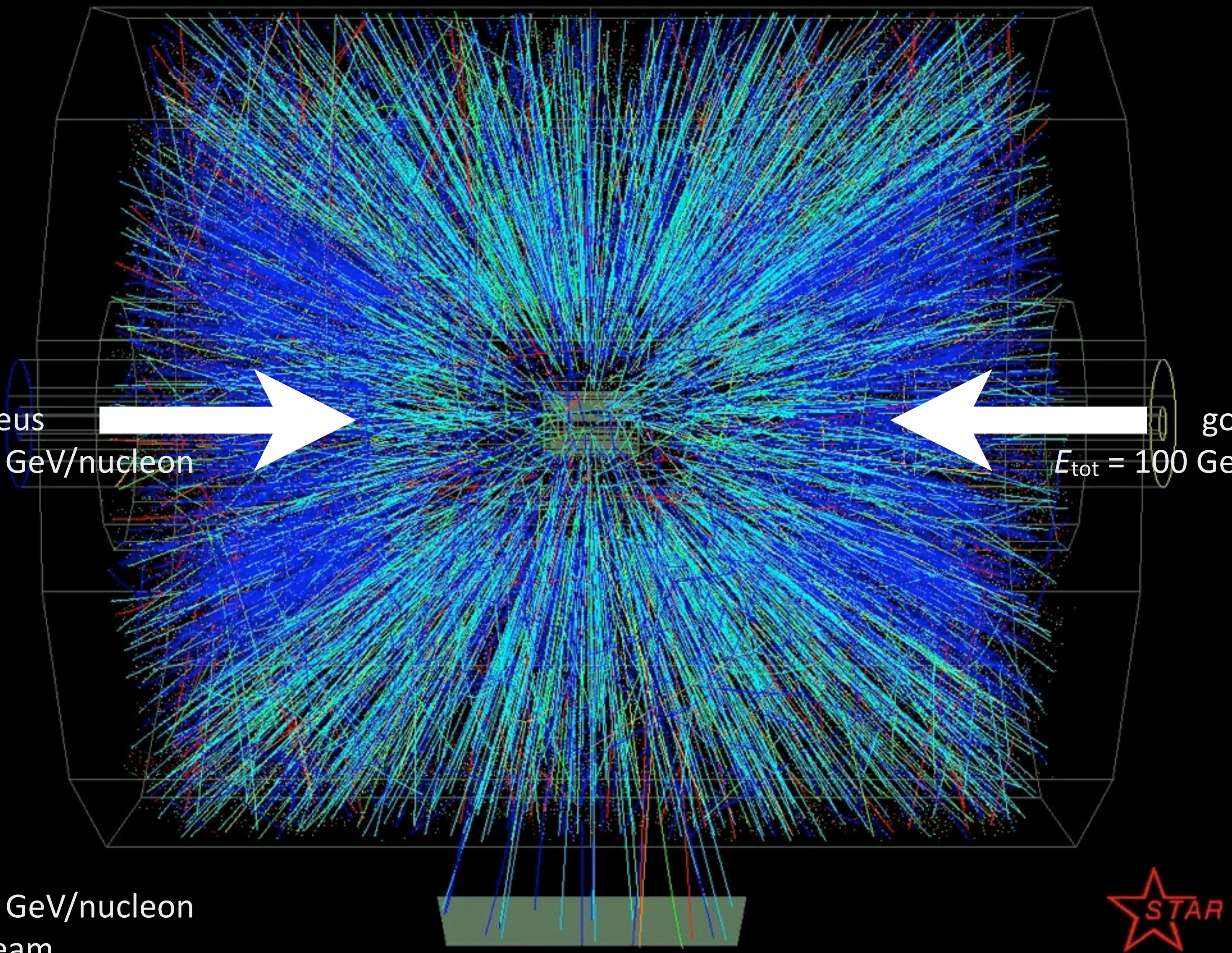
FAIR project at GSI (2014 - ...)

