QGP Physics – From Fixed Target to LHC

6. Space-time Evolution of the QGP

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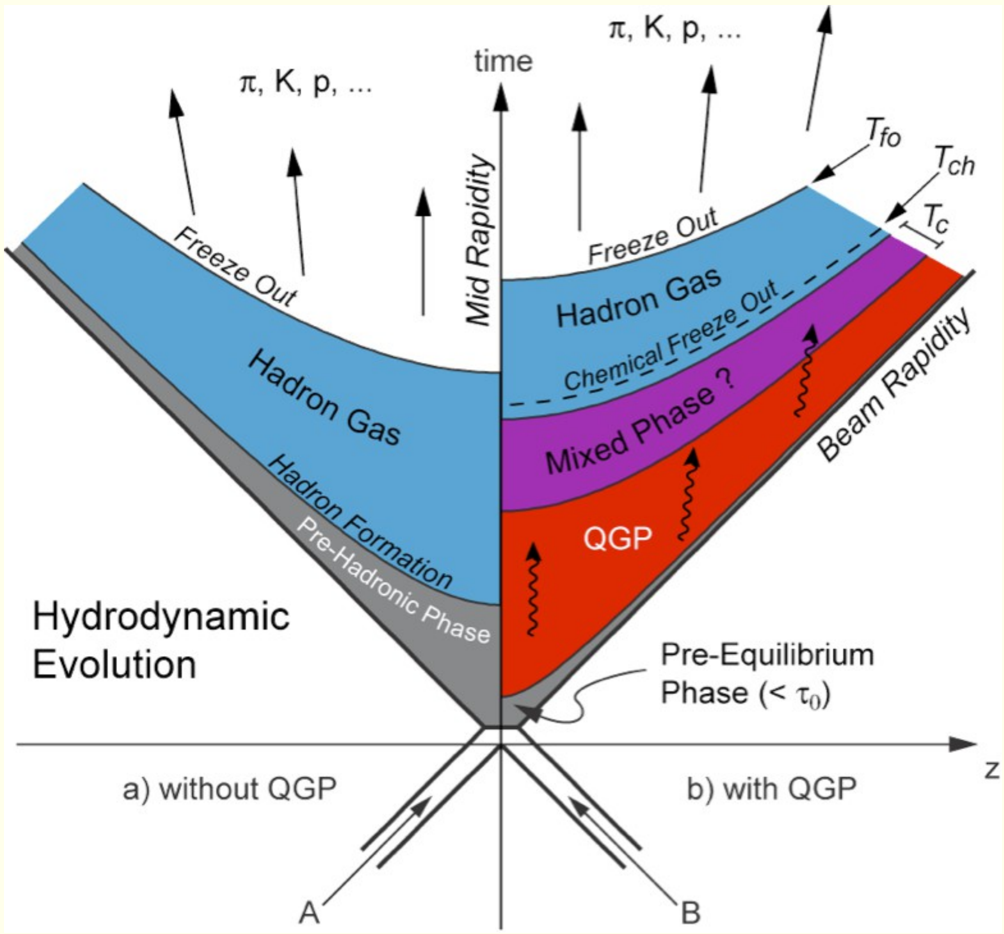
SS 2011

Space-time Evolution of A+A Collisions

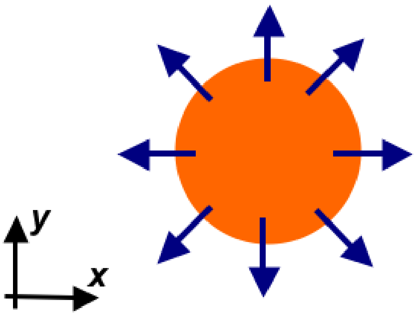


Evolution described by relativistic hydrodynamic models, which need initial conditions as input.

Simplest case: Symmetric collisions (no elliptic flow), ideal gas equation of state (bag model), only longitudinal expansion (1D, Bjorken)

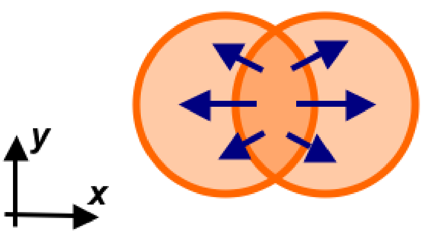
Types of Collective Flow

Radial flow



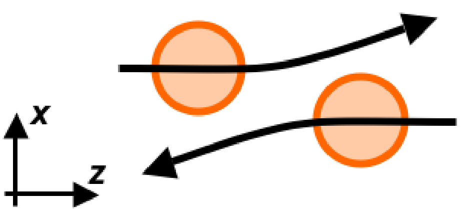
- The only type of collective flow in A+A collisions with impact parameter $b = 0$
- Affects the shape of particle spectra at low p_T

Elliptic flow



- Caused by anisotropy of the overlap zone ($b \neq 0$)
- Requires early thermalization of the medium

Directed flow

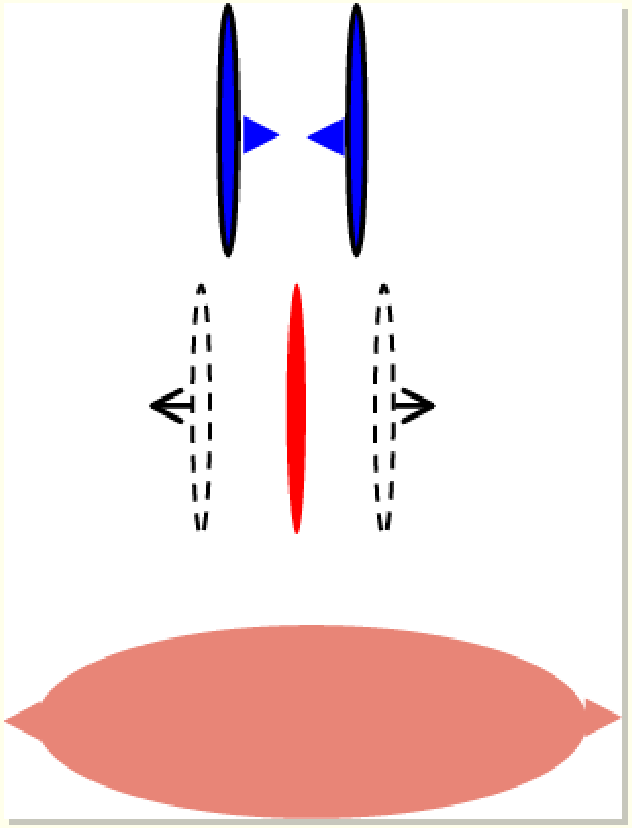


- Is produced in the pre-equilibrium phase of the collision
- Gets smaller with increasing $\sqrt{s_{NN}}$

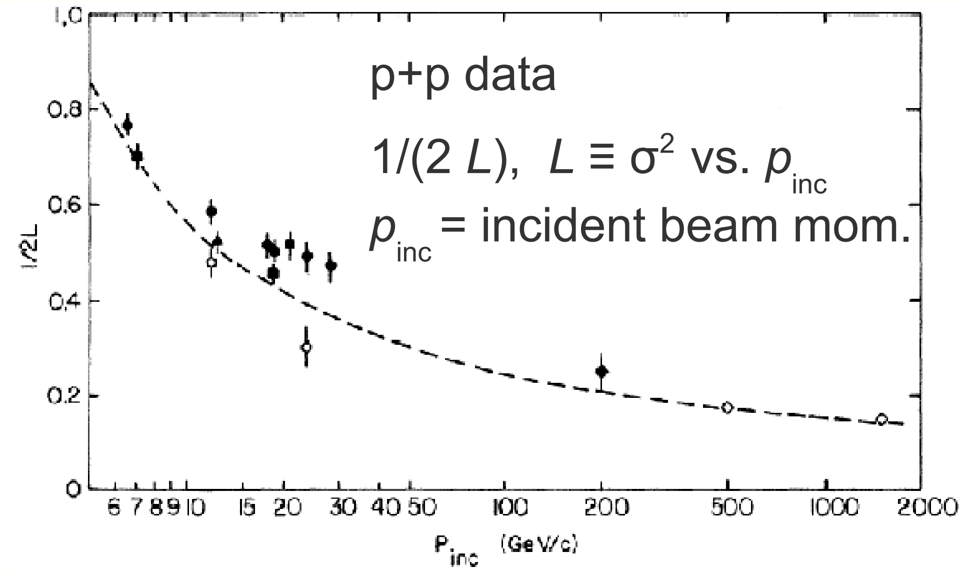
6.1 Longitudinal Expansion

Landau Initial Conditions for the Hydrodynamic Evolution

L. D. Landau, Izv. Akad. Nauk. SSSR 17 (1953) 52
P. Carruthers and M. Duong-Van, PRD8 (1973) 859

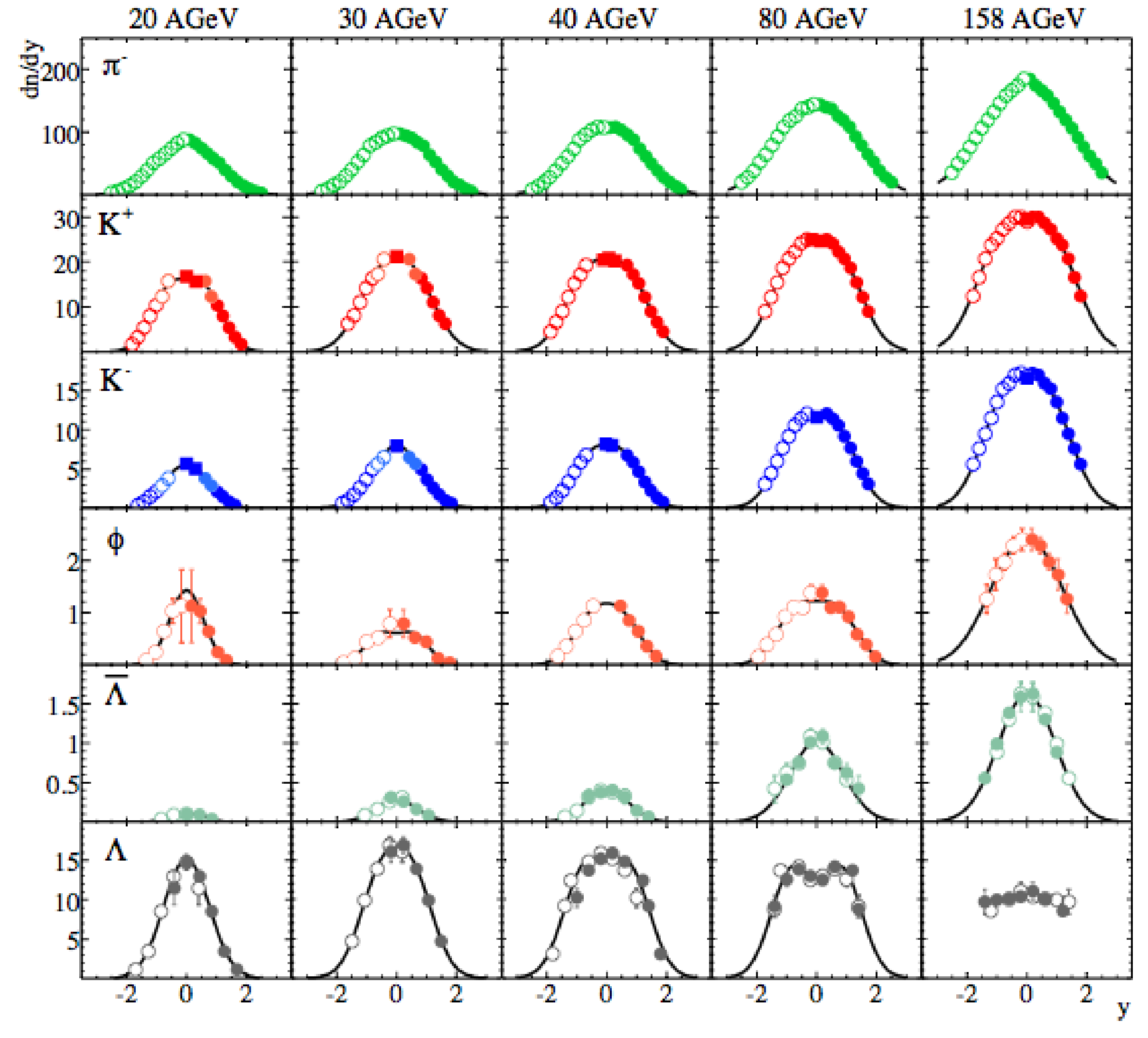


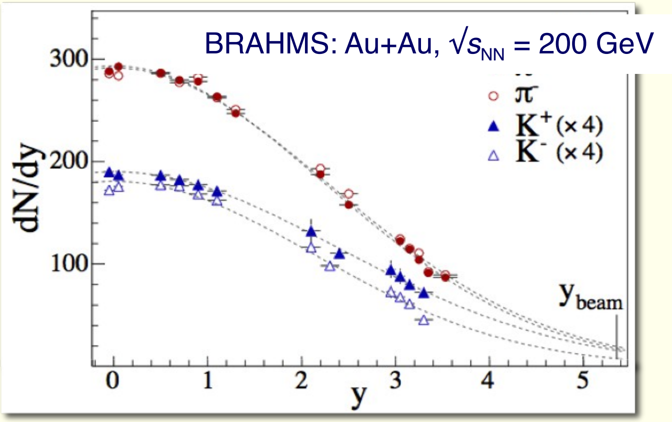
Lev Landau



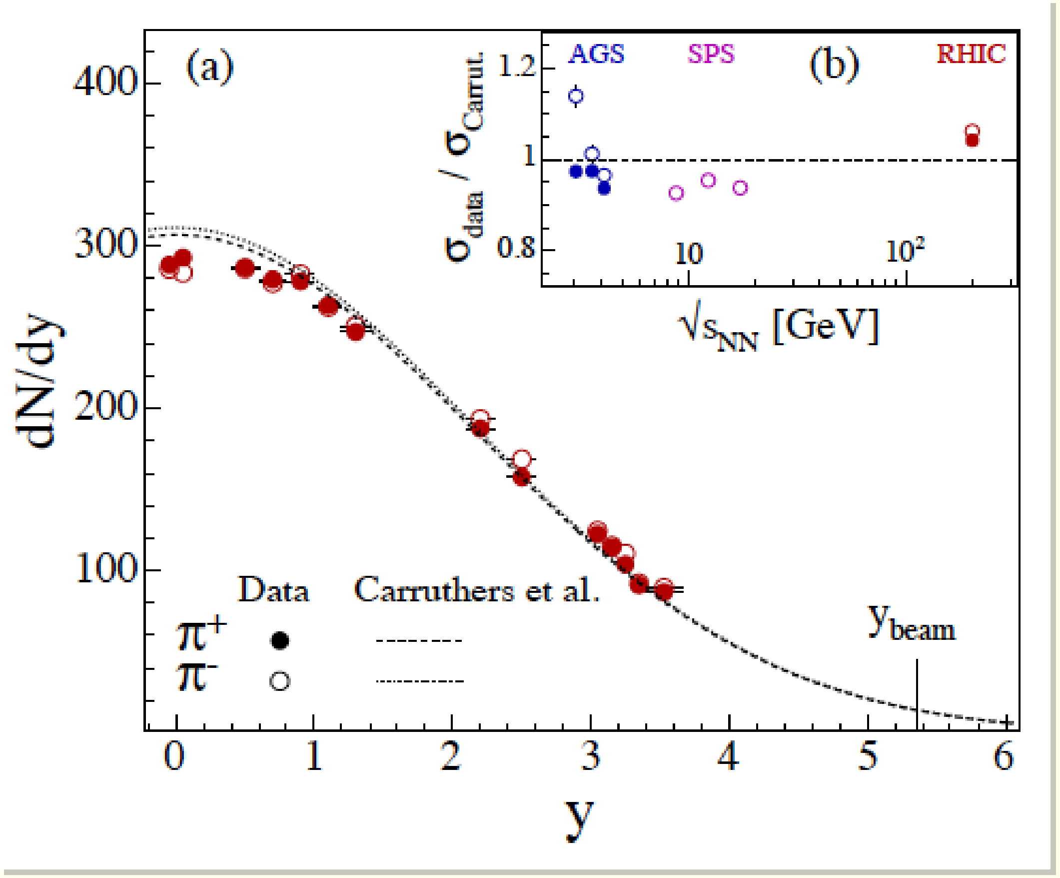
Prediction: d*N*/d*y* is Gaussian with a width given by $\sigma^2 = \ln\left(\frac{\sqrt{s}}{2m_p}\right)$

Rapidity Distributions in A+A



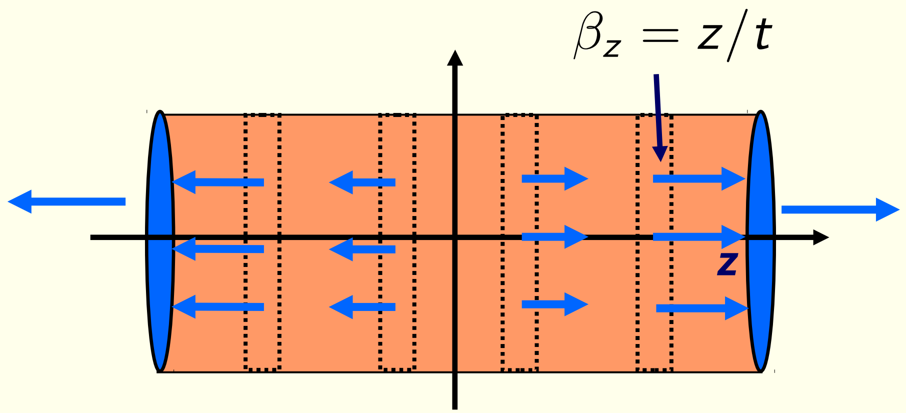


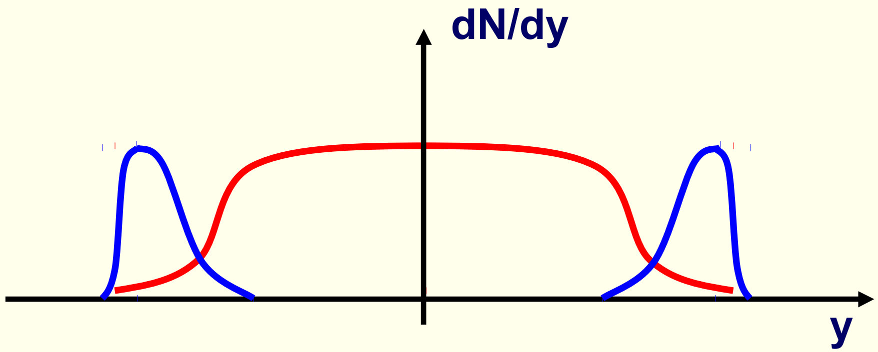
Landau … Also works for Heavy Ions



BRAHMS: PRL94, 162301 (2005)

Reminder (from Chapter 3): Space-Time Evolution: Bjorken Model





Velocity of the local system at position z at time t:

$$\beta_z = z/t$$

Proper time τ in this system:

$$\tau = t/\gamma = t\sqrt{1-\beta^2} = \sqrt{t^2 - z^2}$$

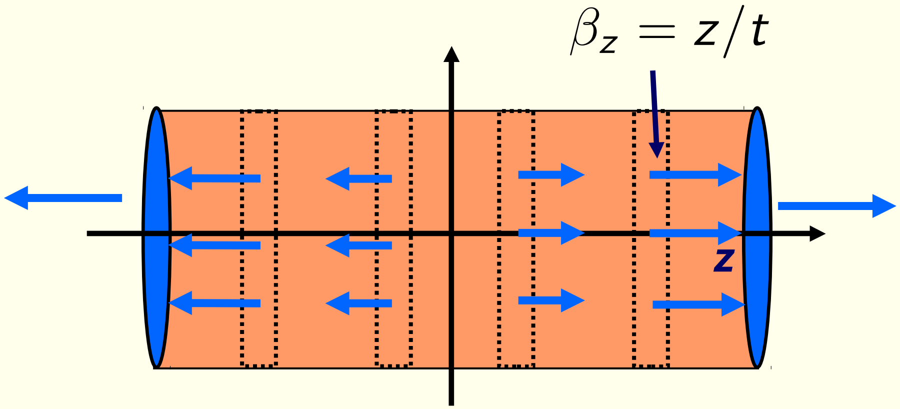
In the Bjorken model all thermodynamic quantities only depend on τ, e.g., the particle density:

$$n(t, z) = n(\tau)$$

This leads to a constant rapidity density of the produced particles (at least at central rapidities):

$$\frac{dN_{ch}}{dy} = \text{const.}$$

1D Bjorken Model (I)



$$\tau = t/\gamma = t\sqrt{1-\beta_z^2} = \sqrt{t^2 - z^2}$$

The 1D Bjorken model is based on the assumption that dN_{ch}/dy ist constant (around mid-rapidity).

