QGP Physics − From SPS to LHC

5. Statistical Hadronization and Strangeness

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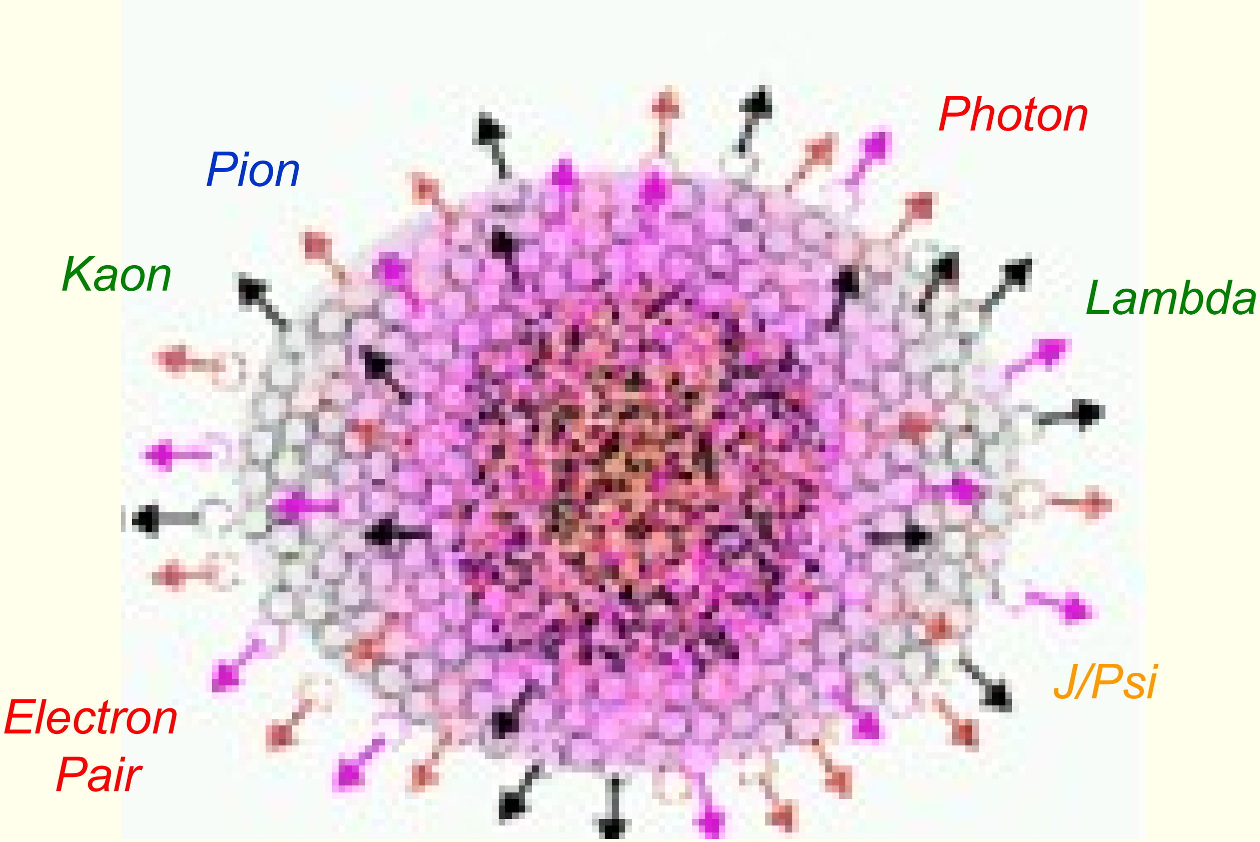
SS 2011

5.1 Hadronization of the nuclear fireball



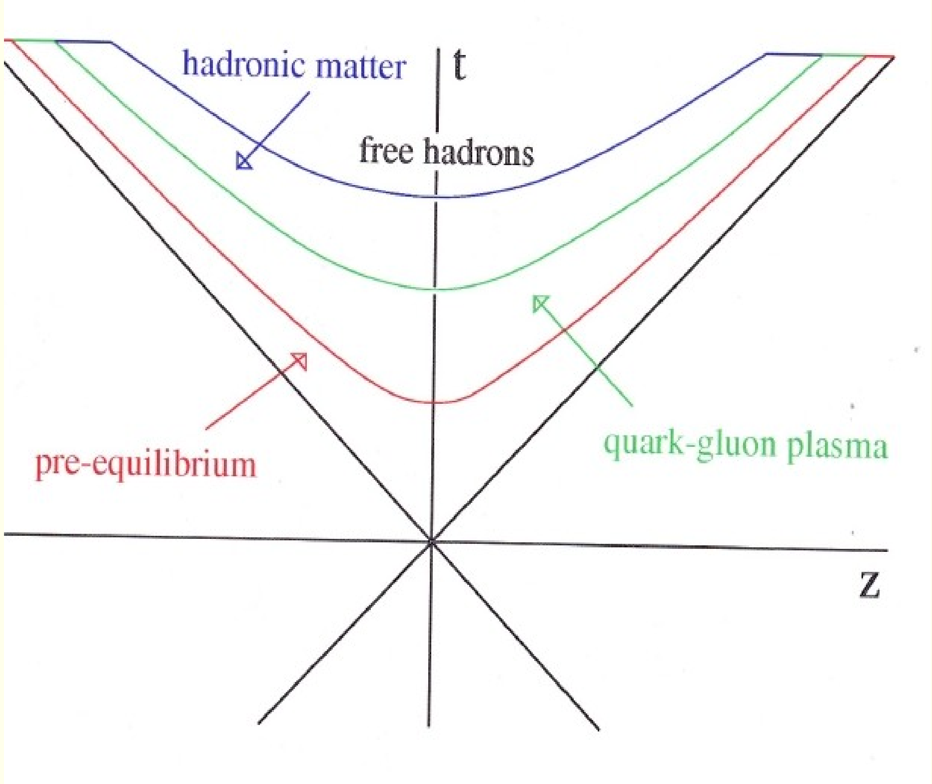
the fireball properties can be determined by measurement of the emitted particles
here as first species: hadrons with up,down,strange constituent quarks

The concept of hadrochemical freeze-out

nuclear fireball evolves
(as sketched in lecture 1)
it cools and expands,
when it hits T_c, it hadronizes
maybe cools and expands further

and finally falls apart
when mean free path large as
compared to interparticle distance
"kinetic" or **"thermal freeze-out"**
momentum distributions are frozen in
no more elastic scattering

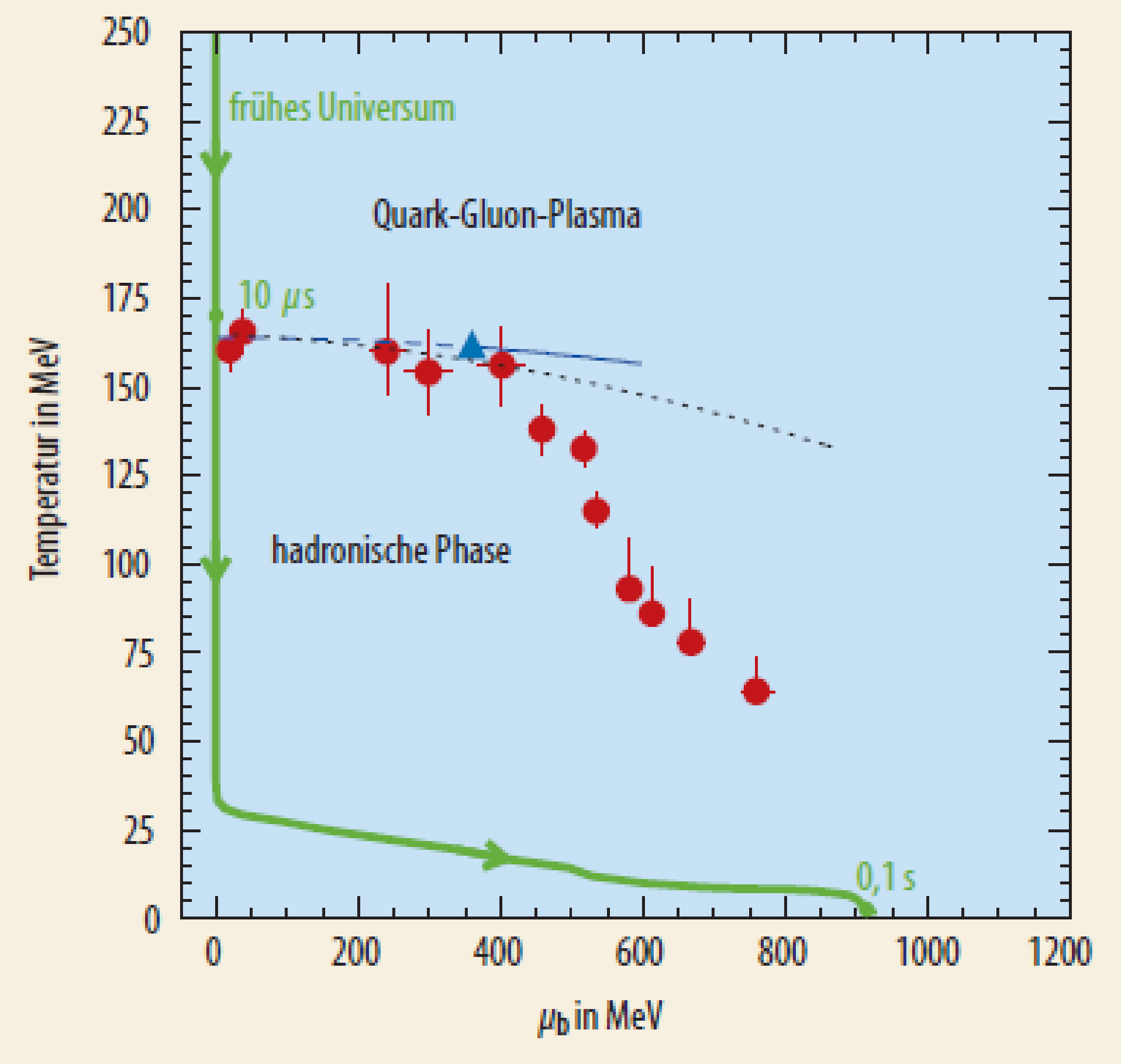
"chemical" or **"hadrochemical freeze-out"**: abundancies of hadrons are frozen in – no more inelastic scattering



Note: chemical freeze-out can happen together with thermal freeze-out or before
the duration of these freeze-out processes if a priori not known

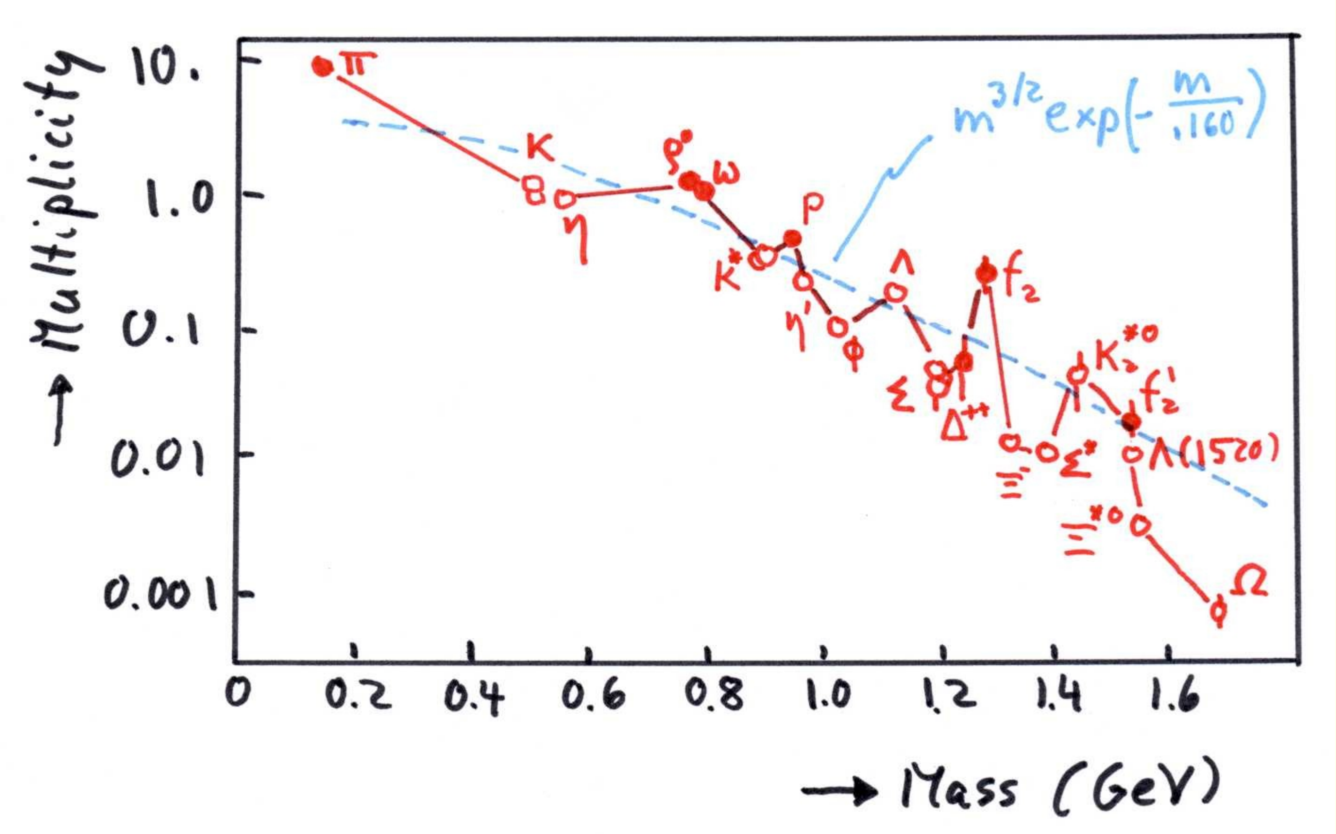
In the early universe freeze-out happened after order of 0.1 s

P.Braun-Munzinger, J. Wambach,
Rev. Mod. Phys. 81 (2009)1031



5.1.1 Hadron production in elementary collisions?

hadron production in e+e- collisions at $\sqrt{s}$ = 91.2 GeV (LEP)

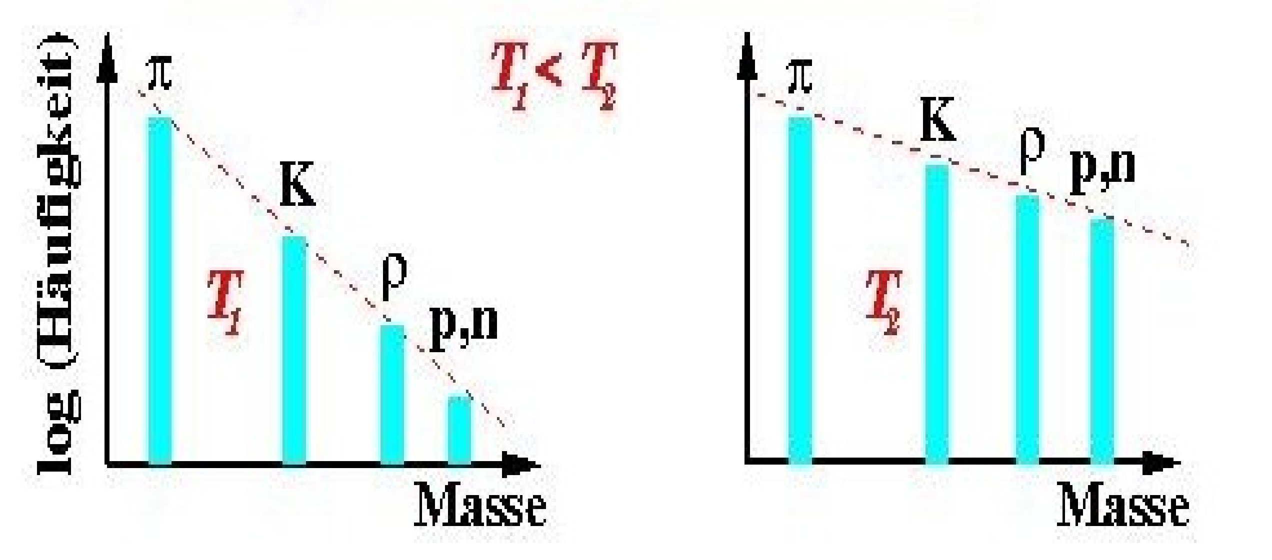


general trend: exponential decrease with mass

in addition: all hadrons with strange valence quarks produced less abundantly
“strangeness suppression”

Thermal energy leads to production of hadrons

equivalence of energy and mass



assume phase space is filled thermally (Boltzmann) at hadronization:
abundance of hadron species

$$\propto \mathrm{m}^{3/2}\exp(-\mathrm{m}/\mathrm{T})$$

determined by temperature (and density) at time of production of hadrons = hadronization

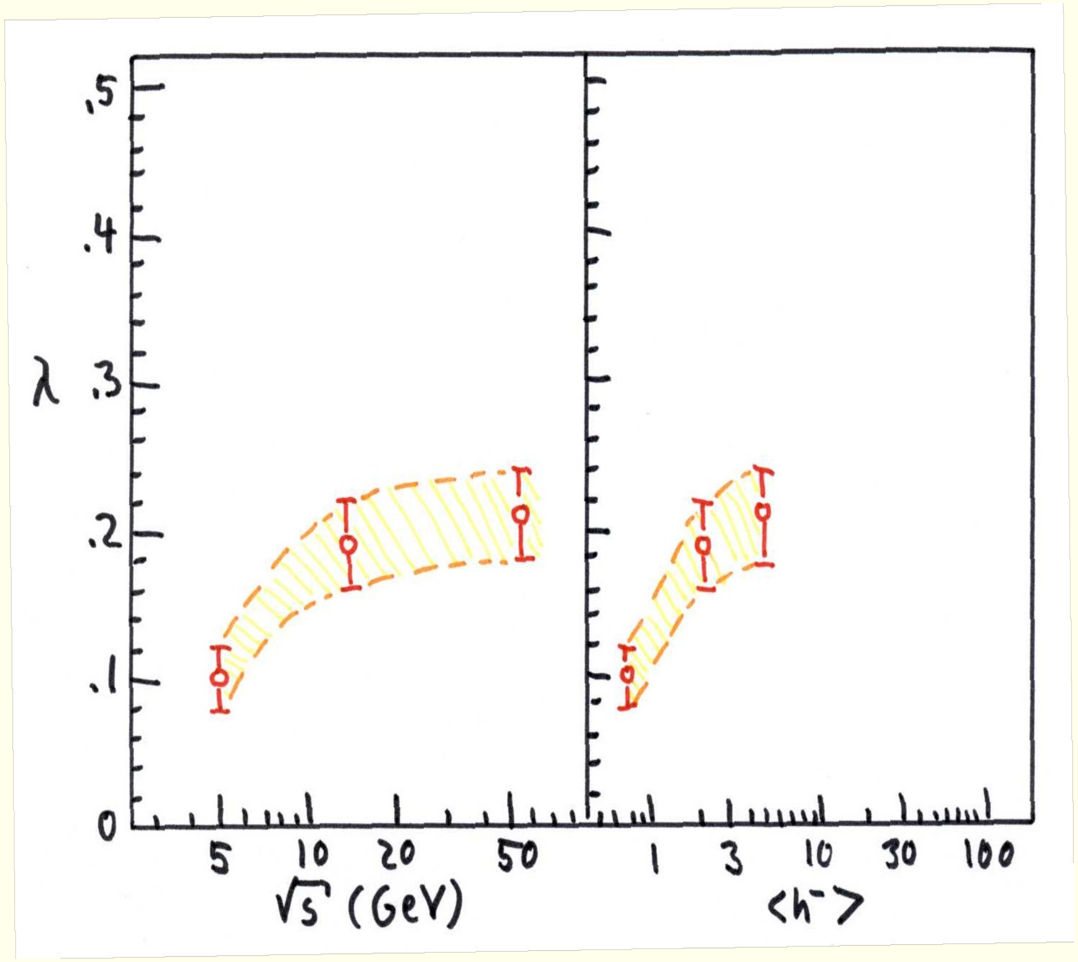
strangeness suppression in hadron-hadron and e+e- collisions

the general exponentially falling trend is superimposed by a characteristic suppression of all hadrons with valence strange quarks

quantified by the so-called Wroblewski factor:

$$\lambda = \frac{2\mathrm{s\bar{s}}}{\mathrm{u\bar{u}} + \mathrm{d\bar{d}}}$$

estimate from measured yields of hadrons the primary yields (before strong decays) and count the valence quarks