Measure of the net baryon density ρ

Ultra-Relativistische Schwerionenkollision



Au+Au Collision at the Relativistic Heavy Ion Collider (RHIC) in the USA

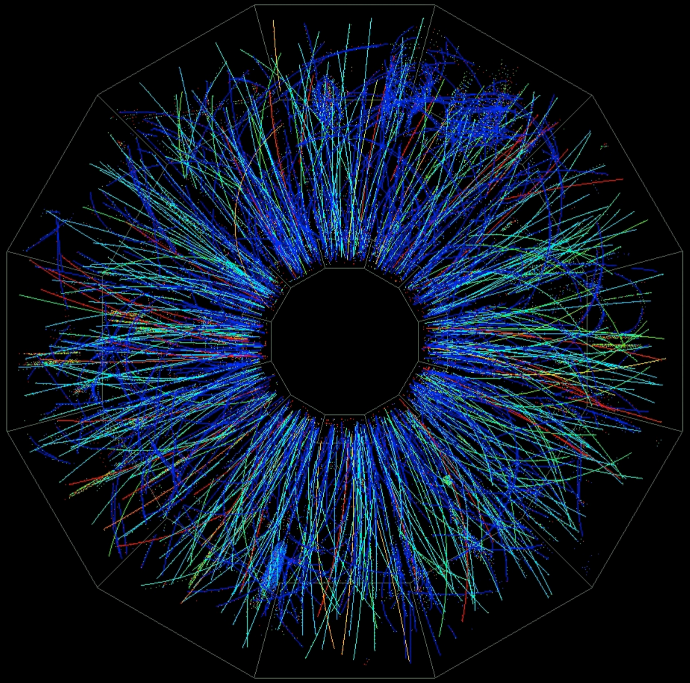


gold nucleus
E_{tot} = 100 GeV/nucleon

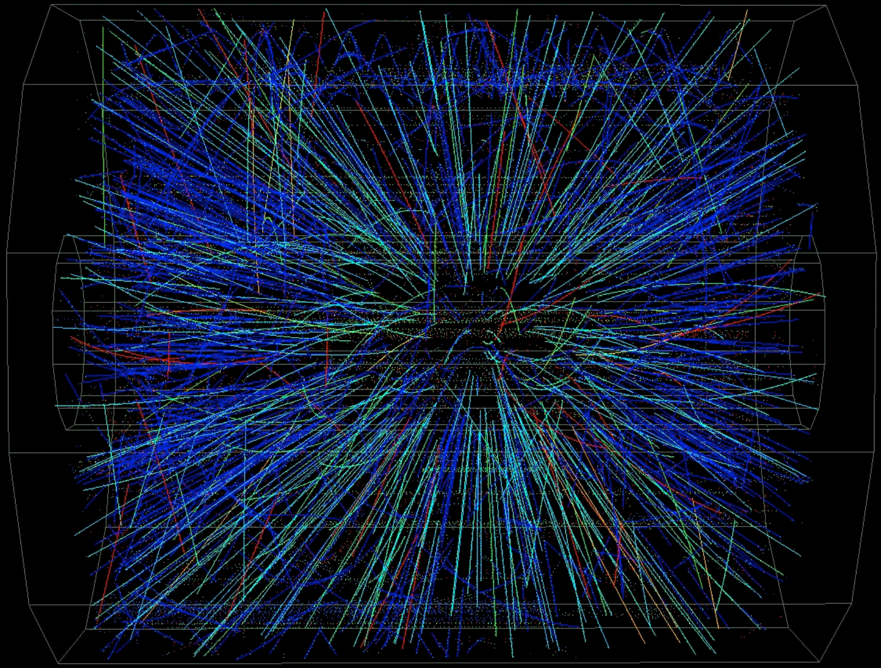
gold nucleus
E_{tot} = 100 GeV/nucleon

LHC:
E_{tot} = 2750 GeV/nucleon
for each beam

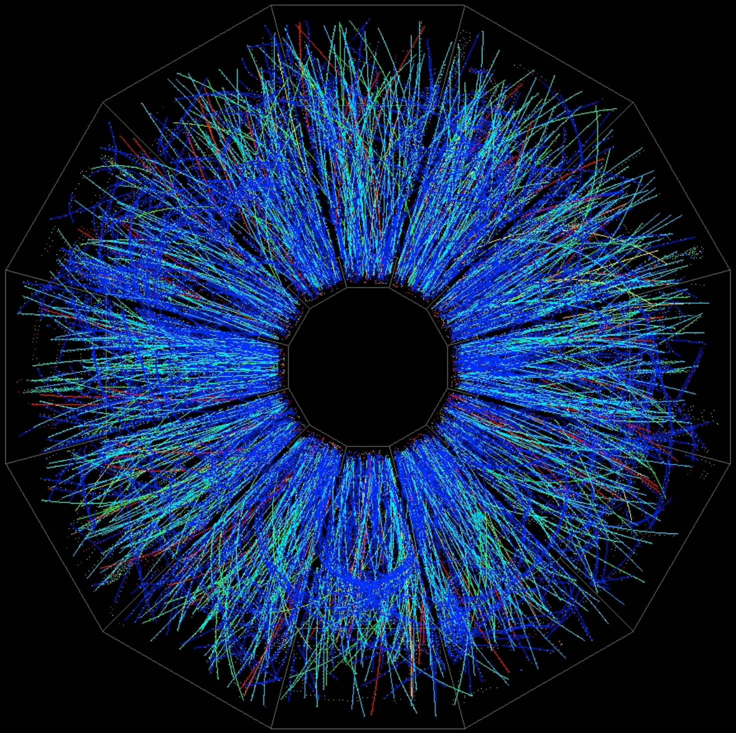
Au + Au Collisions at RHIC



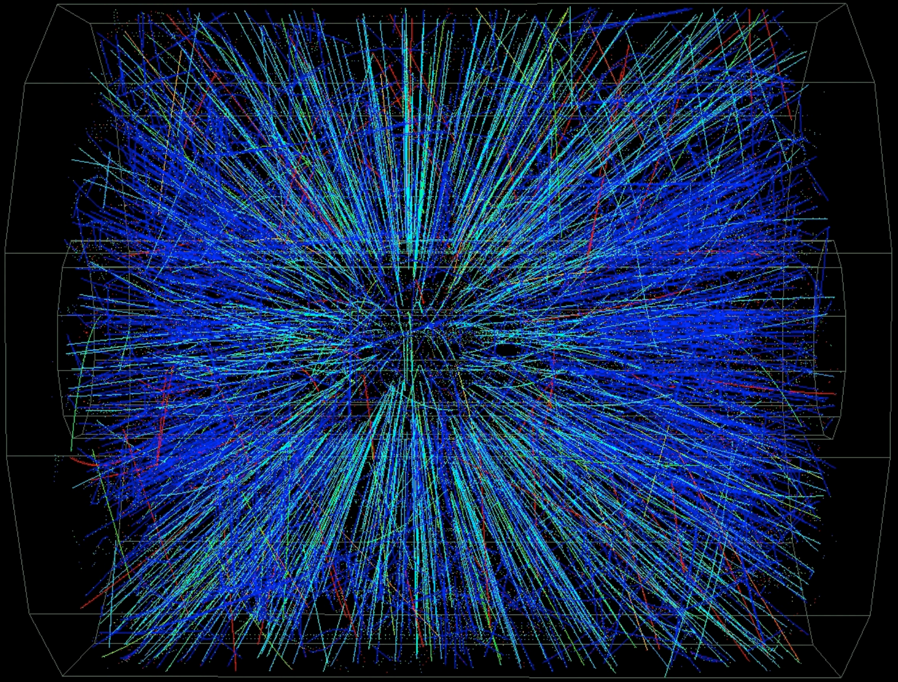
Peripheral Event



Au + Au Collisions at RHIC

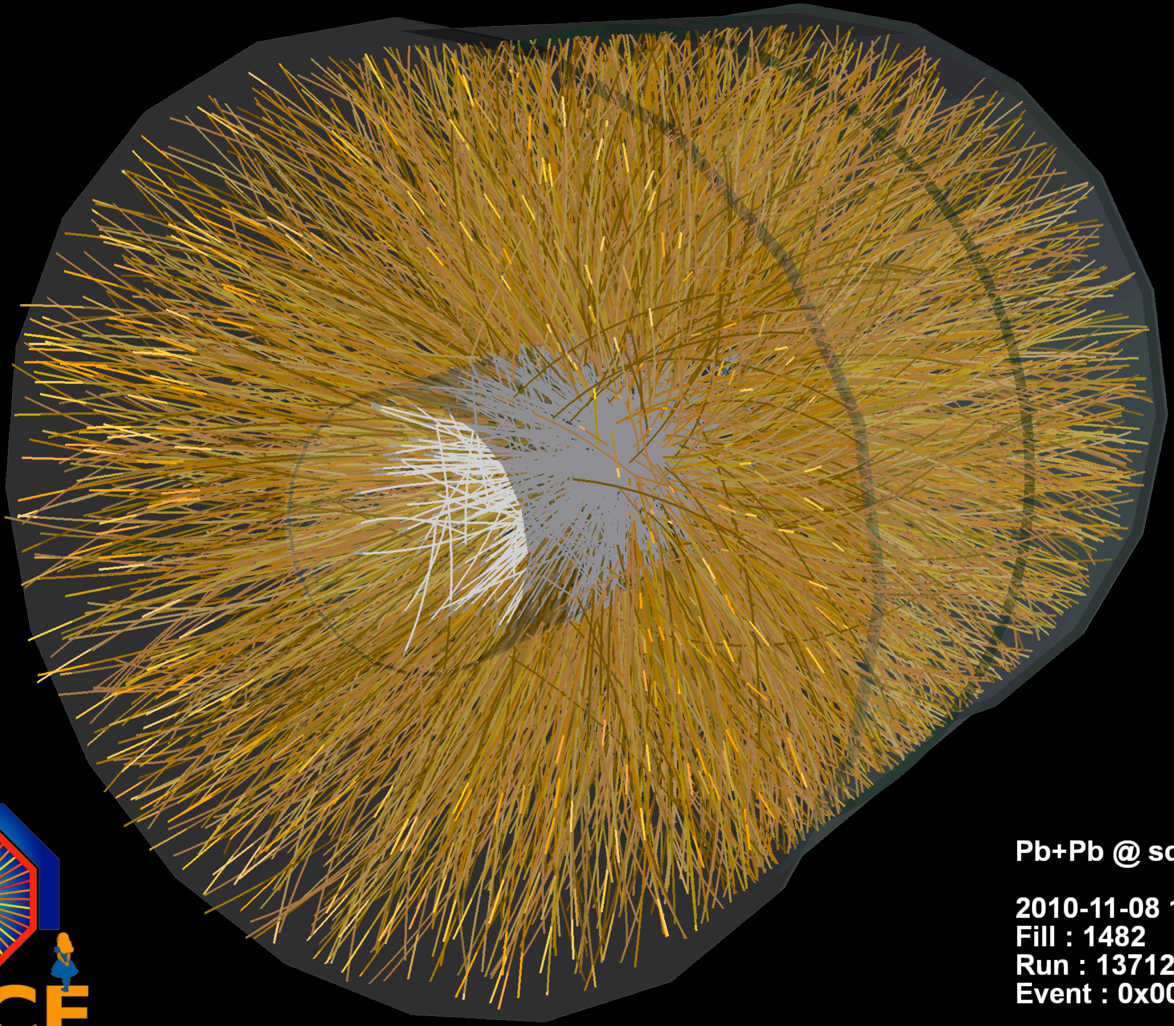


Mid-Central Event



Pb + Pb Collisions at the LHC

about 18 000 charged particles per central collisons

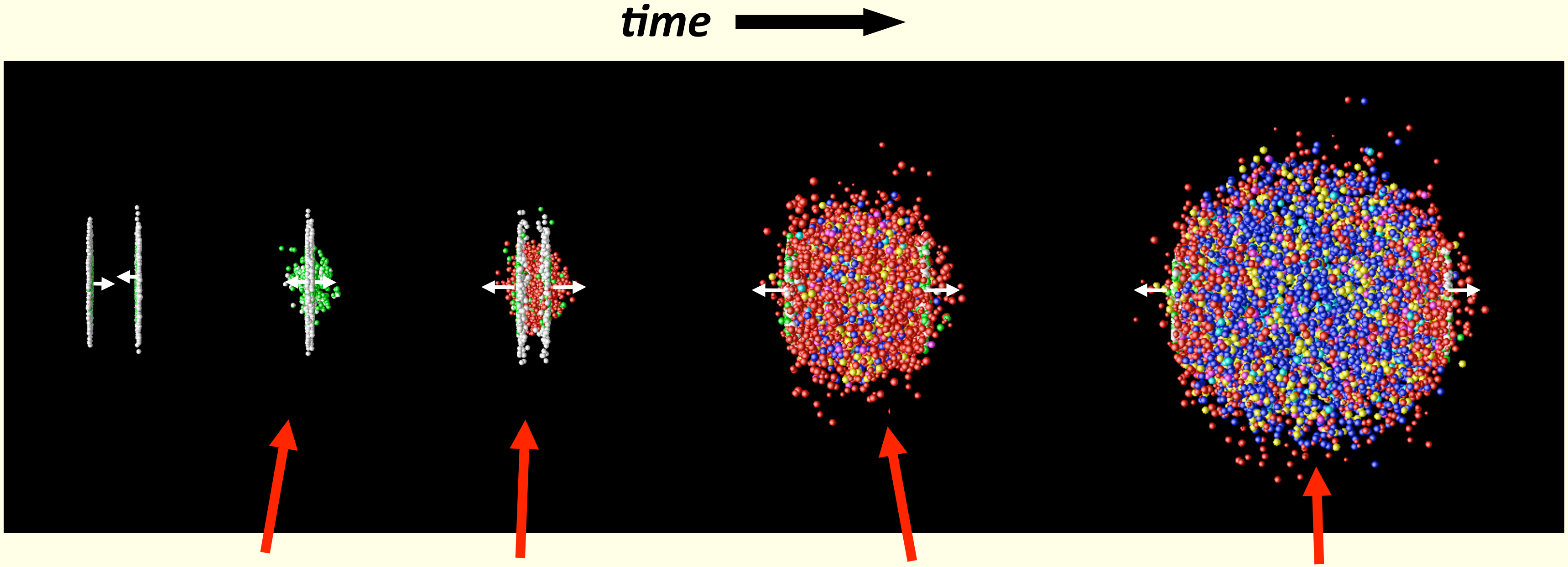


Pb+Pb @ sqrt(s) = 2.76 ATeV

2010-11-08 11:30:46
Fill : 1482
Run : 137124
Event : 0x00000000D3BBE693