This means that the central region is invariant under Lorentz transformation. This implies $\beta_z = z/t$ and that all thermodynamic quantities depend only on the proper time τ

Initial conditions in the Bjorken model:

$$\varepsilon(\tau_0) = \varepsilon_0, \quad u^\mu = \frac{1}{\tau_0}(t, 0, 0, z) = \frac{x^\mu}{\tau_0}$$

Initial energy density

In this case the equations of ideal hydrodynamics simplify to

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + p}{\tau} = 0$$

$\varepsilon = E/V$: energy density
p : pressure
$s = S/V$: entropy density

1D Bjorken Model (II)

For an ideal gas of quarks and gluons, i.e., for

$$\varepsilon = 3p, \quad \varepsilon \propto T^4$$

This leads to

$$\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}, \quad T(\tau) = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}$$

The temperature drops to the critical temperature at the proper time

$$\tau_c = \tau_0 \left(\frac{T_0}{T_c}\right)^3$$

And thus the lifetime of the QGP in the Bjorken model is

$$\Delta\tau_{\mathrm{QGP}} = \tau_c - \tau_0 = \tau_0 \left[\left(\frac{T_0}{T_c}\right)^3 - 1\right]$$

1D Bjorken Model (III)

Entropy conservation in ideal hydrodynamics leads in the case of the Bjorken model (independent of the equation of state) to

$$s(\tau) = \frac{s_i \tau_i}{\tau}$$

If we consider a QGP/pion gas phase transition we have a first oder phase transition and a mixed phase with temperature T_c. The entropy in the mixed phase is given by

$$s(\tau) = s_\pi(T_c)\xi(\tau) + s_{\mathrm{QGP}}(T_c)(1 - \xi(\tau)) = \frac{s_i \tau_i}{\tau}$$

ξ(τ): fraction of fireball in QGP phase

This equation determines the time dependence of ξ(τ) and the time τ_h at which the mixed phase vanishes:

$$\xi(\tau) = \frac{1 - \tau_c/\tau}{1 - g_\pi/g_{\mathrm{QGP}}} \quad \rightsquigarrow \quad \tau_h = \tau_c \frac{g_{\mathrm{QGP}}}{g_\pi}$$

Inserting the number of degrees of freedom we obtain

$$N_f = 2\,(3) \quad \rightsquigarrow \quad g_{\mathrm{QGP}} = 37\,(47.5) \quad \rightsquigarrow \quad \tau_h = 12.3\,(15.8)\tau_c$$

QGP Lifetime in the 1D Bjorken Model

$$\varepsilon_0 = 11\,\mathrm{GeV}/\mathrm{fm}^3 = 11 \cdot 0.197^3\,\mathrm{GeV}^4 \quad \text{for } \tau_0 = 1\mathrm{fm}/c$$

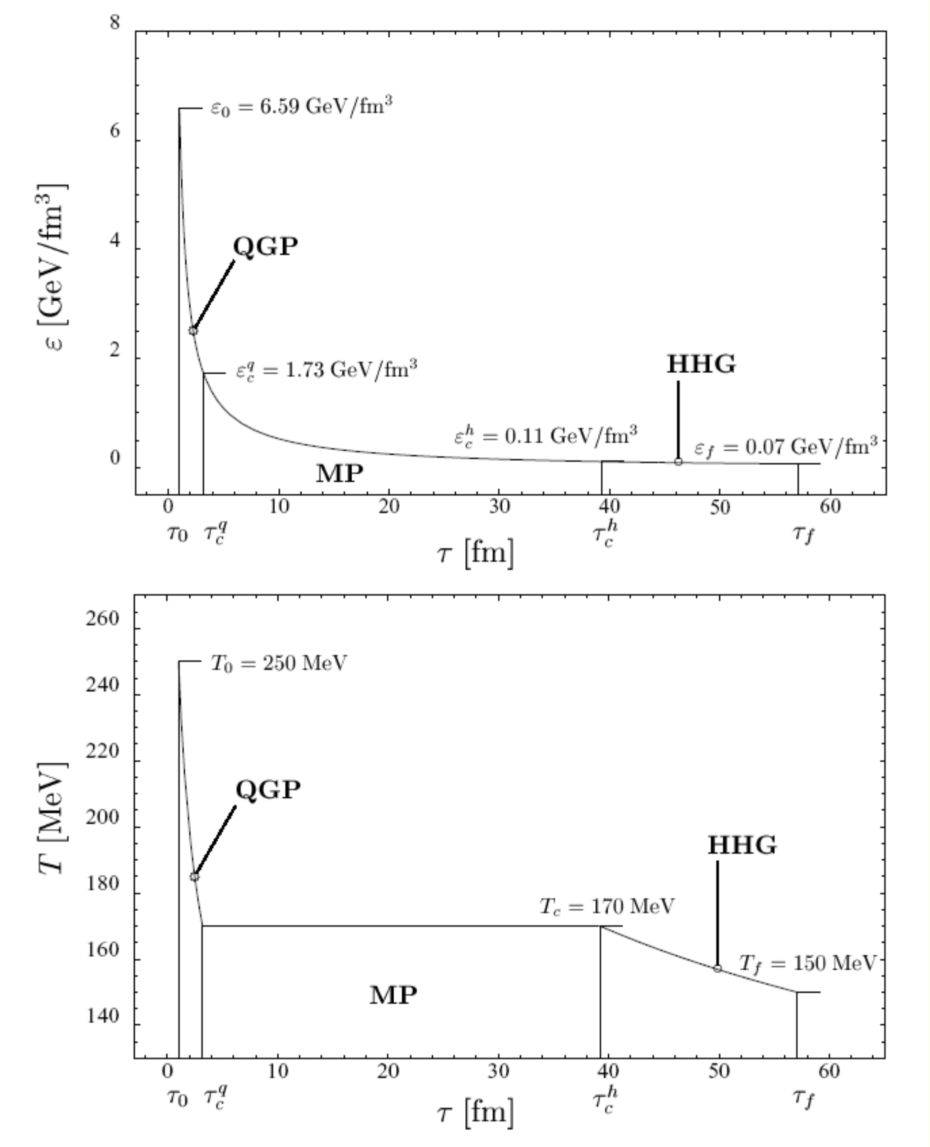
$$1 = \hbar c = 0.197\,\mathrm{GeV} \cdot \mathrm{fm}$$

$$\varepsilon_0 = g_{\mathrm{QGP}} \frac{\pi^2}{30} T^4 \quad \rightarrow \quad T_0 = \left(\frac{30}{\pi^2} \frac{\varepsilon}{g} \right)^{1/4}$$

Parameters	$\Delta\tau_{\mathrm{QGP}}$	$\Delta\tau_{\mathrm{mixed}}$
$\varepsilon_0\,\tau$ = 3 GeV/fm^2	0.84 fm/c	21 fm/c
$\varepsilon_0\,\tau$ = 5 GeV/fm^2	1.70 fm/c	31 fm/c
$\varepsilon_0\,\tau$ = 11 GeV/fm^2	3.9 fm/c	55 fm/c

Fixed parameters: $N_f = 2$, T_c = 170 MeV, τ_0 = 1 fm/c

1D Bjorken Model: Energy Density and Temperature as a Function of Proper Time



Quick First Estimate for the Initial Energy Density in Central Pb+Pb Collisions at the LHC

Bjorken formula: $\varepsilon \cdot \tau_0 = \frac{\langle m_T \rangle}{A} \left. \frac{\mathrm{d}N}{\mathrm{d}y} \right|_{y=0}$

Transverse area in collisions with $b \approx 0$: $A \approx \pi R_{\mathrm{Pb}}^2 = \pi (6.62\,\mathrm{fm})^2 \approx 140\,\mathrm{fm}^2$

Estimate for the mean transverse momentum:

$$\langle p_T \rangle = 0.66\,\mathrm{GeV}/c \rightsquigarrow \langle m_T \rangle \approx \sqrt{(0.138\,\mathrm{GeV})^2 + (0.66\,\mathrm{GeV})^2} = 0.67\,\mathrm{GeV}$$

Measured charged particle multiplicity:

$$dN_{ch}/d\eta \approx 1601 \pm 60 \quad (5\%\ \text{most central})$$

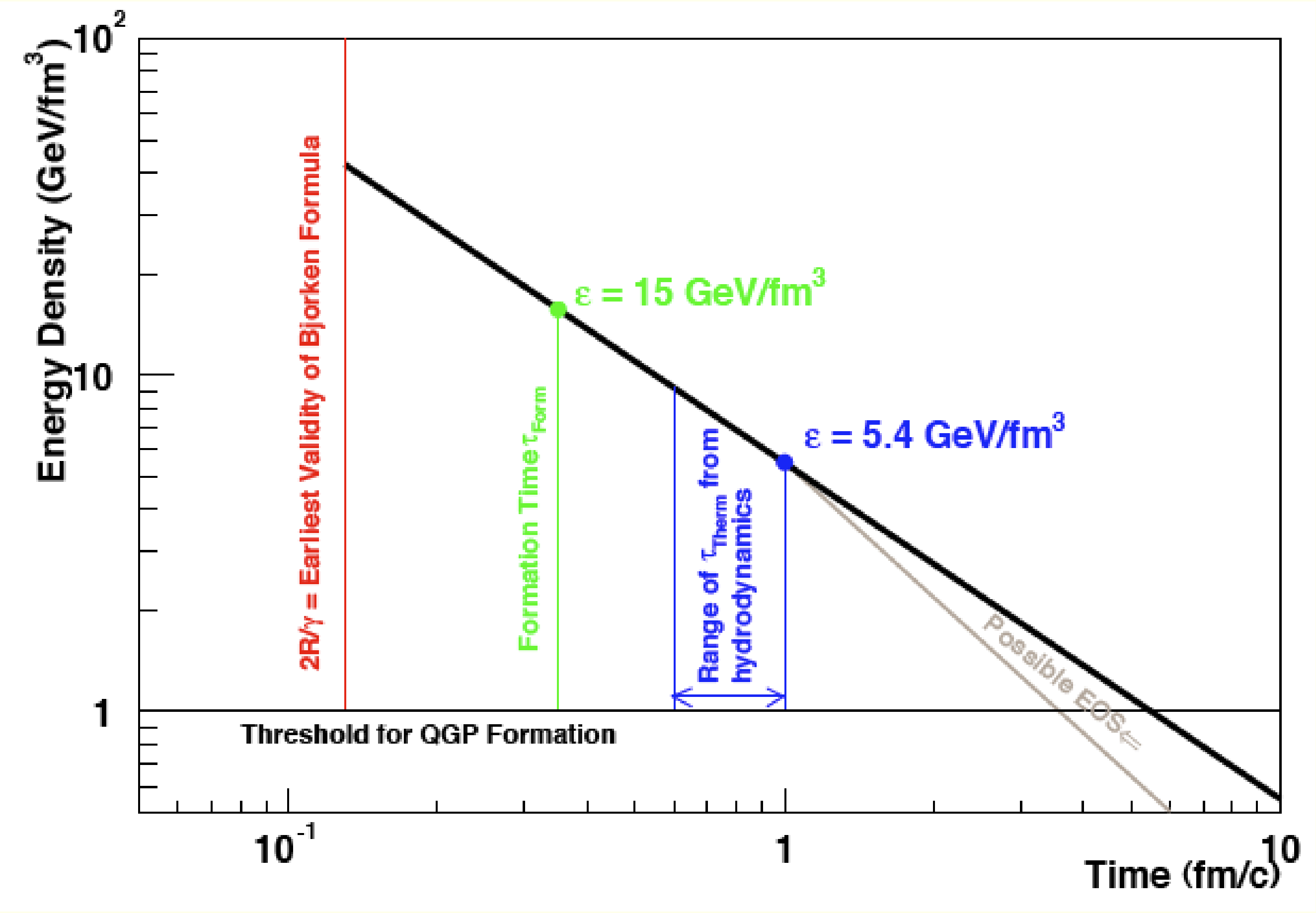
$$\rightsquigarrow \quad \left. \frac{\mathrm{d}N}{\mathrm{d}y} \right|_{y=0} = \frac{3}{2} \cdot \underbrace{\left(1 - \frac{m^2}{\langle m_T \rangle}\right)^{-1/2}}_{1.02} \cdot \left. \frac{\mathrm{d}N_{ch}}{\mathrm{d}\eta} \right|_{\eta=0} = 2450 \pm 92$$

Larger (up to ≈ 1.2) if p and K are taken into account

This leads to : $\varepsilon \cdot \tau_0 = (11.7 \pm 0.43)\,\mathrm{GeV}/\mathrm{fm}^2 \quad (\mathrm{Pb{+}Pb}@\sqrt{s_{NN}} = 2.76\,\mathrm{TeV})$

A factor of two larger than at RHIC: $\varepsilon \cdot \tau_0 \approx 5\,\mathrm{GeV}/\mathrm{fm}^2 \quad (\mathrm{Au{+}Au}@\sqrt{s_{NN}} = 0.2\,\mathrm{TeV})$

Energy Density and Time Scales in the Bjorken Picture



τ_0 = 1 fm/c is generally considered as a conservative estimate for the use in the Bjorken formula. Other estimates yields shorter times (e.g. τ_0 = 0.35 fm/c) resulting in initial energy densities at RHIC of up to 15 GeV/fm^3

6.2 p_T Spectra and Radial Flow

m_T Spectra from a Stationary Thermal Source

Stationary thermal source:

$$E\frac{d^3n}{d^3p} = \frac{1}{m_T} \cdot \frac{dn}{dm_T dy d\phi} = \frac{gV}{(2\pi)^3} E e^{-(E-\mu)/T}$$

$V = \text{volume}$

$g = \text{spin/isospin-degeneracy factor}$

$\mu = b\mu_b + s\mu_s = \text{chemical potential from baryon and strangeness quantum numbers}$

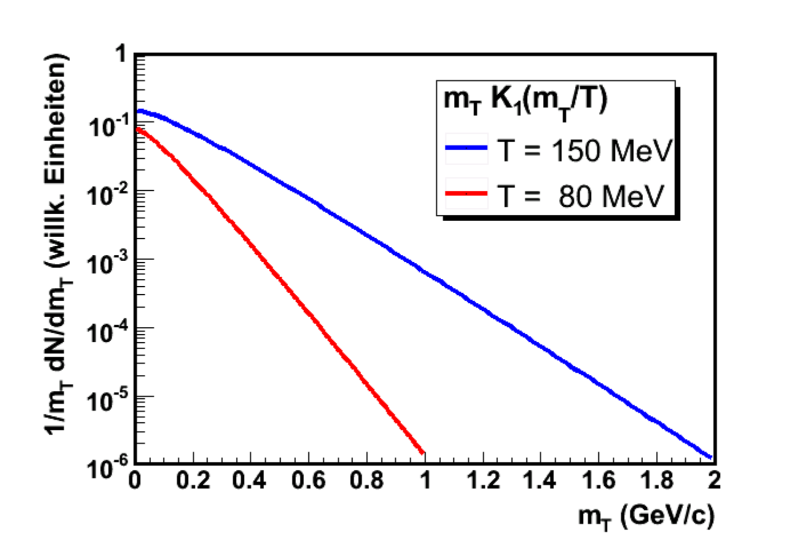
The corresponding transverse mass spectrum can be obtained by integrating over rapidity:

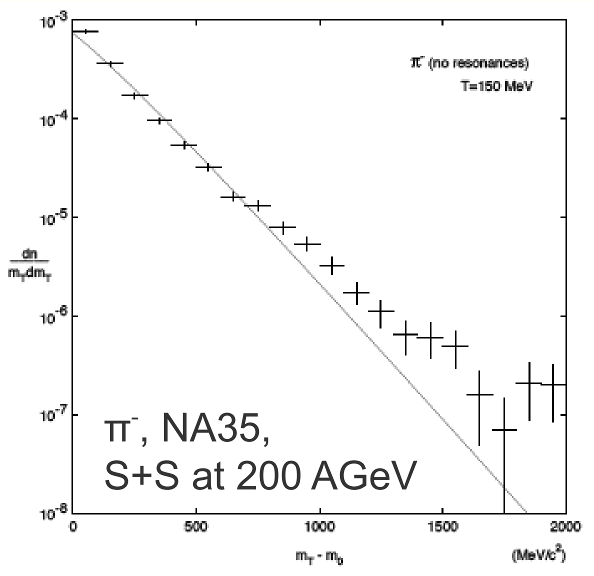
$$\frac{1}{m_T}\frac{dn}{dm_t} = \frac{V}{2\pi^2} m_T K_1\left(\frac{m_T}{T}\right) \stackrel{m_T \gg T}{\longrightarrow} V'\sqrt{m_T}e^{-m_T/T}$$

$K_1 = \text{Modified Bessel functions of 2nd kind}$

Schnedermann, Sollfrank, Heinz, Phys.Rev.C48:2462-2475,1993

Relation between Temperature and Slope





Slope of the m_T (or p_T) spectrum reflects the temperature of the fireball

- However, other effects like collective flow and resonance decays affect the slope as well and make the extraction of the temperature more difficult

- m_T spectra are indeed approximately exponential with an almost uniform slope $1/T$

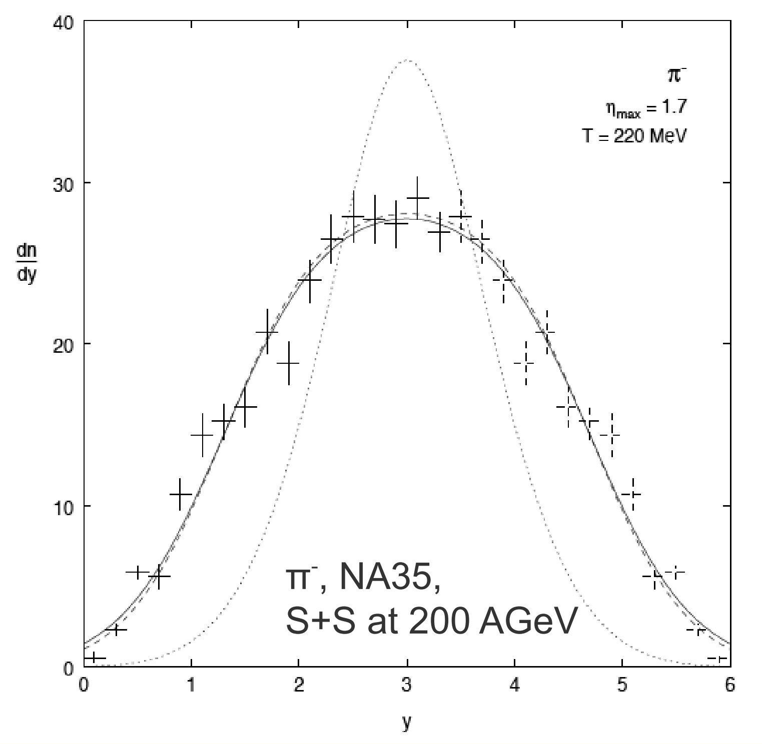
- However, clear deviation are visible: A stationary thermal source clearly is an oversimplification

Schnedermann, Sollfrank, Heinz, Phys.Rev.C48:2462-2475,1993

Rapidity Distribution for a Stationary Fireball

$$\frac{dn_{th}}{dy} = \frac{V}{(2\pi)^2} T^3 \left(\frac{m^2}{T^2} + \frac{m}{T} \frac{2}{\cosh y} + \frac{2}{\cosh^2 y} \right) \exp\left(-\frac{m}{T} \cosh y \right)$$

Reduces for light particles to:
$$\frac{dn}{dy} \propto \frac{1}{\cosh^2(y - y_0)}$$



- Full width at half height for stationary fireball in sharp contrast to the experimental value

$$\Gamma_{\text{th}}^{\text{fwhm}} \approx 1.76 \quad \Gamma_{\text{exp.}}^{\text{fwhm}} \approx 3.3 \pm 0.1$$

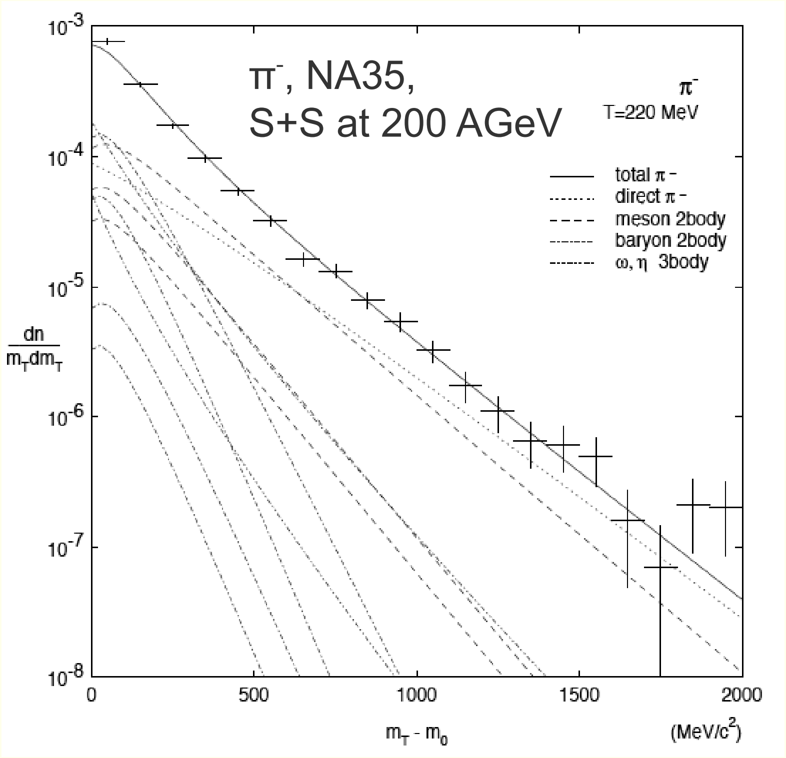
- Superposition of fireballs with different rapidities (following the Bjorken picture) can describe the data

$$\frac{dn}{dy}(y) = \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} d\eta \frac{dn_{th}}{dy}(y - \eta)$$

Effect of Resonance Decays on Transverse Spectra

Apart from directly emitted pions there are also pions which originate from the decay for resonances, e.g.,

$$\rho^0 \to \pi^+\pi^-, \quad \omega \to \pi^+\pi^-\pi^0, \quad \Delta \to N\pi^-$$



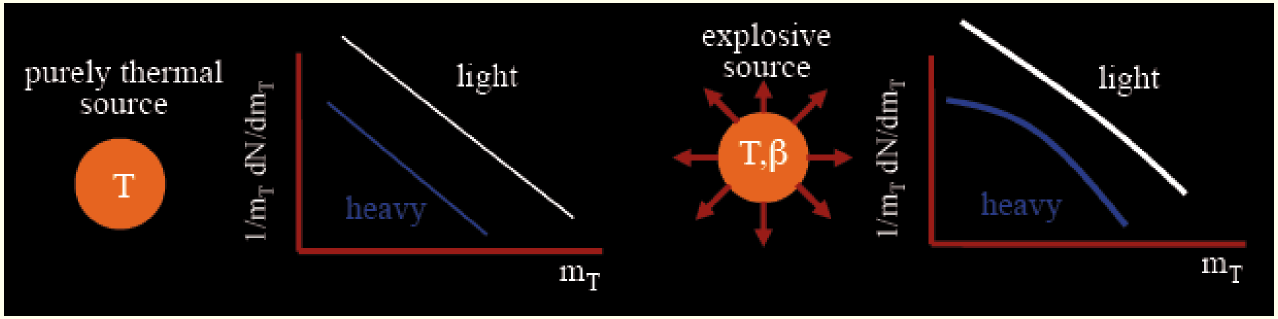
The kinematics of the resonance decays result in very steeply dropping daughter pion spectra and raise considerably the total pion yield at low m_T

It is possible to the describe the spectrum of negative pions over the whole range in m_T , with the temperature T corresponding to the slope at high m_T

