QGP Physics − from Fixed Target to LHC

7. HBT Interferometry

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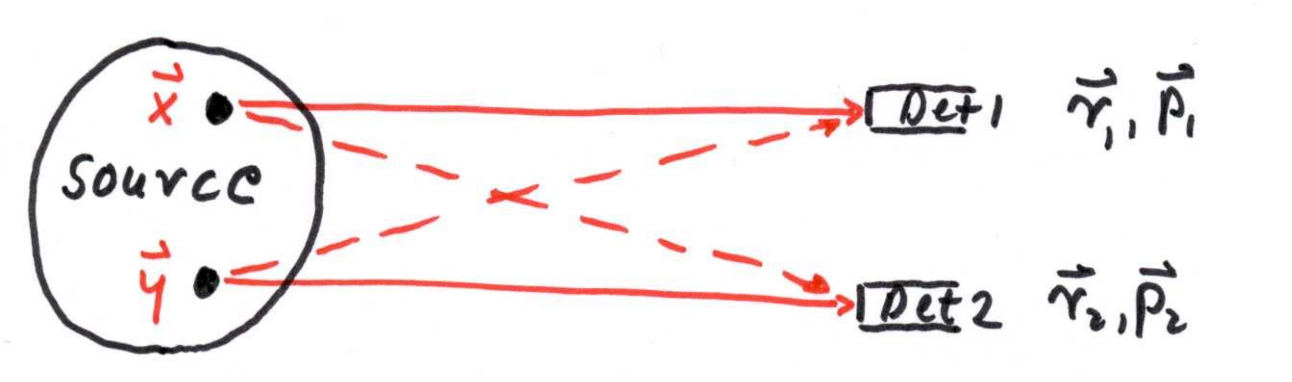
Correlations of identical bosons due to quantum interference

stochastic emission from extended source
consider 2 identical bosons (photons, pions, …)
2 detectors in locations r_1, r_2 observe identical bosons of momenta p_1 and p_2



cannot distinguish solid and dashed paths because of identical particles
→ for plane waves, the probability amplitude for detection of the pair is

$$A_{12} = \frac{1}{\sqrt{2}}\left[e^{ip_1(r_1-x)}e^{ip_2(r_2-y)} + e^{ip_1(r_1-y)}e^{ip_2(r_2-x)}\right]$$

with 4-vectors p,r,x,y (to be general for nonstatic source)

square of amplitude: intensity → "intensity interferometry"

Hanbury-Brown Twiss effect

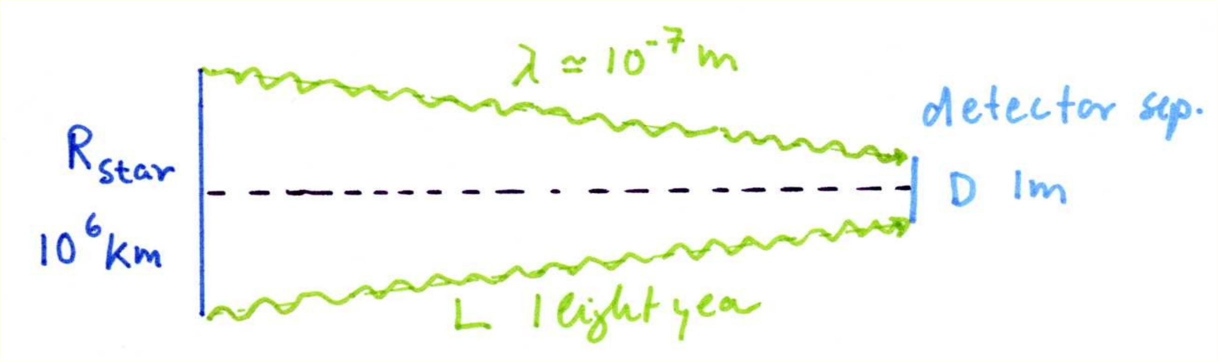
technique of intensity interferometry developed by Hanbury-Brown and Twiss in astrophysics as a means to determine size of distant objects

R. Hanbury-Brown and R.Q. Twiss, Phil. Mag. 45 (1954) 663

radiowaves to determine size of galaxies (Cygnus, Cassiopeia)

and Nature 178 (1956) 1046

visible light to determine the size of stars (Sirius)



condition for interference: $\frac{R_{star}}{L} \cdot \frac{D}{\lambda} \approx 1$

→ with detector separation of the order of 1 m size of star can be determined by intensity interferometry

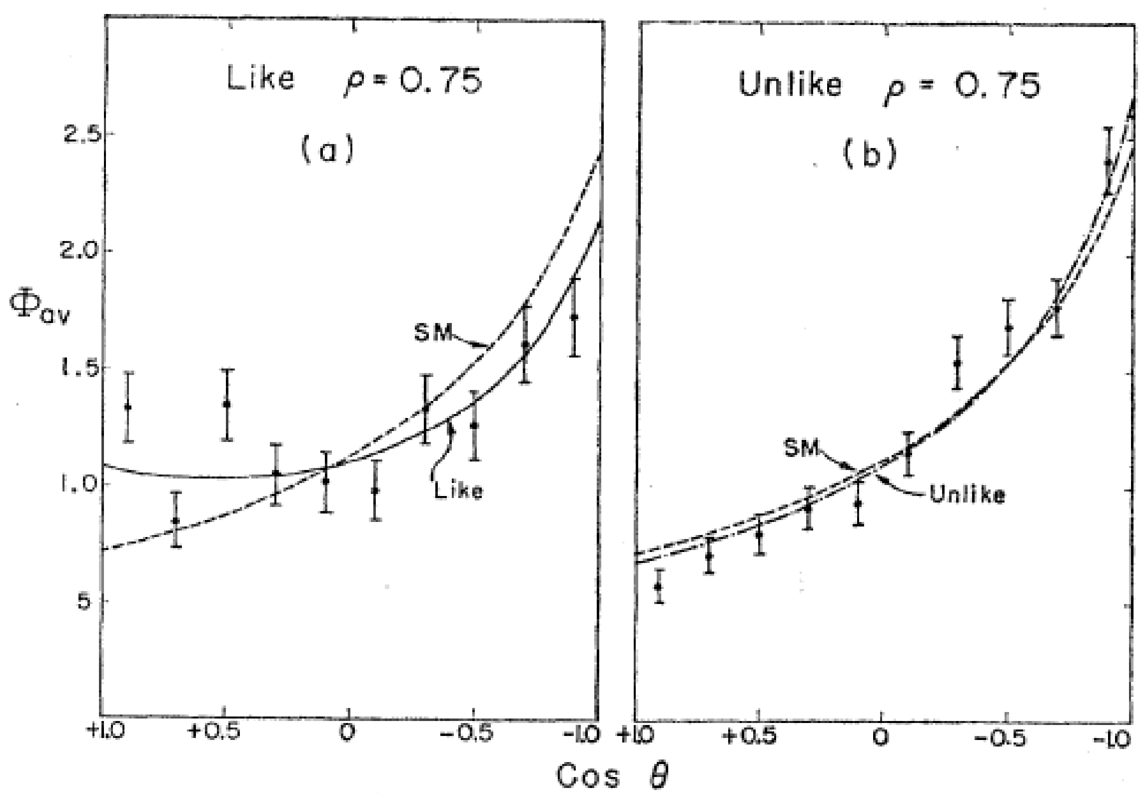
HBT interference in particle physics

When phase space volume smaller than $\Delta p_x \Delta x \approx \hbar$ is considered, chaotic system of identical noninteracting particles exhibits quantum fluctuations following Bose-Einstein (or Fermi-Dirac) statistics

first observation in particle physics: pions with small relative momenta

G. Goldhaber, S. Goldhaber, W.Y. Lee, A. Pais, Phys. Rev. 120 (1960) 300

pion correlations after a pbarp annihilation



angle between 2 pions in the p pbar cm system

look at angular distribution between like- and unlike-sign pions in events with 4 charged pions at sqrt(s) = 2.1 GeV

radius of volume of strong interaction r between 0.5 and 0.75 Compton wavelengths of pion $\frac{\hbar c}{mc^2} = 1.4\,\mathrm{fm}$

i.e. 0.7-1.0 fm

HBT interferometry in nuclear and particle physics

After discovery of GGLP systematically used as tool to determine source size in particular in collisions of high energy nuclei