ALICE

Ultra-Relativistic Nucleus-Nucleus Collisions



Early hard parton-parton scatterings ($Q^2 >> \Lambda^2_{\mathrm{QCD}}$)

Thermalized medium (QGP!?) ($T_0 > T_c$, $T_c \approx 160–190$ MeV)

Transition QGP → hadron gas

Freeze-out

Note:
$$1\ \mathrm{fm}/c = 0.33 \cdot 10^{-23}\ \mathrm{s}$$

- Time scales (RHIC, $\sqrt{s_{\mathrm{NN}}} = 200$ GeV):
  - Thermalization: $\tau_0 < \sim 1$ fm/*c*
  - QGP lifetime (center of a central Au+Au coll.): ~ 5 fm/*c*
- Advantage at the LHC: Longer QGP lifetime

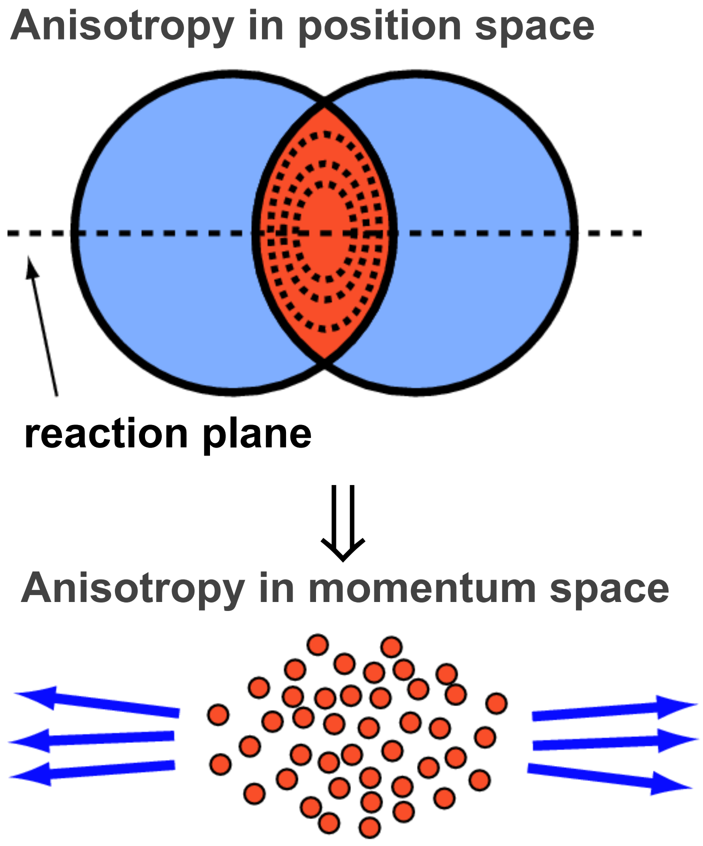
Brief History of Heavy-Ion Physics

Start	Accelerator	Projectile	Max. energy per NN pair ($\sqrt{s_{NN}}$)
~1985	AGS (BNL)	Si	~5 GeV
~1985	SPS (CERN)	O, S	~20 GeV
1994	SPS (CERN)	Pb	17 GeV
2000	RHIC (BNL)	Au	200 GeV
2010 (Nov. 8)	LHC (CERN)	Pb	2760 GeV