Radial Flow

Schnedermann, Sollfrank, Heinz, Phys.Rev.C48:2462-2475,1993

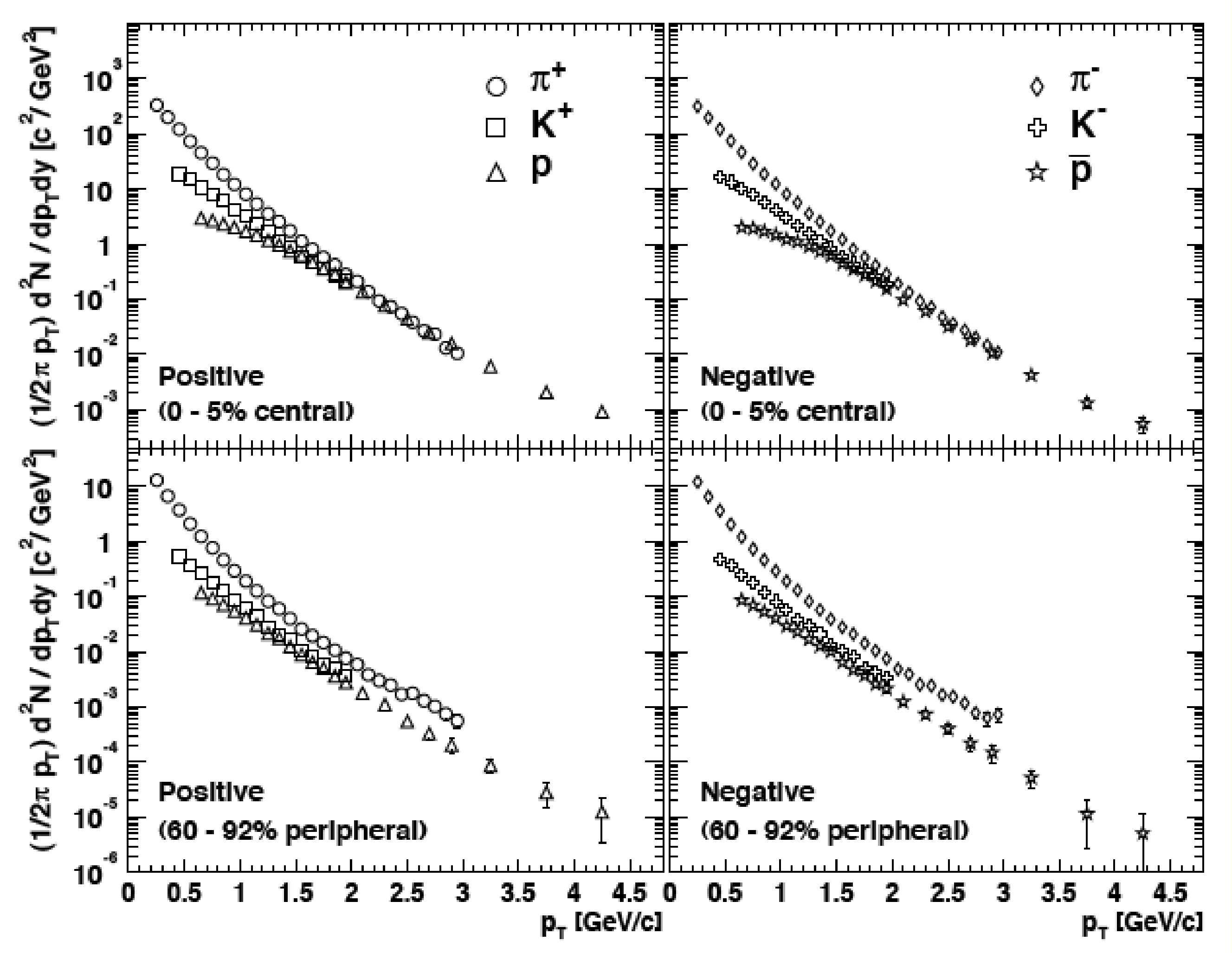
- Arguments for the existence of radial flow
  - At $T \approx 200$ MeV the mean free path of pions in hadronic matter is much less than 1 fm. On the other hand, the size of the fireball is several fm. Consequently, the pions cannot leave the interaction zone at $T \approx 200$ MeV without further collisions, the reaction region cannot decouple thermally and should by continuing expansion force the pions to cool down further.
  - It is inconsistent to assume that a thermalized system expands collectively in longitudinal direction without generating also transverse flow from the high pressures in the hydrodynamic system
  - Experimental argument: Transverse flow flattens, in the region $p_T < m$, the transverse mass spectra of the heavier particles more than for the lighter particles, in agreement with data (next slide)



Heavier particles profit more from collective flow than the light ones:

$$\langle E \rangle \approx \langle E_{\mathrm{th}} \rangle + \frac{m_0}{2} v_{\mathrm{collective}}^2$$

Identified Particle Spectra in Au+Au at $\sqrt{s_{NN}}$ = 200 GeV



Phenix white paper, Nucl.Phys.A757:184-283, 2005 (→ link)

A Simple Model for Radial Flow

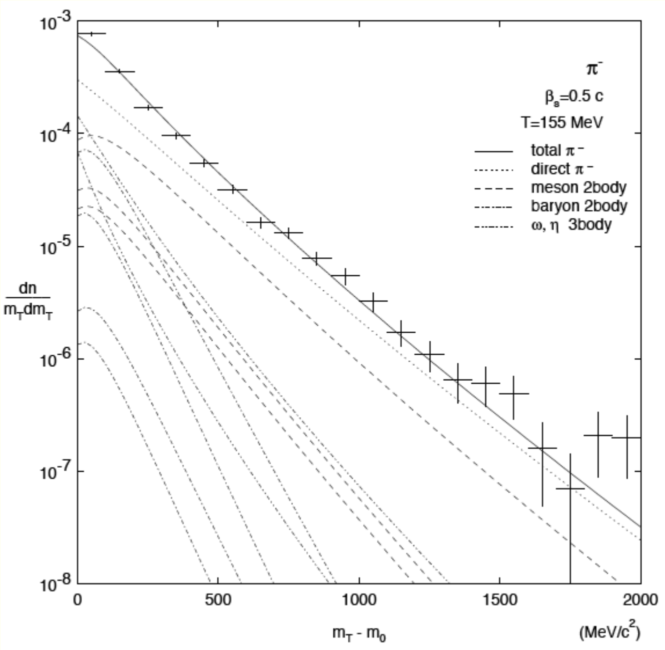
Schnedermann, Sollfrank, Heinz, Phys.Rev.C48:2462-2475,1993

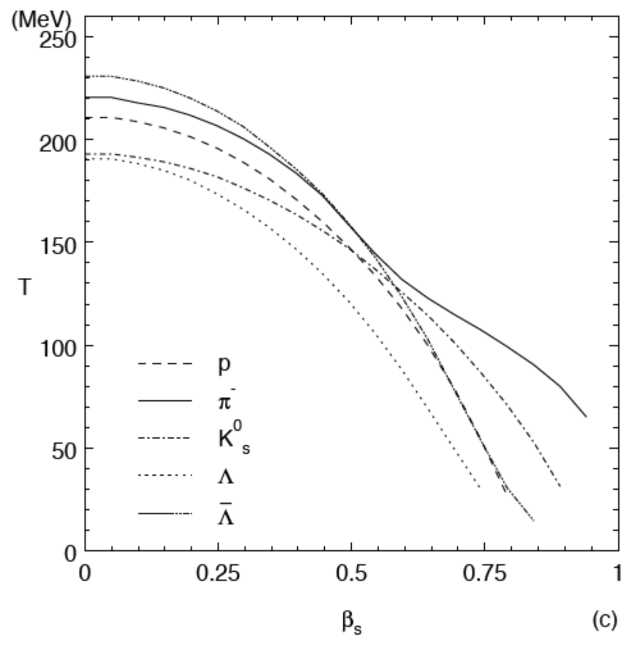
Transverse velocity profile: $\beta_T(r) = \beta_s \left(\frac{r}{R}\right)^n$

This leads to:

$$\frac{1}{m_T}\frac{dn}{dm_T} \propto \int_0^R r\,dr\, m_T I_0\left(\frac{p_T \sinh\rho}{T}\right) K_1\left(\frac{m_T \cosh\rho}{T}\right)$$

$$\rho := \mathrm{arctanh}(\beta_T) \quad \text{"transverse rapidity"}$$

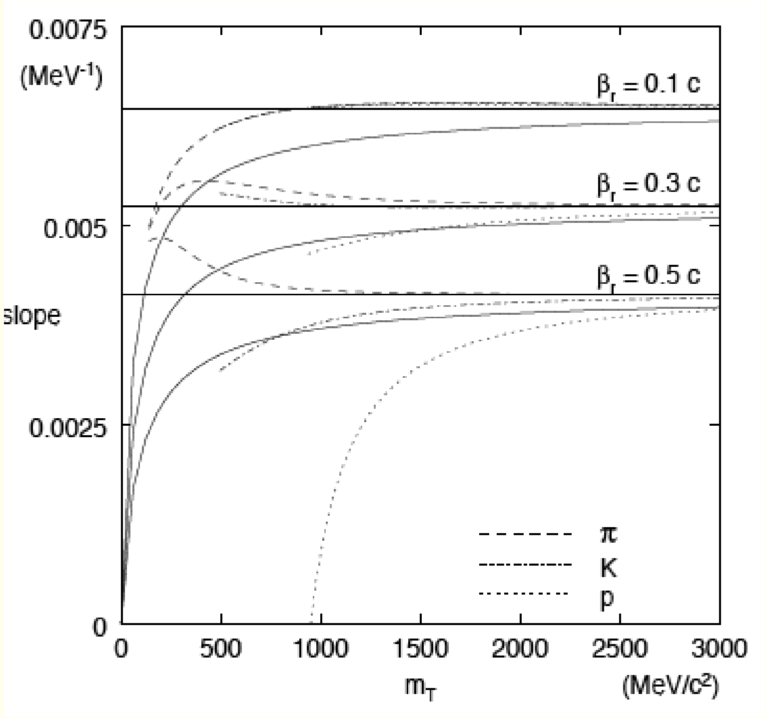




Good description of the NA35 negative pion data with T = 155 MeV and β_s = 0.5 c

However, various (T,β_s) pairs describe the spectrum well.

Local Slope of mT Spectra with Radial Flow



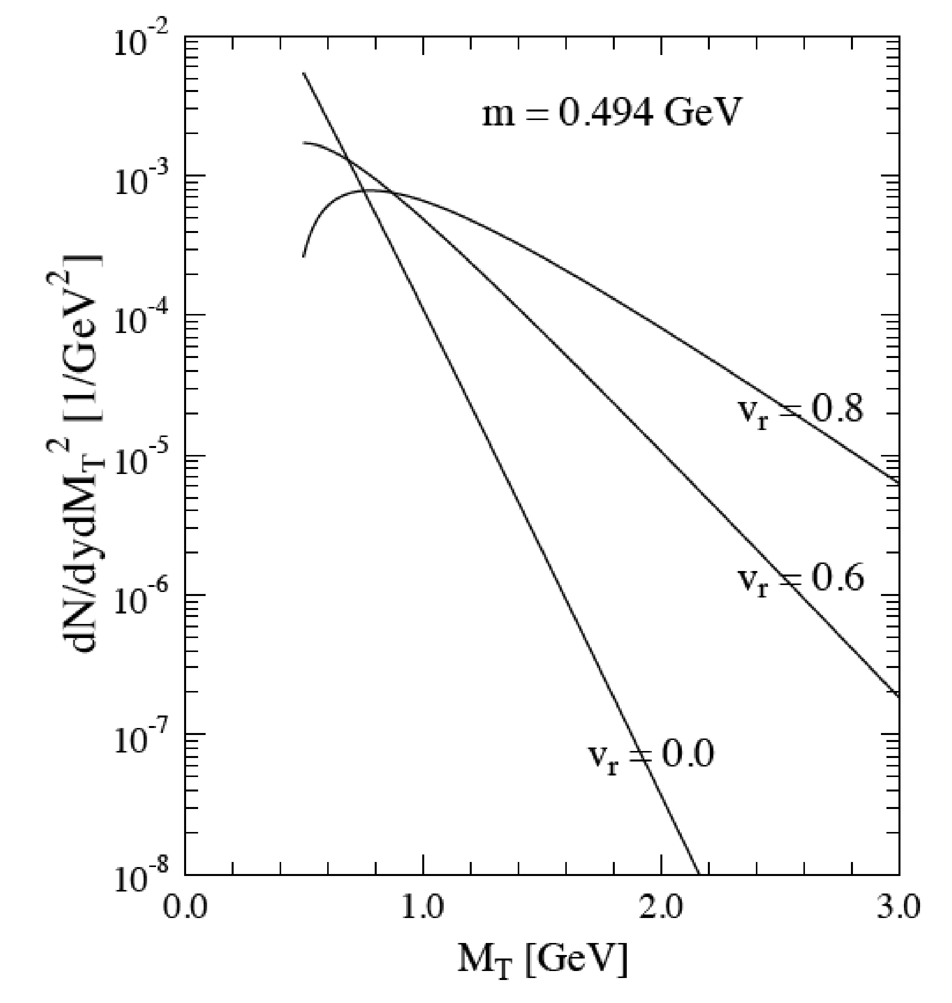
m_T slopes with transverse flow for pions for fixed transverse expansion velocity β_r

$$\lim_{m_t \to \infty} \frac{d}{dm_T} \ln\left(\frac{1}{m_T}\frac{dn}{dm_T}\right) = -\frac{1}{T}\sqrt{\frac{1-\beta_r}{1+\beta_r}}$$

The apparent temperature, i.e., the inverse slope at high m_T, is larger than the original temperature by a blue shift factor:

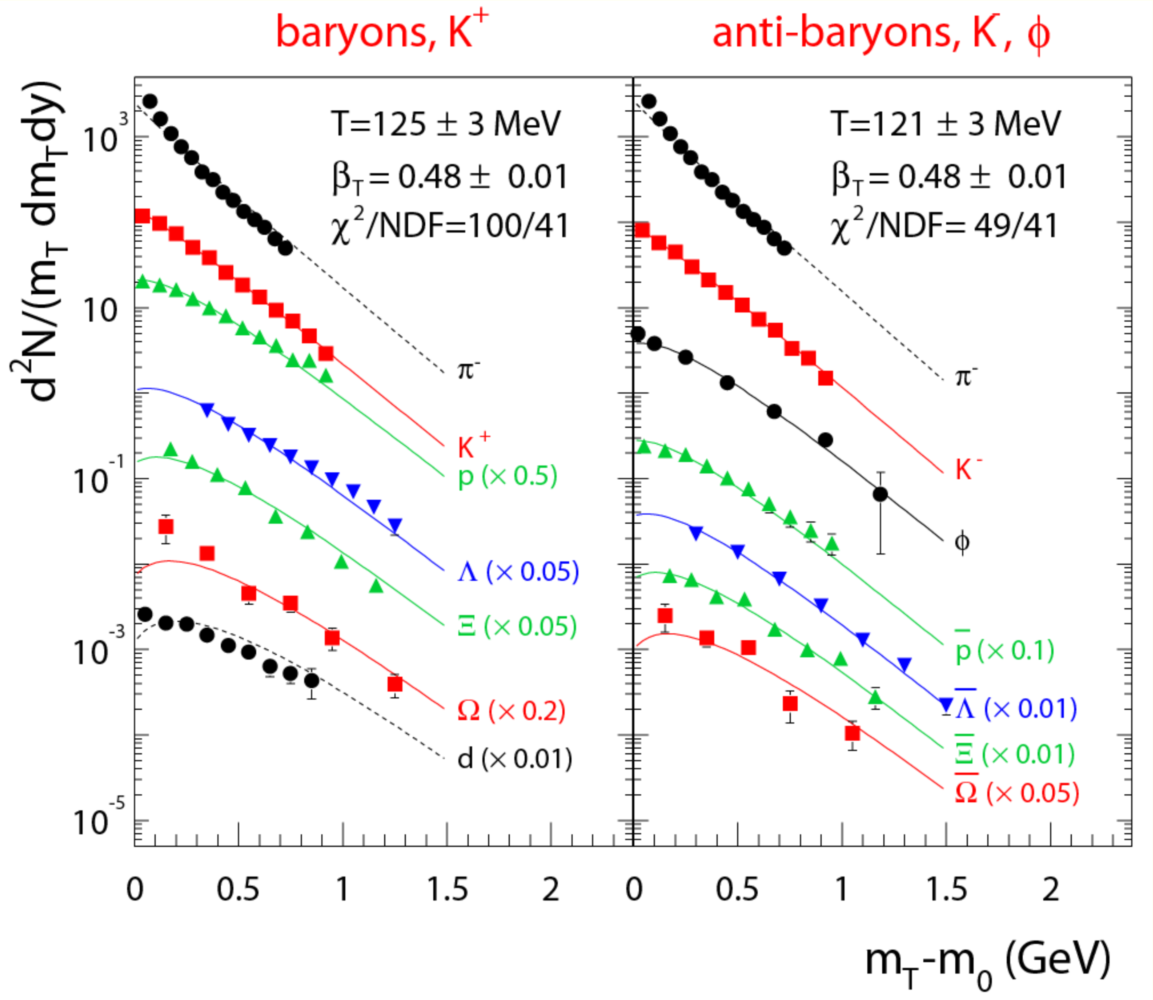
$$T_{\text{eff}} = T\sqrt{\frac{1+\beta_r}{1-\beta_r}}$$

Effect on Radial Flow on m_T Spectra



review: Huovinen, Ruuskanen, arXiv:nucl-th/0605008

Blast-Wave Fits at CERN SPS Energy (NA49)

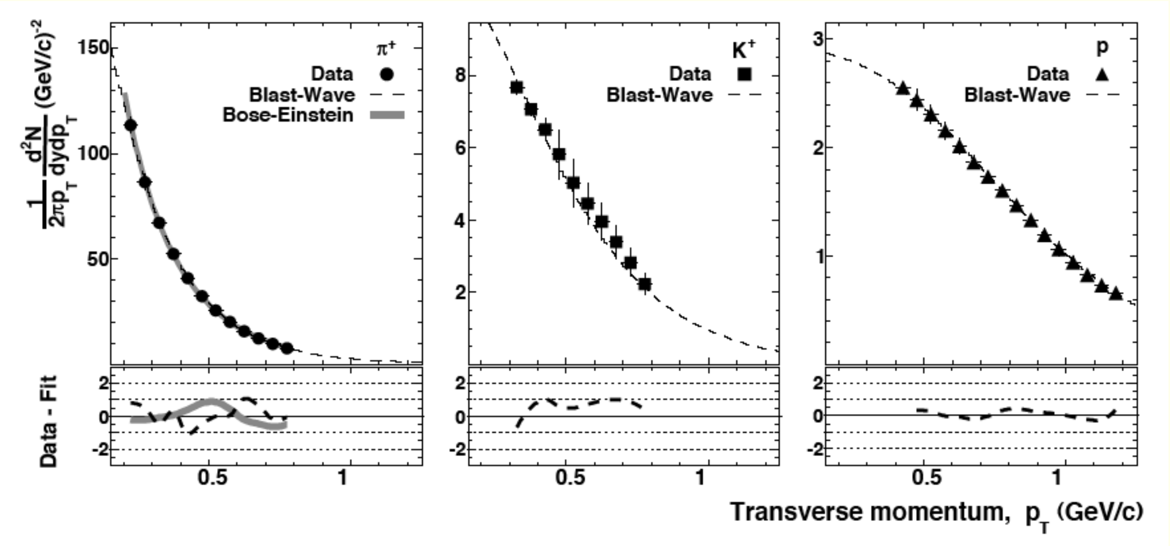


$$\frac{1}{m_T}\frac{dn}{dm_T} \propto \int_0^R r\,dr\,m_T I_0\left(\frac{p_T \sinh\rho}{T}\right) K_1\left(\frac{m_T \cosh\rho}{T}\right)$$

Blast Wave Fits at RHIC Energies (STAR)

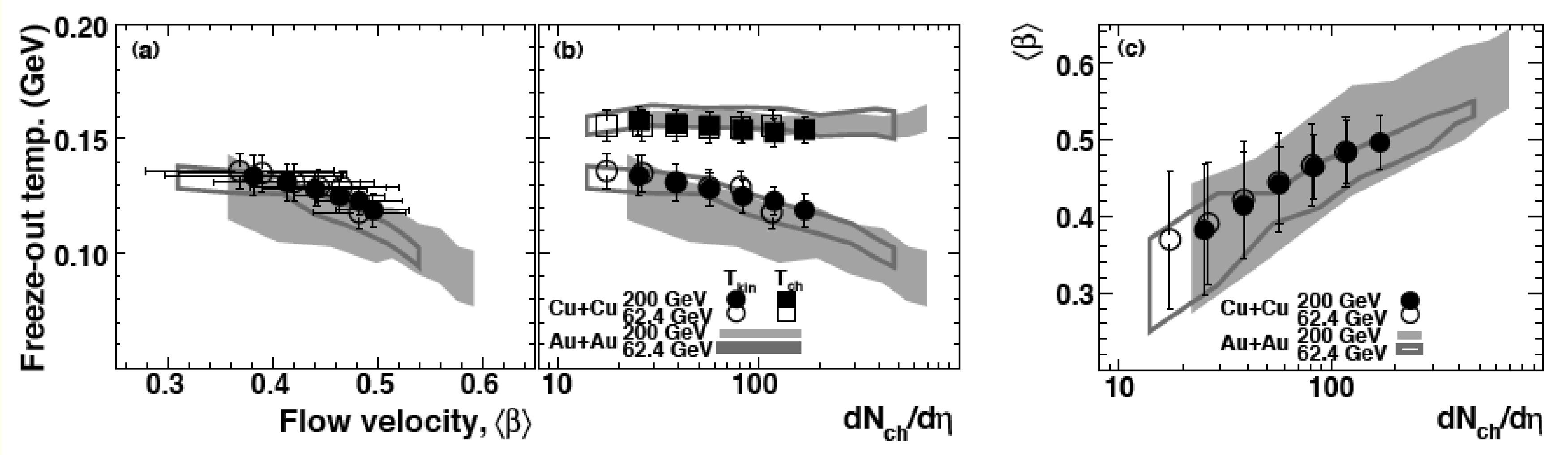
STAR, Phys.Rev.C83:034910,2011

Simultaneous fit to all particle species for given centr. class:



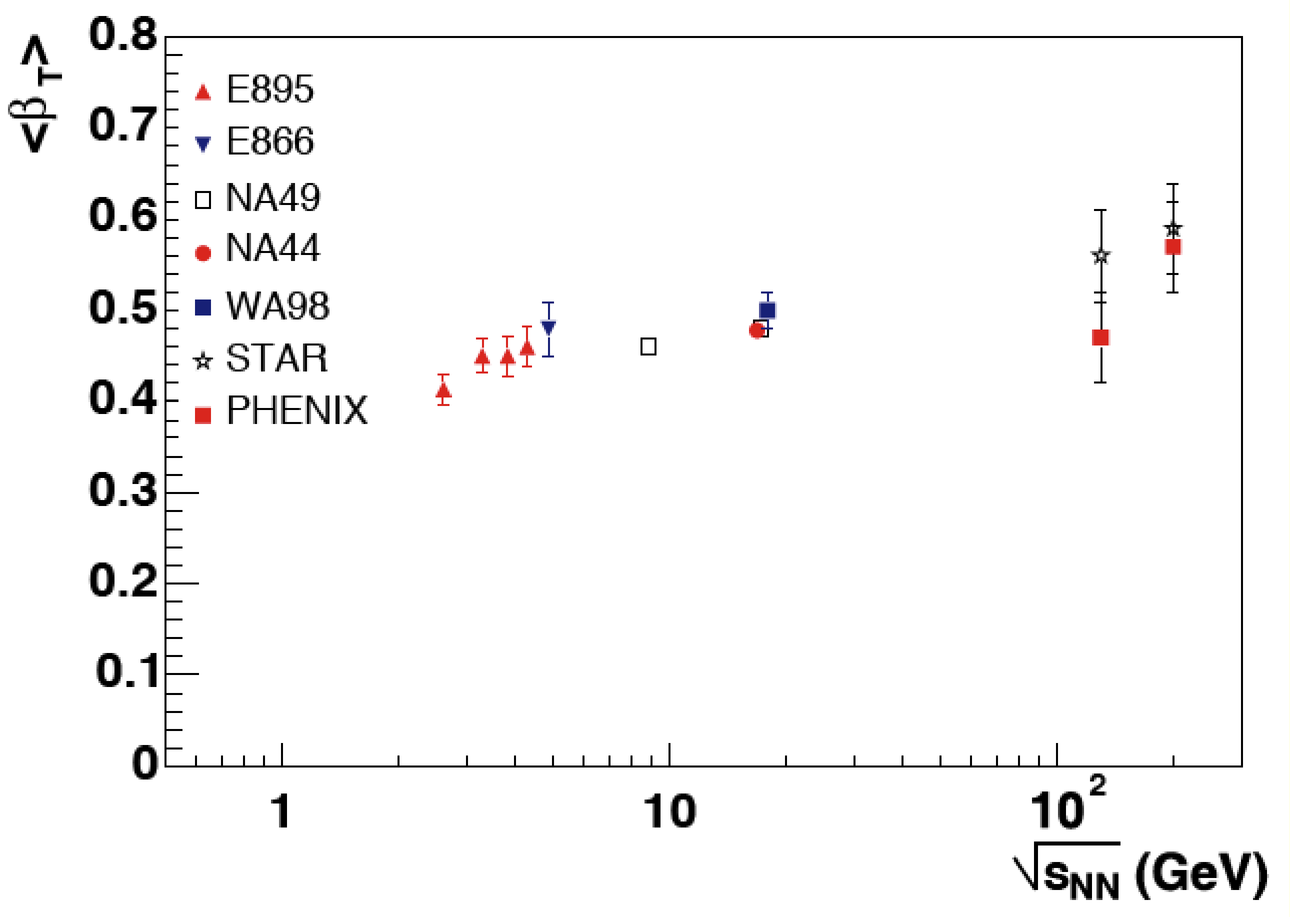
Cu+Cu at 200 GeV, 10% most central

Central A+A collisions at RHIC energies described with
T = 100 – 120 MeV,
<*β*> = 0.45 – 0.6 *c*



Radial Flow Velocities as a Function of $\sqrt{s}_{NN}$

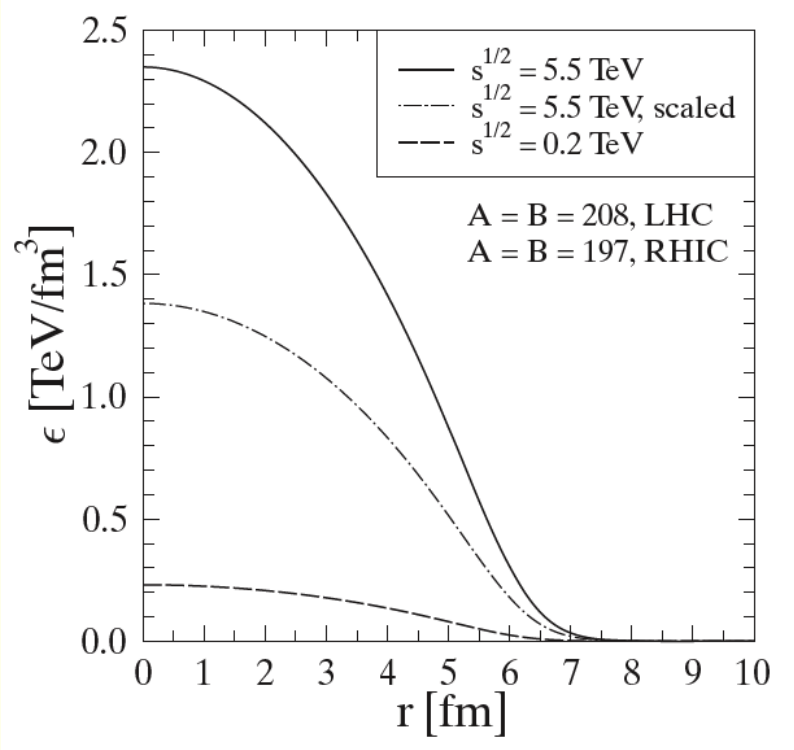
Phenix white paper, Nucl.Phys.A757:184-283, 2005 (→ link)



Radial flow velocity in A+A depends only weakly or not at all on CMS energy

Particle Spectra in Ideal Hydrodynamics (I)

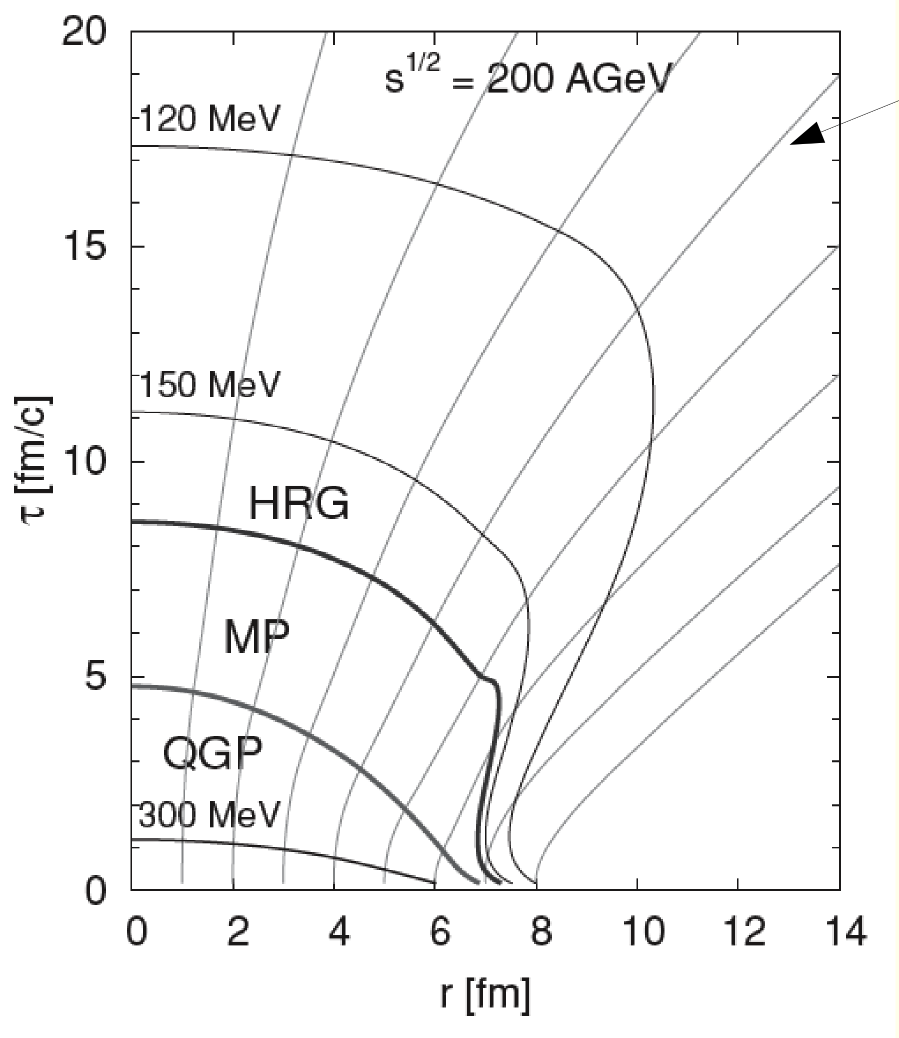
Initial conditions for hydro calc.:



- Fireball evolution treated as flow of an ideal liquid
  - Local thermal equilibrium (mean free path λ = 0)
  - Zero viscosity
- Applicable in case of early thermalization
- Equation of state (EOS) is needed (e.g., form lattice QCD)
- Input: Initial conditions
  - E.g. from Glauber calc.

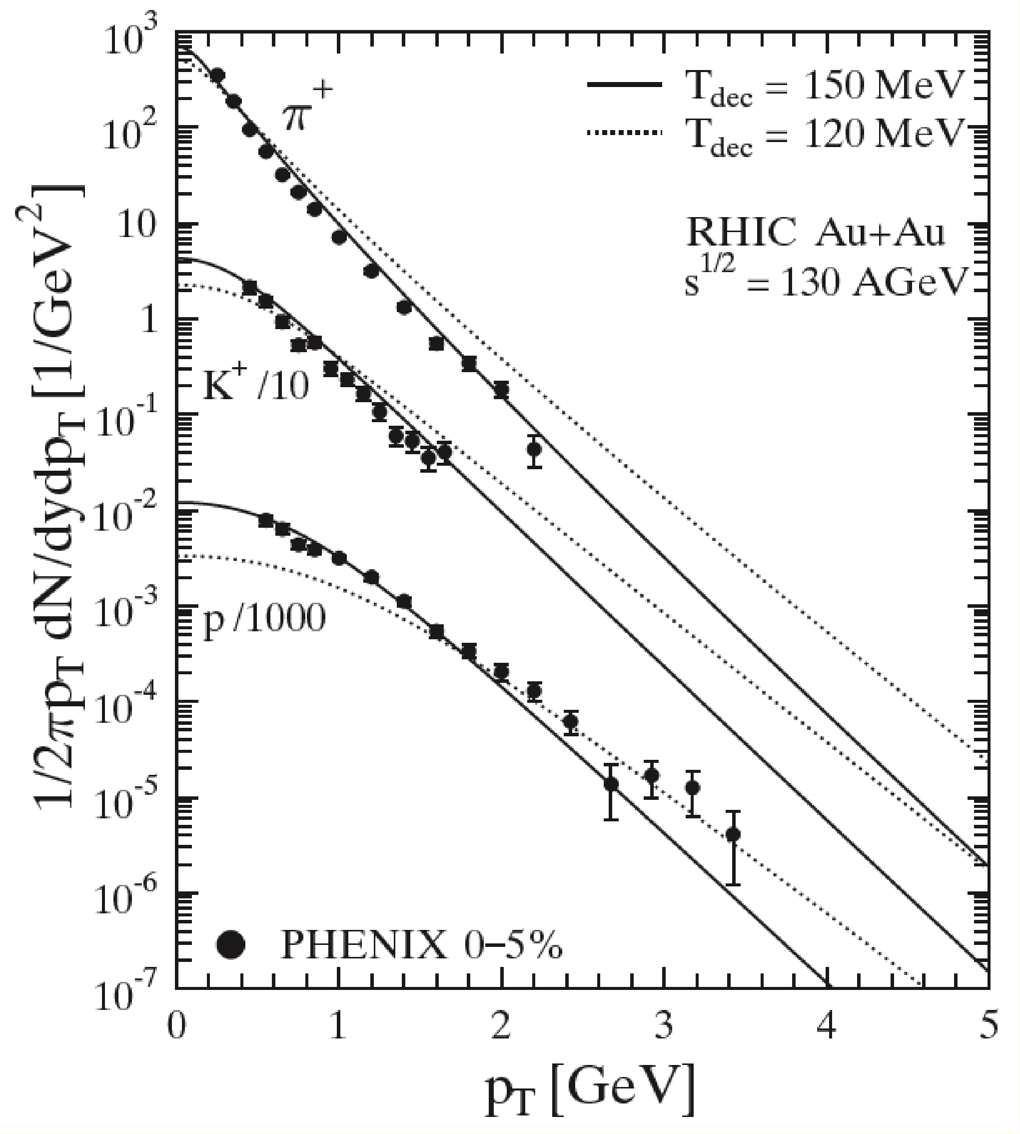
$$\varepsilon(r) \propto \frac{dN_{\text{part}}}{dr} \ (\text{or} \propto \frac{dN_{\text{coll}}}{dr},)$$

Particle Spectra in Ideal Hydrodynamics (II): Temperature Contours and Flow lines



Flow lines indicate the radial flow velocity

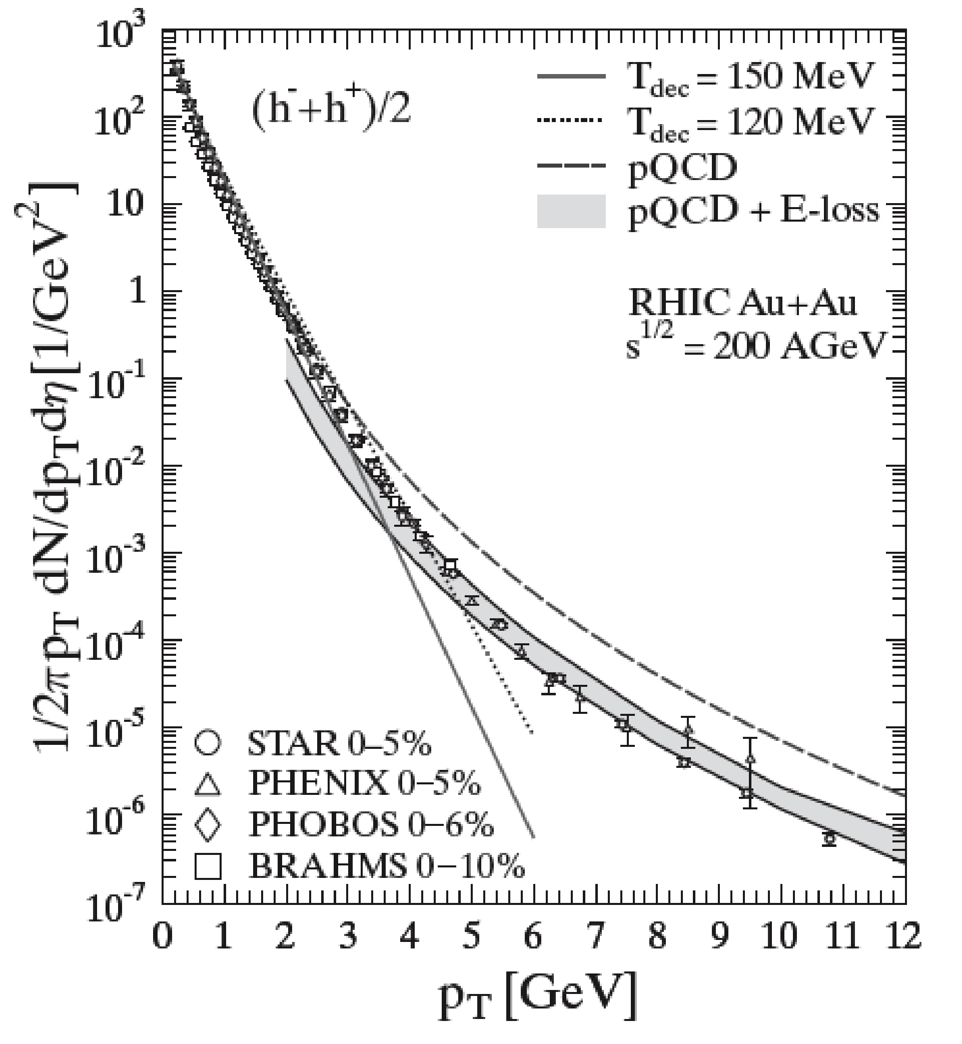
Particle Spectra in Ideal Hydrodynamics (III): Comparison to Data



Unknown decoupling temperature results in model uncertainties

Decoupling temperatures in the range 120 – 150 MeV provide a good description of the data (here: $T_{dec,chem} = T_{dec,kin}$)

Particle Spectra in Ideal Hydrodynamics (III): Validity of the Hydro Description: Up to p_T= 2 - 3 GeV/*c*

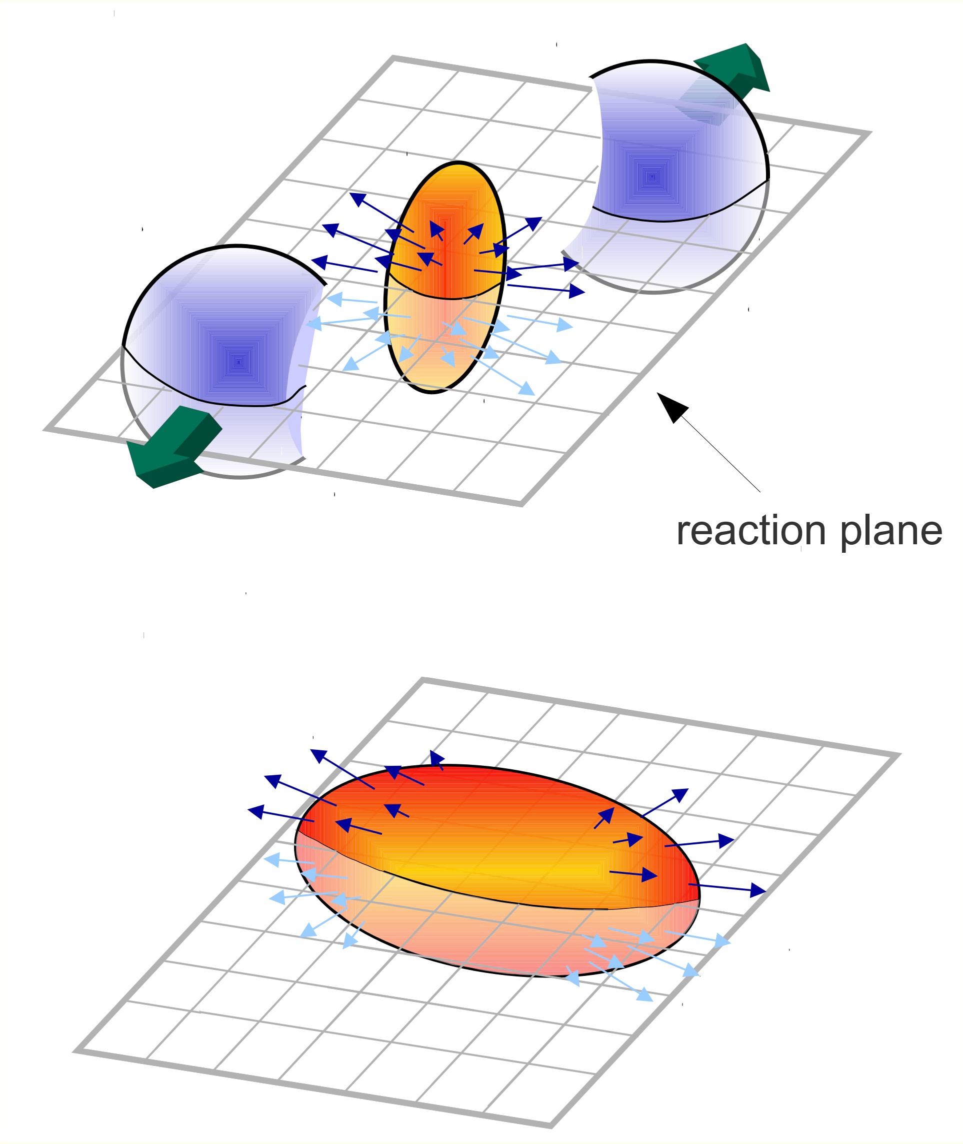


At large p_T the hydro description yields exponential spectra

However, around p_T = 2 – 3 GeV/c the measured spectra start to follow a power law shape

6.3 Directed and Elliptic Flow

The Reaction Plane

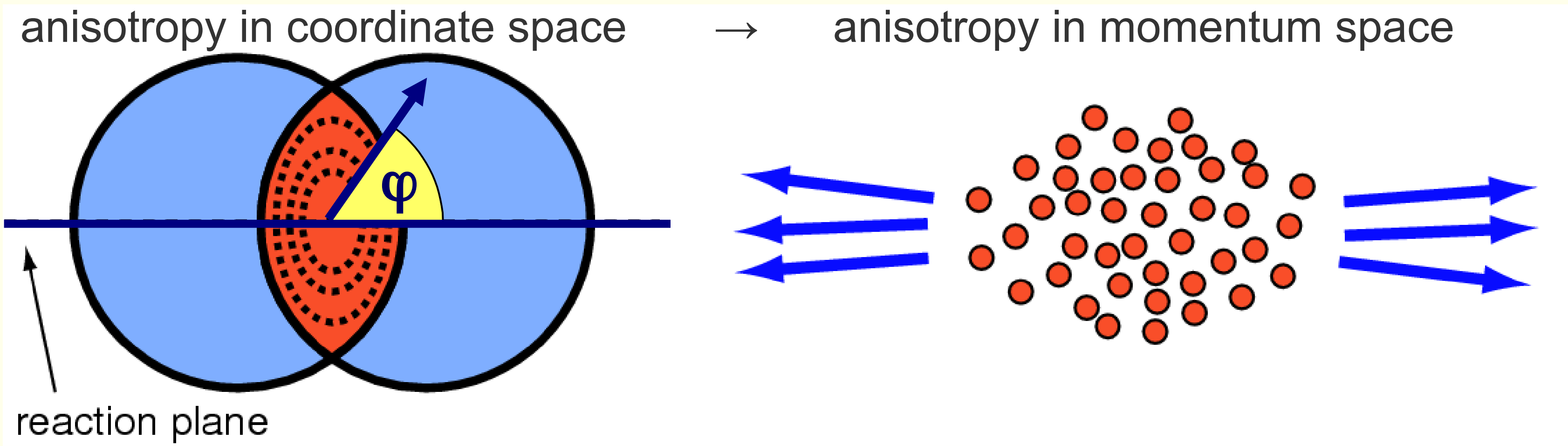


The impact parameter vector ***b*** and the beam axis span the reaction plane

Experimentally, the reaction plane can be measured (with some finite resolution) on an event-by-event basis

One can then study particle production as a function of the emission angle w.r.t. the reaction plane

Fourier Decomposition



$$E\frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi}\frac{d^2N}{p_t\,dp_t\,dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_{RP})]\right)$$

The sine terms in the Fourier expansion vanish because of the reflection symmetry with respect to the reaction plane.