<u>useful references:</u>

G.I. Kopylov, M.I. Podgoretsky, Sov. J. Nucl. Phys. 15 (1972) 219 and
ibid 18 (1974) 336 and PLB50 (1974) 472 - theory of interference of identical pions, lifetime of source

E. Shuryak, Phys. Lett. B44 (1973) 387
introduces time, i.e. duration of emission

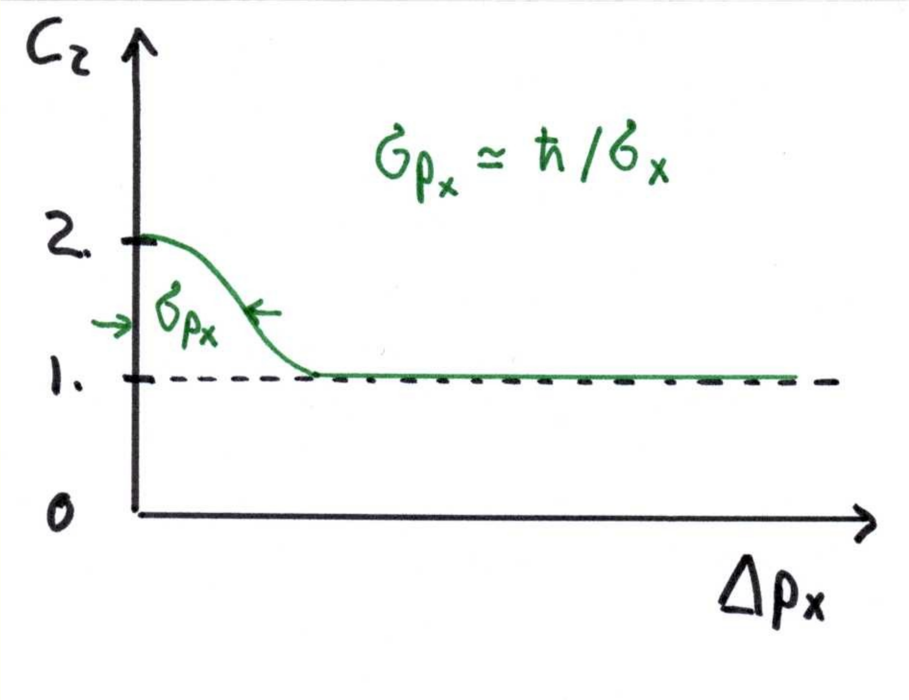
review by:
D. Boal, C.K. Gelbke, B.K. Jennings, Rev. Mod. Phys. 62 (1990) 553

G. Baym, Acta Phys. Polon. B29 (1998) 1839

2-particle correlation function

$$C_2(\vec{p}_1 - \vec{p}_2) = \frac{d^6N/d\vec{p}_1 d\vec{p}_2}{d^3N/d\vec{p}_1 \cdot d^3N/d\vec{p}_2} = 1 + f(\vec{p}_1 - \vec{p}_2)$$

get uncorrelated denominator typically from event mixing



in heavy ion collisions typical dimensions 1-10 fm,
→ leads to interference at momentum differences of 20-200 MeV/c

for a stochastic source we get a 2-particle correlation function

$$C_2(q) = P_{12} = \int d^4x d^4y \mid A_{12} \mid^2 \rho(x)\rho(y)$$

with a 4-momentum difference $\mathrm{q} \equiv \mathrm{p}_1 - \mathrm{p}_2$

and momentum independent source functions $\rho(\mathrm{x}), \rho(\mathrm{y})$

transformation ro relative coordinates and integration leads to

$$C_2(q) = 1 + \mid \tilde{\rho}(q) \mid^2$$

$$\tilde{\rho}(\mathrm{q}) = \int \mathrm{d}^4\mathrm{x}\rho(\mathrm{x})\mathrm{e}^{-\mathrm{iqx}}$$

Fourier transform of space-time density distribution

<u>example:</u> Gaussian space-time density distribution $\rho(\mathrm{x})$

$$\rho(x) = c\ \exp(-\frac{|\vec{x}|^2}{2r_0^2} - \frac{t^2}{2\tau_0^2})$$

$$\longrightarrow \qquad \tilde{\rho}(q) = c'\ \exp(-\frac{|\vec{q}|^2 r_0^2}{2} - \frac{q_0^2\tau_0^2}{2})$$

and $\quad C_2(q) = 1 + \exp(-r_0^2|\vec{q}|^2 - \tau_0^2 q_0^2) \quad$ with norm $\tilde{\rho}(0) = 1$

note that the 3d rms radius is $r_{rms} = \sqrt{3}r_0$

Coupling on spatial information and time

time and space information are coupled, i.e. q_0 and q(vector) are not independent

in the pair rest frame $q^* = (0, \vec{q}) = (0, 2\vec{k}^*)$ with $\vec{k}^* = \vec{p}_1^* = -\vec{p}_2^*$

(in general $\vec{k}^* = \dfrac{m_2 \vec{p}_1^* - m_1 \vec{p}_2^*}{m_1 + m_2}$)

if $\tilde{\mathrm{v}}$ is the velocity of the pair in the overall cm frame, $\gamma^2 = \dfrac{1}{1 - v^2/c^2}$

and $\vec{k}_T^* = \vec{q}_T/2\,,\ k_L^* = q_L/2\gamma$ (here T and L refer to the direction of the pair)

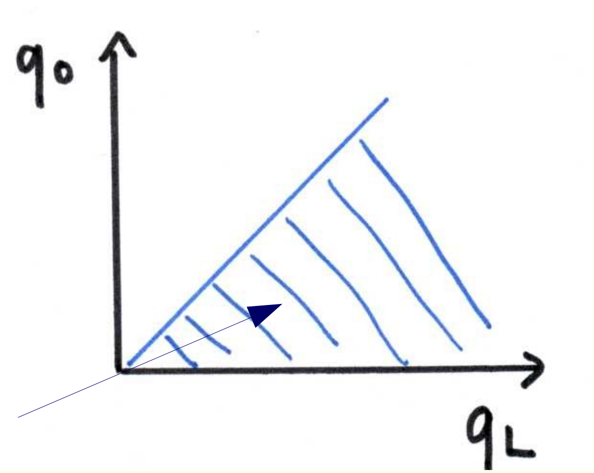
and $\vec{v} \cdot \vec{q} = v\, q_L$ but $\dfrac{\vec{v}}{c} = \dfrac{\vec{p}_1 = \vec{p}_2}{E_1 + E_2}$

→ $\vec{v} \cdot \vec{q} = E_1 - E_2 = q_0$

→ $C_2(q) = 1 + exp(-4 r_0^2 k_T^{*2} - 4\rho^2 \gamma^2 k_L^{*2})$

where $\rho^2 = r_0^2 + (v\tau_0)^2$
in direction of the pair spatial and temporal information
cannot be distinguished for identical particles

also: since $v/c \leq 1 \quad \rightarrow \quad q_0 \leq q_L$



Coulomb correction

in addition to quantum correlations 2 identical pions also experience correlations due to electromagnetic interaction, i.e. Coulomb repulsion