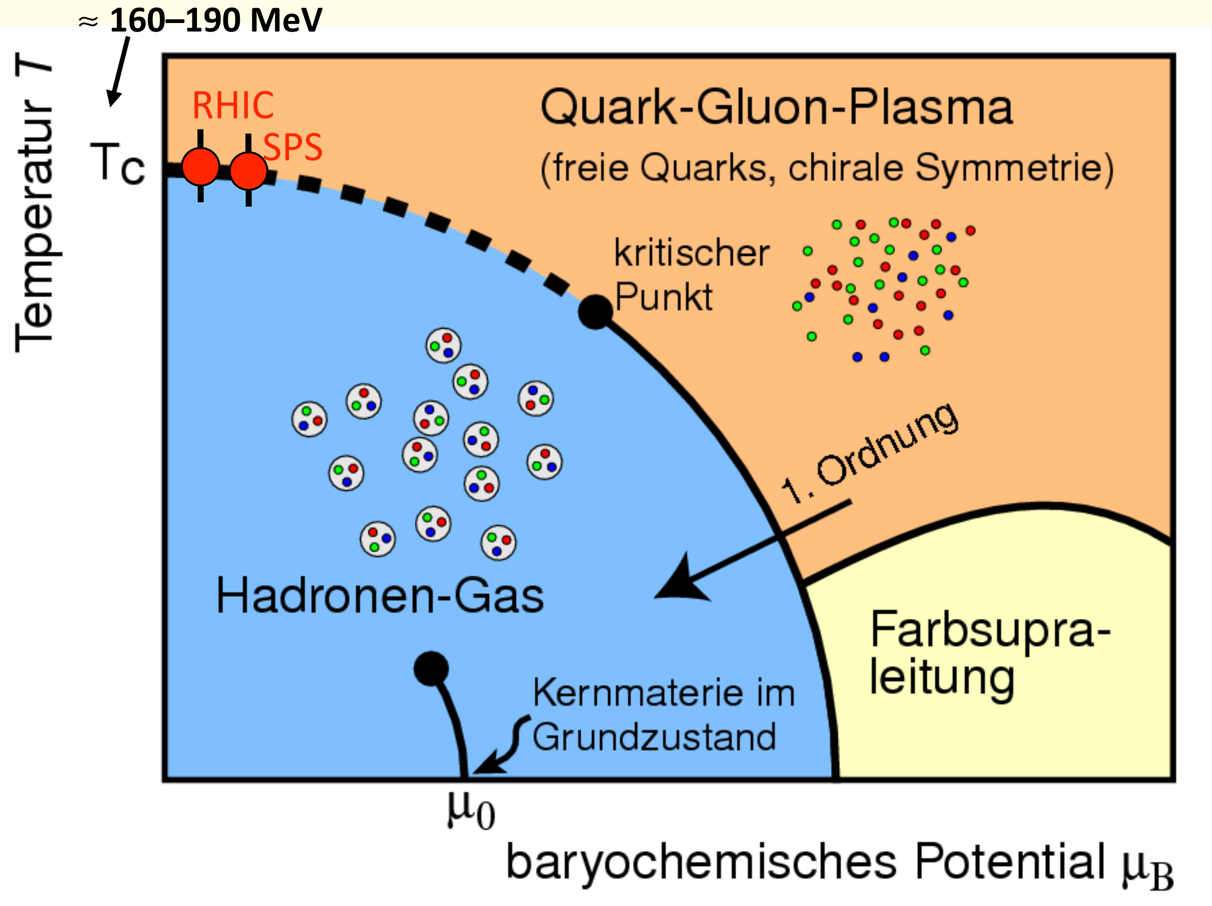
Important Results of the RHIC Heavy-Ion Program

- Hadron suppression at high p_T
  - Medium is to large extent opaque for jets (”jet quenching“)
- Elliptic Flow at low p_T
  - Ideal hydro close to data
    $\Rightarrow$ Small viscosity: ”perfect liquid“
  - Evidence for early thermalization ($\tau \lesssim 1\ \mathrm{fm}/c$)
- All hadron species in chemical equillibrium
  ($T \approx 160$ MeV, $\mu_B \approx 20$ MeV)

Elliptic flow:



Nucleus-Nucleus Collisions: Freeze-out Parameters



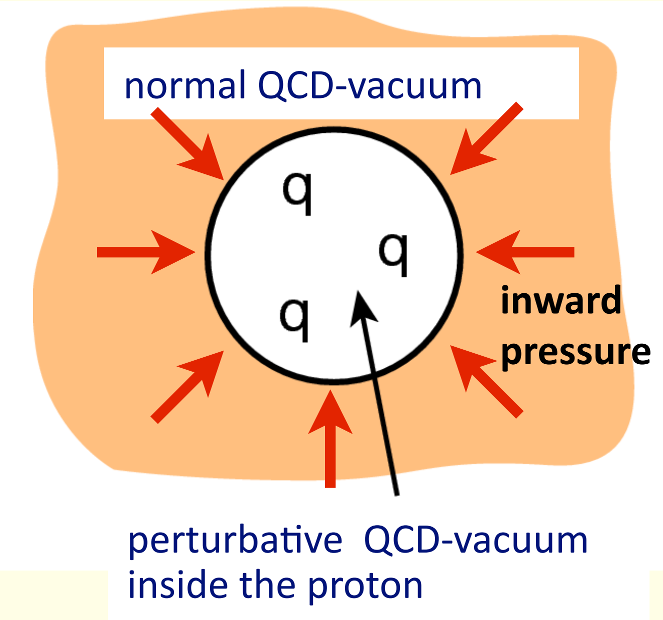
Freeze-out parameters T and μ_B approximately at expected phase boundary

Points to Take Home

- Ultra-relativistic Heavy-Ion Collisions:
  Study of QCD in the non-perturbative regime of extreme temperatures and densities
- Goal: Characterization of the Quark-Gluon Plasma
- Transition QGP $\rightarrow$ hadrons about 10^{-5} s after the Big Bang
- QCD phase diagram: QGP reached
  - at high temperature (about 160-200 MeV [$\sim 2 \cdot 10^{12}$ K])
  - and/or add high baryochemical potential μ_B (maybe realized in neutron stars)
- RHIC/LHC: $\mu_B \approx 0$
- Experiments at FAIR (in a couple of years):
  $\mu_B > 0$ search for critical point

2. Thermodynamics of the QGP

Bag Model



- Hadron = „bag“ filled with quarks
- Two kinds of vacuum
  - Normal QCD-Vacuum outside of the bag
  - Perturbative QCD-Vacuum within the bag

Energy density: $\varepsilon = E / V$

Energy density in the bag is higher than in the vacuum: $\varepsilon_{\text{pert}} - \varepsilon_{\text{vacuum}} =: B > 0$

kinetic energy of *N* particles in a spherical box of radius *R*

Energy of *N* quarks in a bag of radius *R*:

$$E = \frac{2.04 N}{R} + \frac{4\pi}{3} \cdot R^3 \cdot B$$

Condition for stability: d*E*/d*R* = 0 (minimum):

$$B^{1/4} = \left(\frac{2.04 N}{4\pi} \right)^{1/4} \cdot \frac{1}{R} \quad \overset{N=3,\ R=0{,}8\ \text{fm}}{\Longrightarrow} \quad B^{1/4} = 206\ \text{MeV} \quad (\hbar = c = 1)$$

Particles in a Box: Number of States

Number of states between momentum *p* and *p*+d*p*
(each state occupies a volume h^3 in phase space):

physical volume

number of states

$$\mathrm{d}N = \frac{V}{h^3} 4\pi p^2 \mathrm{d}p$$

volume of a spherical shell with radius *p*
and thickness d*p* in momentum space

Fermi-Dirac and Bose-Einstein Distribution

Let‘s consider an ideal gas of bosons and fermions (grand canonical ensemble).