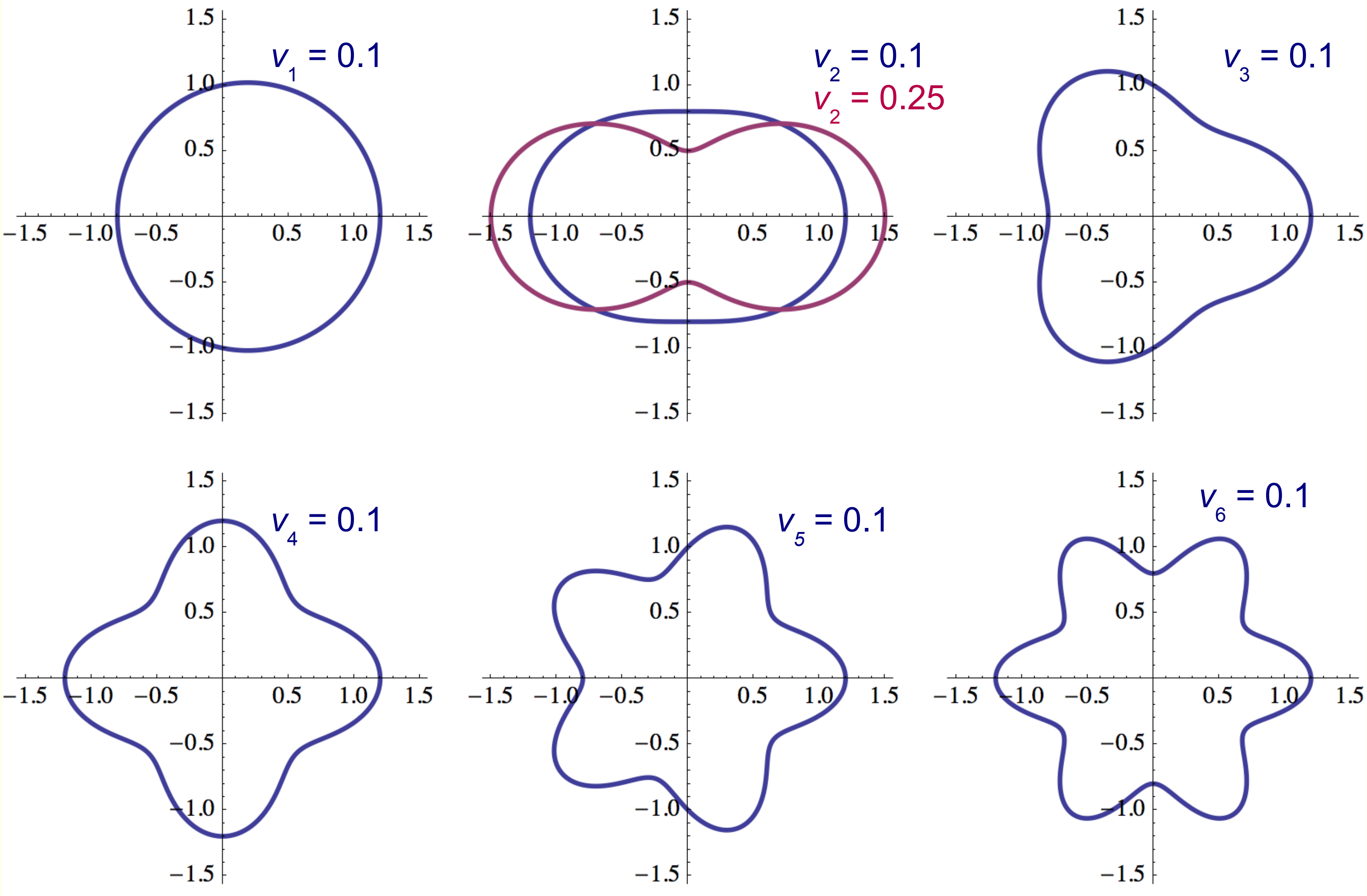
Fourier coefficients: $v_n(p_T, y) = \langle \cos[n(\varphi - \Psi_{RP})] \rangle$

v_1 : Strength of the directed flow (small at midrapidity)

v_2 : Strength of the elliptic flow

Visualization of v_n

$$f(\varphi) = 1 + 2v_n \cos(n\varphi)$$

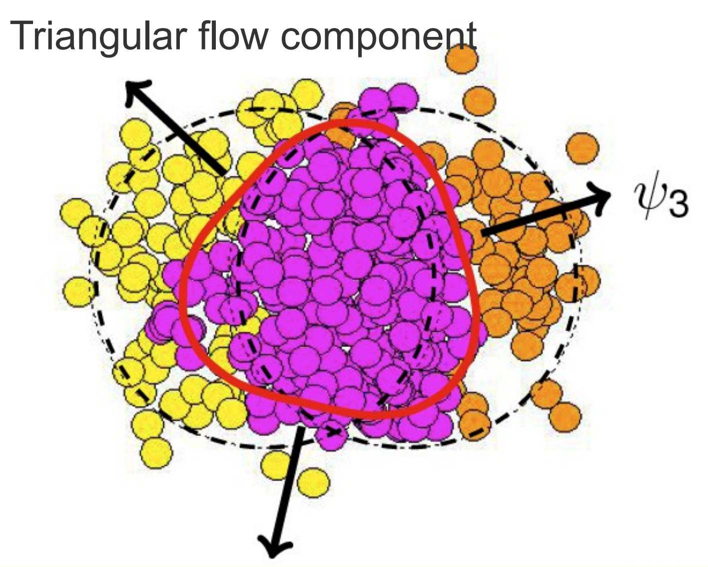


Odd Harmonics

When studying flow w.r.t. to the reaction plane one expects the odd harmonics to be zero due to symmetry reasons:

$$E\frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}\left(1 + 2\sum_{n,\text{ even}} v_n \cos[n(\varphi - \Psi_{RP})]\right)$$

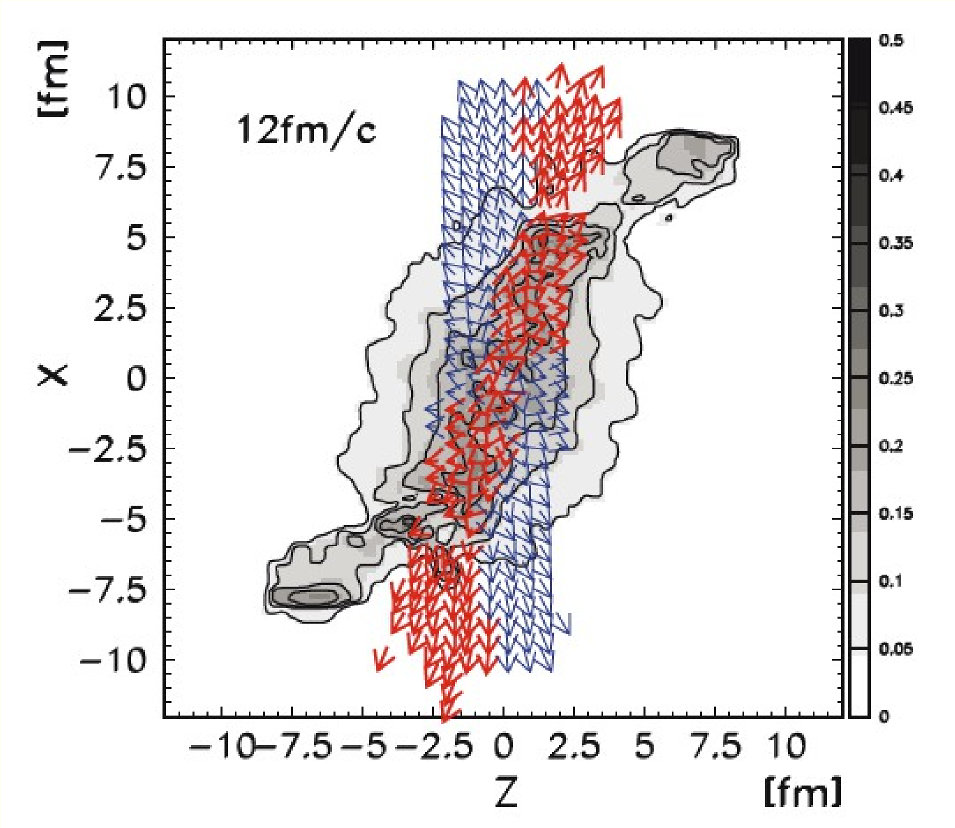
$$\approx \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}\left(1 + 2v_2\cos[2(\varphi - \Psi_{RP})] + 2v_4\cos[4(\varphi - \Psi_{RP})]\right)$$



Recently, it was realized that fluctuations of the overlap zone may lead to flow patterns that need to be described by odd harmonics, e.g., triangular flow (v_3). However, the triangular flow appears to be uncorrelated to the reaction plane:

$$\frac{dN}{d\varphi} \propto 1 + 2\sum_{n=1} v_n \cos[n(\varphi - \Psi_n)]$$

Directed Flow (I)



Net-baryon density at t = 12 fm/c in the reaction plane with velocity arrows for mid-rapidity ($|y| < 0.5$) fluid elements

arXiv:0809.2949

Where the colliding nuclei start to overlap, dense matter is created which deflects the remaining incoming nuclear matter.

The deflection of the remnants of the incoming nucleus at positive rapidity is in the +x direction leading to $p_x > 0$, and the remnants of the nucleus at negative rapidity are deflected in the direction thus having a $p_x < 0$

The deflection happens during the passing time of the colliding heavy-ions. Thus, the system is probed at early times.

Directed Flow (II)

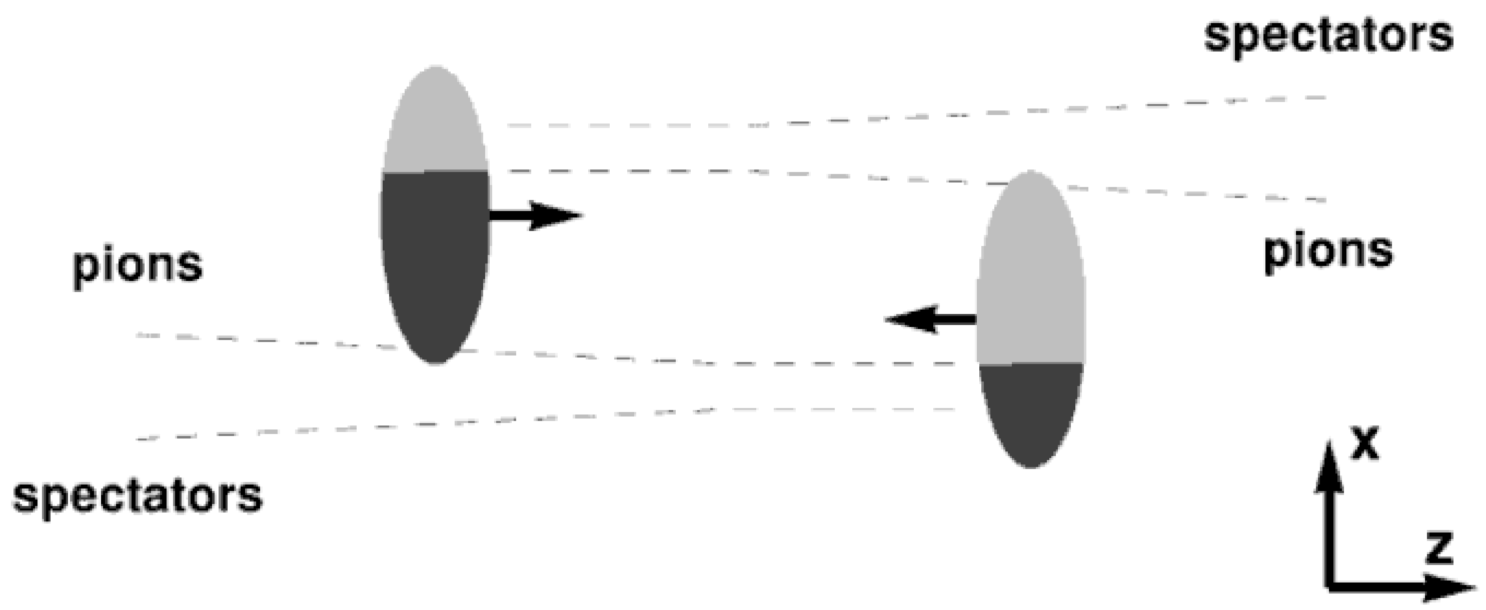
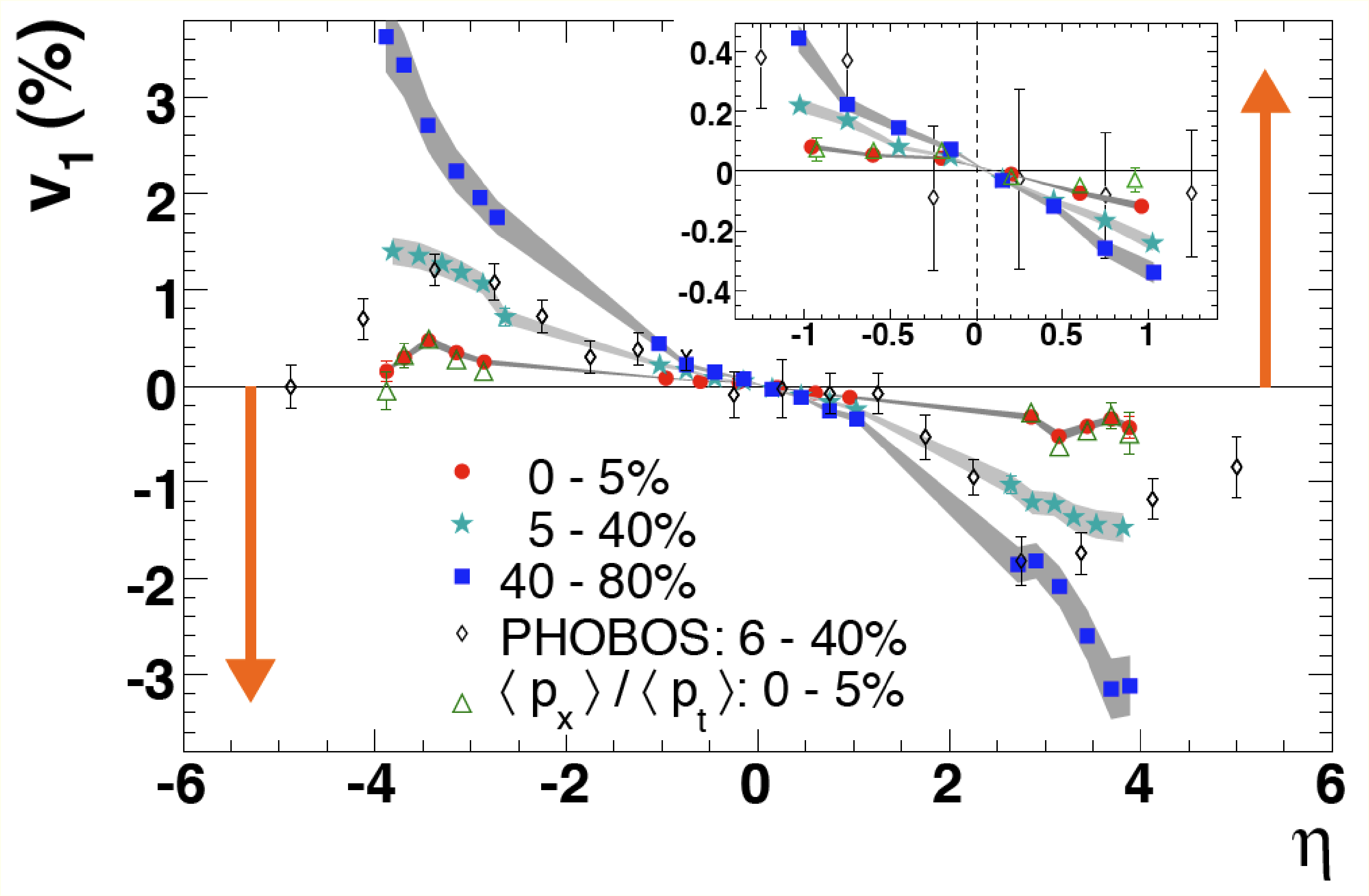


Fig. 2.9 Schematic view of the directed flow observed at relativistic energies. For positive and large rapidities ($y \sim y_P$) the spectators are deflected towards positive values of x. For positive and small rapidities ($y \geq 0$) the produced particles have negative v_1, hence they are deflected towards negative values of x.

The directed flow for spectator nucleons and pions has a different sign.This suggests a different origin of v_1 for protons and pions.

Directed Flow (III)

STAR, Phys.Rev.Lett.101:252301,2008



Directed flow of charged hadrons. The orange arrows indicate the sign of the directed flow (and the rapidity) of spectator neutrons.

Basic Elements of Relativistic Hydrodynamics (Perfect Liquid) (I)

Energy-momentum tensor T^{1o} :

The energy-momentum tensor is the four-momentum component in the μ direction per three-dimensional surface area perpendicular to the ν direction.

$$\Delta \mathbf{p} = (\Delta E, \Delta p_x, \Delta p_y, \Delta p_z) \qquad \Delta \mathbf{x} = (\Delta t, \Delta x, \Delta y, \Delta z)$$

$$\mu = \nu = 0: \quad T_R^{00} = \frac{\Delta E}{\Delta x \Delta y \Delta z} = \frac{\Delta E}{\Delta V} = \varepsilon$$

$$\mu = \nu = 1: \quad T_R^{11} = \frac{\Delta p_x}{\Delta t \Delta y \Delta z}$$

force in x direction acting on an surface $\Delta y\, \Delta z$ perpendicular to the force → pressure

$$T^{\mu\nu} = \begin{pmatrix} \text{energy density} & \text{energy flux density} \\ \text{momentum density} & \text{momentum flux density} \end{pmatrix} \equiv \begin{pmatrix} \varepsilon & \vec{j}_\varepsilon \\ \vec{g} & \Pi \end{pmatrix}$$

Energy-momentum tensor in the fluid rest frame:

$$T_R^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Isotropy in the fluid rest implies that the energy flux T_{0j} and the momentum density T_{j0} vanish and that $\Pi^{ij} = P\,\delta^{ij}$

See also Ollitrault, arXiv:0708.2433 (→ link)

Basic Elements of Relativistic Hydrodynamics (Perfect Liquid) (II)

Energy-momentum tensor (in case of local thermalization) after Lorentz transformation to the lab frame:

$T_{\mu\nu}$ is symmetric. This is a non-trivial consequence of relativity.

$$T^{\mu\nu} = (\varepsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$$

metric tensor diag(1,-1,-1,-1)

Energy density and pressure in the co-moving system

4-velocity: $u^\mu = \mathrm{d}x^\mu / \mathrm{d}\tau = \gamma(1, \vec{v})$

Energy and momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0, \quad \nu = 0, \dots, 3$$

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

in components:

$$\frac{\partial}{\partial t}\varepsilon + \vec{\nabla}\vec{j}_\varepsilon = 0 \quad \text{(energy conservation)}$$

$$\frac{\partial}{\partial t} g_i + \nabla_j \Pi_{ij} = 0 \quad \text{(momentum conservation)}$$

Conserved quantities, e.g., baryon number:

continuity equation

$$j_B^\mu(x) = n_B(x)\, u^\mu(x), \qquad \partial_\mu j_B^\mu(x) = 0 \quad \Leftrightarrow \quad \frac{\partial}{\partial t} N_\mathrm{B} + \vec{\nabla}(N_\mathrm{B}\vec{v}) = 0$$

$$N_\mathrm{B} = \gamma n_\mathrm{B}$$

Hydrodynamic Models

Ingredients of hydrodynamic models

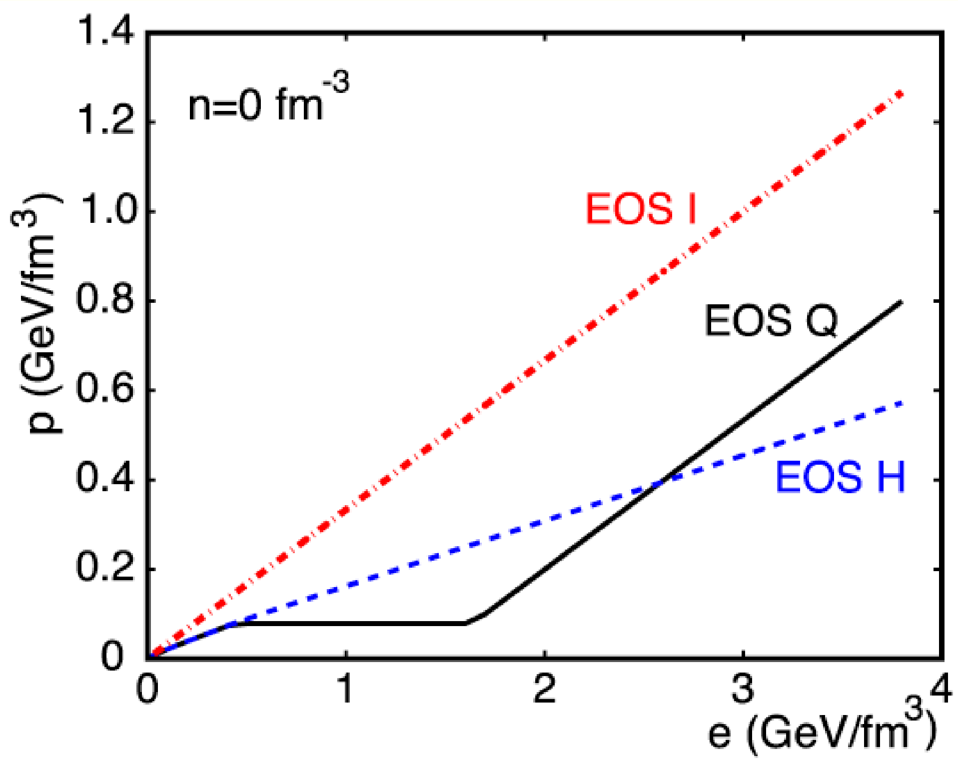
- Equation of motion and baryon number conservation:

  $$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j_B^\mu(x) = 0$$

  5 equations for 6 unknowns:

  $$(u_x, u_y, u_z, \varepsilon, P, n_B)$$

- Equation of state: $P(\varepsilon, n_B)$ (needed to close the system)
- Initial conditions, e.g., from Glauber calculation
- Freeze-out condition