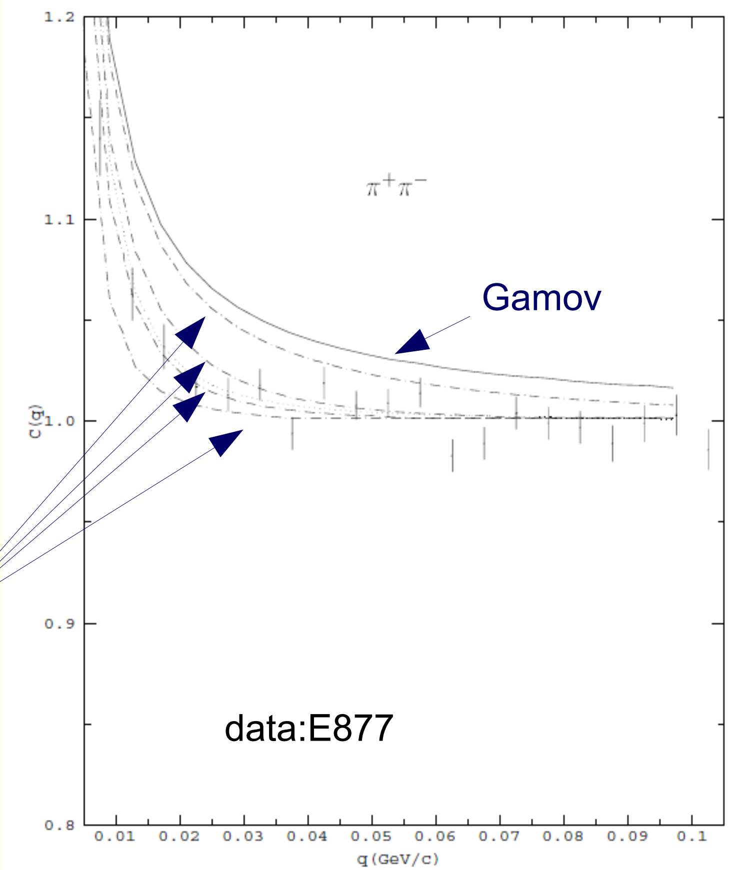
for 2 pions from a point source this problem was already solved for beta-decay (Gamov function)

$$\psi_C(0) = \left(\frac{2\pi\eta}{e^{2\pi\eta} - 1}\right)^{1/2} \quad \text{with} \quad \eta = \frac{zz'e^2}{v_{rel}}$$

was used also in early days of HBT, but approximation is too crude
(P. Braun-Munzinger, G. Baym,
Nucl. Phys. A610 (1996) 286c)
look at $\pi^+\pi^-$ correlation to see Coulomb effect

need to integrate over source of proper size

examples R = 1,5,9,18 fm



a recursive problem, since source size from HBT is determined after Coulomb correction
– typically very few iteration steps

3-dimensional correlations functions in pp and heavy ion collisions

go from momentum difference in cartesian coordinates to so-called Bertsch-Pratt variables (G. Bertsch, Phys. Rev. C37 (1988) 1896)
define z-axis as beam axis

$$q_{long} = \vec{q} \cdot \hat{e}_z = q_z$$

and in transverse direction distinguish direction parallel to pair momentum

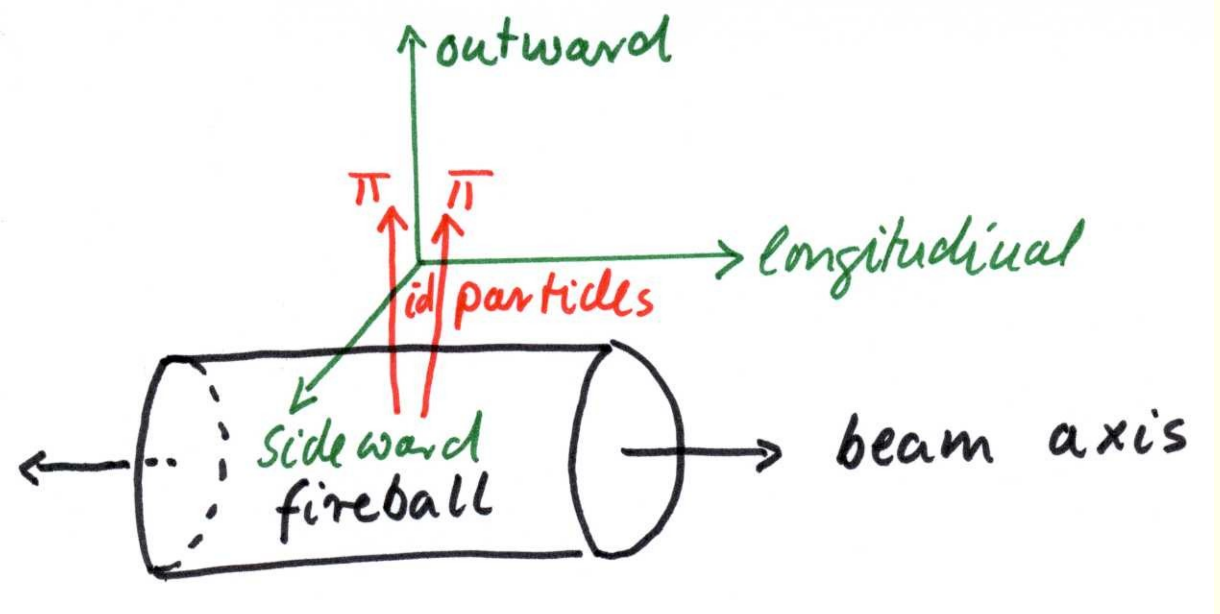
$$\hat{e}_{out} = \frac{\vec{p}_{t_1} + \vec{p}_{t_2}}{|\vec{p}_{t_1} + \vec{p}_{t_2}|}$$

$$q_{out} = \vec{q} \cdot \hat{e}_{out}$$

and perpendicular to it

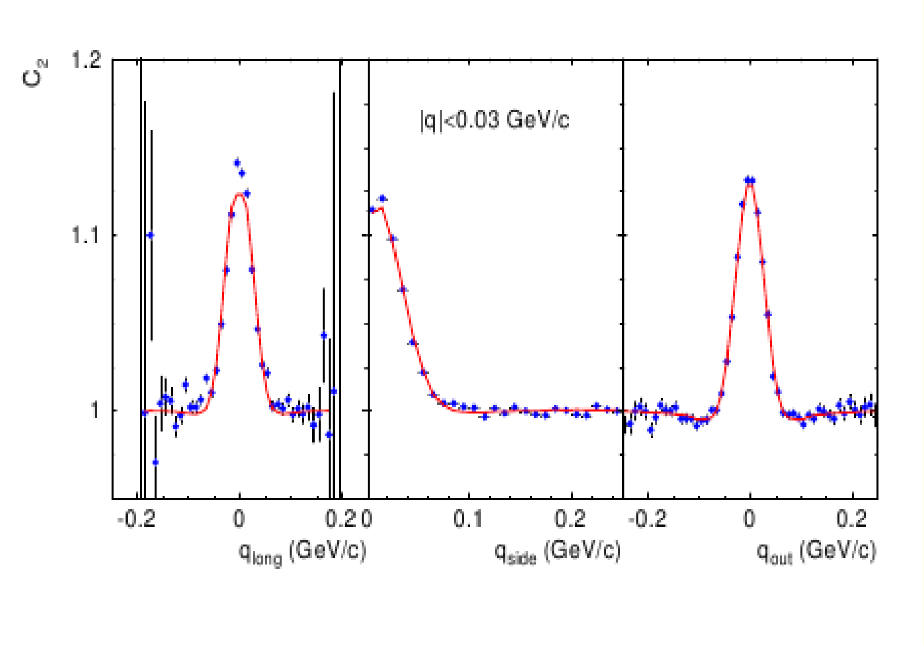
$$\hat{e}_{side} = \hat{e}_{out} \times \hat{e}_z$$