Average occupation number of a state is

$$n(E) = \frac{g}{e^{(E-\mu)/kT} + 1}$$

... for fermions (half-integer spin):

(Fermi-Dirac distribution)

$$n(E) = \frac{g}{e^{(E-\mu)/kT} - 1}$$

... for bosons (integer spin):

(Bose-Einstein distribution)

g : # degrees of freedom (degeneracy)

μ : Chemical potential

T : Temperature

Degeneracy

QGP:

$$g_{\mathrm{Bosons}} = 8_{\mathrm{Color}} \times 2_{\mathrm{Polarisation}} = 16$$

$$g_{\mathrm{Fermions}} = g_{\mathrm{Quarks}} + g_{\mathrm{Antiquarks}} = 2 \times g_{\mathrm{Quarks}}$$

$$= 2 \times 3_{\mathrm{Color}} \times 2_{\mathrm{Flavour}} \times 2_{\mathrm{Spin}} = 24$$

assume only *u* and *d* quarks can be produced in the QGP, the rest too heavy

Pion-Gas:

$$g_{\mathrm{Bosons}} = 3_{\mathrm{Type}} \qquad g_{\mathrm{Fermions}} = 0$$

(π^+, π^-, π^0)

Total Quark Density in the Ideal (= non interacting) QGP at Temperature *T*

Massless quarks, Fermi-Dirac distribution:

$$\mathrm{d}N_{\mathrm{q}} = g_{\mathrm{q}} \cdot \frac{V}{h^3} \cdot 4\pi p^2 \left(\frac{1}{1 + e^{(E-\mu_{\mathrm{q}})/kT}} \right) \mathrm{d}p$$

$$\overset{\hbar=k=c=1}{=} \quad g_{\mathrm{q}} \frac{p^2 V}{2\pi^2} \left(\frac{1}{1 + e^{(p-\mu_{\mathrm{q}})/T}} \right) \mathrm{d}p$$

Quark density:

$$n_{\mathrm{q}}(\mu_{\mathrm{q}}) = \frac{N_{\mathrm{q}}}{V} = g_{\mathrm{q}} \frac{4\pi}{(2\pi)^3} \int_0^\infty \left(\frac{p^2}{1 + e^{(p-\mu_{\mathrm{q}})/T}} \right) \mathrm{d}p$$

holds for massless quarks

Antiquarks ($\mu_{\bar{\mathrm{q}}} = -\mu_{\mathrm{q}}$):

$$n_{\bar{\mathrm{q}}}(\mu_{\bar{\mathrm{q}}}) = \frac{N_{\bar{\mathrm{q}}}}{V} = g_{\mathrm{q}} \frac{4\pi}{(2\pi)^3} \int_0^\infty \left(\frac{p^2}{1 + e^{(p+\mu_{\mathrm{q}})/T}} \right) \mathrm{d}p$$

Quark-Gluon Plasma with μ = 0: Quarks

Quark density
(μ$_q$ = 0):

$$\boxed{n_\mathrm{q} = n_{\bar{\mathrm{q}}} = \frac{N_\mathrm{q}}{V} = \frac{3}{2}\overset{1,20205}{\zeta(3)}\frac{g_\mathrm{q}}{2\pi^2}\frac{\pi^2}{30}T^3}$$

Total energy of the quarks (*E* = *p* for massles quarks):

$$E_\mathrm{q} = \int_0^\infty p\,\mathrm{d}N_\mathrm{q}$$

Energy density and pressure (μ$_q$ = 0):

$$\boxed{\varepsilon_\mathrm{q} = \frac{E_\mathrm{q}}{V} = \frac{7}{8}g_\mathrm{q}\frac{\pi^2}{30}T^4, \qquad p_\mathrm{q} = \frac{1}{3}\varepsilon_\mathrm{q}}$$

(identical result for antiquarks (μ$_q$ = 0))

Example: $T = 200\ \mathrm{MeV}, g_\mathrm{q} = 12 \quad \Rightarrow \quad n_\mathrm{q} = n_{\bar{\mathrm{q}}} = 1,71\,/\,\mathrm{fm}^3$

Quark-Gluon Plasma with μ = 0: Gluons

Gluons, Bose-Einstein distribution:

$$\mathrm{d}N_\mathrm{g} = \frac{V g_\mathrm{g}}{2\pi^2} \cdot \frac{p^2}{e^{p/T} - 1} \mathrm{d}p, \quad n_\mathrm{g} = \frac{N_\mathrm{g}}{V} = \frac{1}{V} \int_0^\infty \mathrm{d}N_\mathrm{g}, \quad E_\mathrm{g} = \int_0^\infty p \, \mathrm{d}N_\mathrm{g}$$

Solution:

Energy density:

$$\varepsilon_\mathrm{g} = \frac{E_\mathrm{g}}{V} = g_\mathrm{g} \frac{\pi^2}{30} T^4,$$

Pressure:

$$p_\mathrm{g} = \frac{1}{3} \varepsilon_\mathrm{g},$$

Particle density:

$$n_\mathrm{g} = \frac{g_\mathrm{g}}{\pi^2} \zeta(3) T^3$$

Example: $T = 200\ \mathrm{MeV},\ g_\mathrm{g} = 16 \quad \Rightarrow \quad n_\mathrm{g} = 2{,}03\ \mathrm{gluons\ /\ fm}^3$

Summary: Quark-Gluon Plasma with μ = 0: Pressure and Energy Density

Pressure and energy density in a Quark-Gluon-Plasma at μ = 0 without particle interactions:

$$p_{\mathrm{QGP}} = \left(g_{\mathrm{g}} + \frac{7}{8}(g_{\mathrm{q}} + g_{\bar{\mathrm{q}}}) \right) \frac{\pi^2}{90} T^4$$

$$= 37 \frac{\pi^2}{90} T^4$$

$$\varepsilon_{\mathrm{QGP}} = 3 p_{\mathrm{QGP}}$$

$$= 37 \frac{\pi^2}{30} T^4$$

Example: $T = 200\ \mathrm{MeV} \quad \Rightarrow \quad \varepsilon_{\mathrm{QGP}}^{\mathrm{id.\ Gas}} = 2{,}54\,\mathrm{GeV/fm^3}$