A.Wroblewski, Acta Phys. Pol. B16 (1985) 379

5.1.2 Hadron production in high energy heavy ion collisions

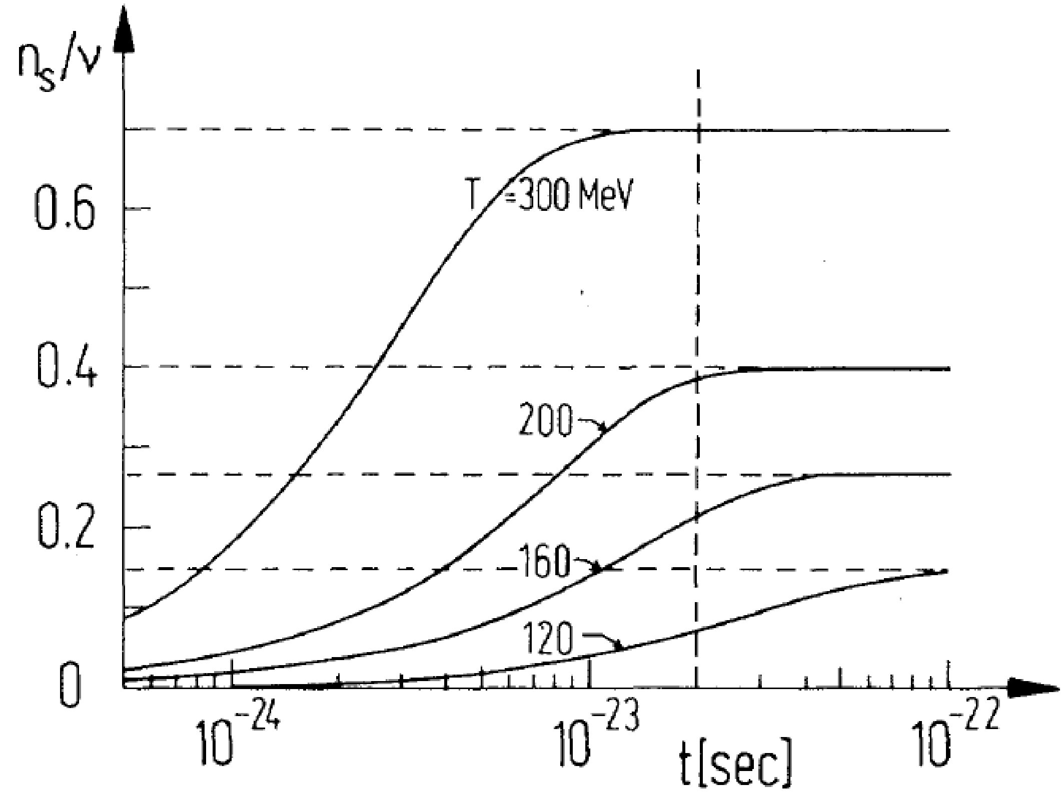
Expectation for strangeness production in heavy ion collisions where QGP is produced:

in QGP strangeness gets into equilibrium on a fast time scale

J. Rafelski, B. Mueller,
Phys. Rev. Lett. 48 (1982) 1066

there should be more strangeness in heavy ion collisions than in elementary collisions if a QGP is formed

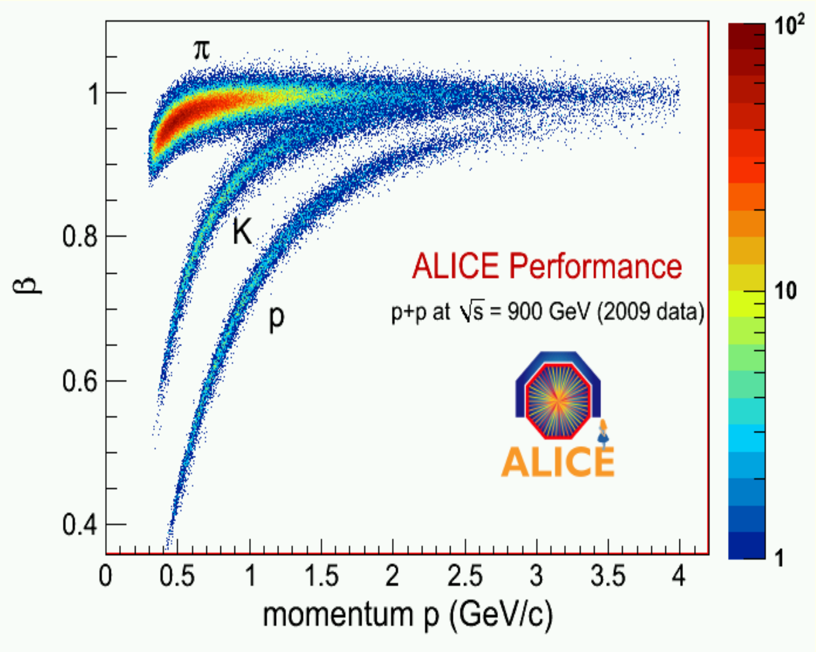
enhanced production of strange hadrons one of the earliest predicted signature of QGP

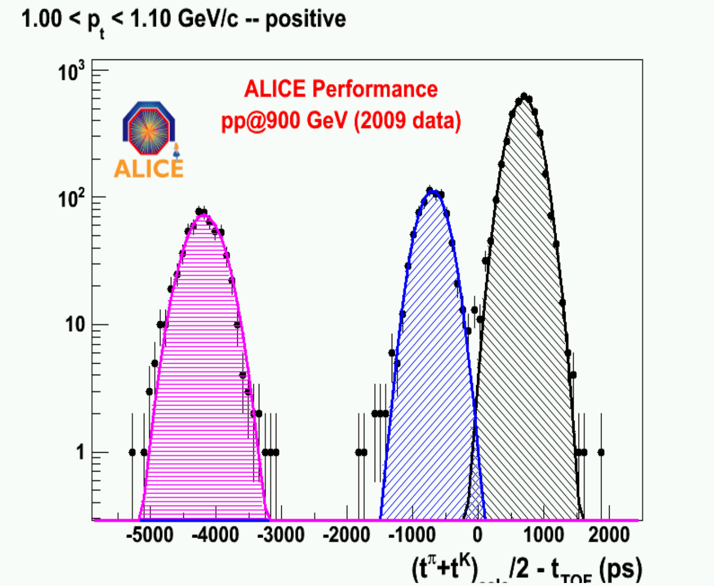


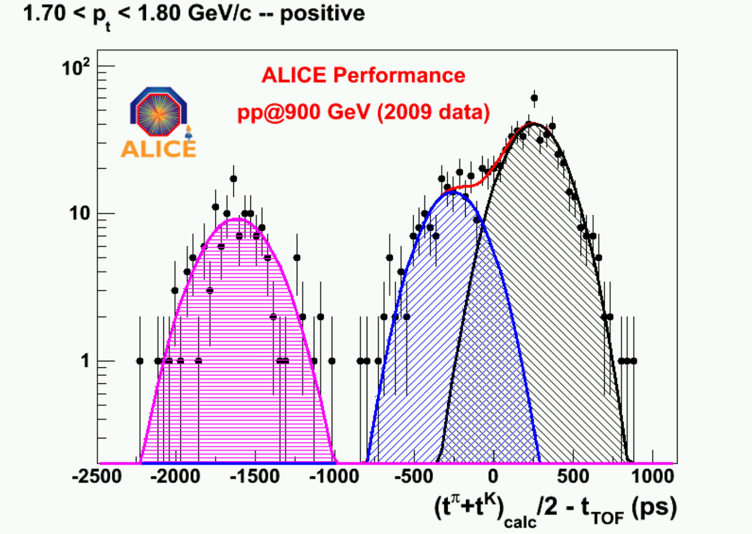
ratio of strange quark to baryon number abundance in a QGP for various temperatures

How to measure production yields of identified hadrons

Identification via time-of-flight plus momentum measurement:





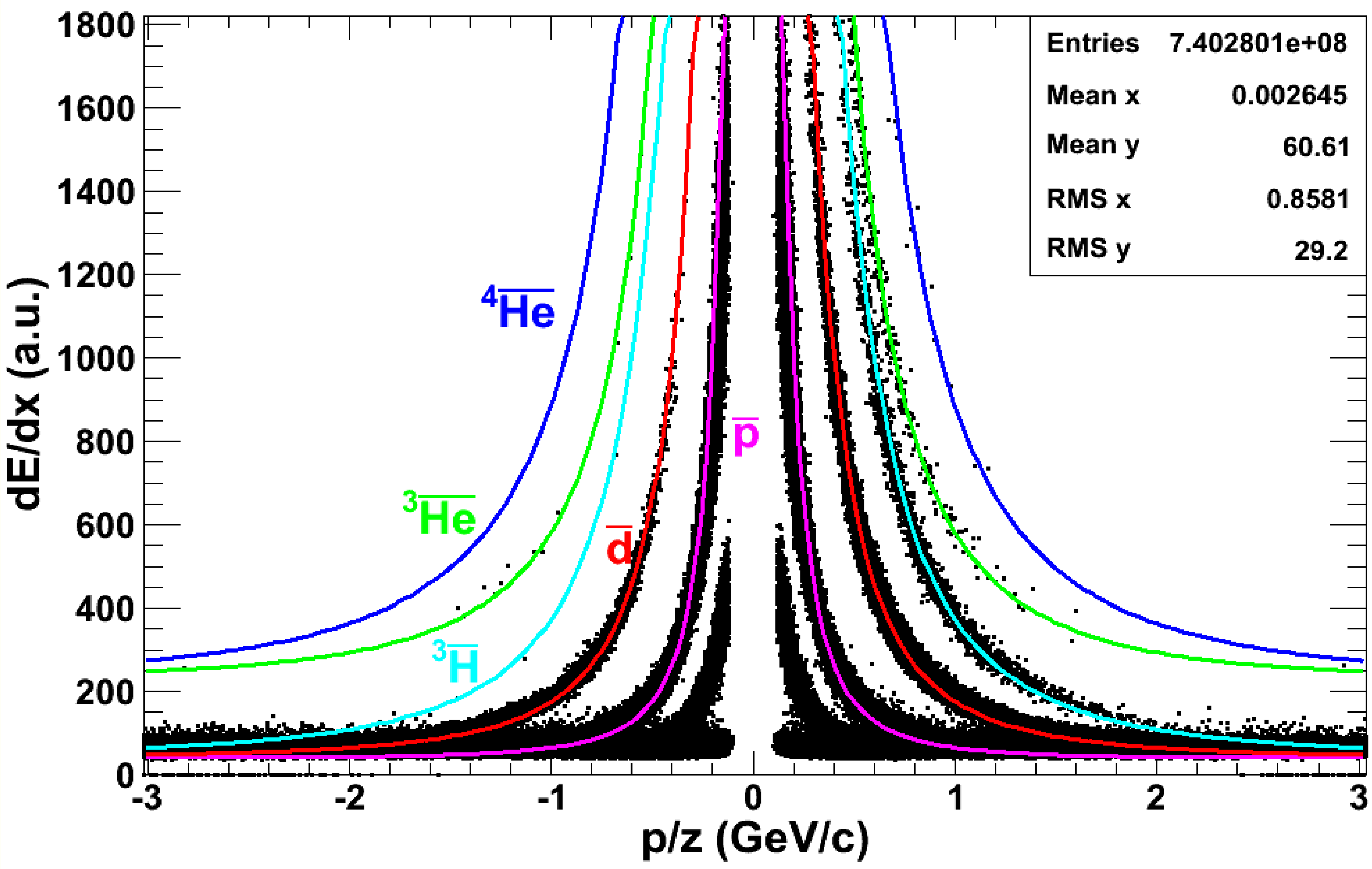


works well as long at β/c < 1, but then fades quickly

How to measure production yields of identified hadrons

Identification via specific energy loss:

example: ALICE TPC 150 space points per track



How to measure production yields of identified hadrons

Identification via invariant mass of decay products (see lecture 2)

$$\begin{aligned} M^2 &= \left[\begin{pmatrix} E_1 \\ \vec{p}_1 \end{pmatrix} + \begin{pmatrix} E_2 \\ \vec{p}_2 \end{pmatrix}\right]^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 \\ &= m_1^2 + m_2^2 + 2E_1E_2 - 2\vec{p}_1 \cdot \vec{p}_2 \\ &= m_1^1 + m_2^2 + 2E_qE_2 - 2p_1p_2\cos\vartheta \end{aligned}$$

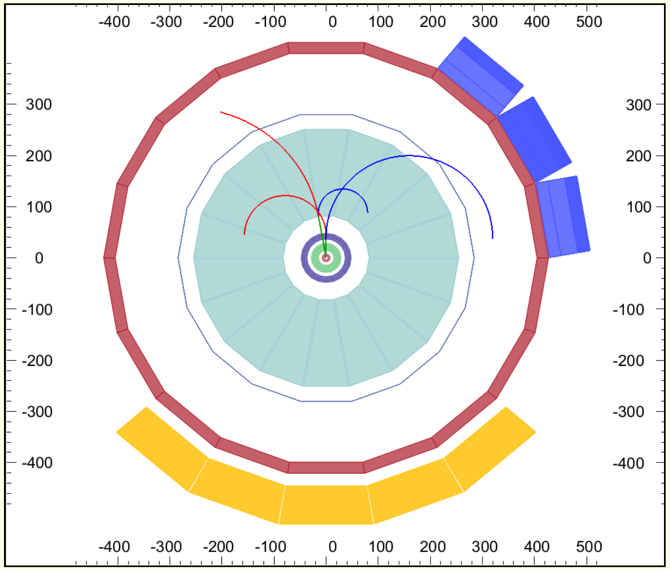
How to measure production yields of identified hadrons

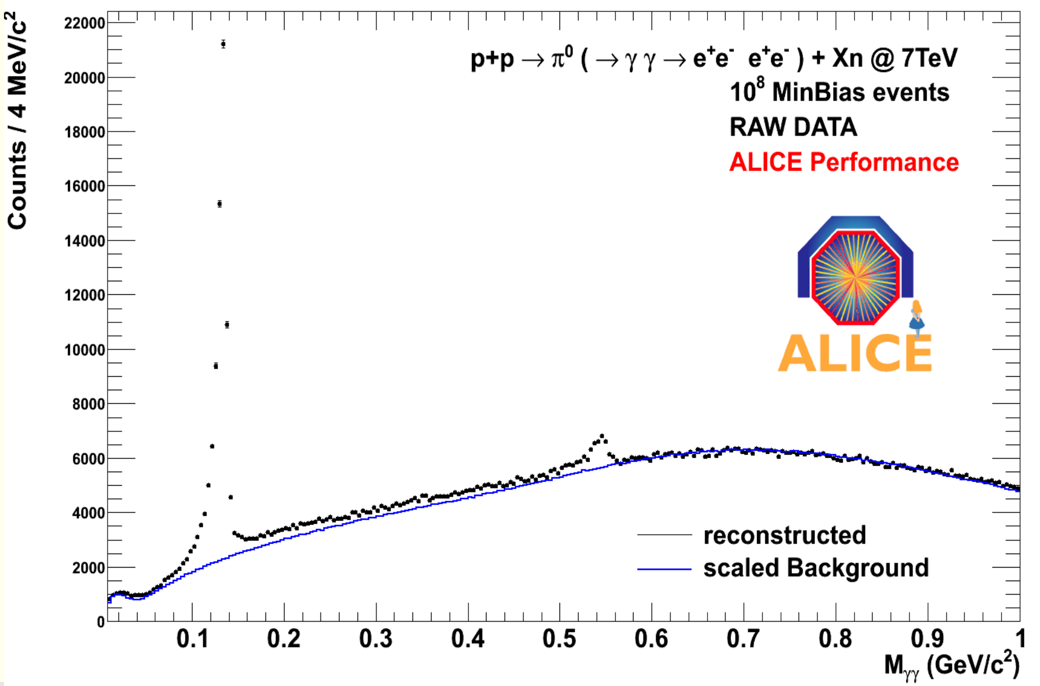
electromagnetic decays:

$$\pi^0 \to \gamma\gamma \qquad \mathrm{m}_{\pi^0} = 0.135\mathrm{GeV},\ \mathrm{BR} = 0.988,\ c\tau = 25.1\,\mathrm{nm}$$
$$\eta \to \gamma\gamma \qquad \mathrm{m}_{\eta} = 0.548\mathrm{GeV},\ \mathrm{BR} = 0.393,\ c\tau = 0.2\,\mathrm{nm}$$

happen practically in the interaction point/target

detect photons in calorimeter
or via e+e- from conversion in detector material





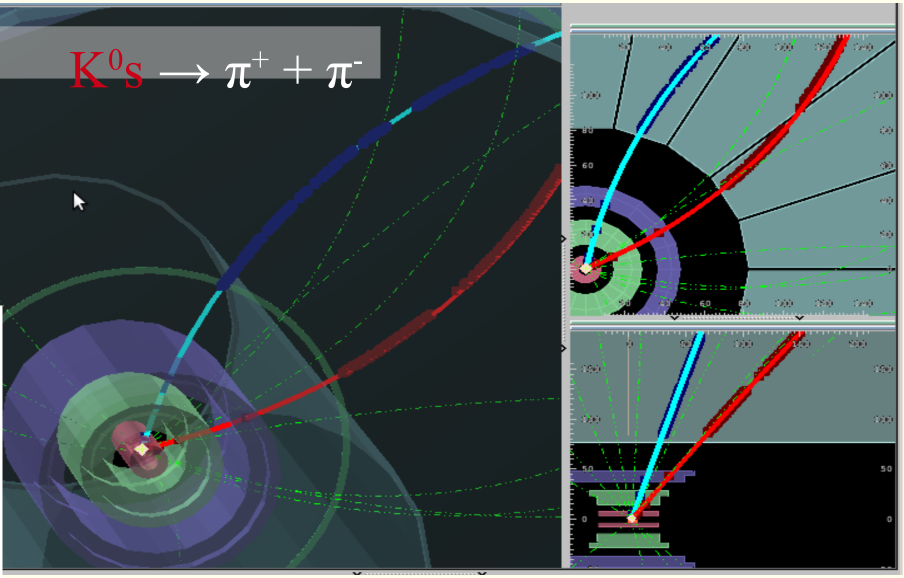
How to measure production yields of identified hadrons

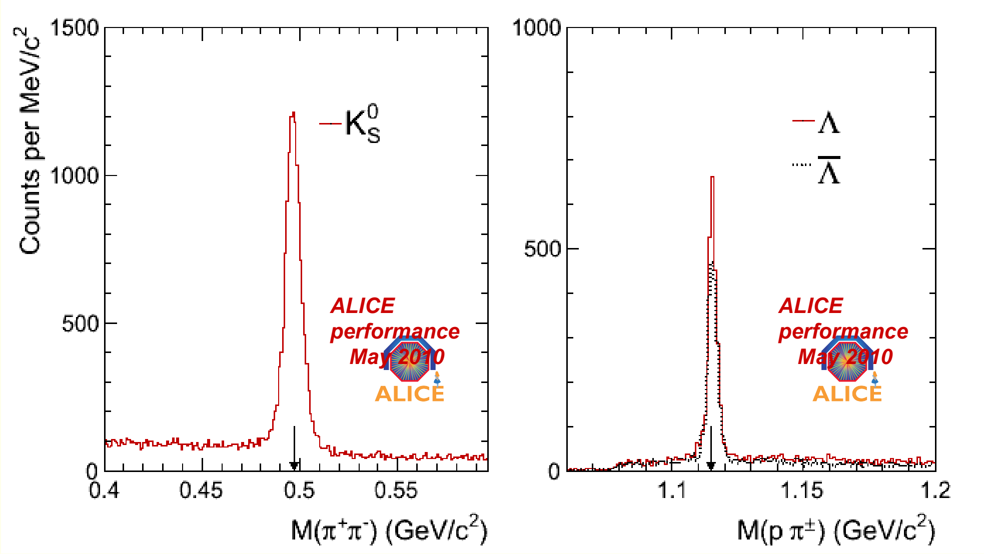
Identification via invariant mass of weak decay products

$$K^0_s \rightarrow \pi^+ + \pi^- \qquad (\mathrm{B.R.}68\%) \qquad c\tau = 2.68\,\mathrm{cm}$$
$$\Lambda \rightarrow p + \pi^- \qquad (\mathrm{B.R.}64\%) \qquad c\tau = 7.89\,\mathrm{cm}$$

look for secondary decay vertex of a neutral object a few 10 cm away from interaction point

works up to very high momentum!