$$q_{side} = \vec{q} \cdot \hat{e}_{side}$$



Example: raw measured pipi correlation function

CERES/NA45 at CERN SPS - central 158 A GeV/c PbAu collisions

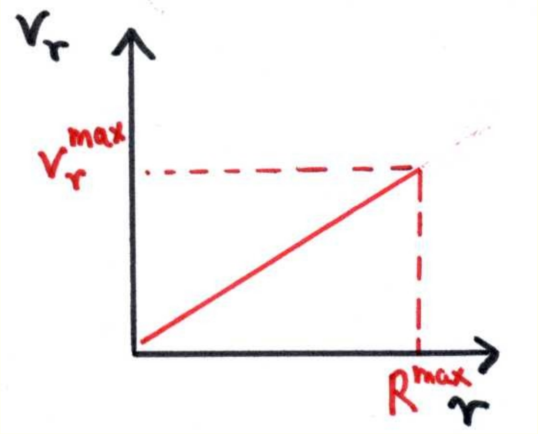


characteristic width order 30 MeV/c

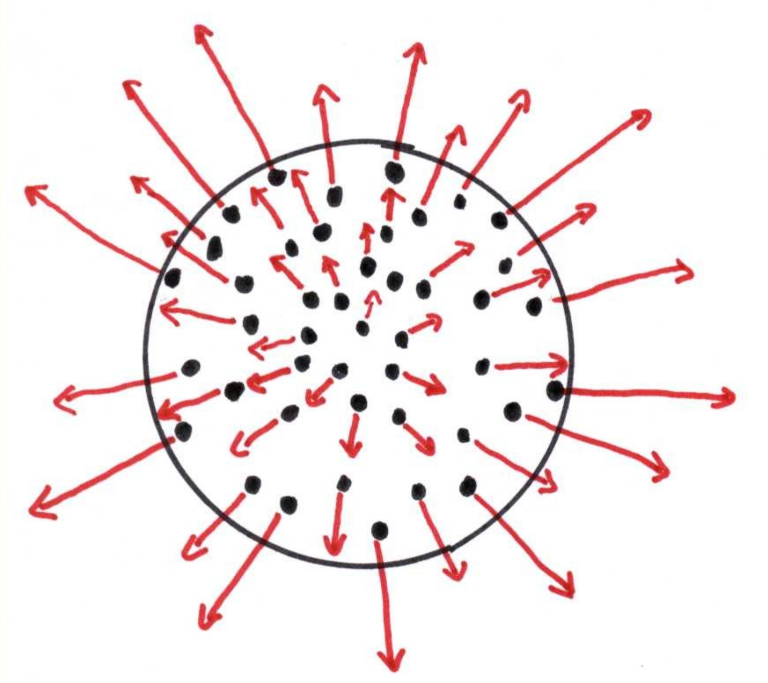
Dynamical source with space-momentum correlations

collective expansion introduces space-momentum correlations leading to an apparent reduction in source radius

Simple example:
source with T=0 emitting particles radially with a Hubble-like velocity profile



the apparent source from
2-particle correlations looks pointlike!
only particles emitted from the same space point have the same momentum, i.e. a small momentum difference



Dynamical source with space-momentum correlations

Effect of hydrodynamic expansion on radius parameters extracted from 2 pion correlations:

S. Pratt, Phys. Rev. Lett. 53 (1984) 1219
A.N. Makhlin, Y.M. Sinyukov, Z. Physik C39 (1988) 69
Y.M. Sinyukov, Nucl. Phys. A498 (1989) 151c

Picture: thermal source that expands hydrodynamically with velocity v(r) and corresponding β(r) and γ(r)

look at 2-pion correlation function for different slices in pair transverse momentum

$$\vec{k} = \frac{1}{2}(\vec{k}_1 + \vec{k}_2) \quad \text{or also} \quad \mathrm{k_t} = \frac{1}{2}(\mathrm{k_{t,1}} + \mathrm{k_{t,2}})$$

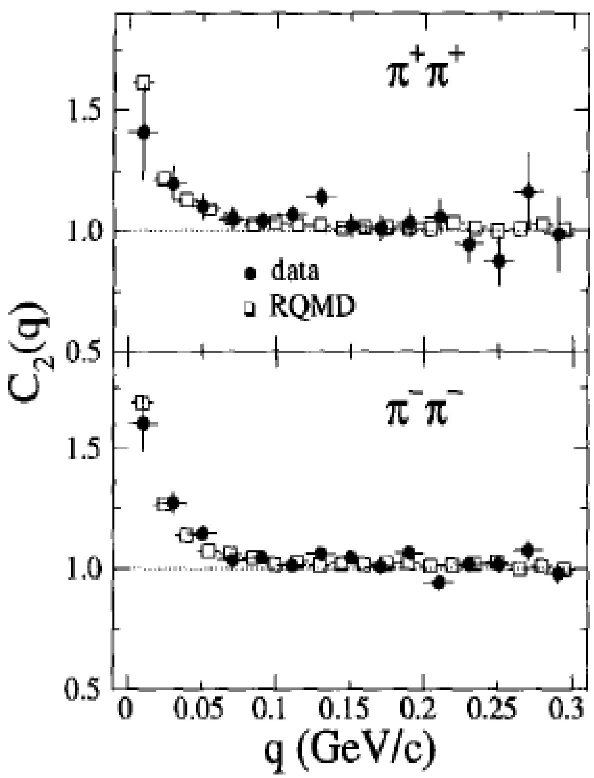
find that R(k) decreases with k, as ratio of energy of collective expansion over thermal energy increases

faster pions are more likely to be emitted near the point on spherical shell expanding with velocity β in direction of k (S.Pratt)

$$R(k) = R[(y \tanh y)^{-1} - \sinh^{-2} y] \quad \mathit{with} \;\; y = k\gamma\beta/T$$

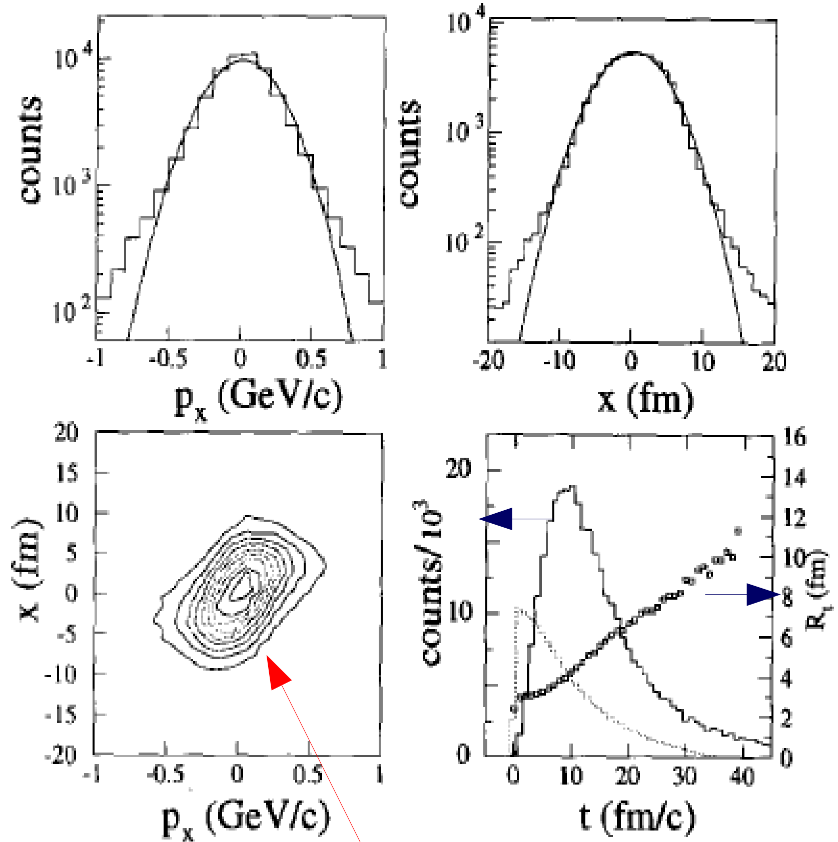
Need to model expansion to extract true source size