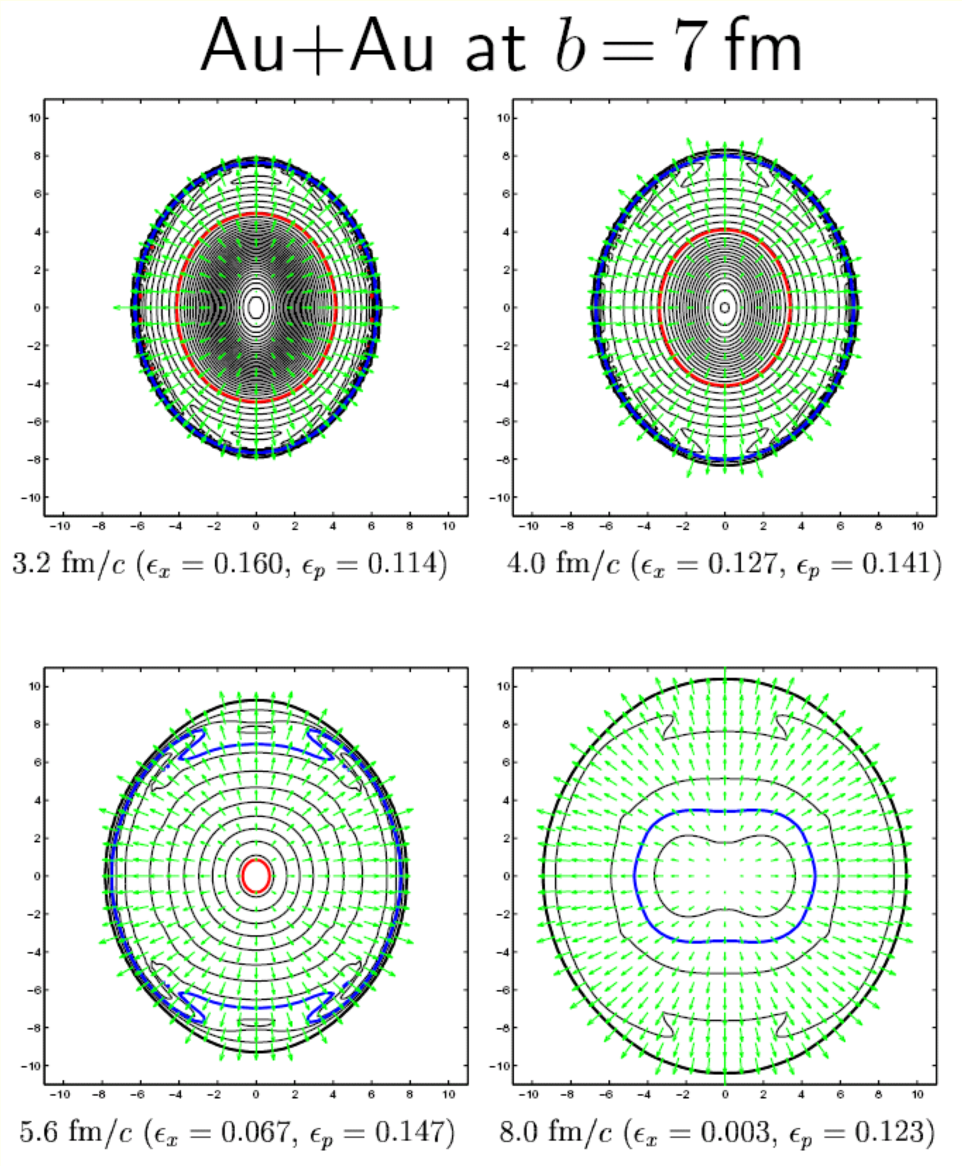


EOS I: ultra-relativistc gas $P = \varepsilon/3$

EOS H: resonance gas, $P \approx 0.15\ \varepsilon$

EOS Q: phase transition, QGP ↔ resonance gas

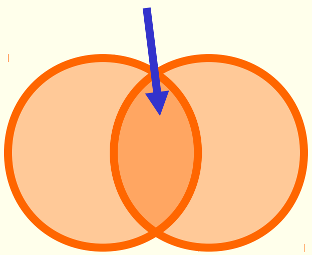
Space-time Evolution of the Fireball in a Hydro Model



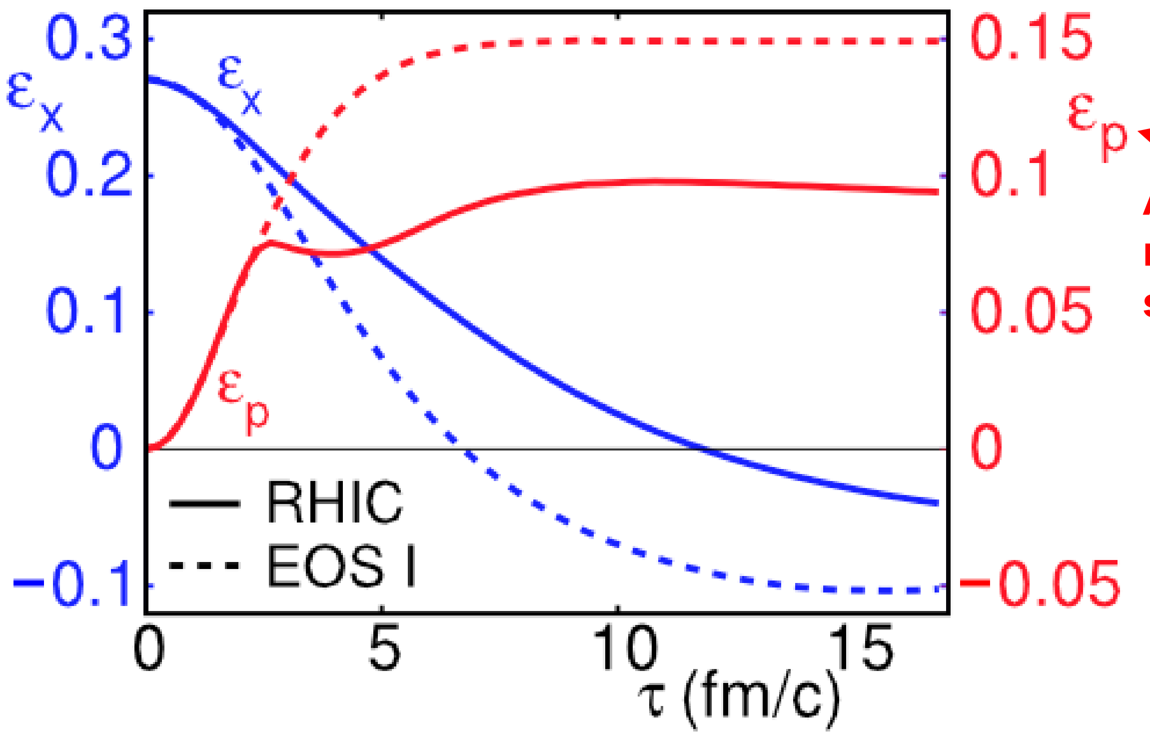
Elliptic flow is “self-quenching”:
The cause of elliptic flow, the initial spacial anisotropy, decreases as the momentum anisotropy increases

Time Dependence of the Momentum Anisotropy

Anisotropy in coordinate space



Ulrich Heinz, Peter Kolb, arXiv:nucl-th/0305084

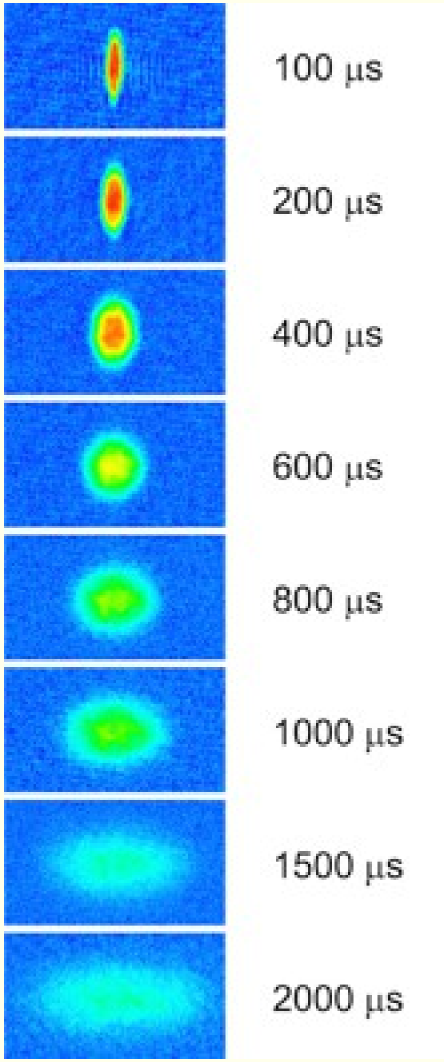


Anisotropy in momentum space

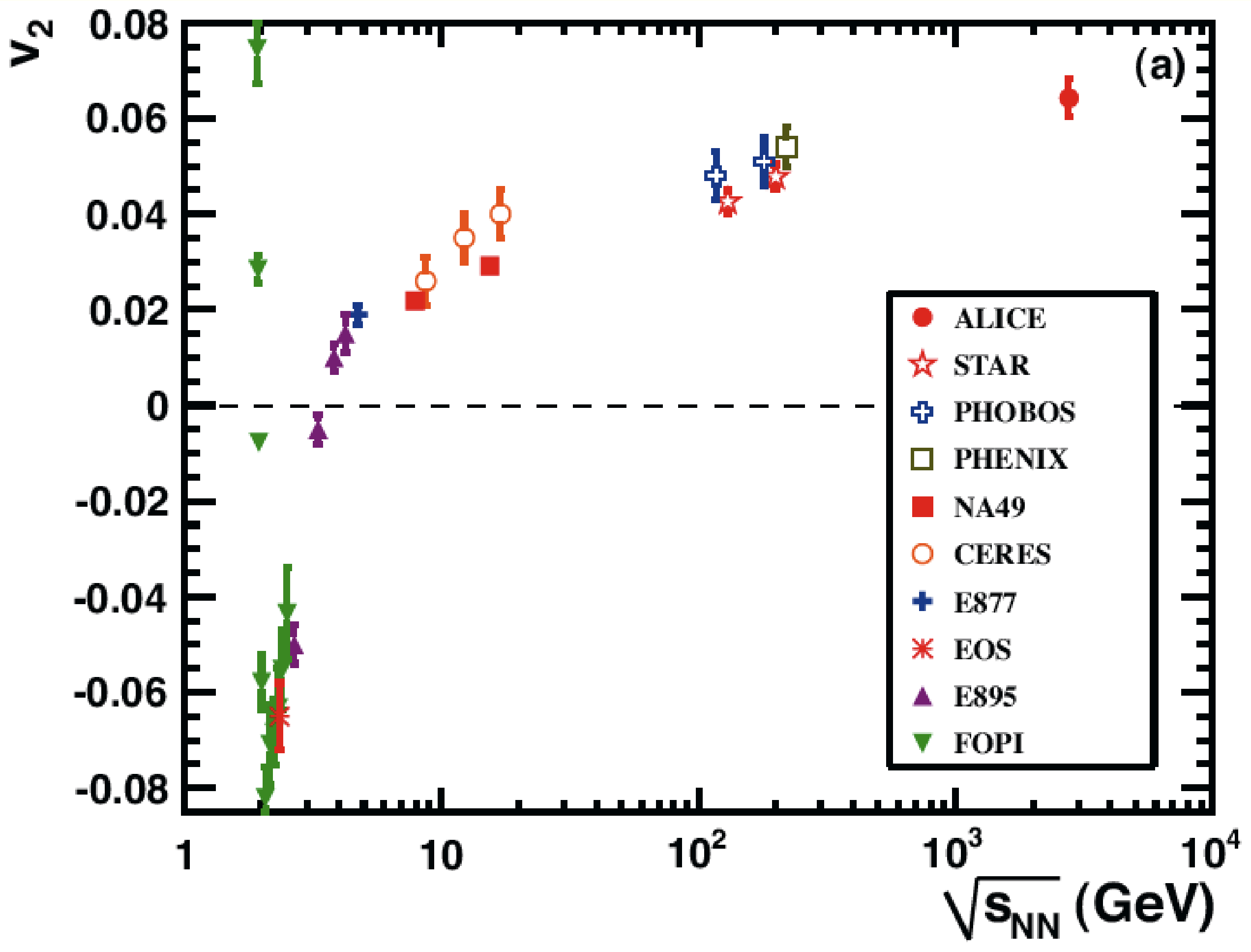
In hydrodynamic models the momentum anisotropy develops in the early (QGP) phase of the collision. Thermalization times of less then 1 fm/*c* are needed to describe the data.

Elliptic Flow of Cold atoms

- 200 000 Li-6 atoms in an highly anisotropic trap (aspect ratio 29:1)
- Very strong interactions between atoms (Feshbach resonance)
- Once the atoms are released the one observed a flow pattern similar to elliptic flow in heavy-ion collisions



v_2 as a Function of $\sqrt{s_{NN}}$

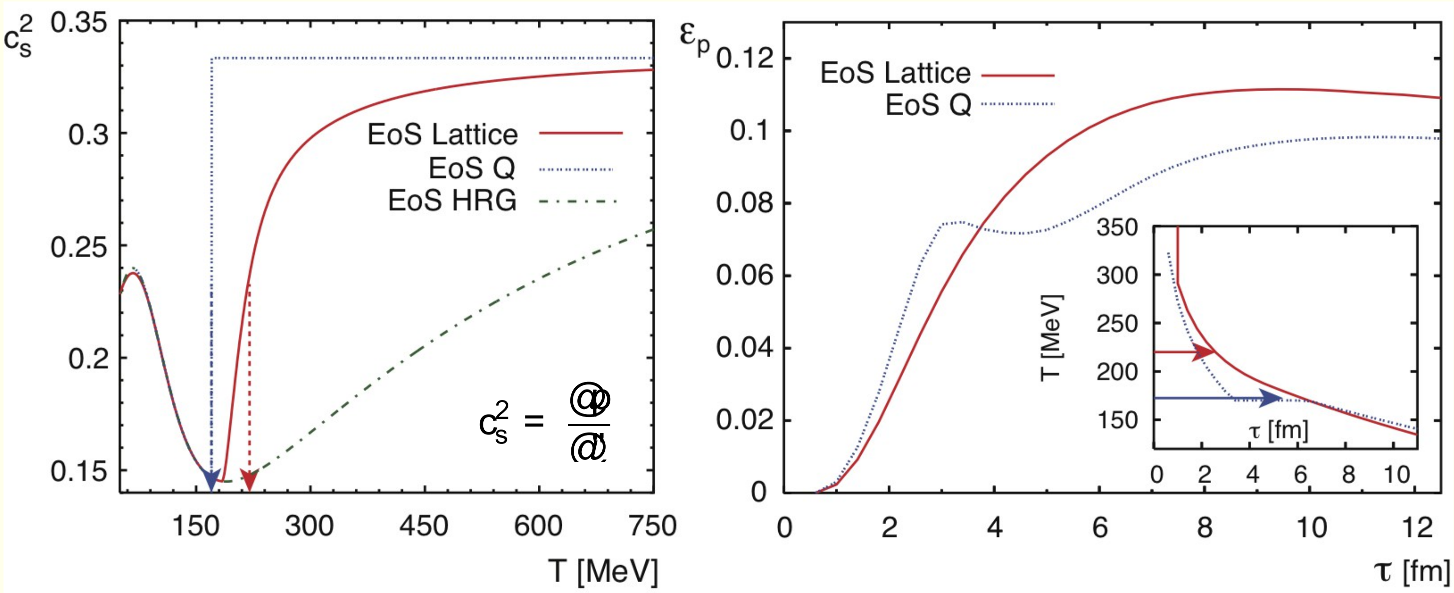


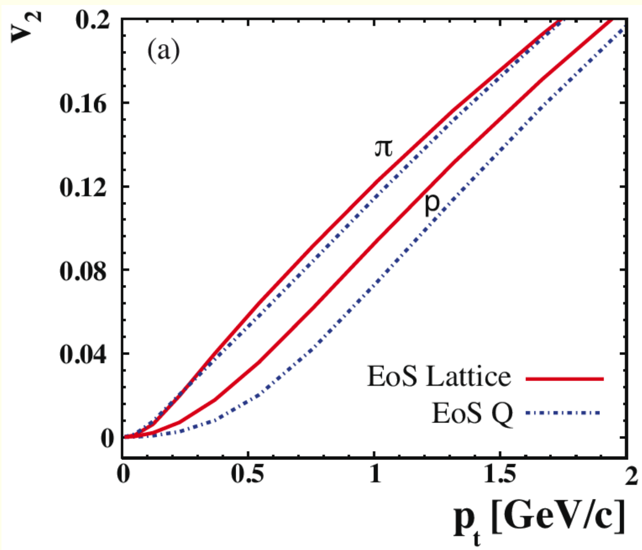
R. Snellings, arXiv:1102.3010,
P. Sorensen, arXiv:0905.0174

- $\sqrt{s_{NN}} > \sim 4$ GeV: initial excentricity leads to pressure gradients that cause positive v_2
- $2 < \sqrt{s_{NN}} < 4$ GeV: velocity of the nuclei is small so that presence of spectator matter inhibits in-plane particle emission ("squeeze-out")
- $\sqrt{s_{NN}} < 2$ GeV: rotation of the collision system leads to fragments being emitted in-plane

Sensitivity to the Equation-of-State

R. Snellings, arXiv:1102.3010

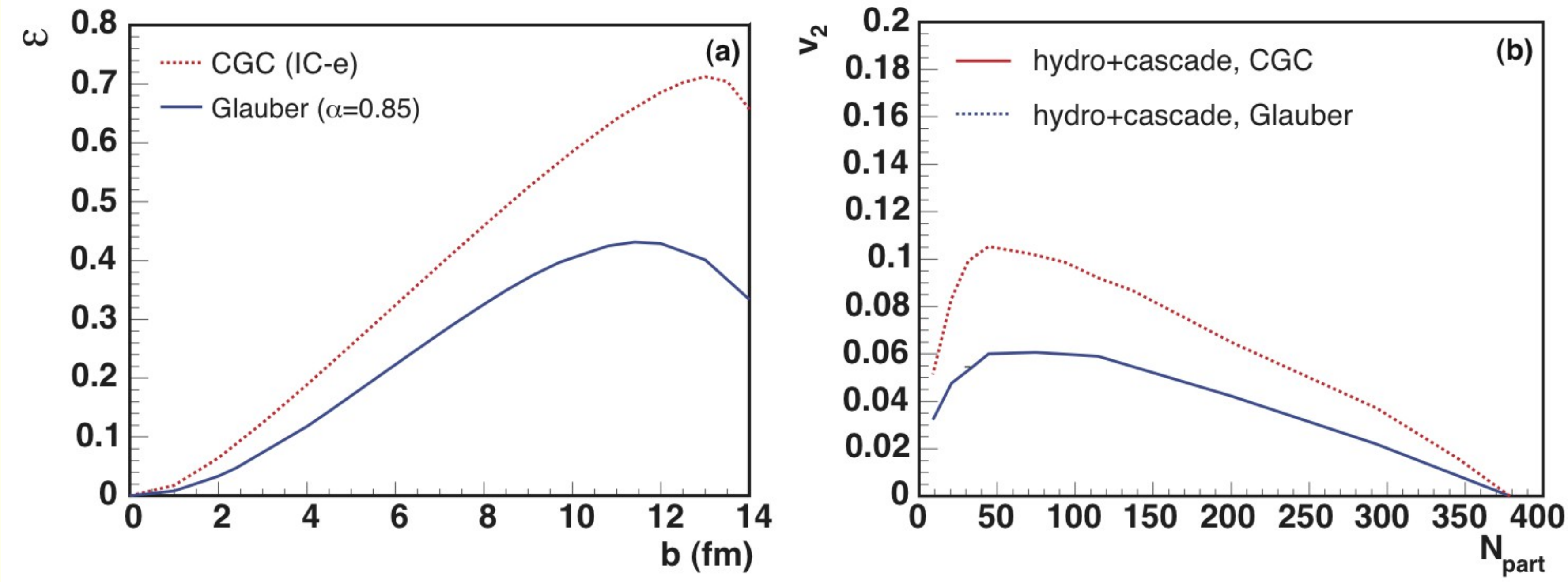


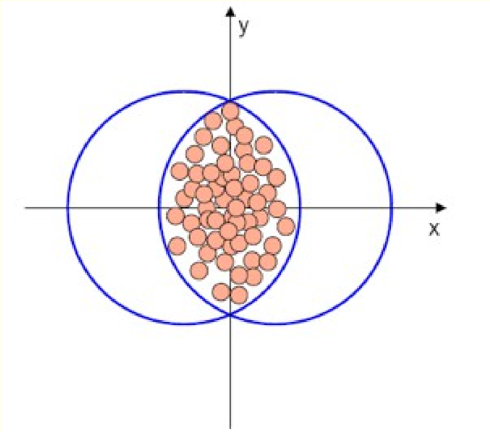


The sensitivity to the EoS corresponds to the fact that v_2 is sensitive to the sound velocity.

The measurement of v_2 for different particle species helps constrain the EoS.