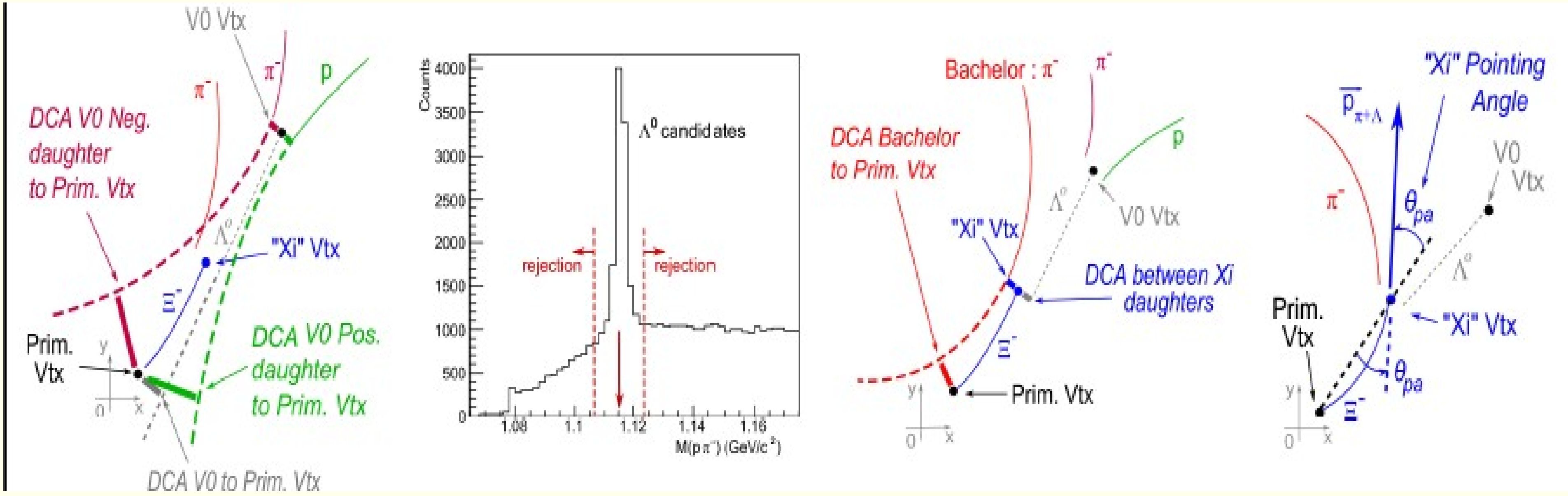


How to measure production yields of identified hadrons

Identification via invariant mass of weak decay products

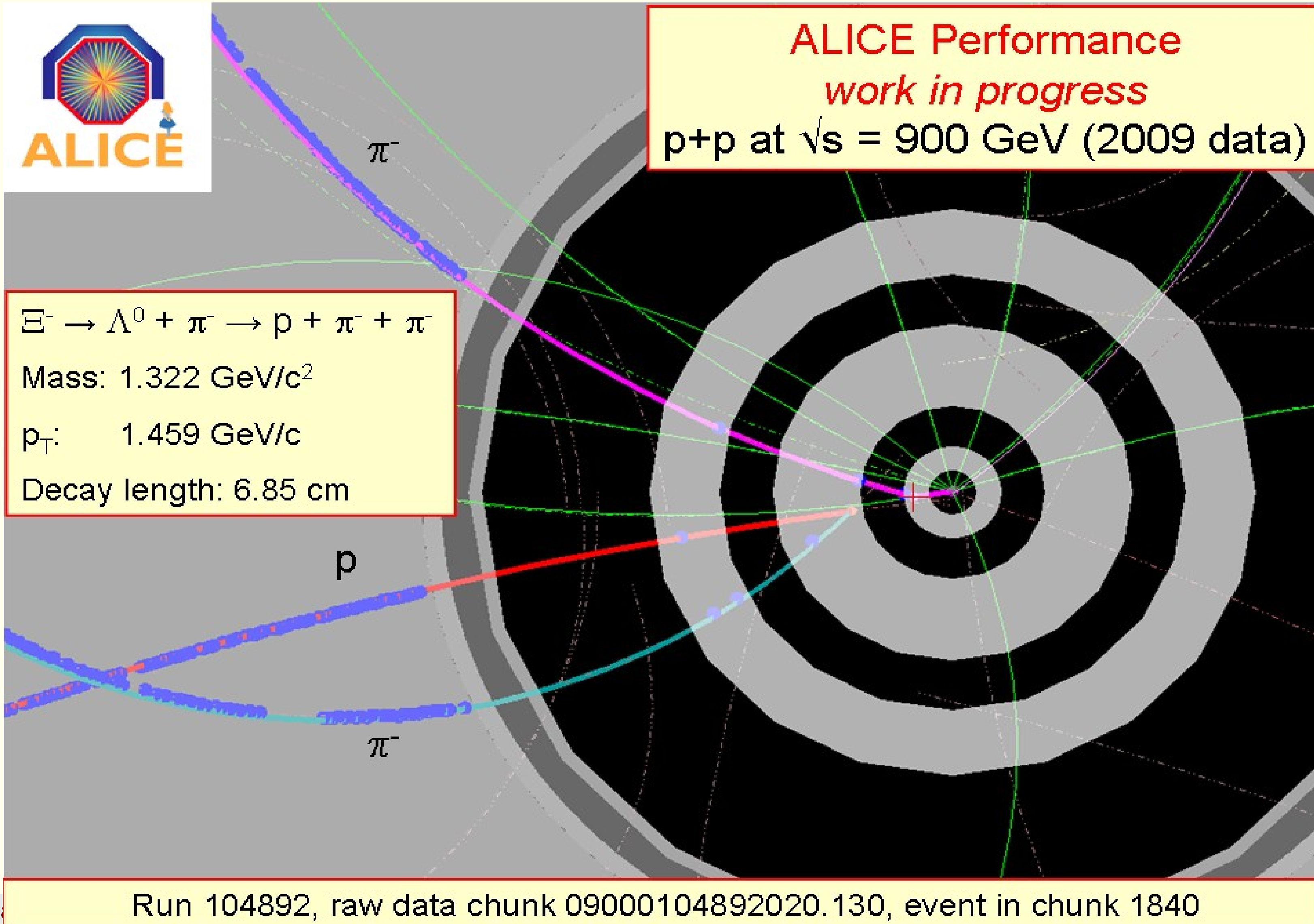
$$\left.\begin{aligned} \Xi^- &\to \Lambda^0 + \pi^- \to p^+ + \pi^- + \pi^- && \text{(B.R. 63.9\%)} \\ \bar{\Xi}^+ &\to \bar{\Lambda}^0 + \pi^+ \to \bar{p}^- + \pi^+ + \pi^+ && \text{(B.R. 63.9\%)} \end{aligned}\right\} \; c\tau = 4.9\,\mathrm{cm}$$

$$\left.\begin{aligned} \Omega^- &\to \Lambda^0 + K^- \to p^+ + \pi^- + K^- && \text{(B.R. 43.3\%)} \\ \bar{\Omega}^+ &\to \bar{\Lambda}^0 + K^+ \to \bar{p}^- + \pi^+ + K^+ && \text{(B.R. 43.3\%)} \end{aligned}\right\} \; c\tau = 2.5\,\mathrm{cm}$$



How to measure production yields of identified hadrons

Identification via invariant mass of weak decay products



ALICE Performance

work in progress

p+p at √s = 900 GeV (2009 data)

$\Xi^- \rightarrow \Lambda^0 + \pi^- \rightarrow p + \pi^- + \pi^-$

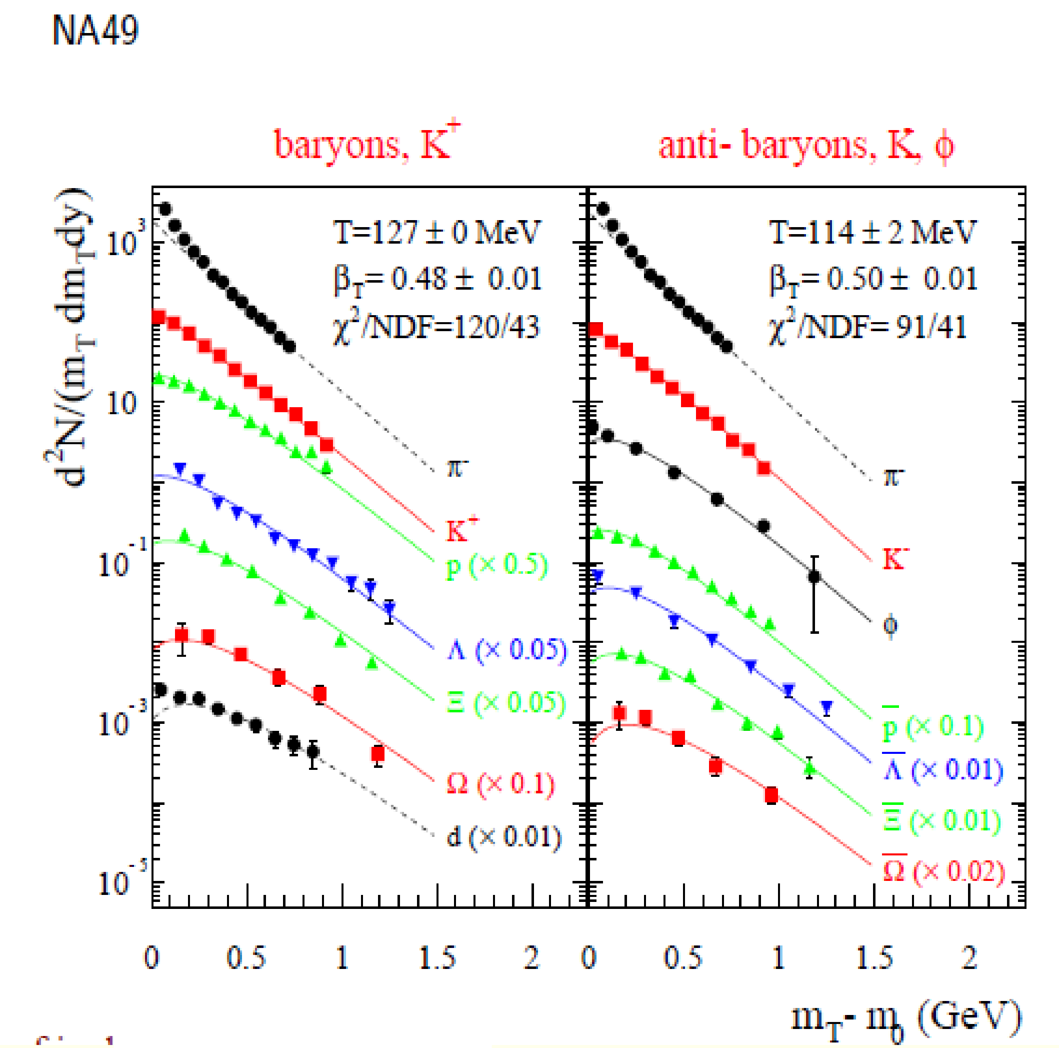
Mass: 1.322 GeV/c^2

p_T: 1.459 GeV/c

Decay length: 6.85 cm

Run 104892, raw data chunk 09000104892020.130, event in chunk 1840

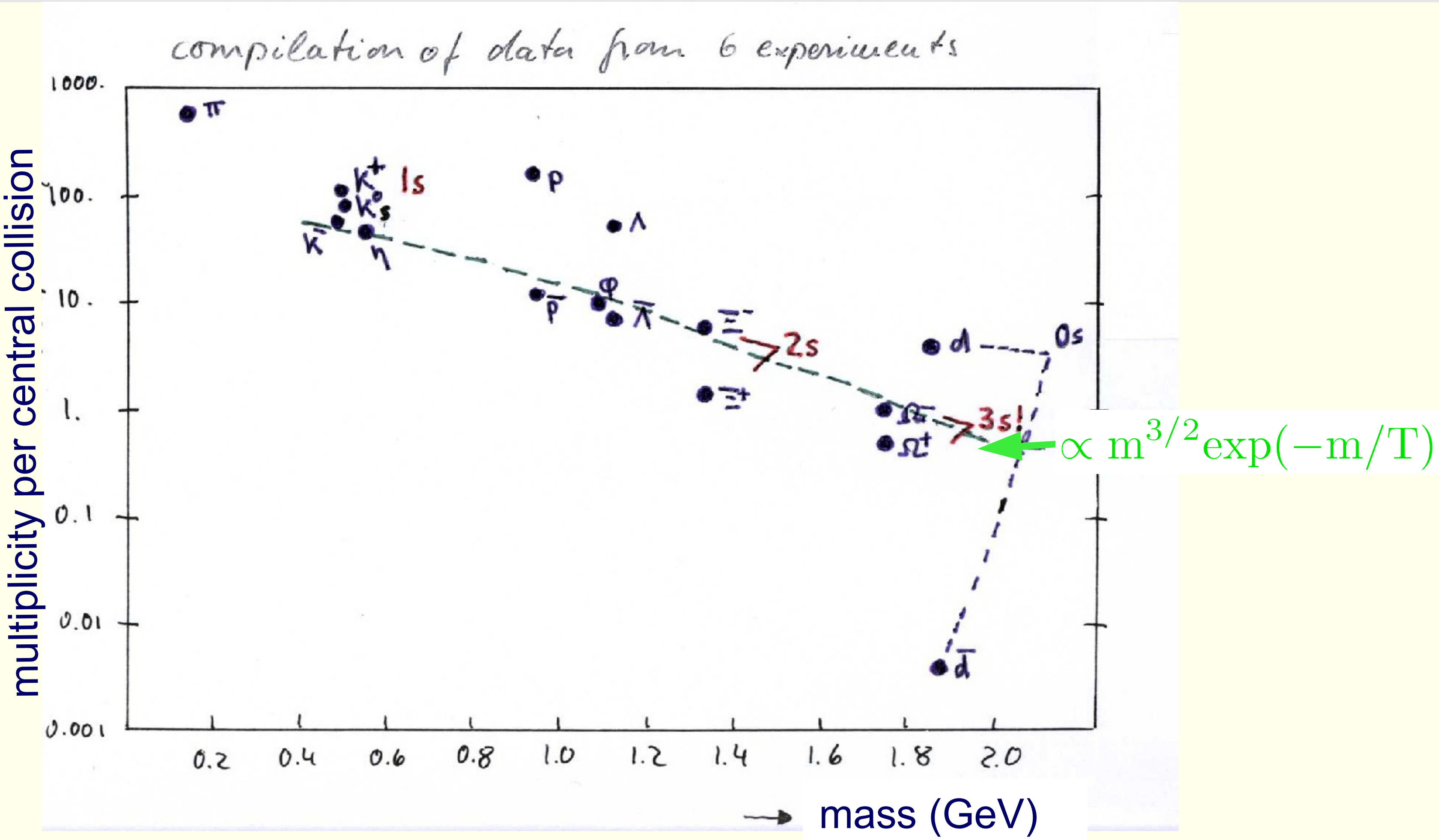
hadron production in central PbPb collisions at the CERN SPS



measure p_t spectra and integrate/extrapolate yield over all values of p_t from 0 to infinity

between 5 different experiments a comprehensive data set for 158 A GeV PbPb collisions