Quark-Gluon Plasma with μ = 0: Critical Temperature (I)

Accounting for the QCD-vacuum:

$$\varepsilon_{\mathrm{QGP}}^{\text{QCD-Vac.}} = \varepsilon_{\mathrm{QGP}} + B$$

$$p_{\mathrm{QGP}}^{\text{QCD-Vac.}} = p_{\mathrm{QGP}} - B$$

$$E = T\,S - pV \qquad (\mu = 0)$$

$$\Rightarrow \; p = Ts - \varepsilon, \qquad s := \frac{S}{V}$$

So we have:

$$p_{\mathrm{HG}} = 3aT^4 \qquad \varepsilon_{\mathrm{HG}} = 9aT^4$$

$$p_{\mathrm{QGP}}^{\text{QCD-Vac.}} = 37aT^4 - B \qquad \varepsilon_{\mathrm{QGP}}^{\text{QCD-Vac.}} = 111aT^4 + B$$

$$a := \frac{\pi^2}{90}$$

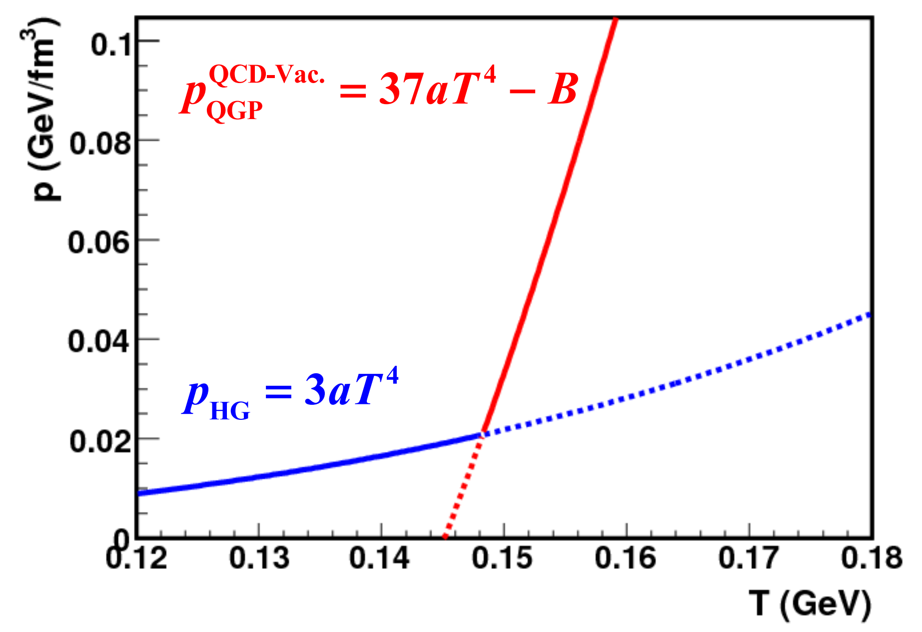
Gibbs criterion for the phase transition:

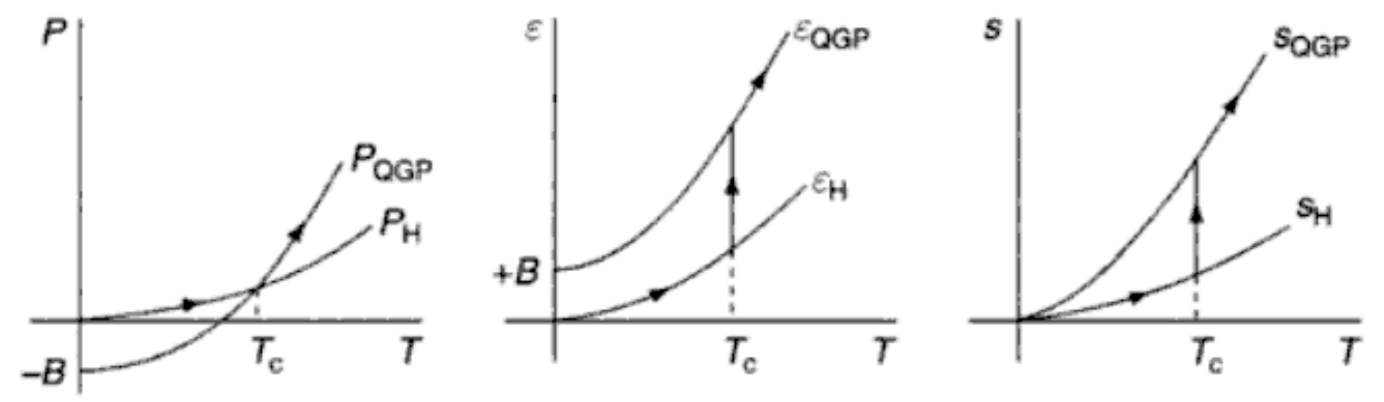
$$p_{\mathrm{HG}} = p_{\mathrm{QGP}}^{\text{QCD-Vac.}}, \; T_{\mathrm{HG}} = T_{\mathrm{QGP}} = T_{\mathrm{c}} \quad \Rightarrow \quad T_{\mathrm{c}} = \left(\frac{B}{34a}\right)^{1/4} \approx 150\,\mathrm{MeV}$$

Phase transition in the bag model is of first order. Latent heat:

$$\varepsilon_{\mathrm{QGP}}^{\text{QCD-Vac.}}(T_c) - \varepsilon_{\mathrm{HG}}(T_c) = 102aT_c^4 + B = 4B$$

Quark-Gluon Plasma with μ = 0: Critical Temperature (II)





Quark-Gluon Plasma with μ = 0: Entropy

Entropy density for constant temperature and pressure:

$$\varepsilon = T s + \mu N - p \quad \overset{\mu=0}{\Rightarrow} \quad s = \frac{\varepsilon + p}{T} = 4\frac{p}{T}$$

Ratio of entropy density (QGP / pion gas):

$$s_{\mathrm{QGP}} = 148\, a T^3, \quad s_{\mathrm{HG}} = 12\, a T^3 \quad \Rightarrow \quad \frac{s_{\mathrm{QGP}}}{s_{\mathrm{HG}}} \approx 12,3$$

Entropy per particle:

Pion gas:

$$\frac{s_{\mathrm{HG}}}{n_\pi} = \frac{12\pi^2 / 90 \cdot T^3}{g_\pi \cdot 1,202 / \pi^2 \cdot T^3} = 3.6$$

QGP:

$$\frac{s_{\mathrm{q}}}{n_{\mathrm{q}}} = 1,4 \qquad \frac{s_{\mathrm{g}}}{n_{\mathrm{g}}} = 1,2$$

Quark-Gluon Plasma with $\mu \neq 0$: Energy and Particle Number Density of the Quarks

For $\mu_\mathrm{q} \neq 0$ a solution in closed form can be found for $\varepsilon_\mathrm{q} + \varepsilon_{\bar{\mathrm{q}}}$ but not for ε_q and $\varepsilon_{\bar{\mathrm{q}}}$ separately:

$$\varepsilon_\mathrm{q} + \varepsilon_{\bar{\mathrm{q}}} = g_\mathrm{q} \left(\frac{7\pi^2}{120} T^4 + \frac{1}{4} \mu_\mathrm{q}^2 T^2 + \frac{1}{8\pi^2} \mu_\mathrm{q}^4 \right)$$