E814 Collaboration / Physics Letters B 333 (1994) 33–38



Exp corr function compared to
RQMD model simulation

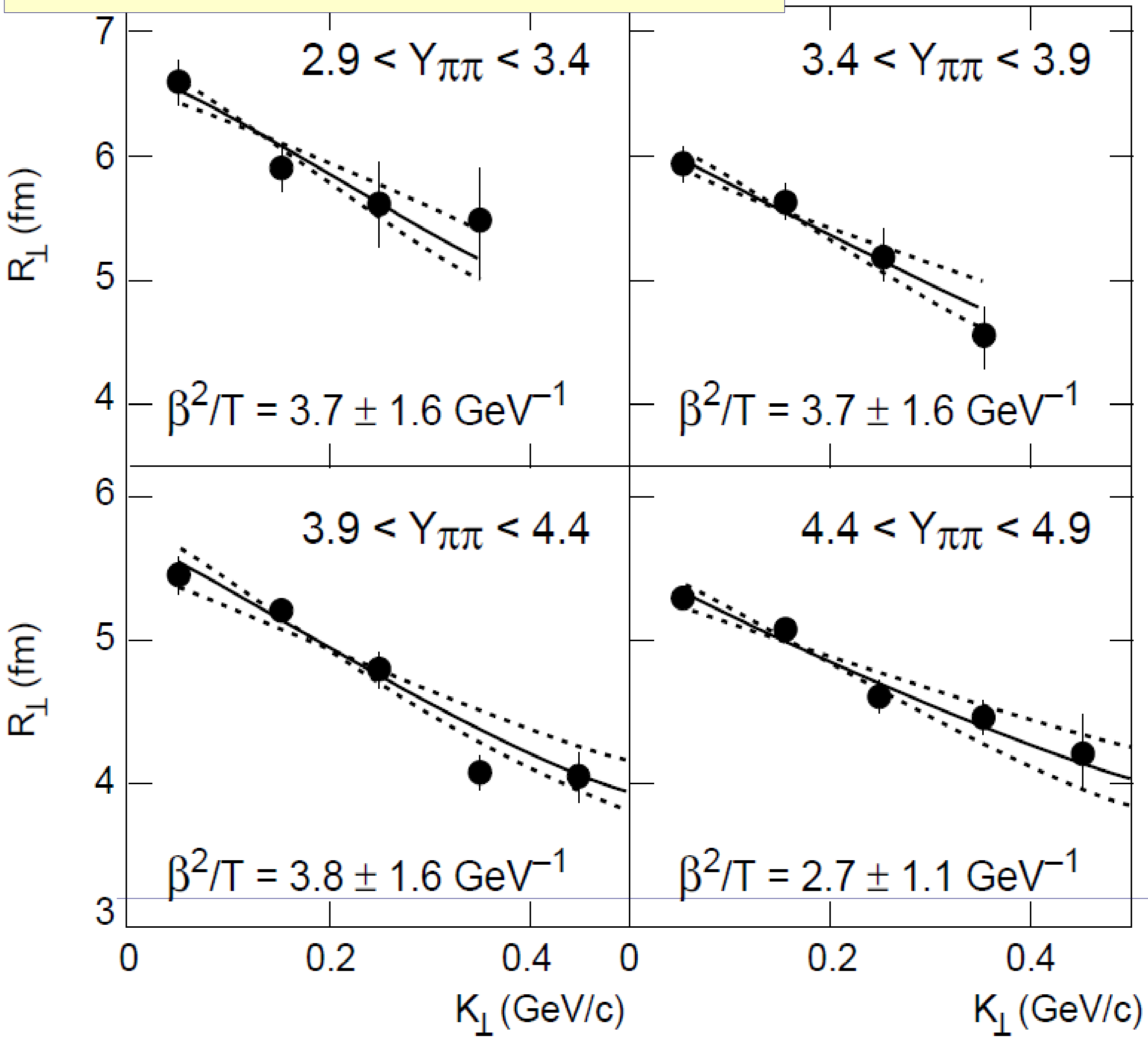
RQMD model



space-momentum correlation
leads to apparent reduction in source size
1992 claim by E802:: pion source radius is 2 fm!

First successful hydrodynamic description of HBT data

data: NA49, thesis H. Appelshäuser
Eur. Phys. J. C2 (1998) 661



$$R_\perp = \sigma_x (1 + \frac{m_t \beta^2}{T\, cosh(y_{YPK} - y_{\pi\pi})})^{-1/2}$$

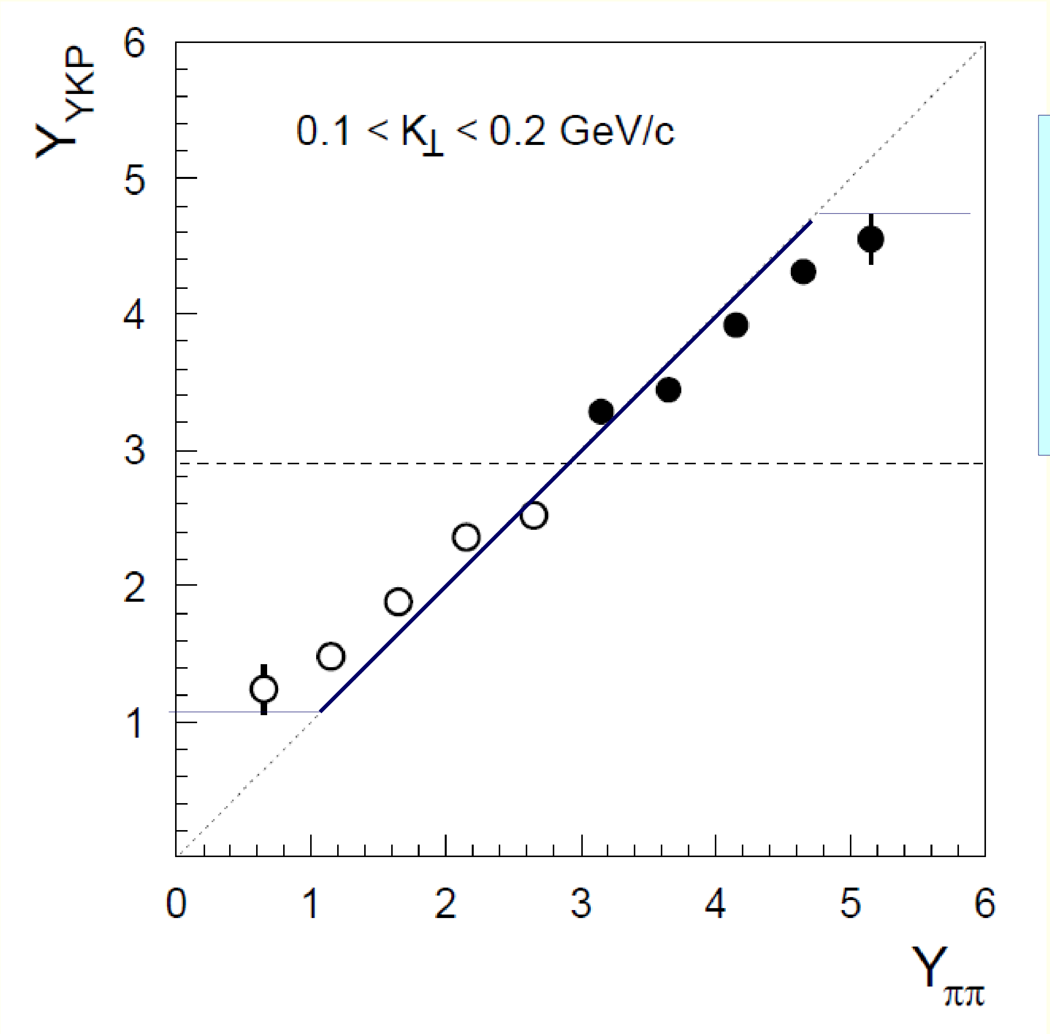
fit function U.Heinz et al.
expanding source with
expansion velocity β,
temperature T and
pair transverse mass

$$m_t^2 = m_\pi^2 + k_t^2$$

→ consistent description w.
β = 0.5, T = 120 MeV
σ_x = 8.2 fm

Velocity of source emitting pions rel. to pair velocity

data: NA49, Eur. Phys. J. C2 (1998) 661



consistent with boost invariant expansion over +- 2 units of y (source rapidity equal pair rapidity) central fireball with Bjorken expansion 4 units long