First look at particle multiplicities for CERN SPS PbPb central collisions



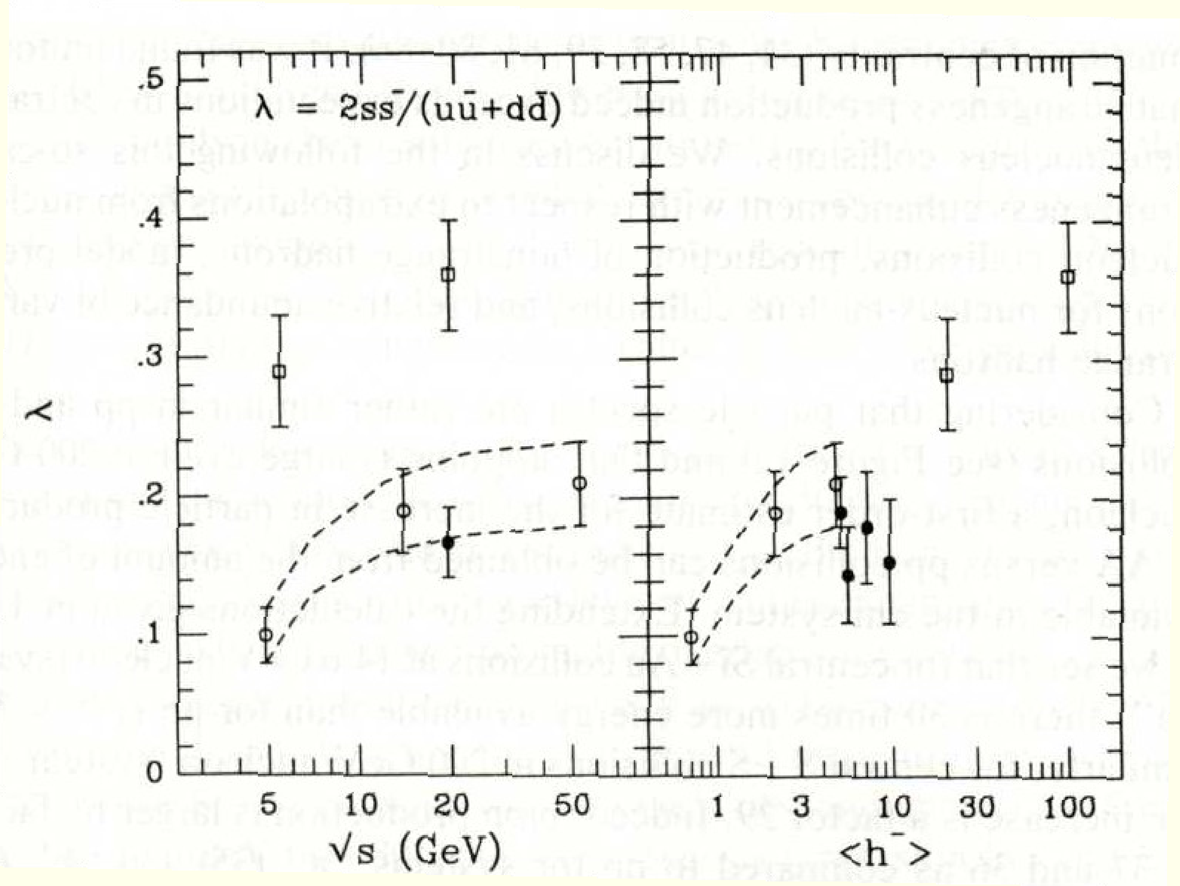
$\propto \mathrm{m}^{3/2}\exp(-\mathrm{m}/\mathrm{T})$

Measured multiplicities can be understood under assumption that all particles are produced simultaneously at temperature of 170 MeV

(will come to characteristic splitting of baryons and antibaryons later)

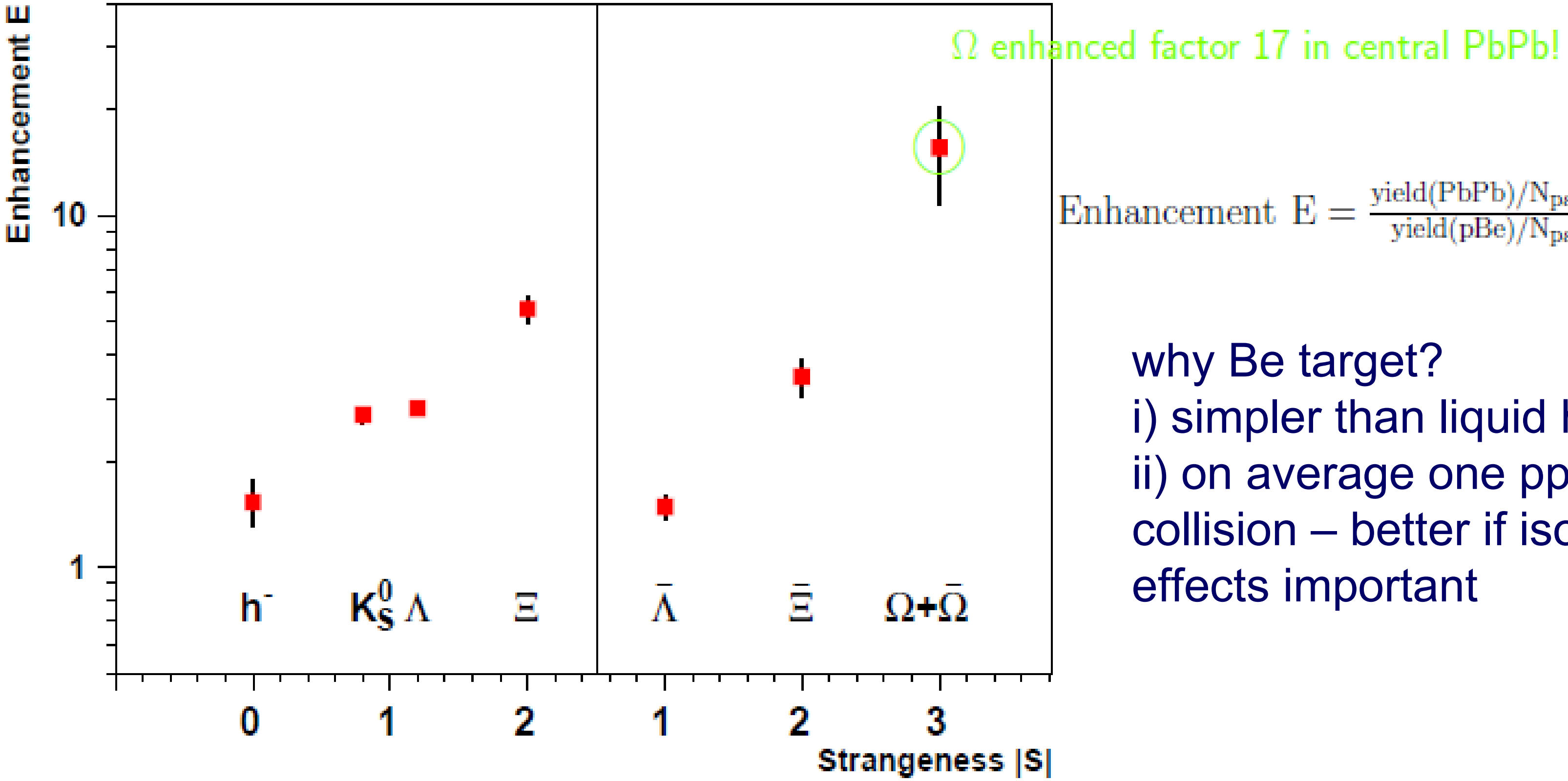
First indication from SiSi and SS collisions:

In nuclear collisions about factor of 2 more strangeness produced than in pp



Strangeness enhancement in PbPb collisions relative to pp

WA97, 158 A GeV/c Pb + Pb Collisions, Phys. Lett. B449 (1999) 401



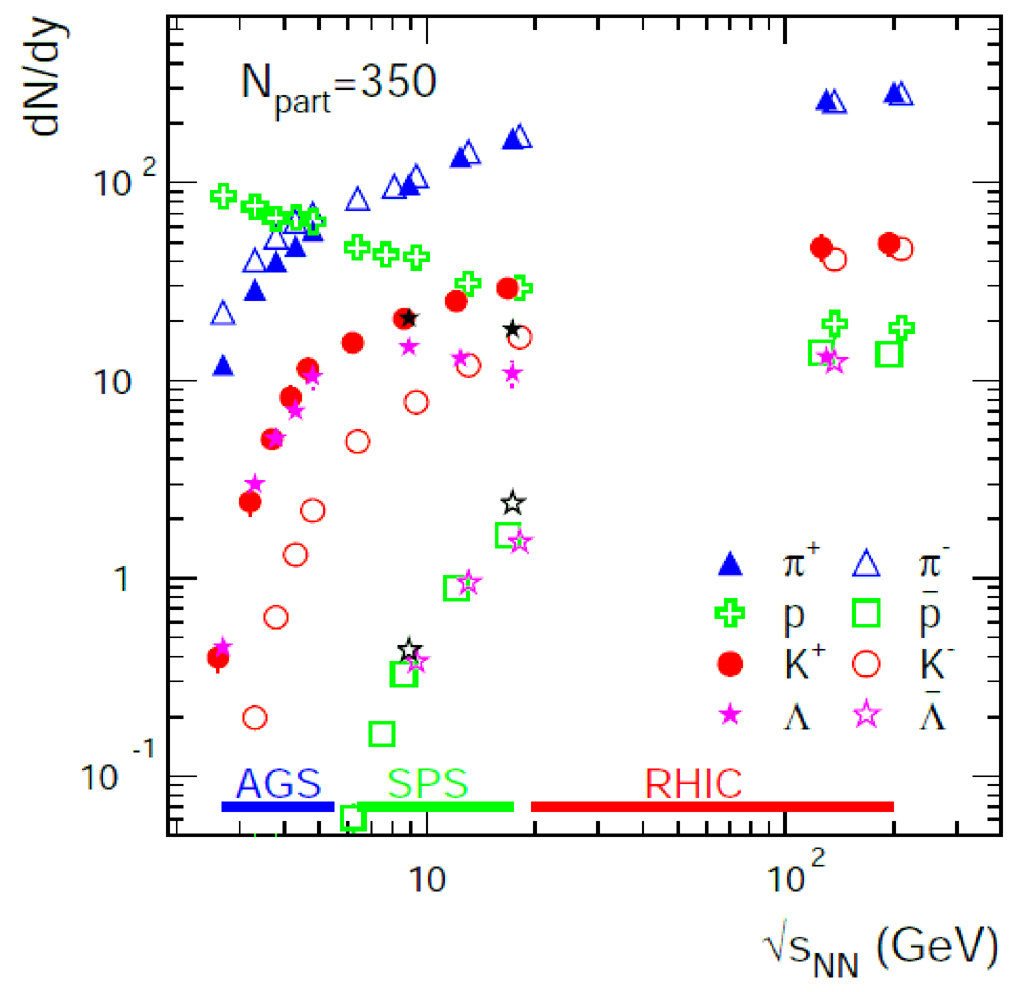
$$\text{Enhancement } E = \frac{\text{yield(PbPb)}/N_{\text{part}}(\text{PbPb})}{\text{yield(pBe)}/N_{\text{part}}(\text{pBe})}$$

why Be target?
i) simpler than liquid hydrogen
ii) on average one pp or pn collision – better if isospin effects important

general feature: in high energy nuclear collisions hadrons with strange quarks produced more abundantly than in pp collisions

the enhancement grows with the number of strange valence quarks

Particle production in central AA collisions



a summary of 15 years of experimental research

systematic trends with beam energy
mesons rise and level off
baryons drop
antibaryons rise steeply

can we understand all of these?

5.2 Statistical model description of hadron yields

5.2.1 Choice of statistical ensemble

Grand Canonical Ensemble (GC): in large system, with large number of produced particles, conservation of additive quantum numbers (B, S, I_3) can be implemented on average by use of chemical potential μ

→ **asymptotic realization of exact canonical approach**
much simpler to compute

Canonical Ensemble (C): in small system, with small particle multiplicity, conservation laws must be implemented locally on event-by-event basis (Hagedorn 1971, Shuryak 1972, Rafleski/Danos 1980, Hagedorn/Redlich 1985)

severe phase space reduction for particle production **“canonical suppresssion”**

Results of C and GC can be related in a simple way: (Tounsi/Redlich 2001)

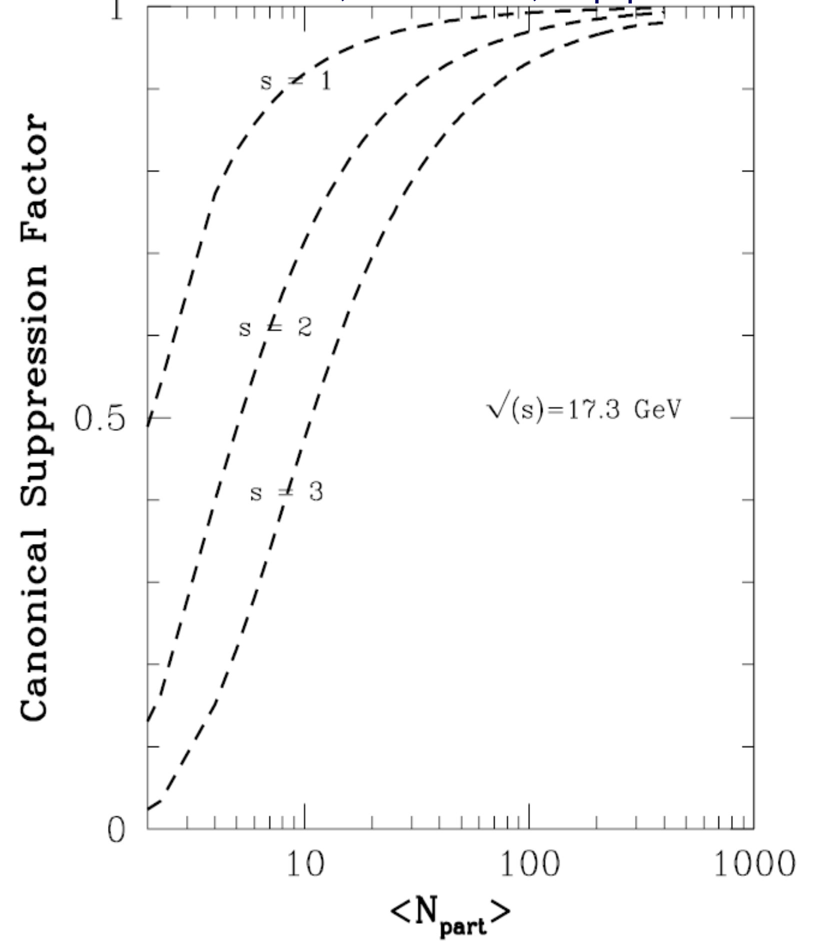
$$\langle \mathrm{N_K} \rangle^{\mathrm{C}} = \langle \mathrm{N_K} \rangle^{\mathrm{GC}} \underbrace{\frac{\mathrm{I_1}(2\langle \mathrm{N_K}\rangle^{\mathrm{GC}})}{\mathrm{I_0}(2\langle \mathrm{N_K}\rangle^{\mathrm{GC}})}}_{\mathrm{F_{CS}}}$$

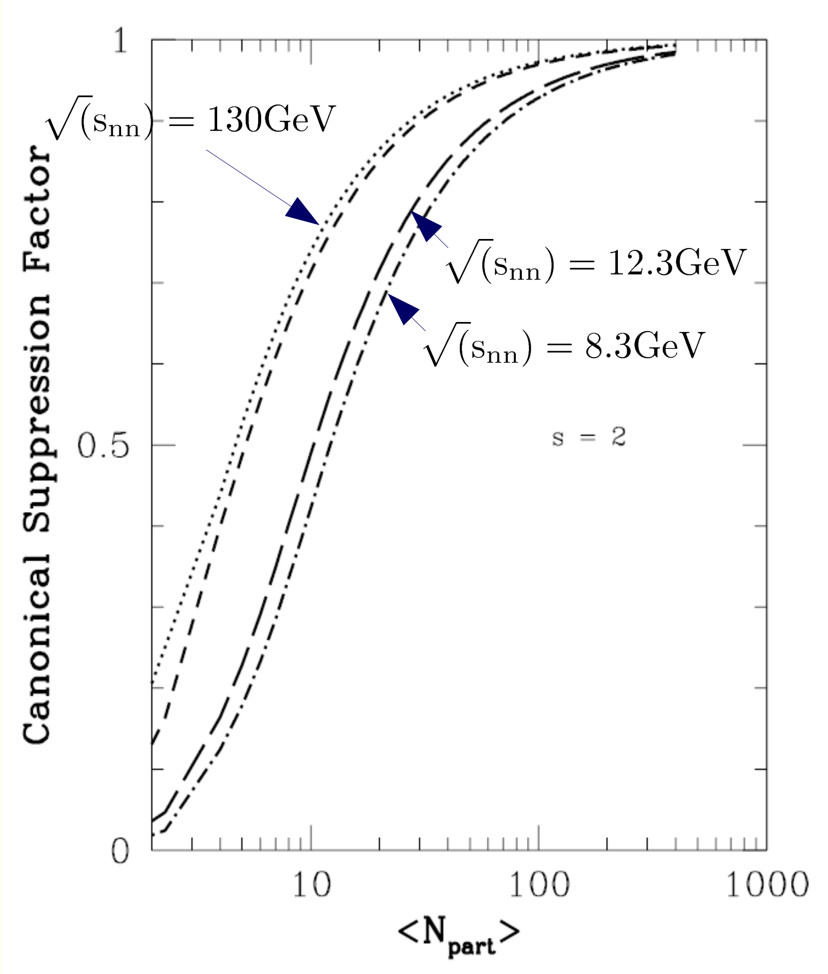
here 'K' stands generically for all hadrons with S = -1
and analogously for S = -2 and S = -3

$$\mathrm{F_{CS,2}} = \frac{\mathrm{I_2}}{\mathrm{I_0}} \qquad \mathrm{F_{CS,3}} = \frac{\mathrm{I_3}}{\mathrm{I_0}}$$

Difference between computations in the canonical and grand canonical ensemble

A. Tounsi, K. Redlich, hep-ph/0111159





centrality of the collision

already for moderately central PbPb collisions (100 of possible 416 nucleons in overlap region) deviations small – 10% level for SPS energy

C relevant in:

- low energy HI collisions (Cleymans/Redlich/Oeschler 1998/1999)
- very peripheral HI collisions (Hamieh, Redlich, Tounsi 2000)
- high energy hh or e+e- collisions (Becattini/Heinz 1996/1997)
- considering heavy quarks in HI collisions (see later)

5.2.2 Grand canonical ensemble and application to data from high energy heavy ion collisions

partition function: $\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$

particle densities: $n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp((E_i - \mu_i)/T) \pm 1}$

for every conserved quantum number there is a chemical potential:

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

but can use conservation laws to constrain V, μ_S, μ_{I_3}

baryon number: $\mathrm{V} \sum_{\mathrm{i}} \mathrm{n_i B_i} = \mathrm{Z + N}$ $\rightarrow \mathrm{V}$

strangeness: $V \sum_i n_i S_i = 0$ $\rightarrow \mu_S$

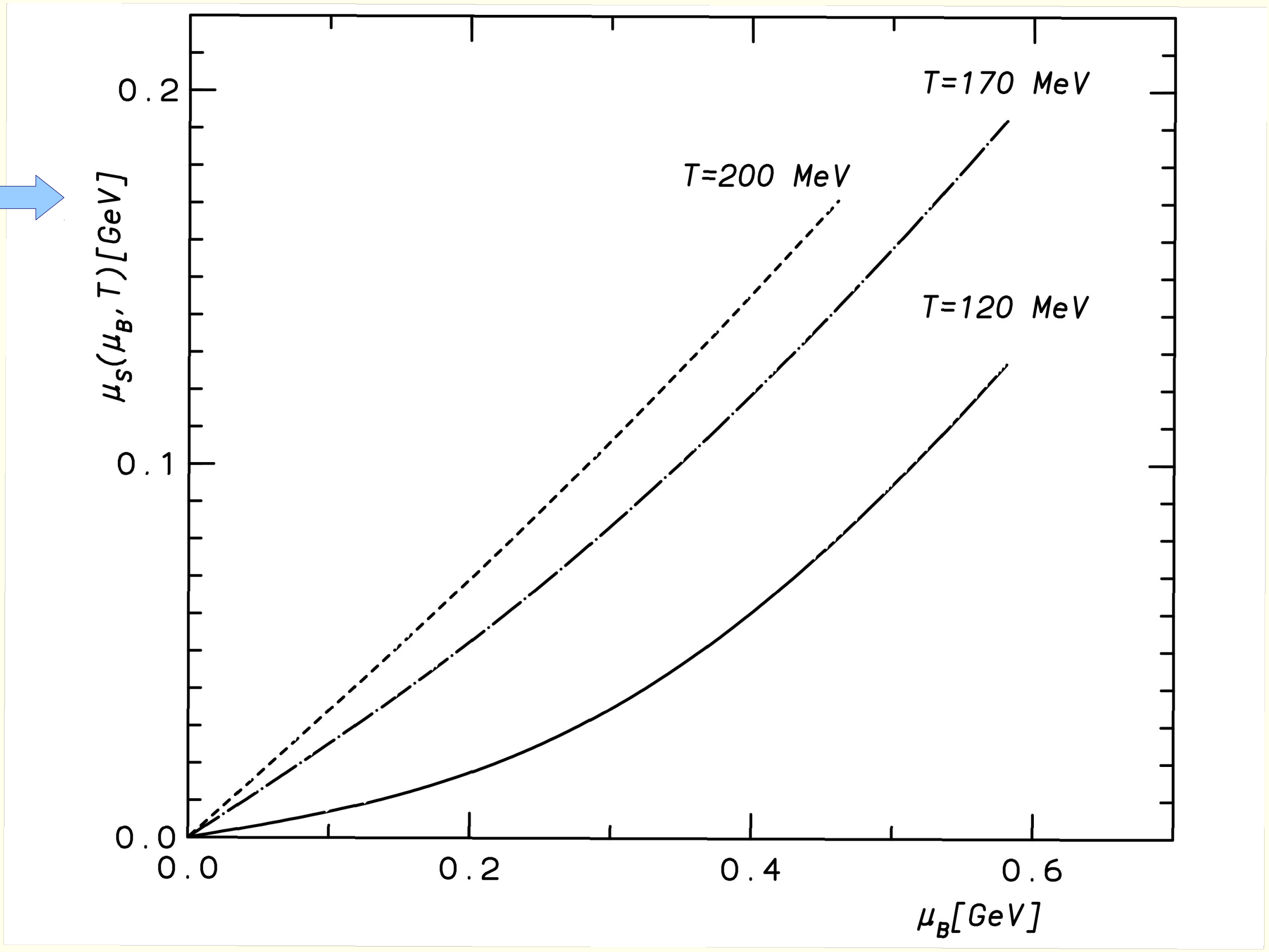
charge: $V \sum_i n_i I_i^3 = \frac{Z - N}{2}$ $\rightarrow \mu_{I_3}$

only 2 free parameters left ➡

Fit at each energy provides values for T and μ_b

Dependence of μ_s on T,μ_b

for every pair of T, μ_b
there is only one value of μ_s that ensures total strangeness S = 0
not a free parameter at all



Some technical details:

van der Waals type interaction via excluded volume correction
a volume is assigned to each hadron and densities have to be corrected
good guess: r=0.3 fm to account for hard core nucleon-nucleon repulsion
Thermodynamically consistent treatment following Rischke, Gorenstein, Stöcker, Greiner, 1991

$$p^{excl.}(T,\mu) = p^{id.gas}(T,\hat{\mu}); \qquad \text{with } \hat{\mu} = \mu - v_{eigen}\, p^{excl.}(T,\mu)$$

finite volume correction a la Balian and Bloch

$$f = 1 - \frac{3\pi}{4pR} + \frac{1}{(pR)^2}$$

width of all resonances included by integrating over Breit-Wigner distributions
Weinhold, Friman, Nörenberg, 1996

$$\ln Z_R = N \frac{V d_R}{2\pi^2} T \exp[\mu/T] \int_{s_{min}}^{s_{max}} ds\, s\, K_2(\sqrt{s}/T)\, \frac{1}{\pi} \frac{m_R \Gamma_R}{(s - m_R^2)^2 + m_R^2 \Gamma_R^2}$$

For a review see: Braun-Munzinger, Redlich, Stachel, in QGP3,
R. Hwa ed. (Singapore 2004) 491-599; nucl-th/0304013

Comparison to experimental data

compute primary thermal occupation probability for each particle species

spectrum of hadrons involves for state-of-the-art calculation 426 hadronic species (PDG2008)
beyond 3 GeV mass knowledge still very incomplete (effects see later)

implement all strong decays according to PDG
(example: for T=160 MeV, 80% of all pions come from strong decays)
do experimental data include weak decays? If yes, do same in calculation

compute for a grid of (T,μ_b) χ^2 between statistical ensemble calculation and data
note: ratios of particle yields may have smaller systematic errors
data sets from different experiments may not correspond to exactly the same collisions centrality, correct for this

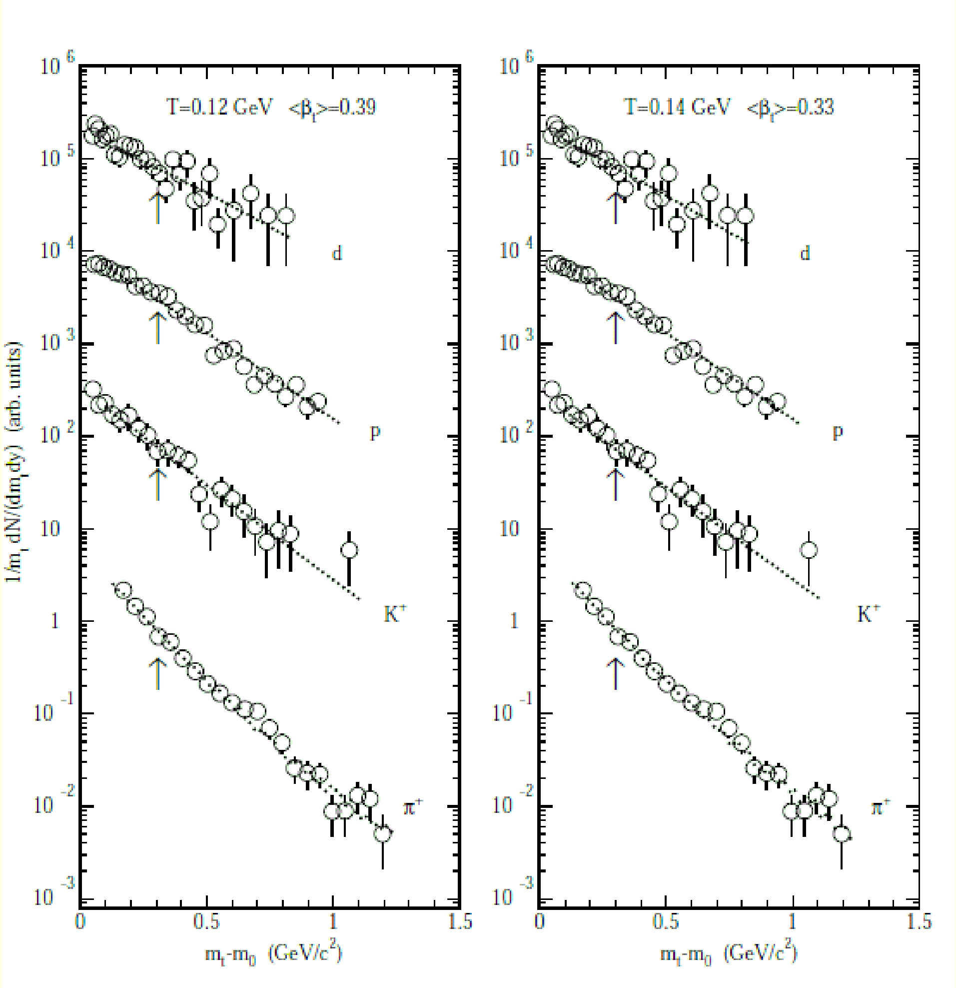
minimize χ^2 to obtain for each beam energy and collision system best set (T,μ_b)

First thermal model results in 1994 for AGS Si+Au data

Integrate hadron spectra over p_t and rapidity

pion spectra exhibit increase at low p_t due to decay of Delta resonance

understanding of spectral shapes in chapter 6
(interplay of temperature, collective expansion and decays)

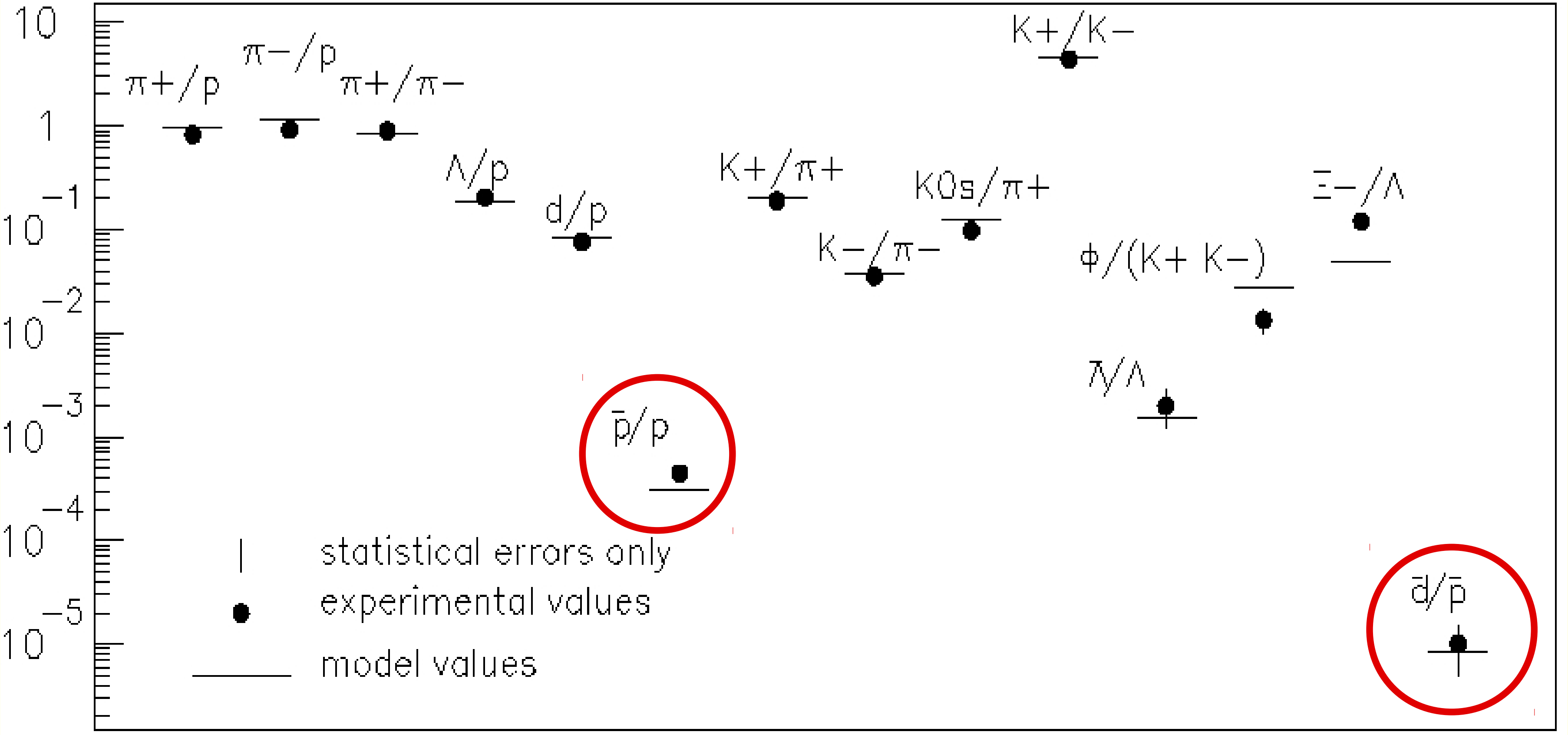


Fit to AGS Data – reproduces yields for strange and nonstrange hadrons down to dbar

14.6 A GeV/c central Si + Au collisions and GC statistical model

T = 120 MeV, μ_b = 540 MeV

P. Braun-Munzinger, J. Stachel, J.P. Wessels, N. Xu, PLB 344 (1995) 43; nucl-th/9410026

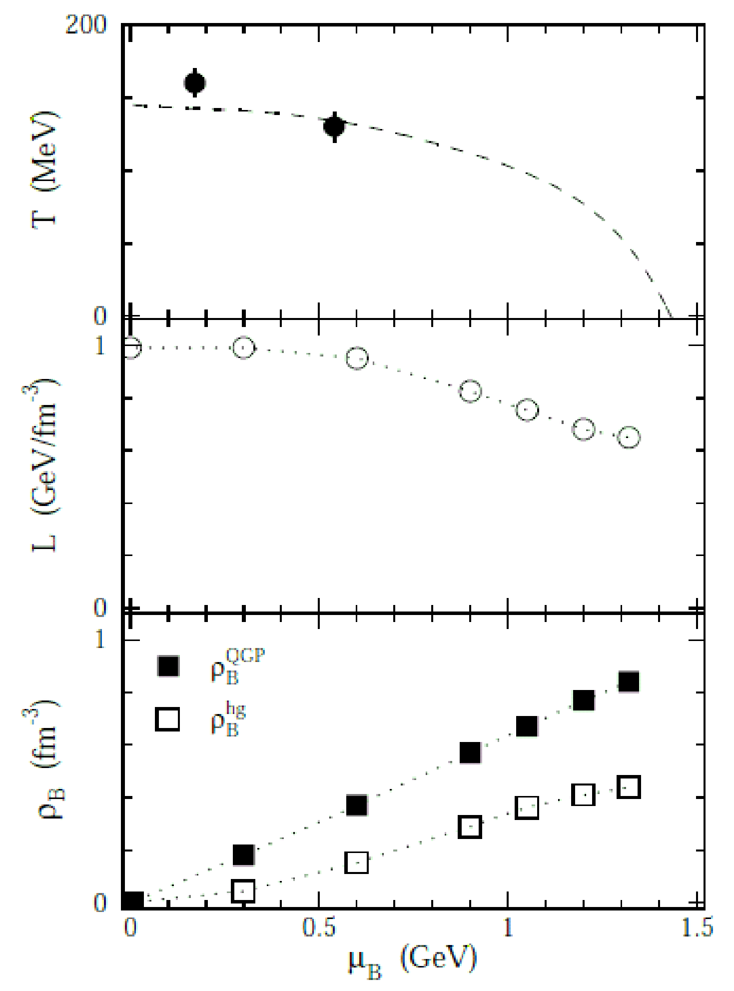


dynamic range: 9 orders of magnitude!

First attempt to establish connection to phase boundary

equation of state from a bag model, transition 1st order by construction (see above)

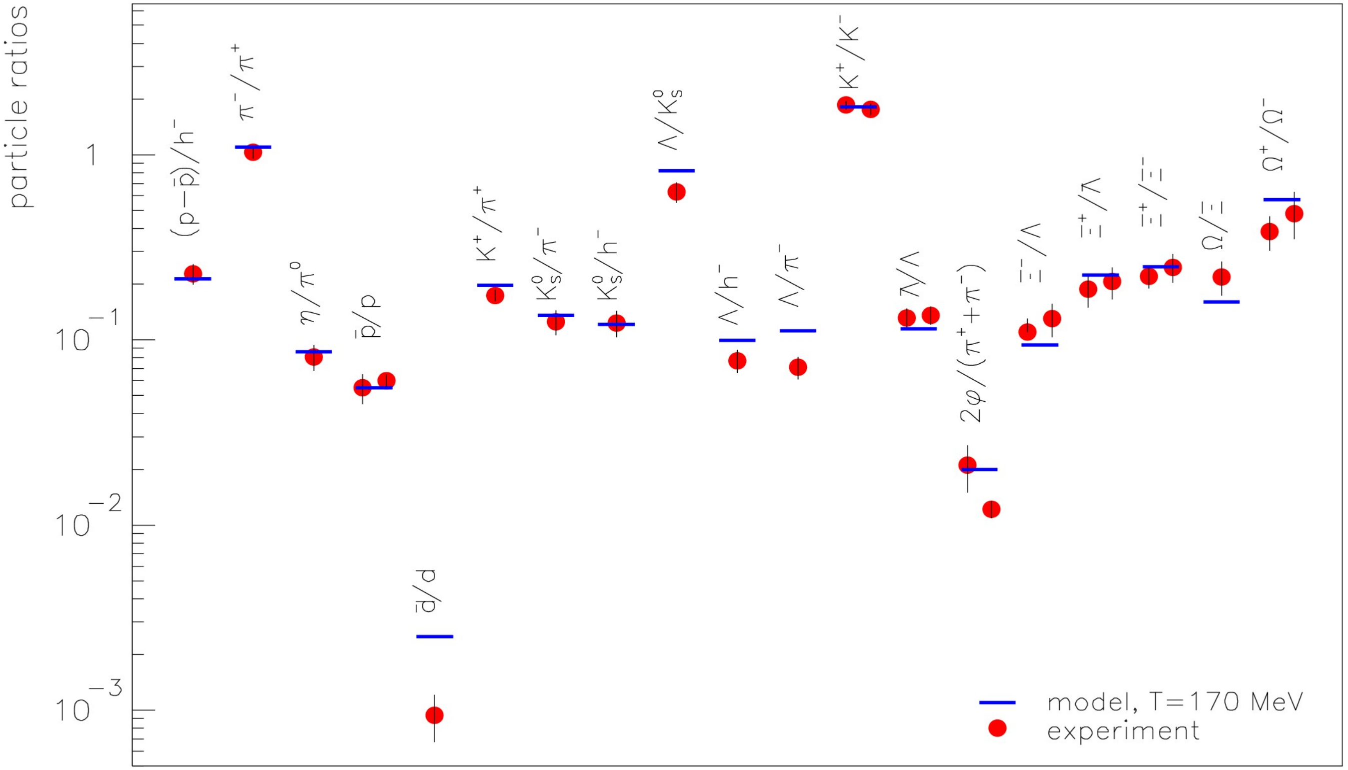
only 2 data points at AGS and SPS looks at this for many more collisions systems in the following