Sensitivity to Initial Condition





Eccentricity:

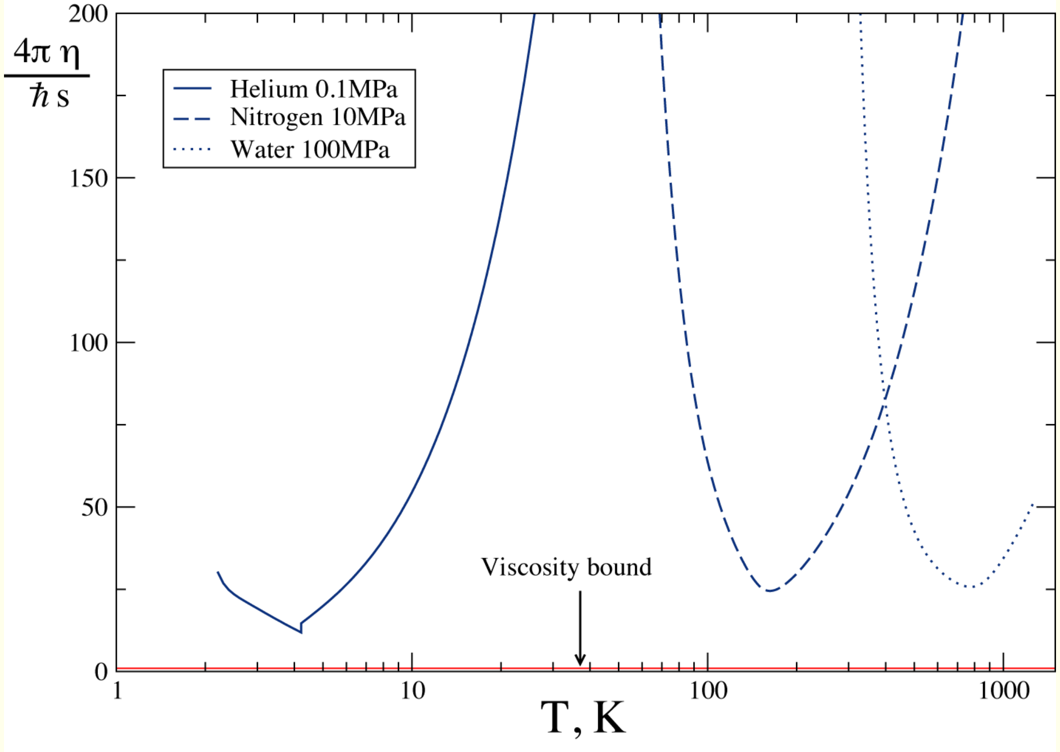
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

In hydrodynamic models: $v_2 \propto \varepsilon$.

The initial eccentricity results from the 2D profile of produced gluons. Gluon saturation models predict larger eccentricities than Glauber calc's (in which the gluon profile follows the wounded nucleon profile, or $d^2N_{coll}/dxdy$)

Sensitivity to Viscosity

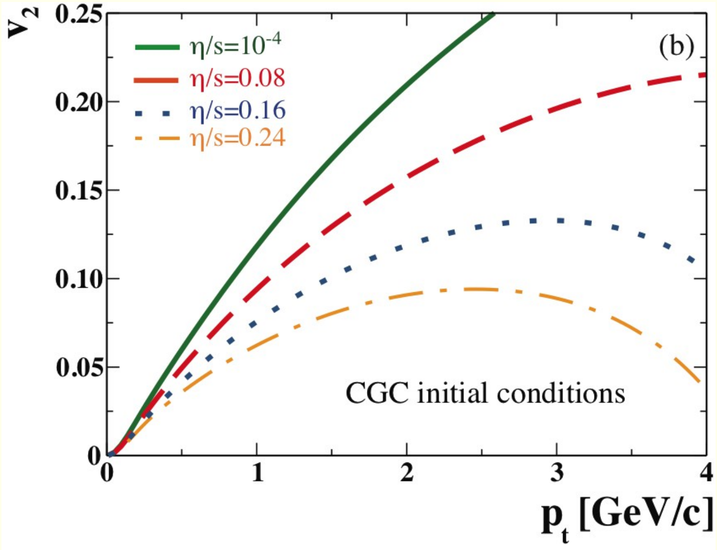
Kovtun, Son, Starinets,
PRL 94 (2005) 111601



Based on a correspondence between string theory and quantum field theory ("AdS/CFT correspondence") Kovtun, Son, and Starinets argued that there is a lower limit for the viscosity of any fluid:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

v_2 is sensitive to the viscosity of the quark-gluon plasma. The larger η/*s*, the smaller is the resulting v_2



Event Plane Method (I)

S. A. Voloshin, A. M. Poskanzer, R. Snellings, arXiv:0809.2949

Event flow vector Q_n:

$$Q_{n,x} = \sum_i w_i \cos(n\phi_i) = Q_n \cos(n\Psi_n)$$

$$Q_{n,y} = \sum_i w_i \sin(n\phi_i) = Q_n \sin(n\Psi_n)$$

The optimal choice for w_i is to approximate $v_n(p_T; y)$. $w_i = p_{T,i}$ is often used as a good approximation.

Event plane angle:

$$\Psi_n = \frac{1}{n} \mathrm{atan2}(Q_{n,y}, Q_{n,x})$$

$\mathrm{atan2}(y, x)$ is defined such that $(r, \mathrm{atan2}(y, x))$ are the polar coordinates of the cartesian coordinates (x, y); $r := \sqrt{x^2 + y^2}$. atan2 is a C/C++ function.

Event Plane Method (II)

Fourier coefficient w.r.t. the event plane (not the reaction plane):

$$v_n^{\text{observed}}(p_T, y) = \langle \cos[n(\varphi - \Psi_{\text{RP}})] \rangle$$

To remove auto-correlations one has to subtract the Q-vector of the particle of interest from the total event Q-vector, obtaining ψ_n to correlate with the particle.

Alternatively, one determines the reaction plane at forward rapidities and correlates this event plane with particles measured at mid-rapidity.

Since finite multiplicity limits the estimation of the angle of the reaction plane, the v_n have to be corrected for the event plane resolution for each harmonic:

$$v_n = \frac{v_n^{\text{observed}}}{R_n}, \qquad R_n = \langle \cos[n(\Psi_n - \Psi_{\text{RP}})] \rangle$$

To estimate the event plane resolution one divides the full event up into two independent sub-events of equal multiplicity

$$R_n = \sqrt{\langle \cos[n(\Psi_n^A - \Psi_n^B)] \rangle}$$

Two Particle Correlations

The correlation of all particles with the reaction plane implies a 2-particle correlation:

$$\frac{dN^{\text{pairs}}}{d\Delta\varphi} \propto (1 + \sum_n 2 v_n^2 \cos(n\Delta\varphi))$$

Hence, the vn can also be determined from a fit to the 2-particle azimuthal distribution.

A related method is the two-particle cumulant method in which the coefficients are calculated as:

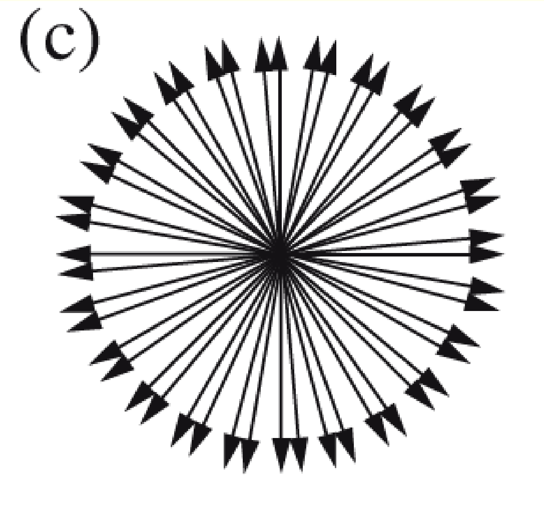
$$v_n\{2\}^2 = \langle \cos[n(\varphi_1 - \varphi_2)] \rangle$$

Non-Flow Effects

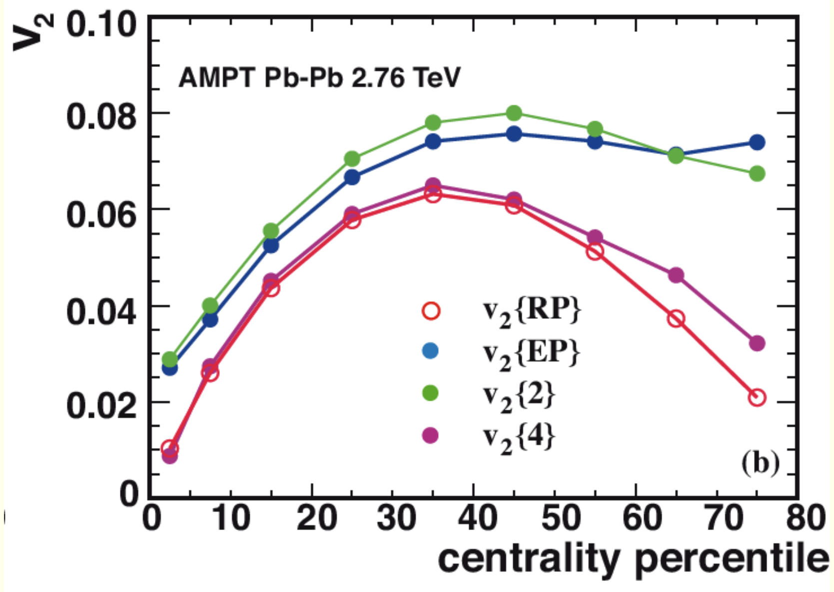
Not only flow leads to azimuthal correlations. Examples: resonance decays, jets, …
The extracted Fourier coefficients thus have a nonflow component, e.g., for the two-particle correlations:

$$v_n\{2\}^2 = \langle v_n^2 \rangle + \delta_n$$

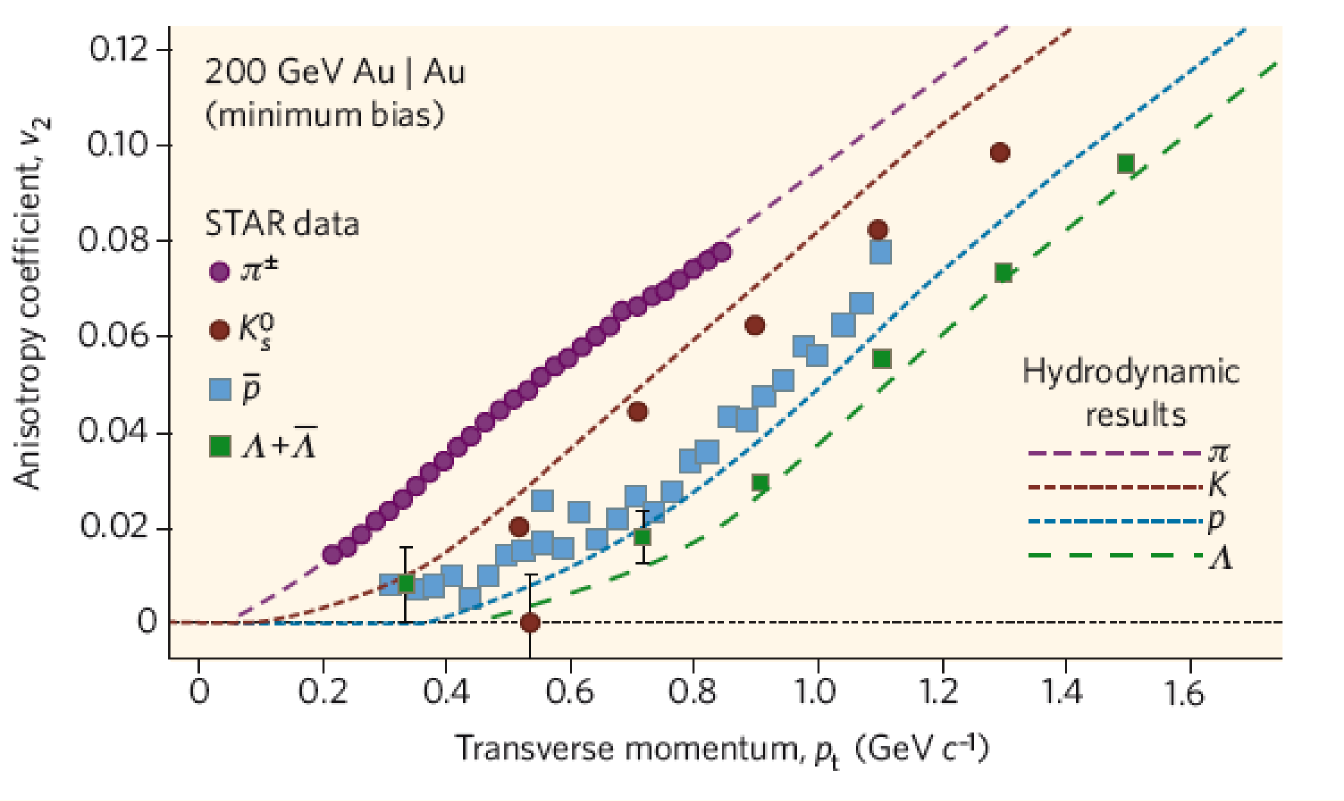
Different methods have different sensitivities to nonflow effects. For instance, the 4-particle cumulant method is significantly less sensitive to nonflow effects than the 2-particle cumulant method



Example:
$v_2 = 0$, $v_2\{2\} > 0$



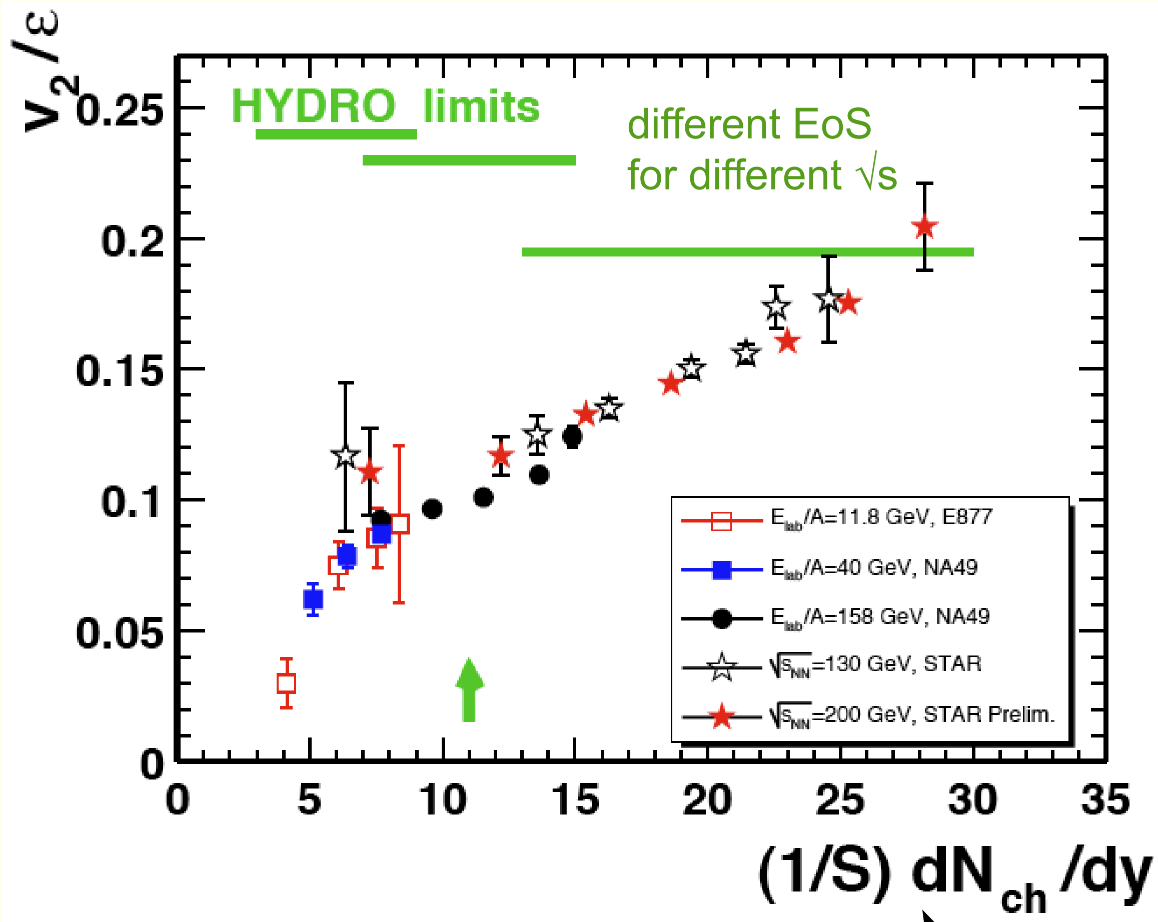
Elliptic Flow at RHIC



Plot from
Braun-Munzinger, Stachel,
Nature 448:302-309,2007

- Measured v_2 in good agreement with ideal hydro
- Hydro predicts mass ordering: $v_2 \sim \frac{1}{T}(p_T - v m_T), \quad v = \text{average flow velocity}$
- Indeed observed!
- “Perfect liquid” created at RHIC

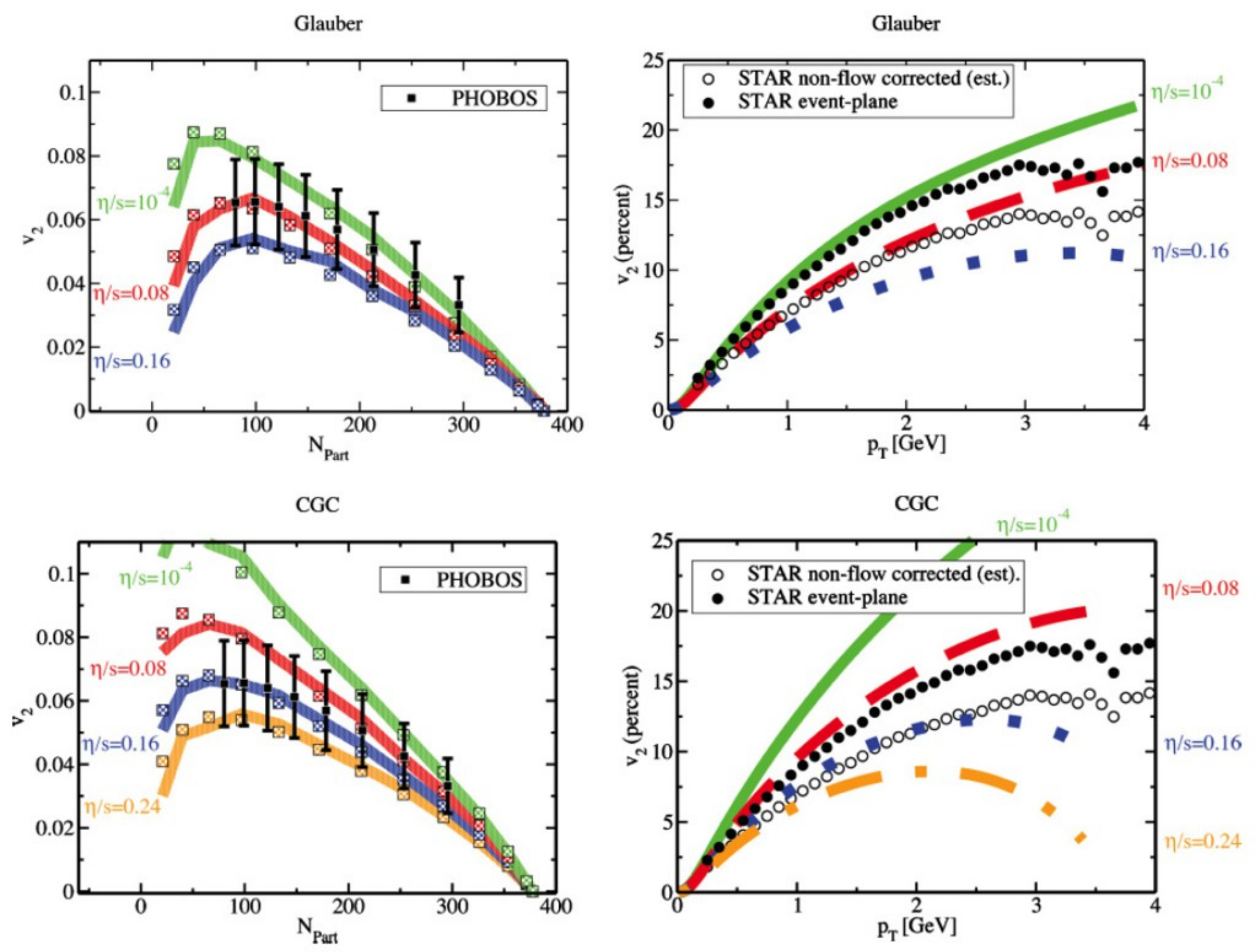
Maximum v_2 from Hydro



charged particle multiplicity per unit of rapidity per transverse area S of the source

Hydro limit only reached at RHIC energies

How Perfect is the QGP Fluid at RHIC?



Luzum, Romatschke, Phys.Rev.C78:034915,2008

Glauber initial cond.

$$\Rightarrow \quad 0 < \eta/s < 0.1$$

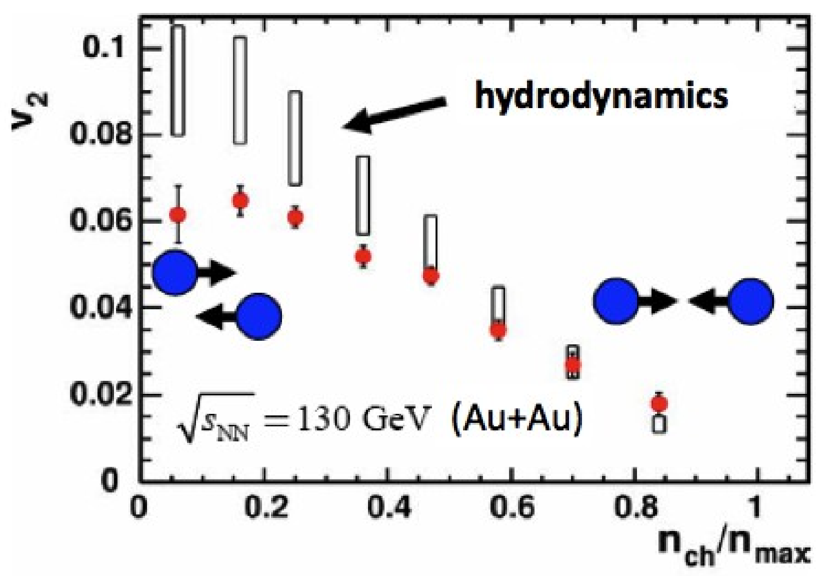
CGC initial cond.