Accordingly one finds for the quark density

$$n_\mathrm{q} - n_{\bar{\mathrm{q}}} = g_\mathrm{q} \left(\frac{1}{6} \mu_\mathrm{q} T^2 + \frac{1}{6\pi^2} \mu_\mathrm{q}^3 \right), \quad g_\mathrm{q} = 12$$

From this the net baryon density can be determined as:

$$n_\mathrm{B} = \frac{n_\mathrm{q} - n_{\bar{\mathrm{q}}}}{3} = \frac{2}{3} \mu_\mathrm{q} T^2 + \frac{2}{3\pi^2} \mu_\mathrm{q}^3 = \frac{2}{9} \mu_\mathrm{B} T^2 + \frac{2}{81\pi^2} \mu_\mathrm{B}^3 \qquad (\mu_\mathrm{B} = 3\mu_\mathrm{q})$$

Quark-Gluon Plasma with $\mu \neq 0$: Critical Temperature and Critical Quark Potential

Energy density in a QGP with $\mu \neq 0$ (without particle interactions):

$$\varepsilon_{\mathrm{QGP}} = \frac{37}{30}\pi^2 T^4 + 3\mu_{\mathrm{q}}^2 T^2 + \frac{3}{2\pi^2}\mu_{\mathrm{q}}^4$$

Condition for QGP stability:

$$p_{\mathrm{QGP}} = \frac{1}{3}\varepsilon_{\mathrm{QGP}} \overset{!}{=} B \quad \Rightarrow \quad T_{\mathrm{c}}(\mu_{\mathrm{q}})$$

Condition for QGP:
QGP-pressure ≥ pressure of the QCD-vacuum (similar, but not identical, to the previous condition $p_{\mathrm{HG}} = p_{\mathrm{QGP}}$)

Critical temperature / quark potential:

$$T_{\mathrm{c}}(\mu_{\mathrm{q}} = 0) = \left(\frac{90B}{37\pi^2}\right)^{1/4}$$

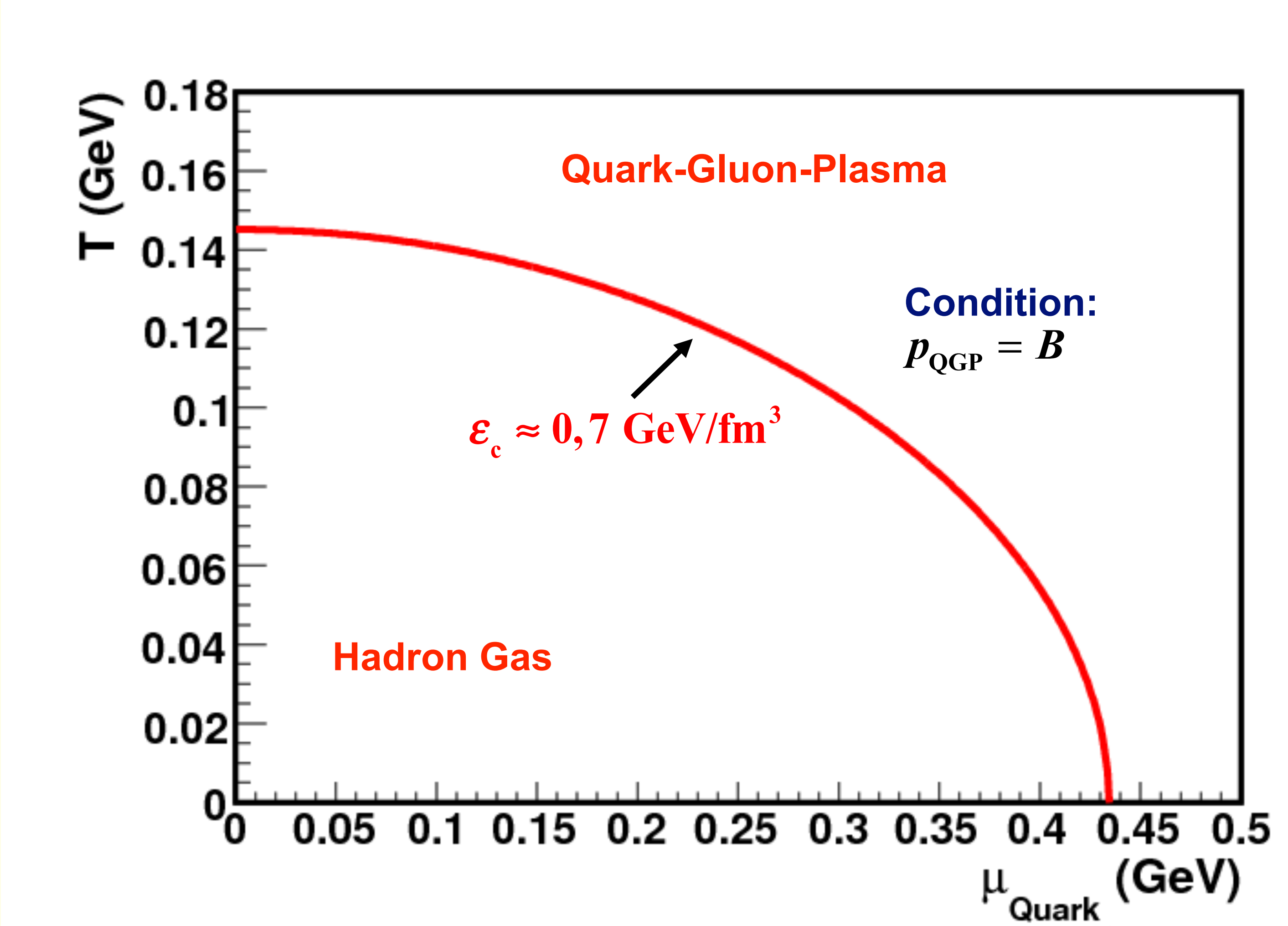
$$\mu_{\mathrm{q}}^{\mathrm{c}}(T=0) = \left(2\pi^2 B\right)^{1/4} = 0{,}43\ \mathrm{GeV}$$

$$n_{\mathrm{B}}^{\mathrm{c}}(T=0) = \frac{2}{3\pi^2}\left(2\pi^2 B\right)^{3/4}$$