Braun-Munzinger, Stachel, Phys. Lett. B365 (1996) 1

CERN SPS data: 158 A GeV/c Pb Pb collisions

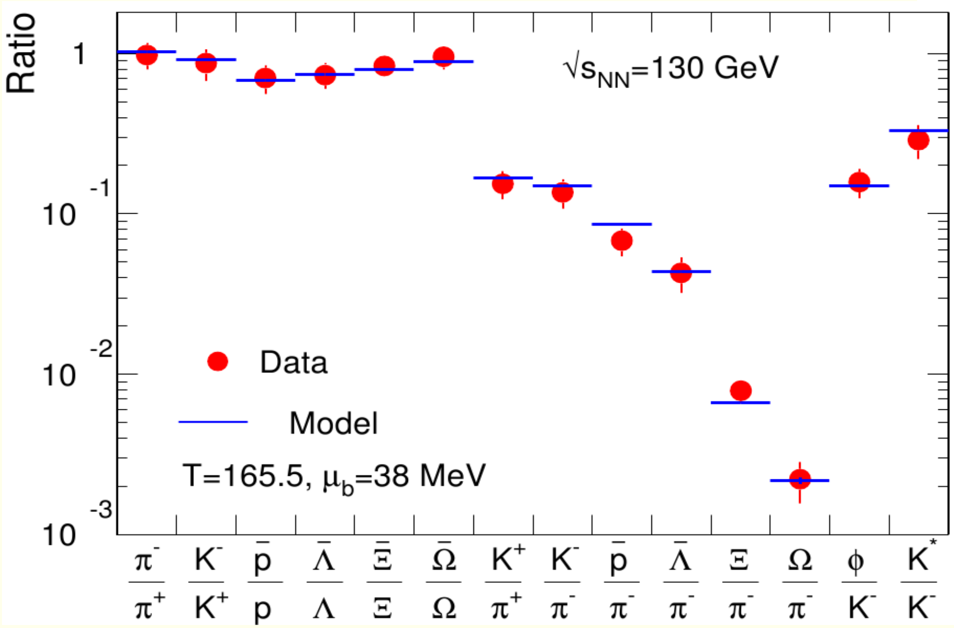
good fit with: $\mathrm{T} = 0.170 \pm 0.005\,\mathrm{GeV} \quad \mu_\mathrm{b} = 0.255 \pm 0.010\,\mathrm{GeV}$



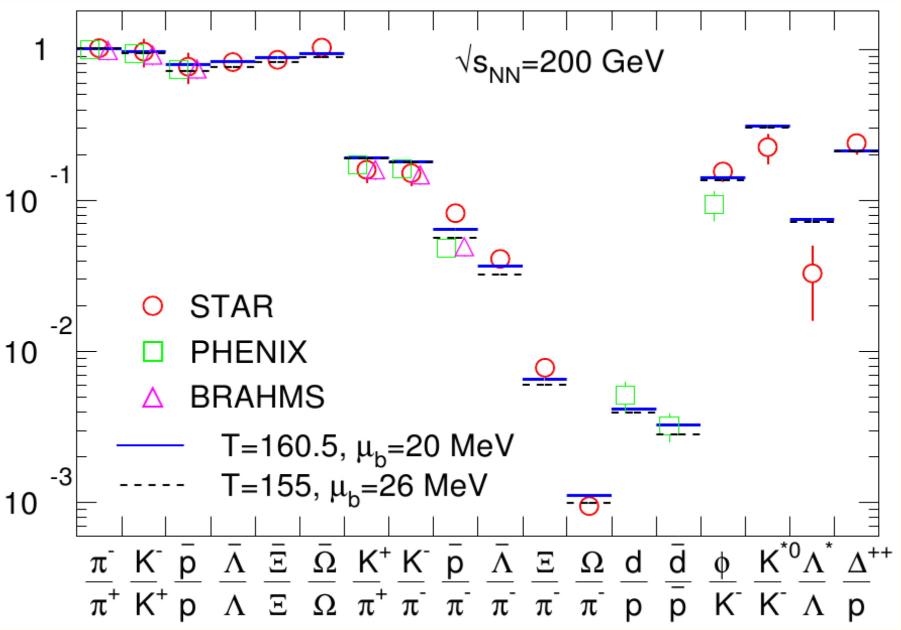
P. Braun-Munzinger, I. Heppe, J. Stachel, PLB 465 (1999) 15
and reanalysis 2004 with more data

hadron yields at RHIC compared to statistical model (GC)

130 GeV data in excellent agreement
with thermal model predictions



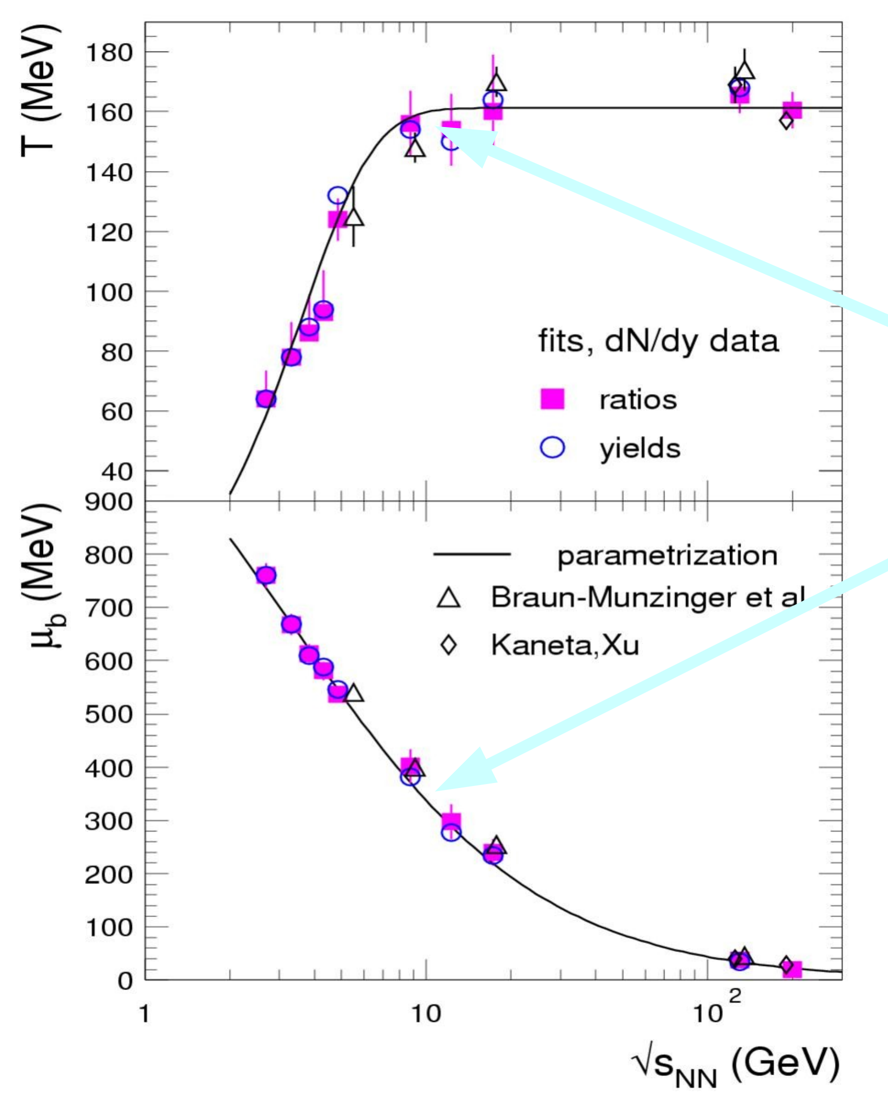
prel. 200 GeV data fully in line
still some experimental discrepancies



chemical freeze-out at: T = 165 ± 5 MeV

P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel, Phys. Lett. B518 (2001) 41
A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167

systematic beam energy dependence of hadro-chemical freeze-out parameters



T_{chem} saturates at $\sqrt{s_{NN}}$ of about 10 GeV

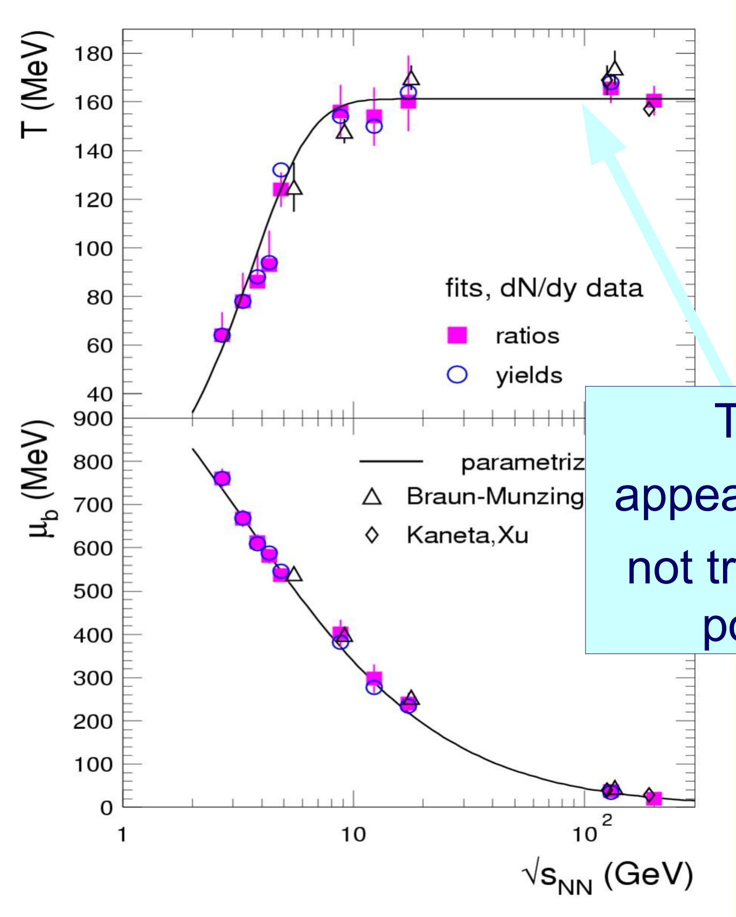
baryon chemical potential decreases monotonically with beam energy

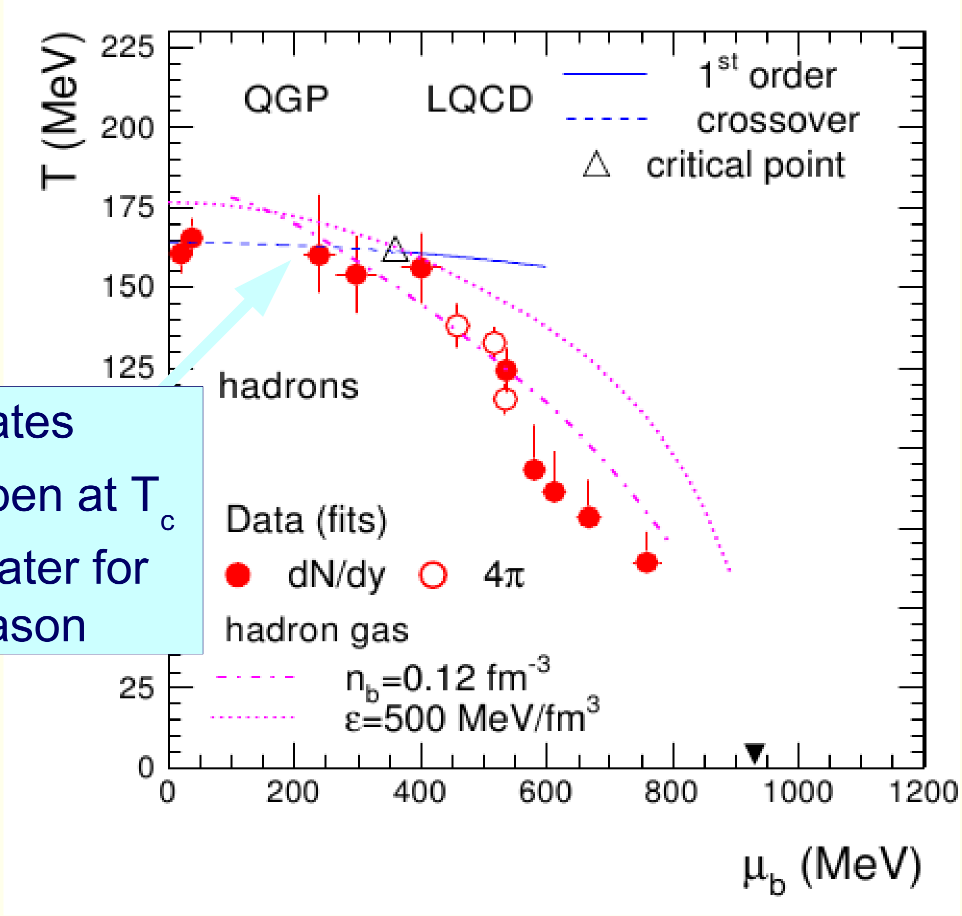
- nuclei become transparent
  empirically rapidity shift of 2 units for a nucleon in pA and AA collisions
- increasing importance of pbarp production

A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167

hadrochemical freeze-out points and the phase diagram

A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167





T_{chem} saturates appears to happen at T_c

not trivial; see later for possible reason

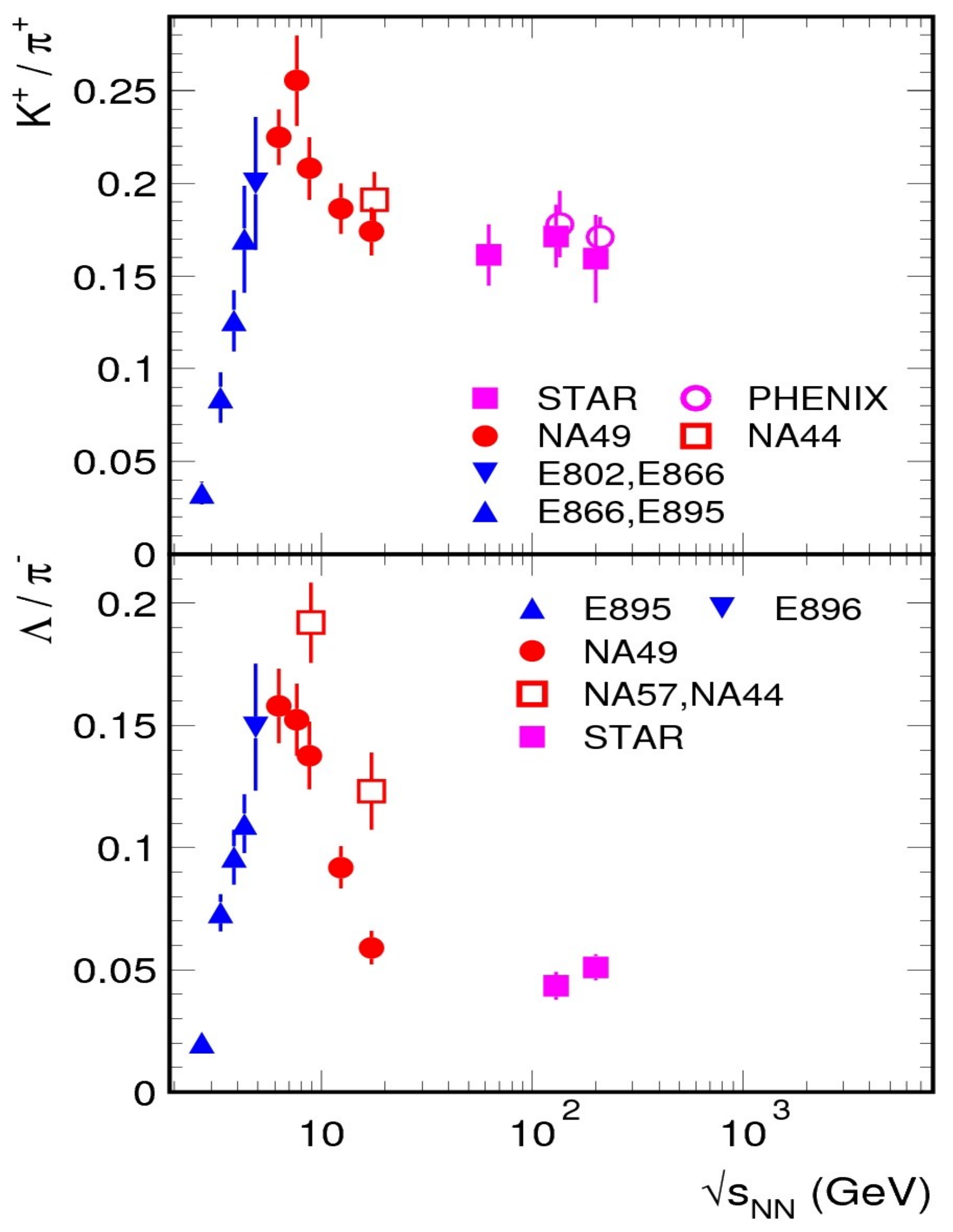
in general good fits to all central heavy ion collision data

The horn

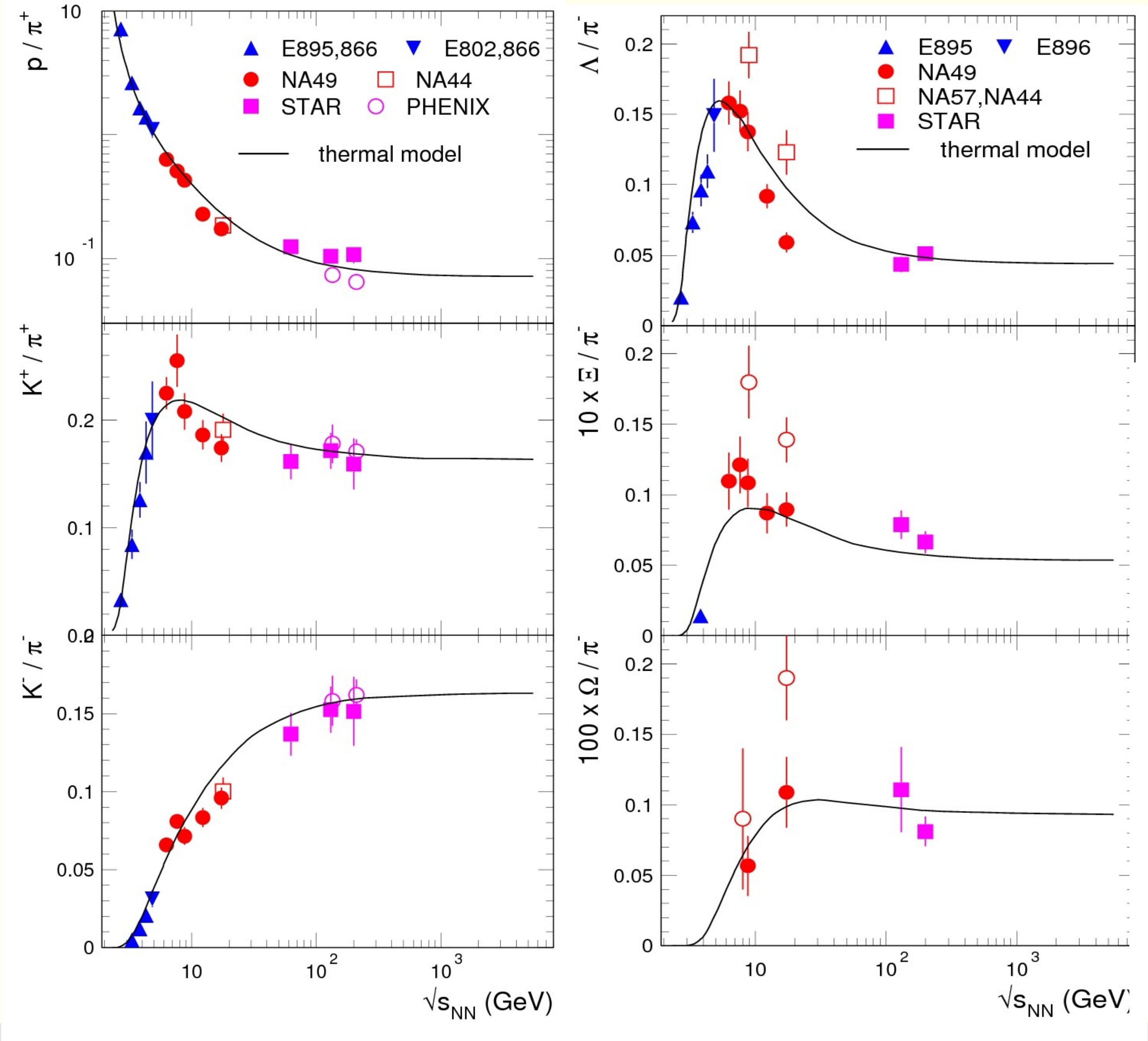
pronounced maxima in some hadronic yield ratios of specific beam energy in region of low SPS energies

origin?

anything new happening here?