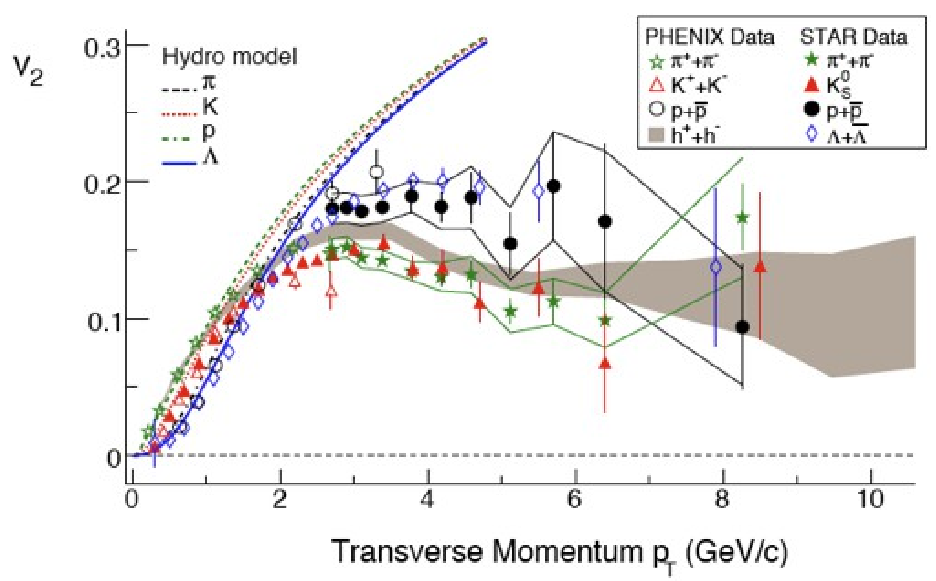
$$\Rightarrow \quad 0.08 < \eta/s < 0.2$$

Conservative estimate for the QGP (taking into account e.g. effects of EOS variations, bulk viscosity, ...):

$$\eta/s \quad < \quad 5 \times \left.\frac{\eta}{s}\right|_{\mathrm{KSS}} \quad = \quad 5 \times \frac{1}{4\pi}$$

Breakdown of Ideal Hydro

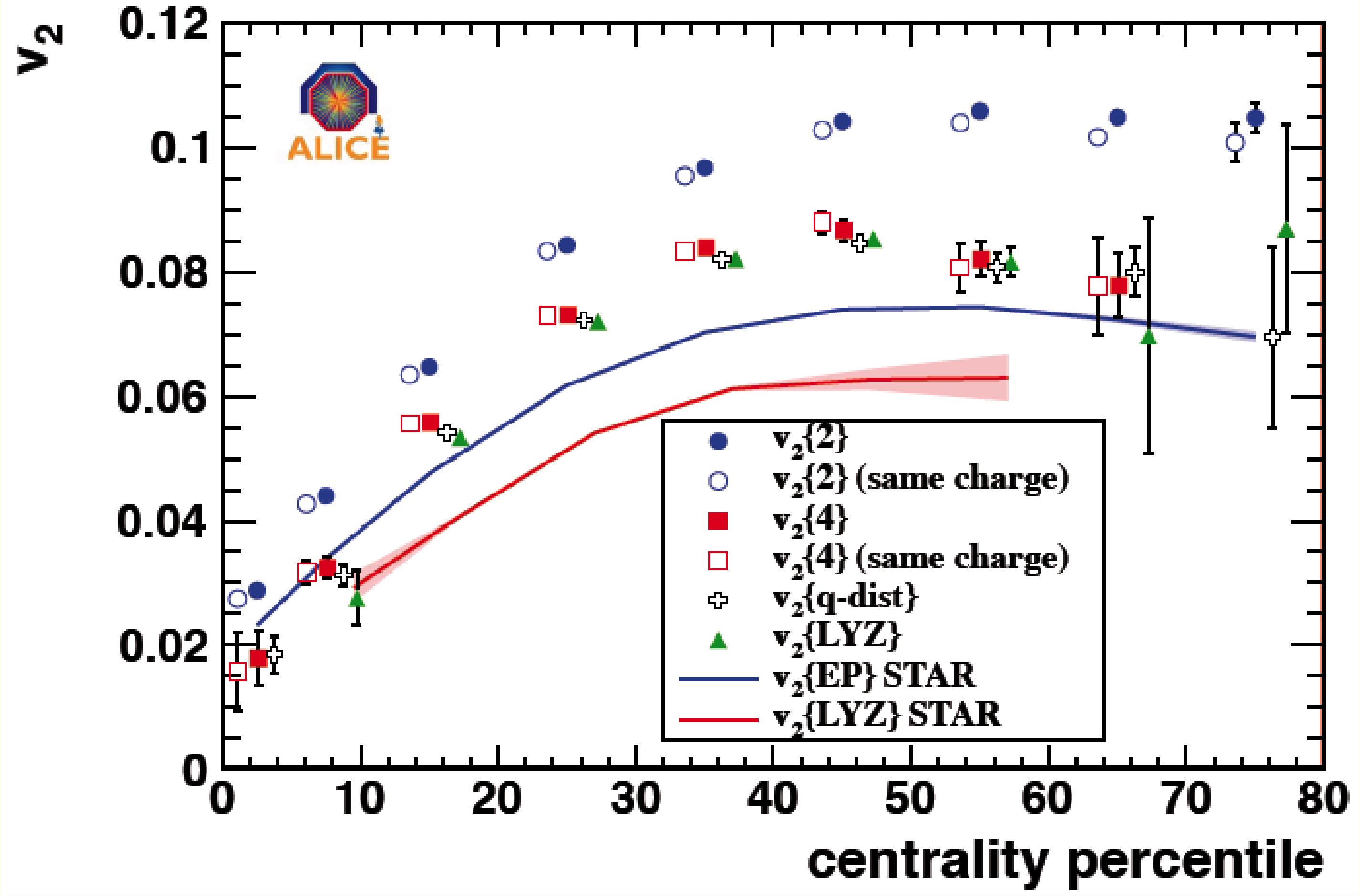




Hydro description for Au+Au at RHIC only works in central collisions and for p_T < 1.5 GeV/c

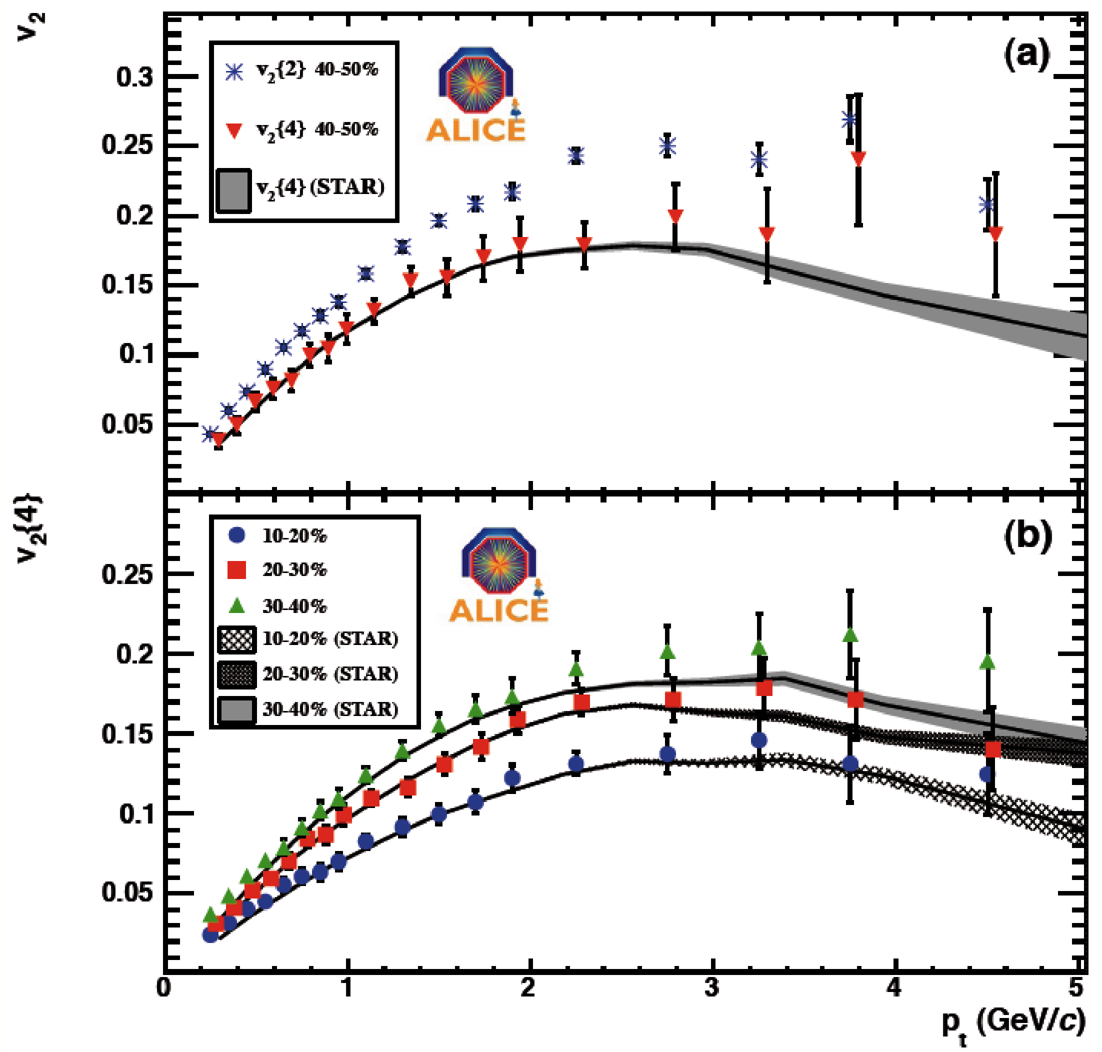
$v_2(p_T)$ in Pb+Pb at $\sqrt{s_{NN}}$ = 2.76 TeV from ALICE Compared to v_2 at RHIC (I)

ALICE, Phys. Rev. Lett. 105, 252302 (2010)



v_2 increases up to 30% (for more peripheral collisions)

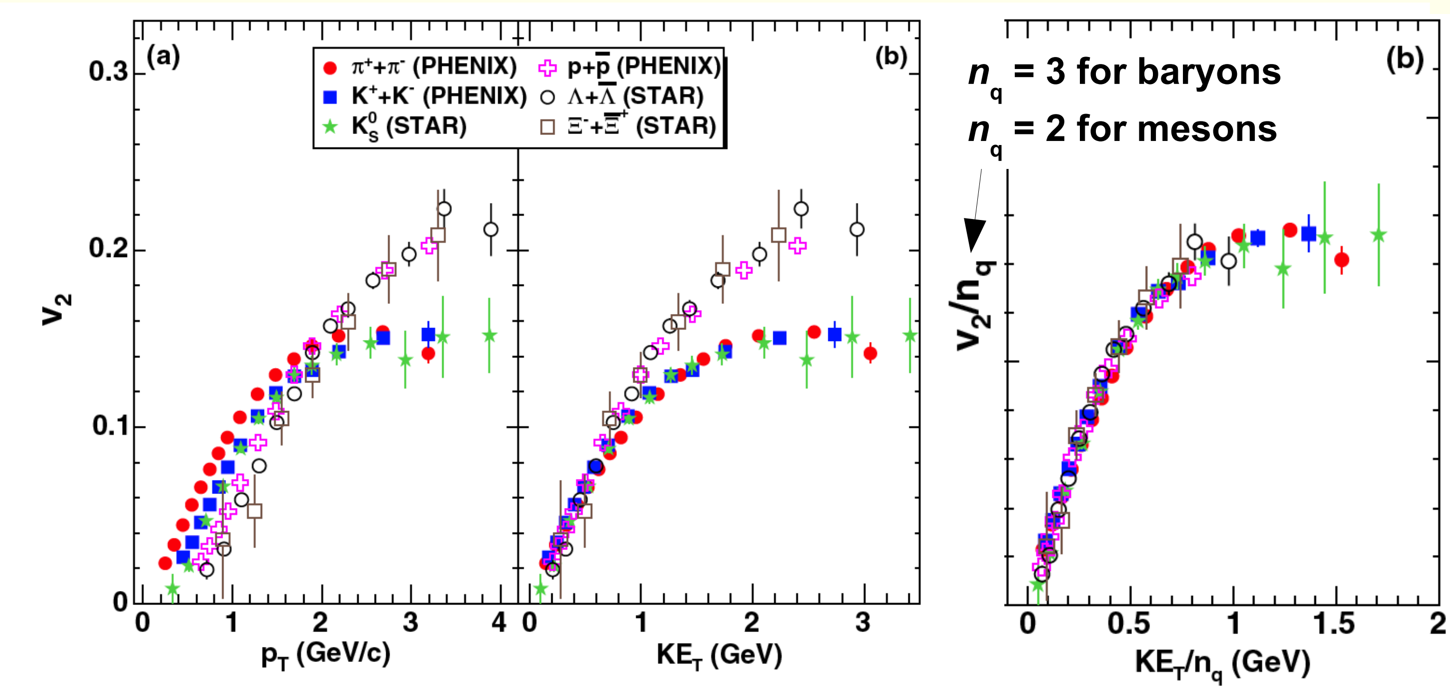
ALICE, Phys. Rev. Lett. 105, 252302 (2010)



$v_2(p_T)$ at LHC and RHIC is virtually identical.

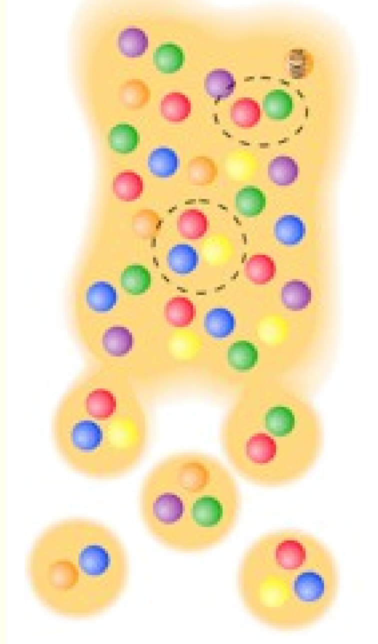
The increase of the mean p_T at the LHC can explain the increase of the p_T-integrated v_2 value.

An Interesting Observation: Quark Number Scaling



PHENIX, PRL, 98, 162301

Quark recombination:



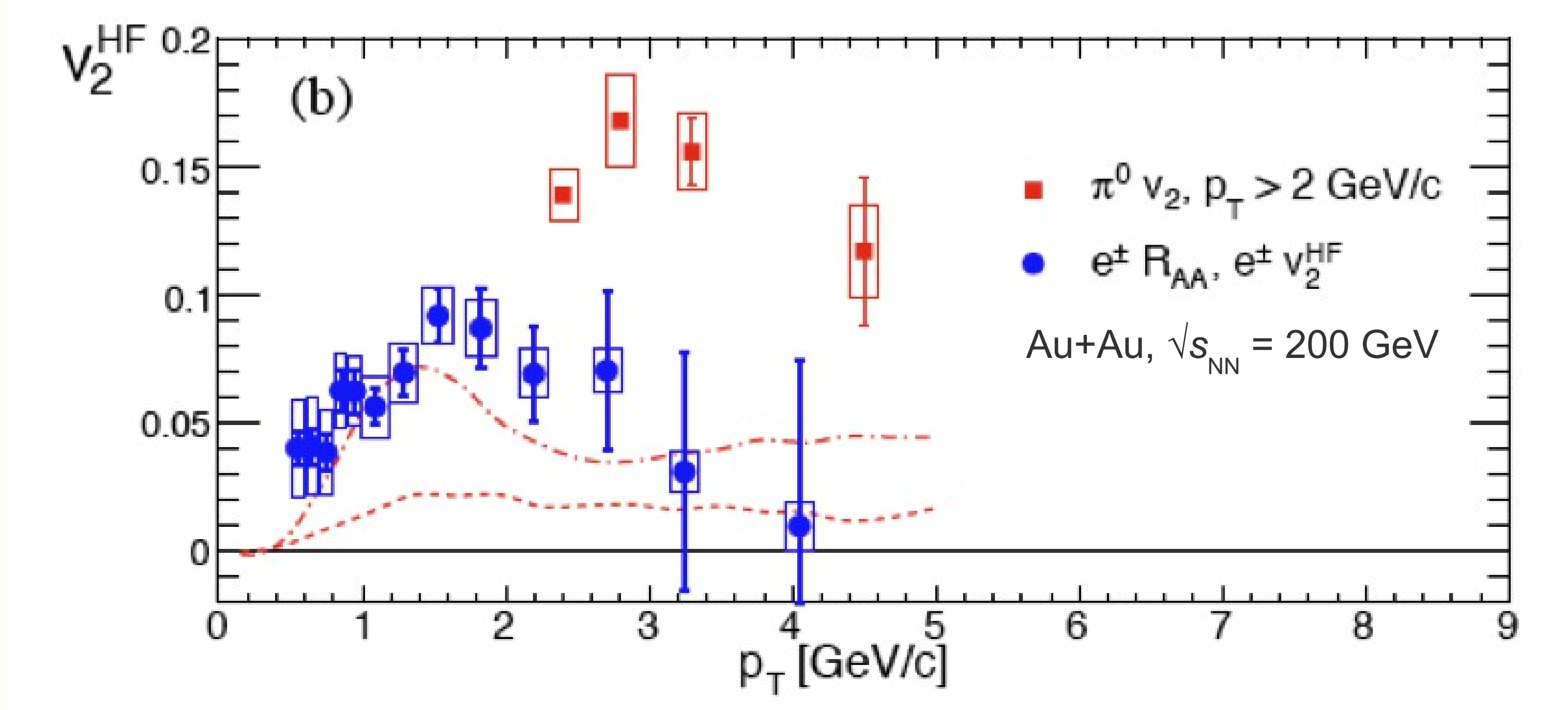
$\mathrm{KE}_T = \text{kin. energy in the transverse direction} = m_T - m_0$

- Scaling of v2 with n_q suggests that the flowing medium at some point consists of constituent quarks (in line with recombination models)
- Is there a transition from massless u and d quarks to constituent quarks ($m_u \approx m_d \approx 300$ MeV)?

Heavy Quarks Apparently Take Part in the Flow

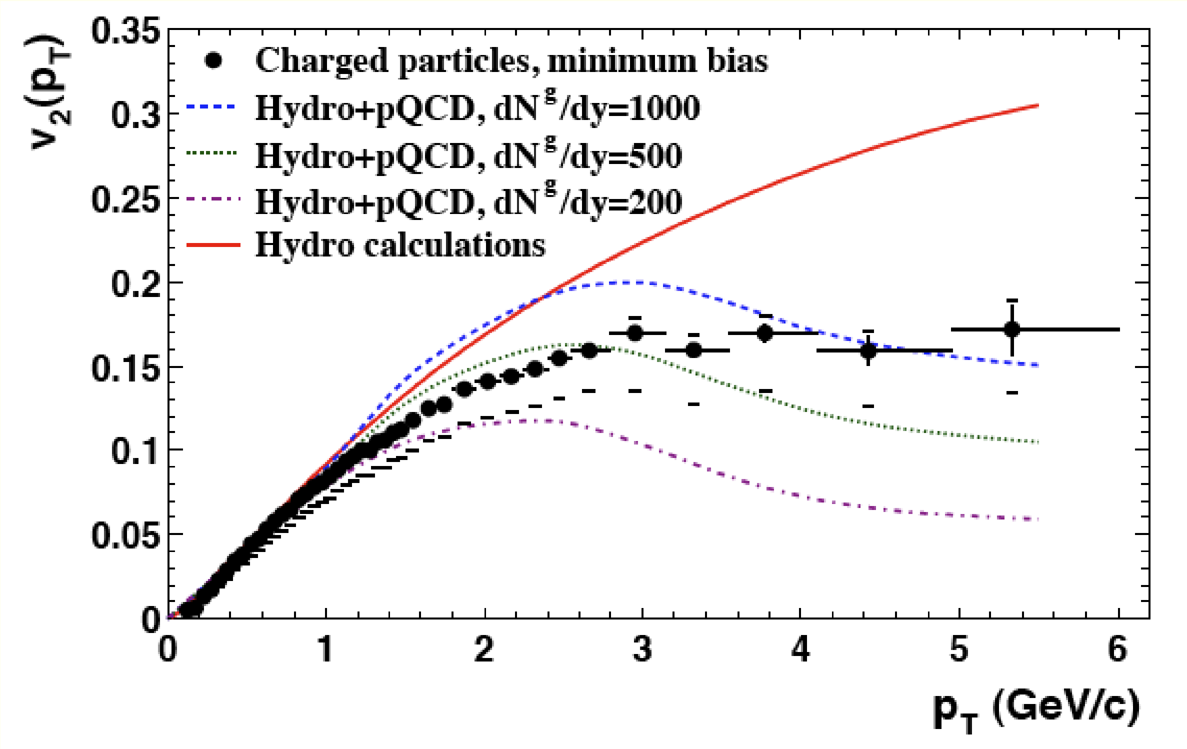
measured

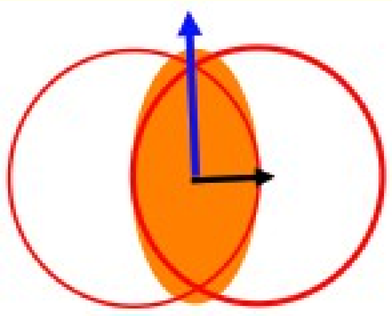
Example for a semi-leptonic heavy quark decay: $D^0(c\bar{u}) \rightarrow K^-(s\bar{u}) + e^+ + \nu_e$



- Current masses: $m_u \approx m_d \approx 4$ MeV, $m_c \approx 1270$ MeV, $m_b \approx 4200$ MeV
- Even though $m_{heavy, quark} > 200 \cdot m_{light,quark}$ heavy and light quarks exhibit a similar flow strength

v_2 and Jet Quenching





For p_T > 4-6 GeV/c particle production is dominated by jet fragmentation. Jets, i.e, energetic quark and gluons, are expected to lose energy in the QGP ("jet quenching"). The shorter path length for jets in the reaction plane compared to jets perpendicular to the reaction plane is expected to result in a positive v_2 at hight p_T.

Points to Take Home

- QGP at RHIC and LHC is close to an ideal fluid (close to KSS bound)
- Elliptic flow coefficient v_2 sensitive to viscosity of the QGP (viscosity reduces v_2)
- Largest systematic uncertainty in the extraction of η/s is the unknown initial eccentricity ($\varepsilon_{CGC} > \varepsilon_{Glauber}$)
- Similar η/s for RHIC and LHC
- Upper limit from data/theory comparison (ca. 2009):

$$\eta/s < 5 \times \left.\frac{\eta}{s}\right|_{\mathrm{KSS}} = 5 \times \frac{1}{4\pi}$$

- At Quark Matter 2011 somewhat tighter bounds of $\eta/s < 3/(4\pi)$ were reported