R_{long} - Longitudinal Expansion of Fireball

Duration of expansion (lifetime) τ of the system can be estimated from the transverse momentum dependence of R_{long}

$$R_{long} \approx \tau \sqrt{T_f / m_t} \qquad \text{Y.Sinyukov}$$

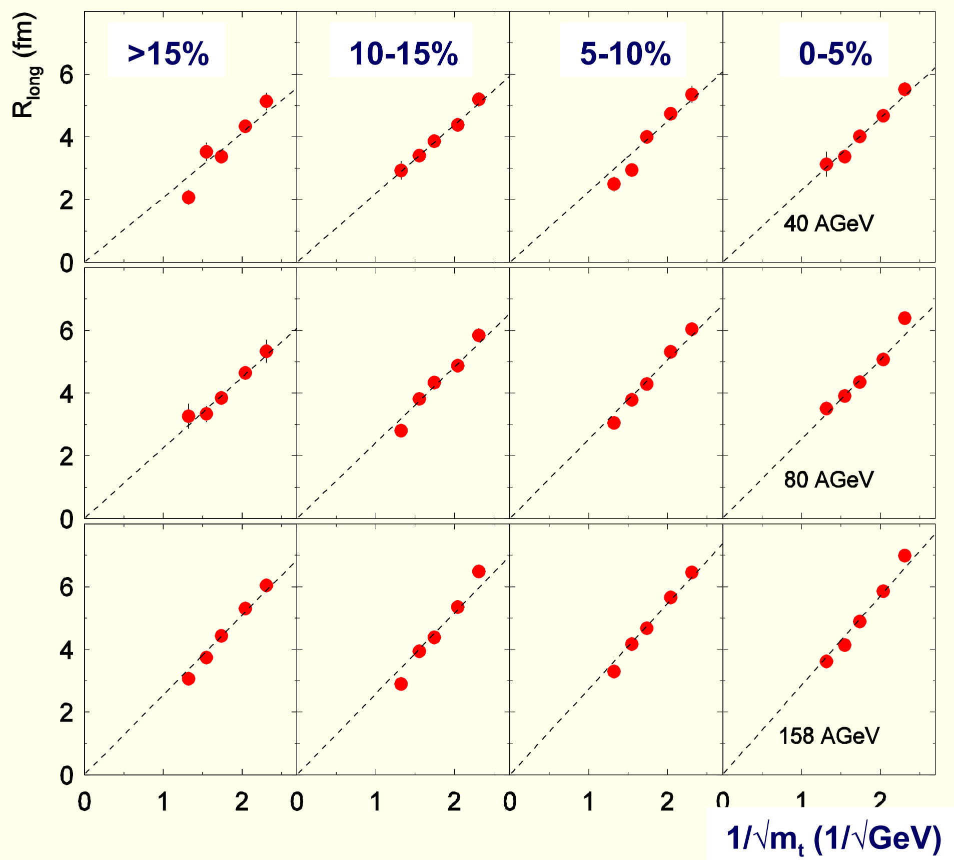
thermal velocity

$$\tau = 6 - 8 \text{ fm/c}$$
$$\text{for } T_f = 120 - 160 \text{ MeV}$$

due to finite T, pions are correlated over an interval in rapidity and due to expansion this grows linearly with duration of expansion

R_{long} is longitudinal correlation length

CERES Pb-Au *Nucl. Phys. A714 (2003) 124*



Hubble plot of nuclear fireball

R_{side} – transverse expansion and geometry

$$R_{side} = R_{geo}/\sqrt{1+\eta_f^2 m_t/T_f}$$

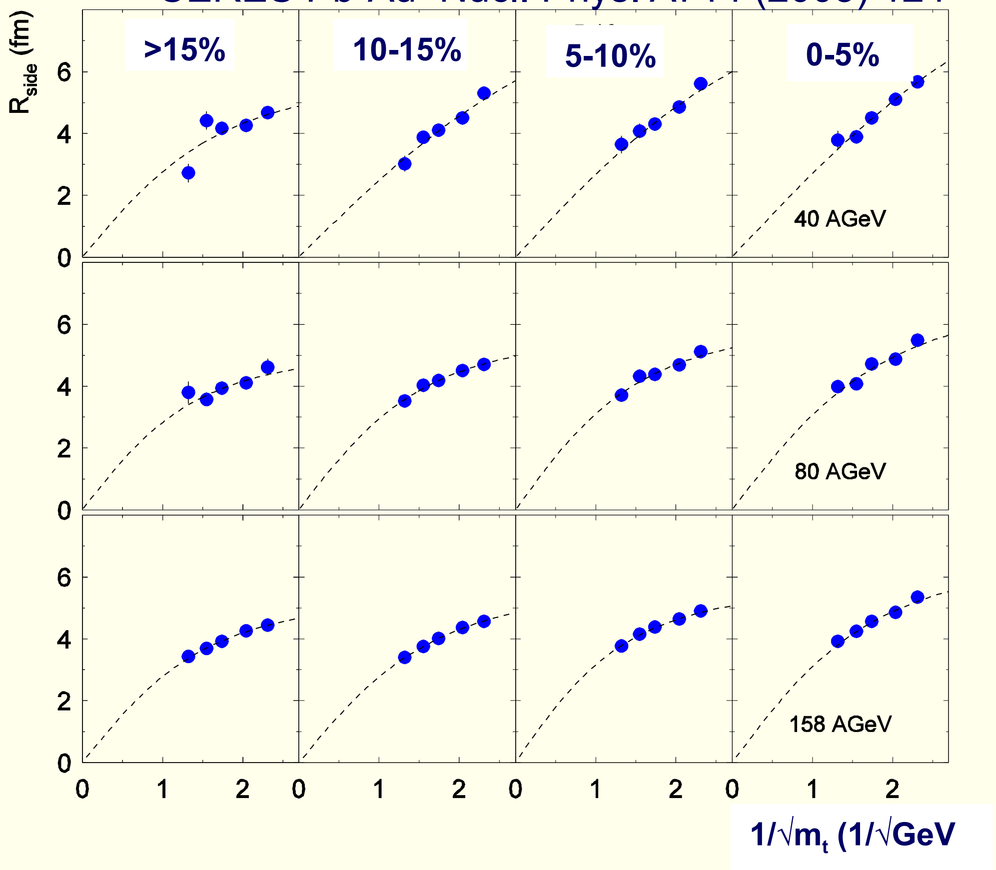
η_f^2 :

strength of transverse expansion
(U. Heinz, B. Tomasik, U.Wiedemann)

$$\langle\beta_t\rangle = 0.5 - 0.6$$
$$\text{for } T_f = 160\text{-}120 \text{ MeV}$$