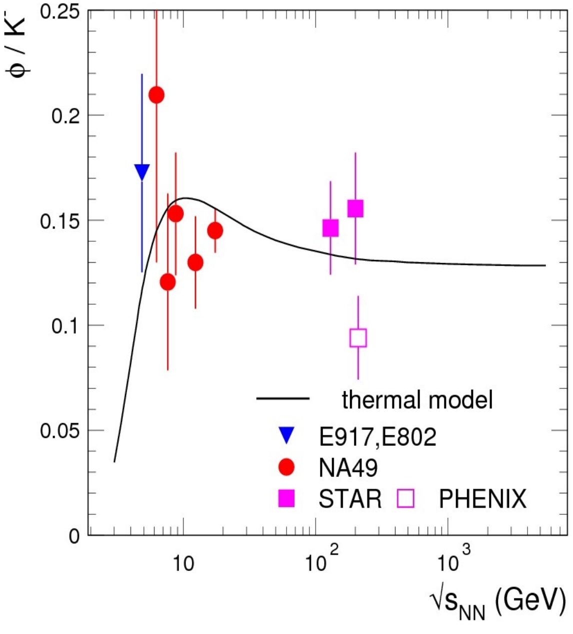


Specific beam energy dependence of hadron production well reproduced



origin of maxima:
increase and saturation
of T and monotonic
decrease of μ_b

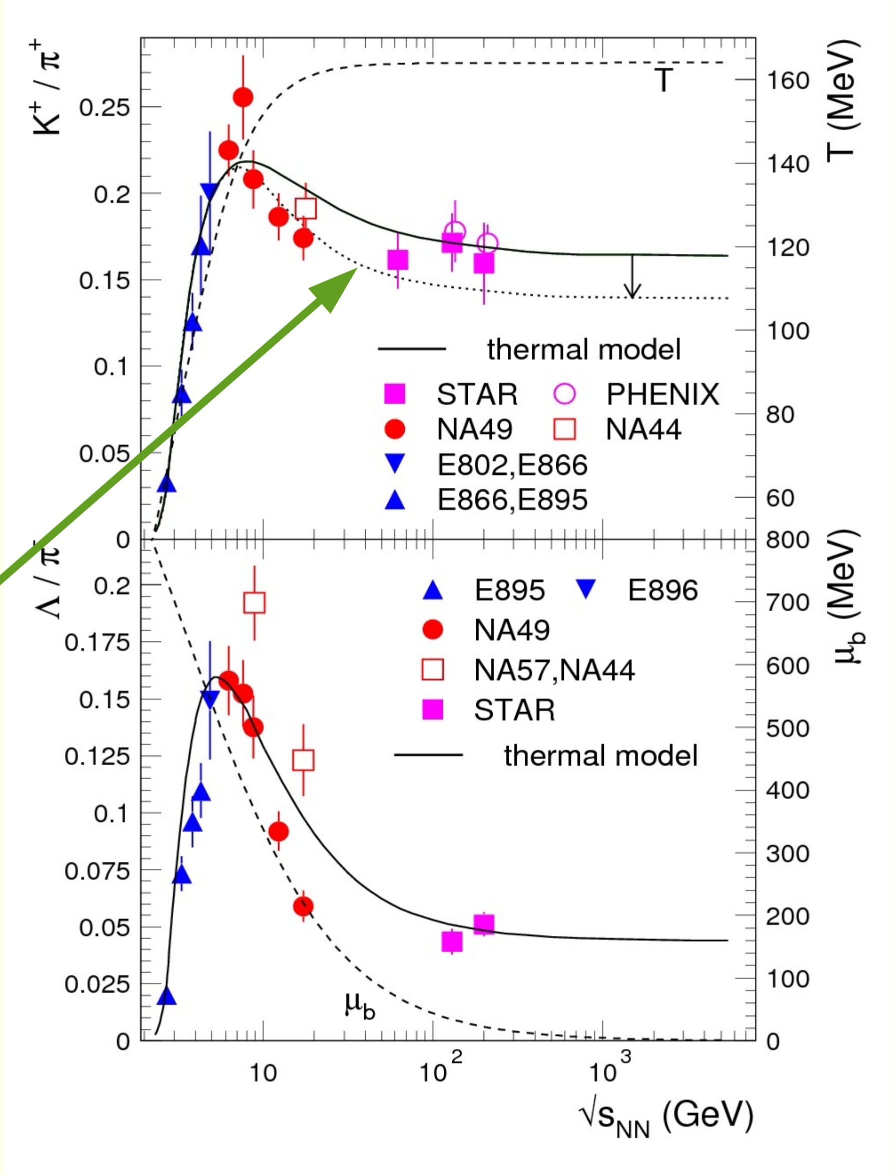
very sensitive to location
of freeze-out curve!



Effect of still incomplete knowledge of hadron spectrum

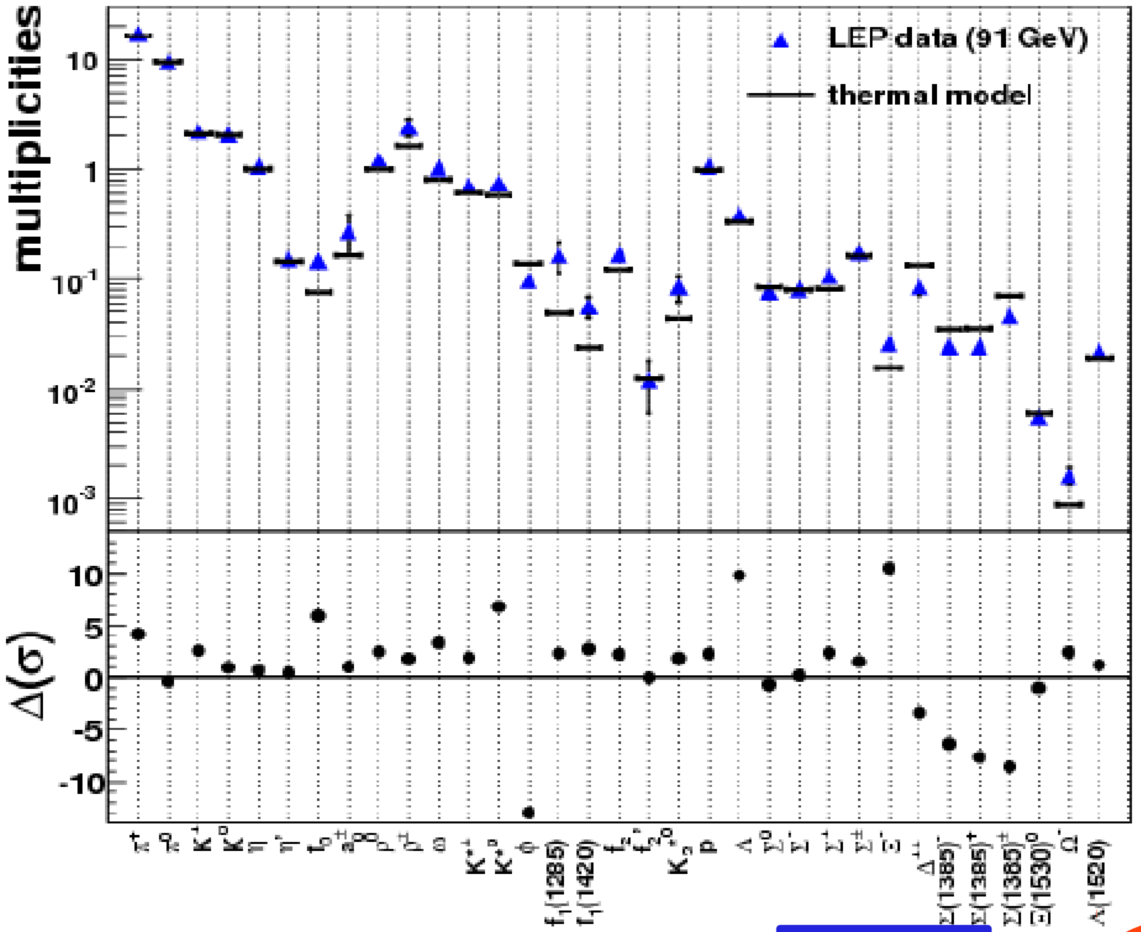
based on Hagedorn spectrum:
estimate effect by extending mass spectrum beyond 3 GeV based on $T_{Hagedorn}$ = 200 MeV and
assumption how states decay

strongest contribution to kaon from K* producing one K
all high mass resonances produce multiple pions
-> further reduction of K^+/π^+
similar expectation for p/π



Revisit e+e- collisions: initialize thermal model with u,d,s,c,b – jets according to measurement (weak isospin)

A. Andronic, P. Braun-Munzinger, F. Beutler, K. Redlich, J. Stachel, Phys. Lett. B678 (2009) arXiv 0804.4132



parameter set: T=164 MeV, V=20 fm^3, $\boxed{\gamma_s=0.72}$ with $\chi^2=718/30$

Message: in e+e-some thermal features in fragmentation of jets when quark abundancies used as input
but strangeness is supressed
– fit still not good!

Production of light nuclei and antinuclei in nuclear collisions at the AGS

data cover 10 oom!

addition of every nucleon

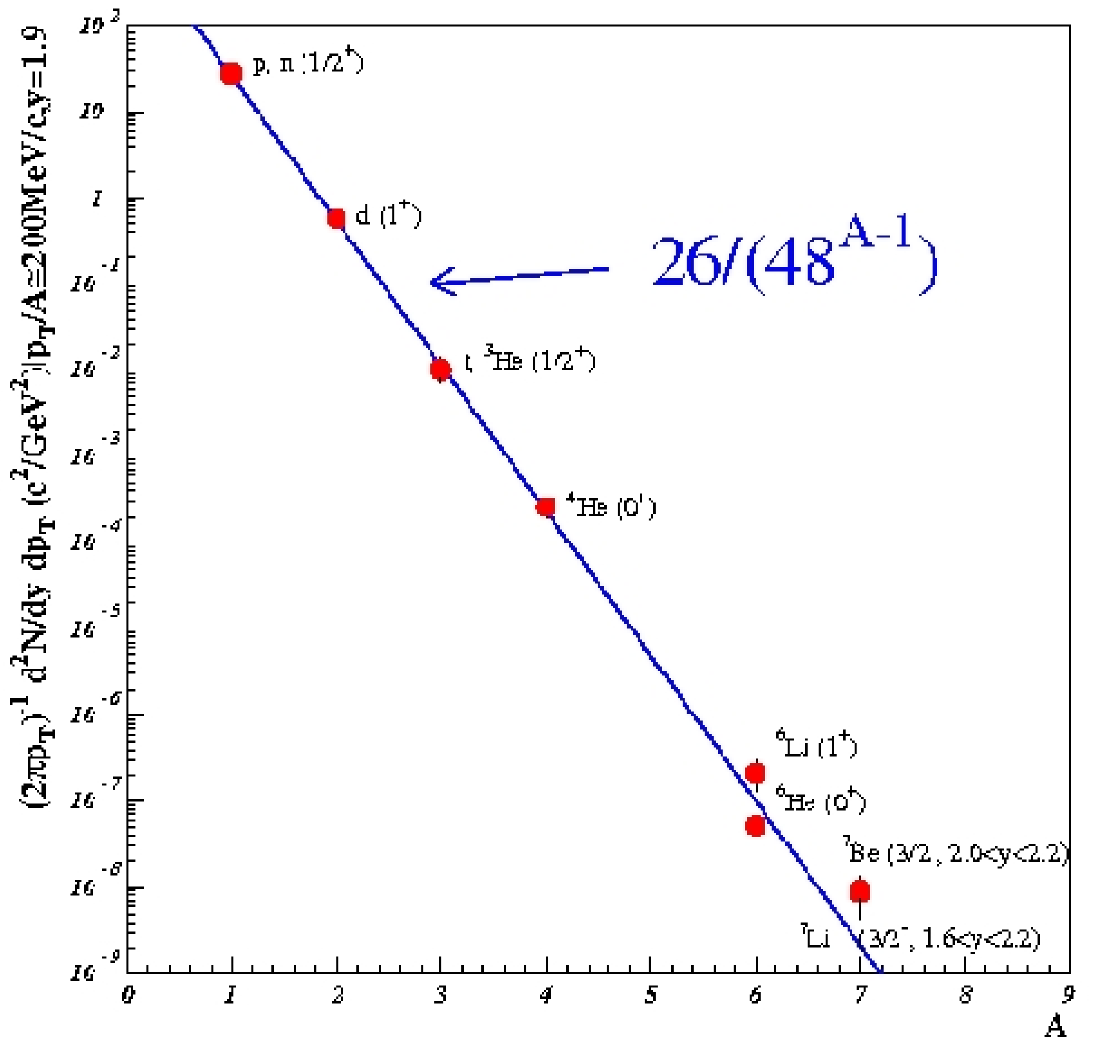
-> penalty factor R_p = 48

but data are at very low pt

use m-dependent slopes following systematics up to deuteron

-> R_p = 26

E864 Coll., Phys. Rev. C61 (2000) 064908



GC statistical model:

$$R_p \approx \exp[(m_n \pm \mu_b)/T]$$

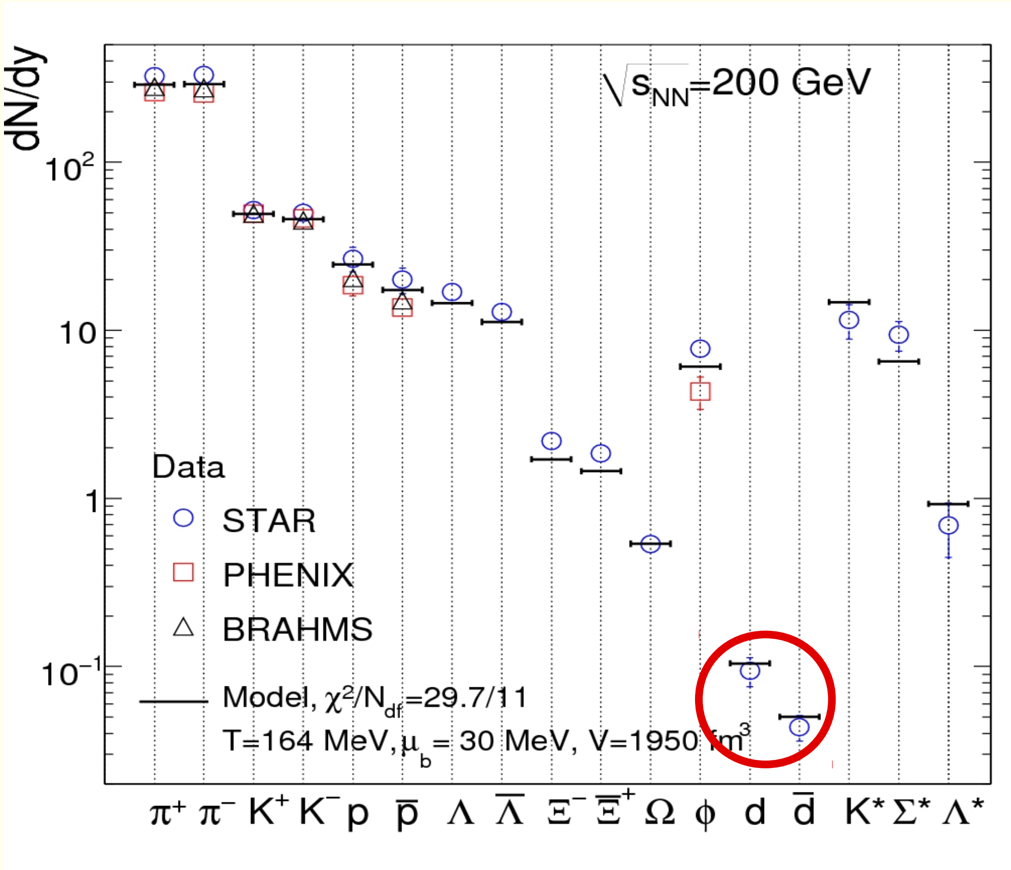
for T=124 MeV and μ_b= 537 MeV

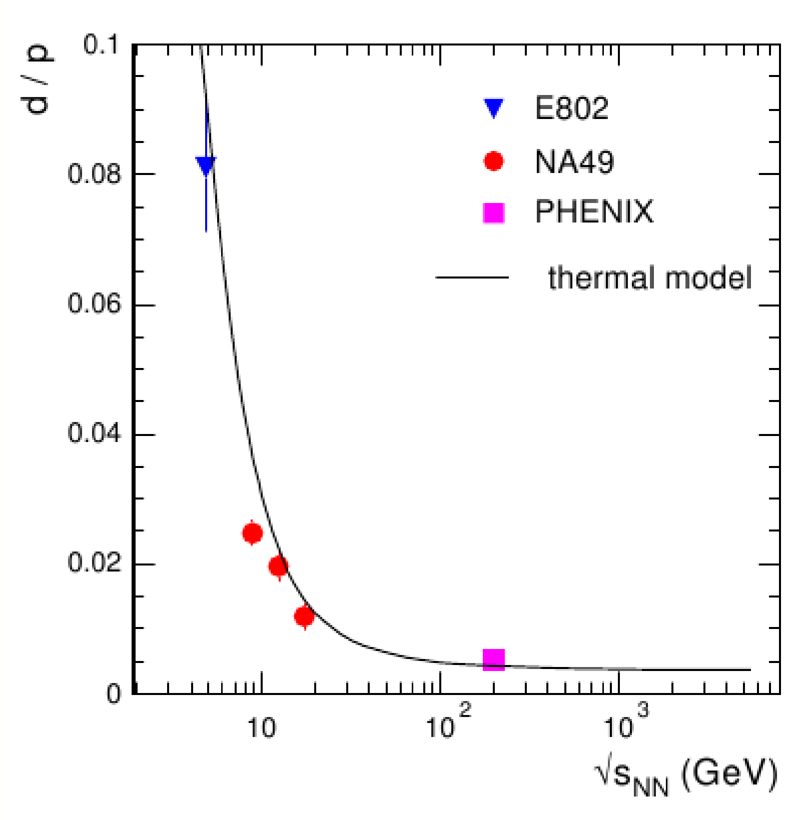
R_p = 24 good agreement

also good for **antideuterons**:

data: $R_p = 2\pm1\cdot10^5$ SM: $1.3\cdot10^5$

at RHIC description equally good where data exist
- beam energy dependence driven by μ_b