$$= 0{,}72\ \mathrm{fm}^{-3} \approx 5 \times n_{\mathrm{nucleus}}$$

Possibly reached in neutron stars

Quark-Gluon Plasma with $\mu \neq 0$: Phase Diagram of the Non-Interacting QGP



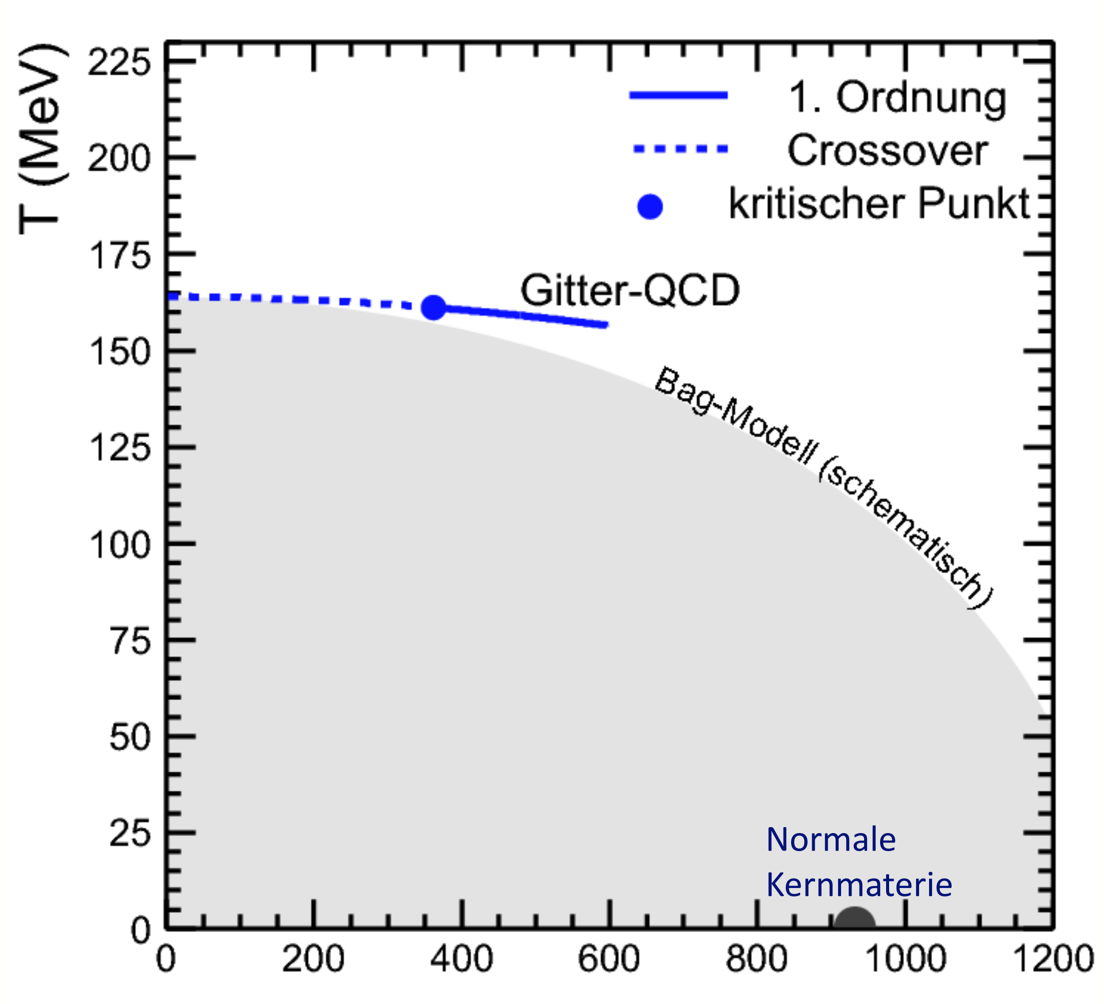
$\varepsilon_c \approx 0{,}7\ \text{GeV/fm}^3$

Condition:

$p_{\text{QGP}} = B$

$$\mu_B = 3 \cdot \mu_{\text{Quark}}$$

Quark-Gluon Plasma with $\mu \neq 0$: Phase Diagram from Lattice-QCD



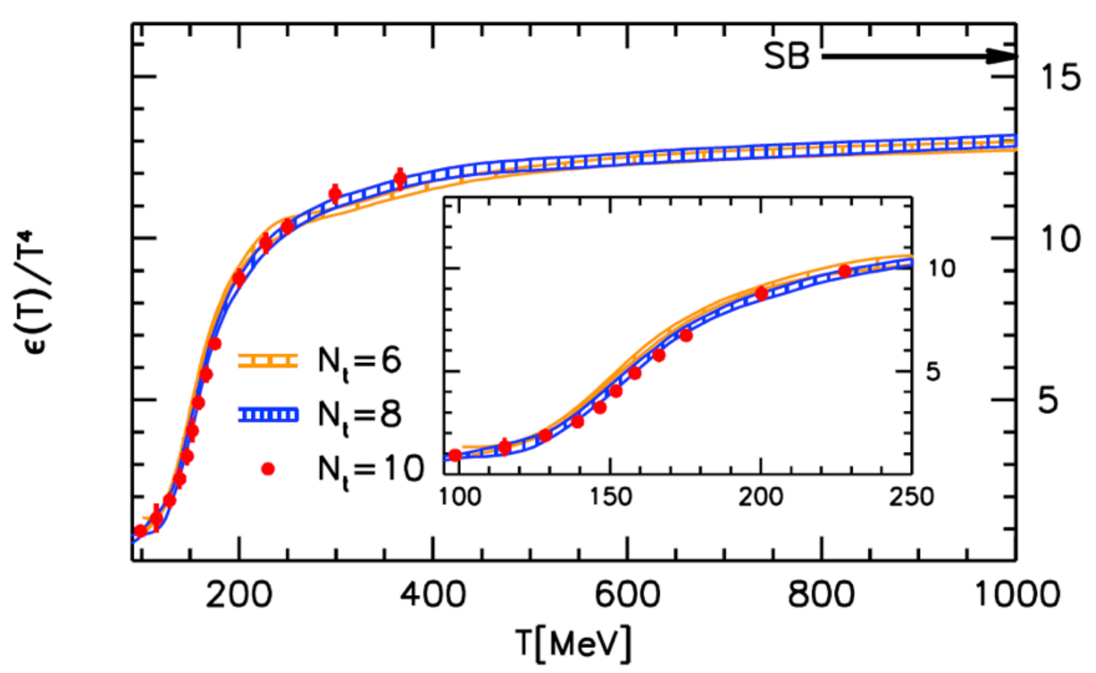
Z. Fodor, S.D. Katz,
JHEP 404 (2004) 50 [hep-lat/0402006].

- Lattice-calculations for $\mu_b \neq 0$
  - numerically very expensive
- Some calculations suggest a critical point (with large theoretical uncertainties):
  - T = 162 MeV
  - μ_b = 340 MeV

The existence and exact position of the critical point remains an open question

Latest Lattice Results

- Latest lattice results for $T_c\,(\mu = 0)$:
  - T_c = 147 - 157 MeV (Wuppertal-Budapest collaboration)
  - T_c = 157 ± 4 ± 3 ± 1 MeV (HotQCD collaboration, preliminary)
- Example: ε/T^4 vs. T from Wuppertal-Budapest collaboration:



Borsanyi et al., JHEP 1011 (2010) 077

Points to Take Home

- When treated as a relativistic ideal gas, parameters for the transition Hadron Gas ↔ QGP are:
    - $T_c\ (\mu_b{=}0) \approx 150$ MeV
    - $\mu_{b,c}(T{=}0) = 3\ \mu_{\mathrm{Quark},c}(T{=}0) \approx 1{,}3$ GeV (this is approximately five times the density of „normal“ nuclear matter)
- Lattice QCD calculations show that for temperatures up to several times T_c the assumption of an ideal gas is a poor approximation
- Transition temperature from Lattice QCD (as of 2011):
  $T_c\ (\mu_b{=}0) = 150 - 165$ MeV