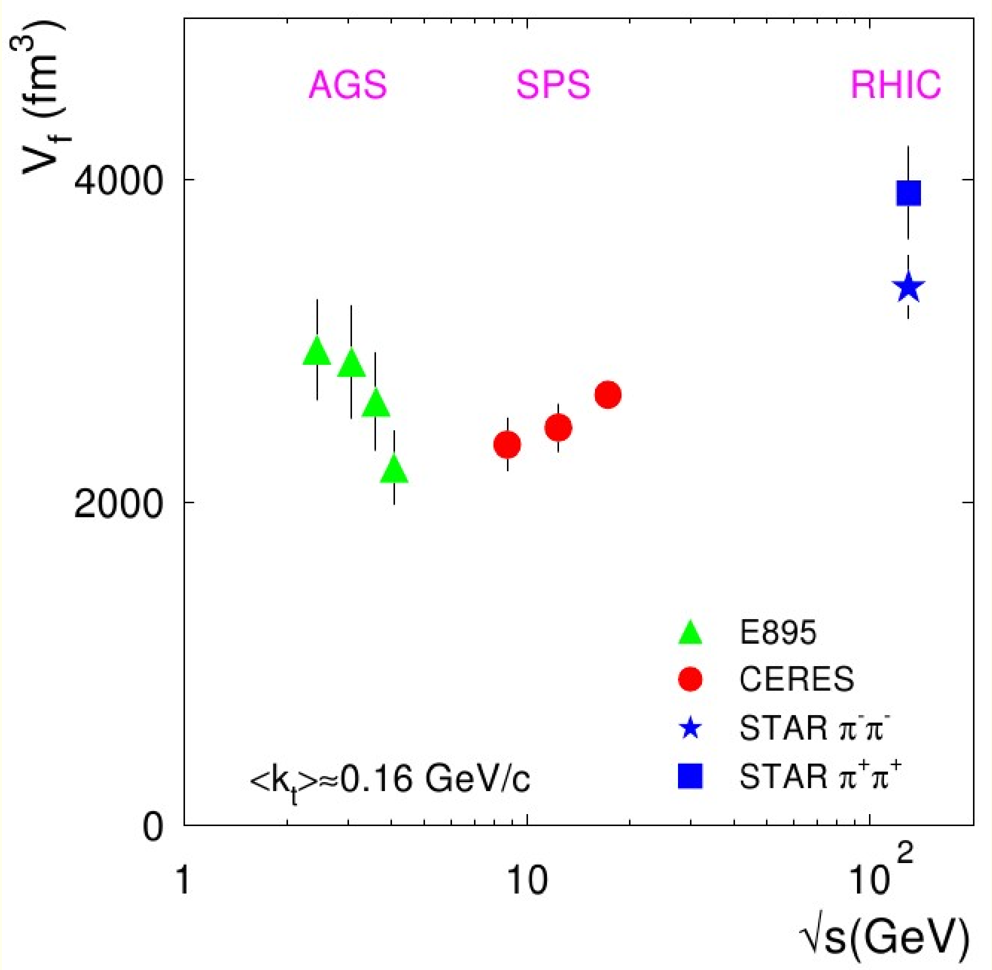
$$R_{geo} = 5.5 - 6 \text{ fm}$$

CERES Pb-Au Nucl. Phys. A714 (2003) 124



Freeze-out volume vs. beam energy

estimate freeze-out volume V_f: $V_f = (2\pi)^{3/2} R_{side}^2 R_{long}$



note: this is volume of 0.88 units of rapidity

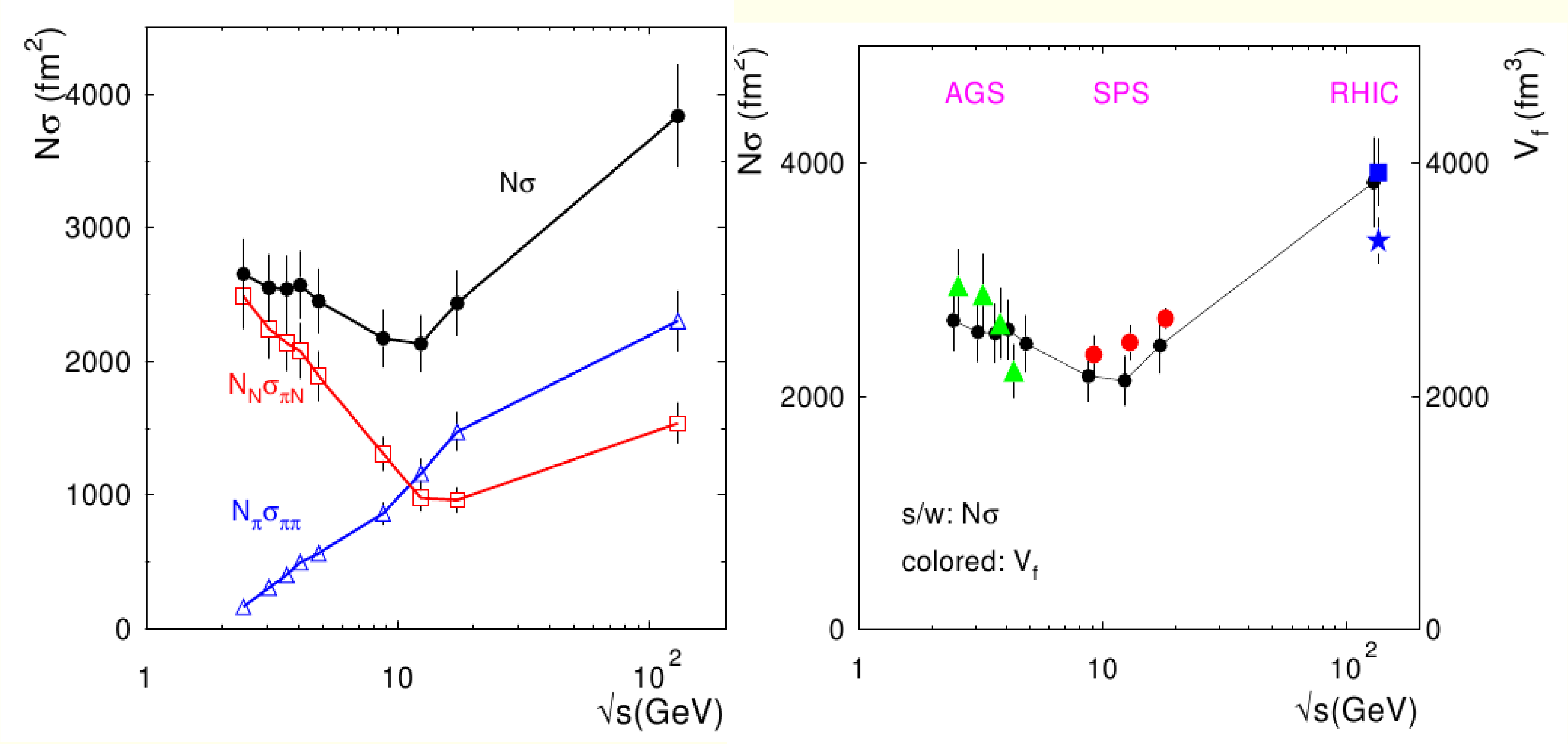
- surprise: non-monotonic behaviour
- minimum between AGS and SPS
- rules out freeze-out at constant density

what governs pion freeze-out?

pion mean free path: $\lambda_f = 1/(\rho_f \cdot \sigma) = V_f/(N \cdot \sigma)$