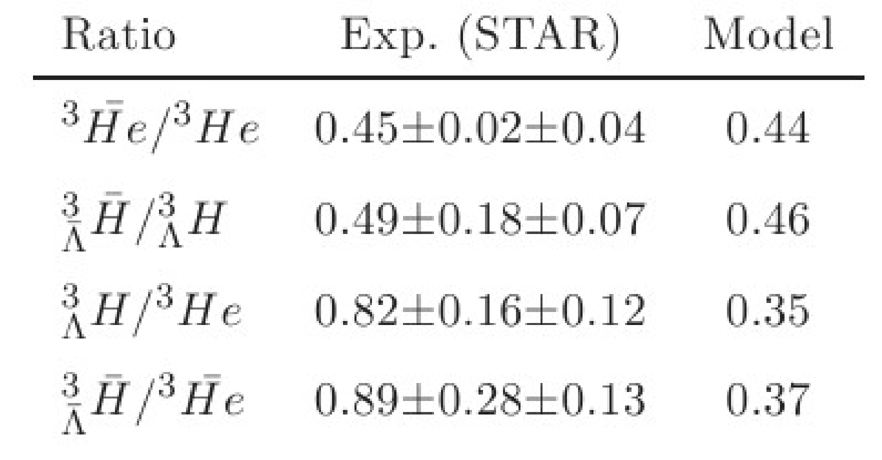
can statistical model reproduce mass 3 antinuclei and antihypernuclei?

comparing mass 3 nuclei and antinuclei -> extreme sensitivity to μ_b

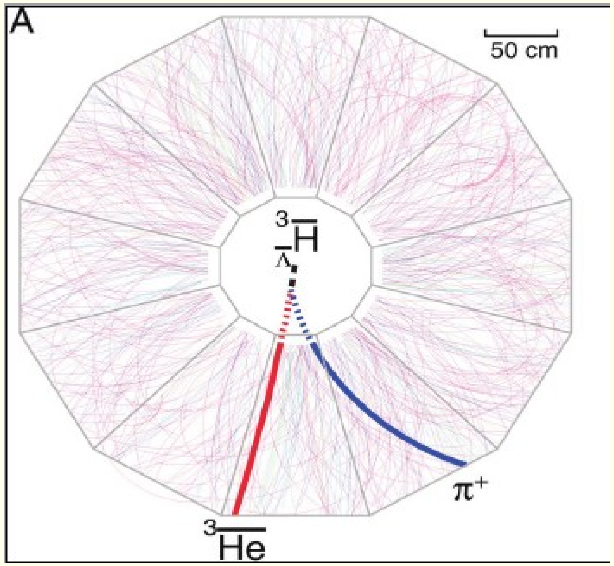
$$\mathrm{Y}(^3\bar{\mathrm{H}}\mathrm{e})/\mathrm{Y}(^3\mathrm{He}) = \exp(-6\mu_\mathrm{b}/\mathrm{T})$$

revisit fit: $\mu_b = 24$ +-2 MeV (instead of 30 +-4 MeV without mass 3)

most precise determination of baryon chemical potential!



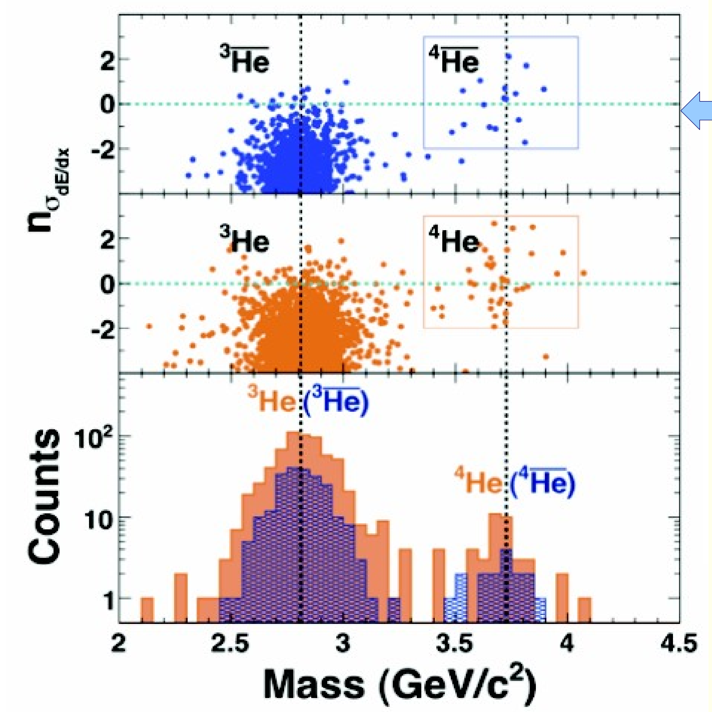
Ratio	Exp. (STAR)	Model
$^3\bar{He}/^3He$	0.45±0.02±0.04	0.44
$^3_{\bar{\Lambda}}\bar{H}/^3_{\Lambda}H$	0.49±0.18±0.07	0.46
$^3_{\Lambda}H/^3He$	0.82±0.16±0.12	0.35
$^3_{\bar{\Lambda}}\bar{H}/^3\bar{He}$	0.89±0.28±0.13	0.37



Data: STAR, Science March 2010

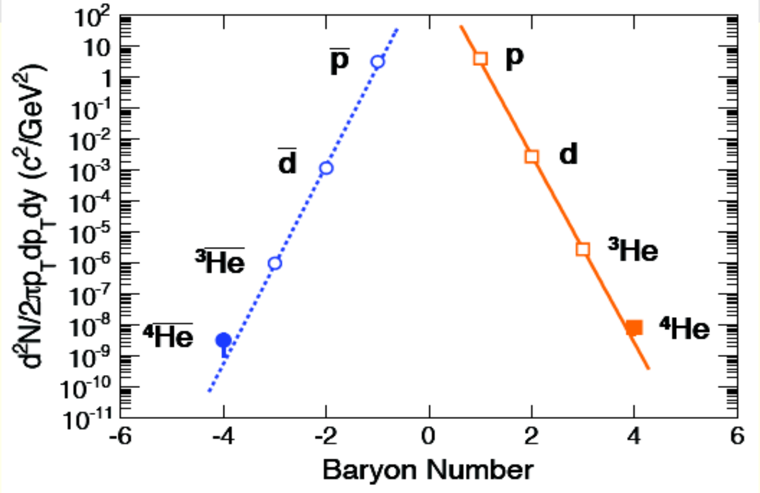
Model: A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker, PLB (2011)

Recently in STAR at RHIC and in ALICE at LHC discovery of anti 4Helium

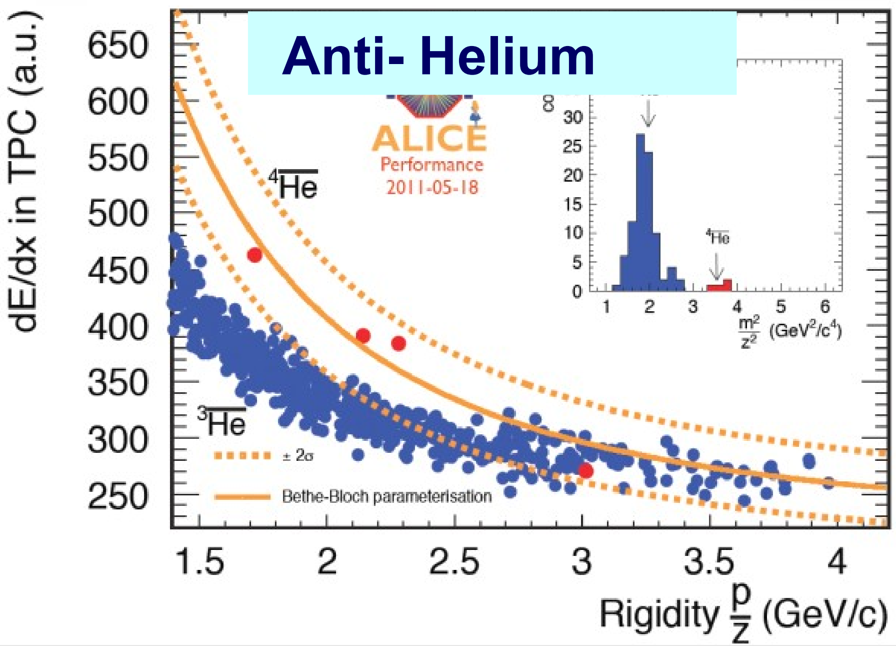


STAR

Should cost factor 245 per nucleon
4Hebar/pbar
3.6 10^{-9}
Like data



Nature 473, 353-356, (19 May 2011)
doi:10.1038/nature10079



no precise comparison yet to statistical model
need to integrate over spectrum
extreme sensitivity to (unknown) cross section of antimatter in detector material

How is chemical equilibration achieved?

2-particle collisions not enough – takes about one order of magnitude too long

even when system is initialized in equilibrium at T = 170 MeV, it falls out of equilibrium quickly

<u>simple example:</u>
use a data driven estimate of rate of cooling
near chemical freeze-out (can be explained later) $|\dot{T}/T| = \tau_T^{-1} = (13 \pm 1)\%/\mathrm{fm}$
typical densities at $T_{ch} : \rho_\pi = 0.174/\mathrm{fm}^3 (\mathrm{incl.res.}), \rho_K = 0.030/\mathrm{fm}^3 \rho_\Omega = 0.0003/\mathrm{fm}^3$
to maintain equlibrium during 5 MeV temperature drop need a relative rate of change of densities of $|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}| = \tau_\Omega^{-1} - \tau_K^{-1} = 1.10 - 0.55/\mathrm{fm} = 0.55/\mathrm{fm}$

so Ω density needs to change by 100 % in 1 fm/c
typical reactions with large cross section (10 mb) and rel. velocities of 0.6 give

$$\Omega + \bar{K} \to \Xi + \pi \qquad \to \qquad \bar{r}_\Omega / n_\Omega = n_{\bar{K}} \langle v\sigma \rangle = 0.018/\mathrm{fm}$$

$$\pi + \pi \to K + \bar{K} \quad (\sigma = 3\mathrm{mb}) \qquad \bar{r}_K / n_K = 0.18/\mathrm{fm}$$

much too slow to maintain equlibrium even over drop of T of 5 MeV!
much harder to get into equilibrium!

A possible scenario for rapid equilibration

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004) 61

near phase boundary multiparticle reactions become important
dynamics associated with collective excitations
(key word: critical opalescence at phase transition)
Propagation and scattering of these collective excitations expressed in form of multi-hadron scattering

will see: this drives the system into equilibrium very rapidly

Evaluation of multi-strange baryon yield as most challenging test case

consider situation at T_{ch} = 176 MeV first

rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

with
$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

the phase space factor φ depends on √s

needs to be weighted by the probability *f*(s) that multiparticle scattering occurs at a given value of √s

evaluate numerically in Monte-Carlo using thermal momentum distribution

typical reaction $\Omega + \bar{N} \rightarrow 2\pi + 3K$

assume cross section equal to the measured one for $p + \bar{p} \rightarrow 5\pi$

at proper energy above threshold, i.e. √s = 3.25 GeV ➡ 6.4 mb

compute matrix element and use for rate of $2\pi + 3K \rightarrow \bar{N} + \Omega$

$$r_\Omega = n_\pi^5 (n_K / n_\pi)^3 |\mathcal{M}|^2 \phi$$

Evaluation of multi-strange baryon yield

reaction $2\pi + 3K \to \bar{N} + \Omega$ leads to

$$r_\Omega = 0.00014 \mathrm{fm}^{-4} \quad \text{or} \quad r_\Omega / n_\Omega = 1/\tau_\Omega = 0.46/\mathrm{fm}$$

can achieve final density starting from only pions and kaons at t=0 in 2.2 fm/c

similarly one obtains

for $3\pi + 2K \to \Xi + \bar{N} \quad \text{or} \quad \tau_\Xi = 0.71\,\mathrm{fm}$

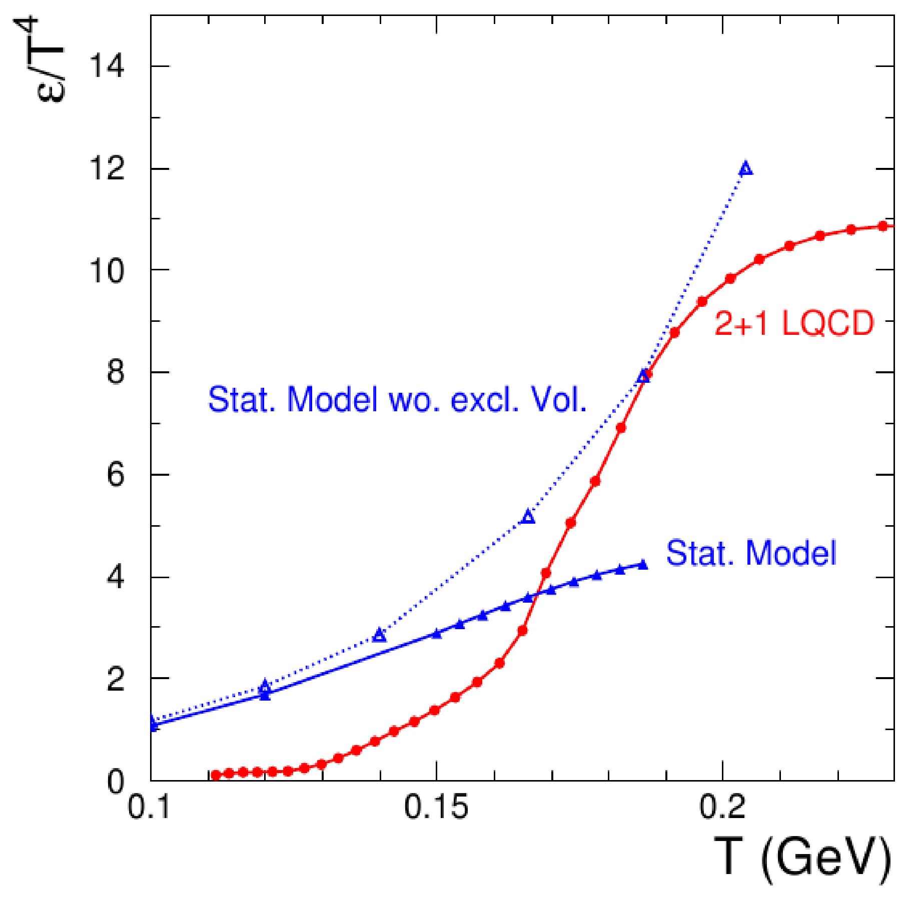
and

for $4\pi + K \to \Lambda + \bar{N} \quad \text{or} \quad \tau_\Lambda = 0.66\,\mathrm{fm}$

why do all particle yields show one common freeze-out T?

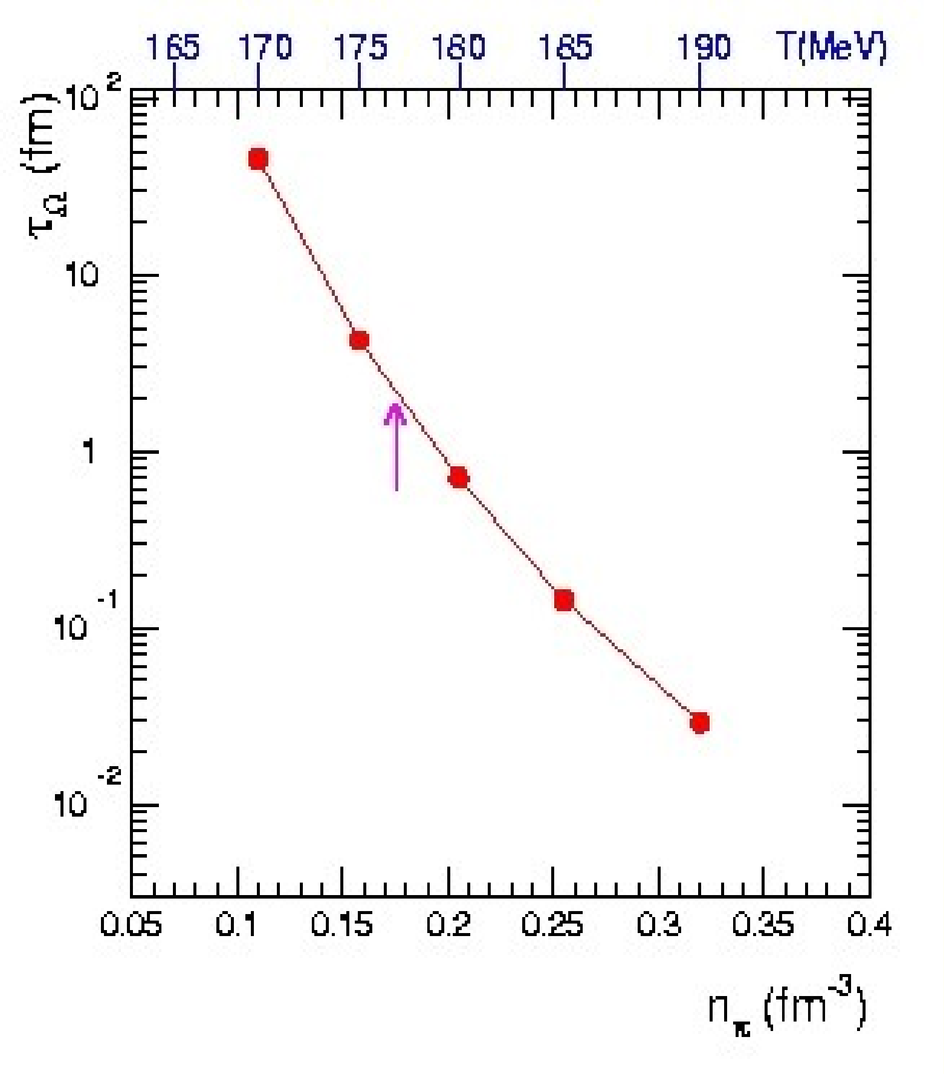
the density of particles varies rapidly (factor 2 within 8 MeV) with T near the phase transition due to increase in degrees of freedom.
also: system spends time at Tc -> volume has to triple (entropy cons.)
Multi-particle collisions are strongly enhanced at high density and lead to chem. equilibrium very near to T_c
independently of cross section all particles can freeze out within narrow temperature interval

natural consequence that chemical freeze-out takes place at T_c!



Lattice QCD by F. Karsch et al.

density dependence of characteristic time for strange baryon production



near phase transition particle density varies rapidly with T

for small μ_b, reactions such as $2\pi + KKK \rightarrow \Omega\,\bar{N}$ bring multi-strange baryons close to equilibrium.

in region around T_c equilibration time $\tau_\Omega \propto T^{-60}$!

increase ρ_π by 1/3 or 8 MeV: $\tau = 0.2$ fm/c

decrease ρ_π by 1/3: $\tau = 27$ fm/c

all particles freeze out within a very narrow temperature window.