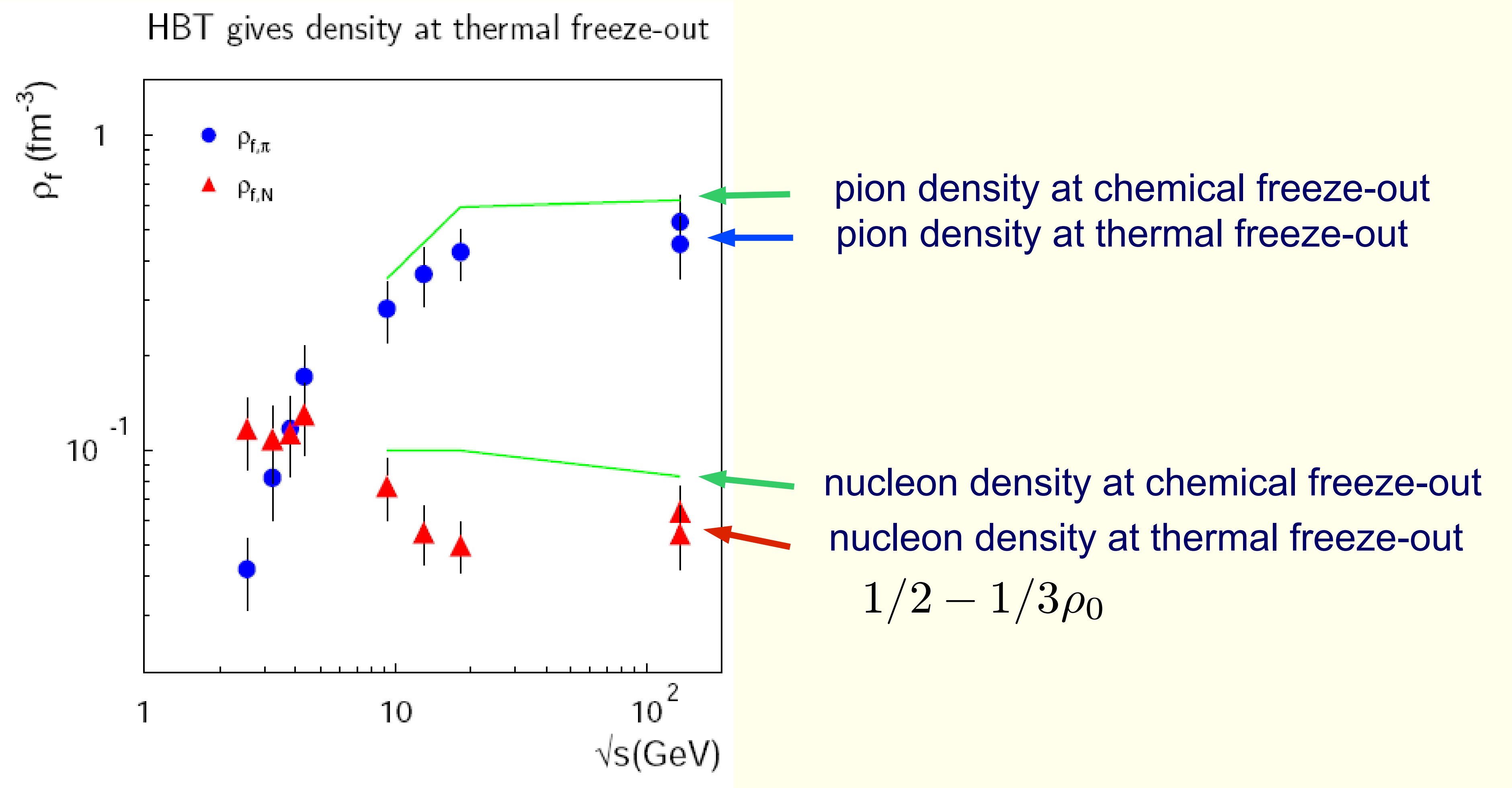
$$N \cdot \sigma \approx N_N \cdot \sigma_{\pi N} + N_\pi \cdot \sigma_{\pi\pi}$$

CERES Phys. Rev. Lett. 90 (2003) 022301



Universal freeze-out at mean free path $\lambda_f \approx 1$ fm - small vs system size

Densities at chemical and thermal freeze-out



$$1/2 - 1/3\rho_0$$

volume appears only to grow 30 % between chemical and thermal freeze-out
get from the 2 densities, the velocity and final radii: isentropic expansion for 0.9-2.3 fm
T_f = 158 – 132 MeV, rate of cooling near T_c: (13 ± 1) % per fm/c

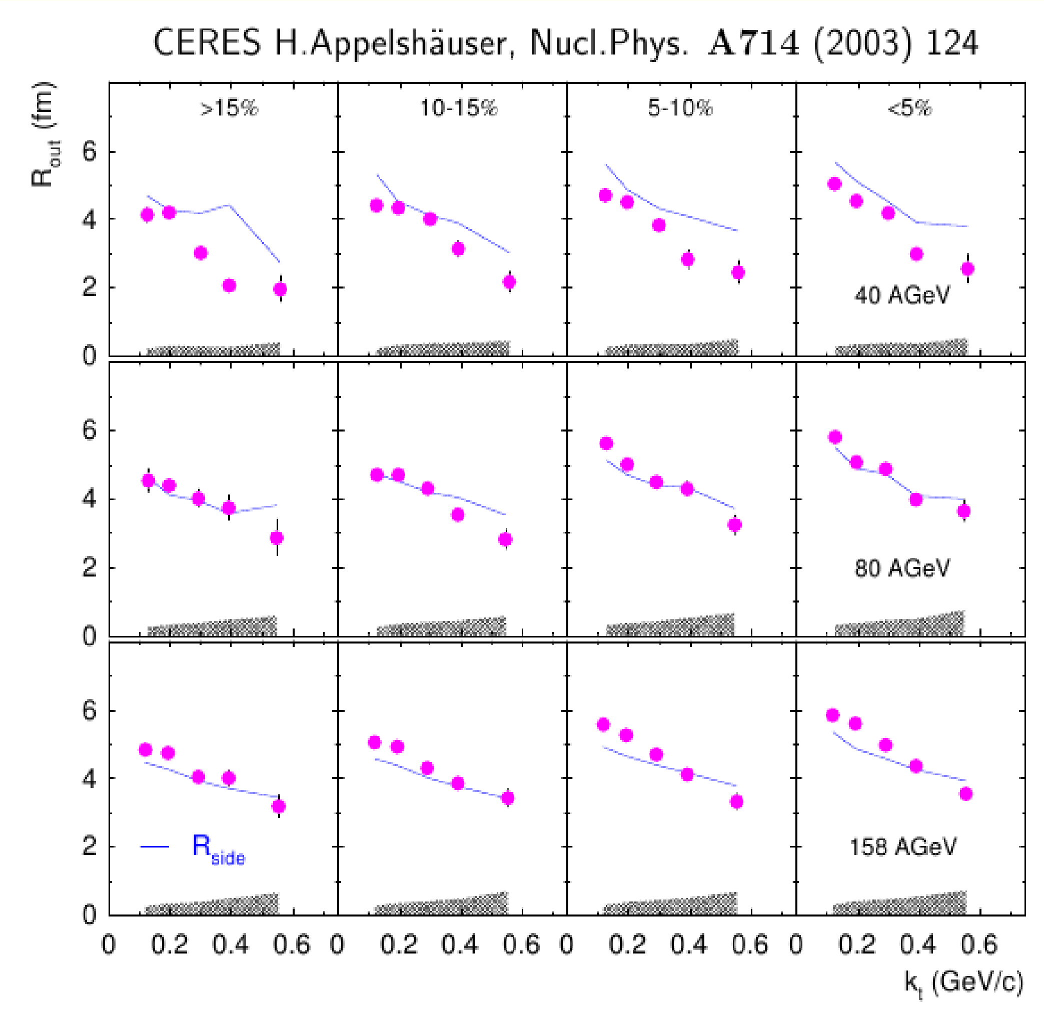
R_{out} – duration of pion emission

generally: $R_{out} \approx R_{side}$

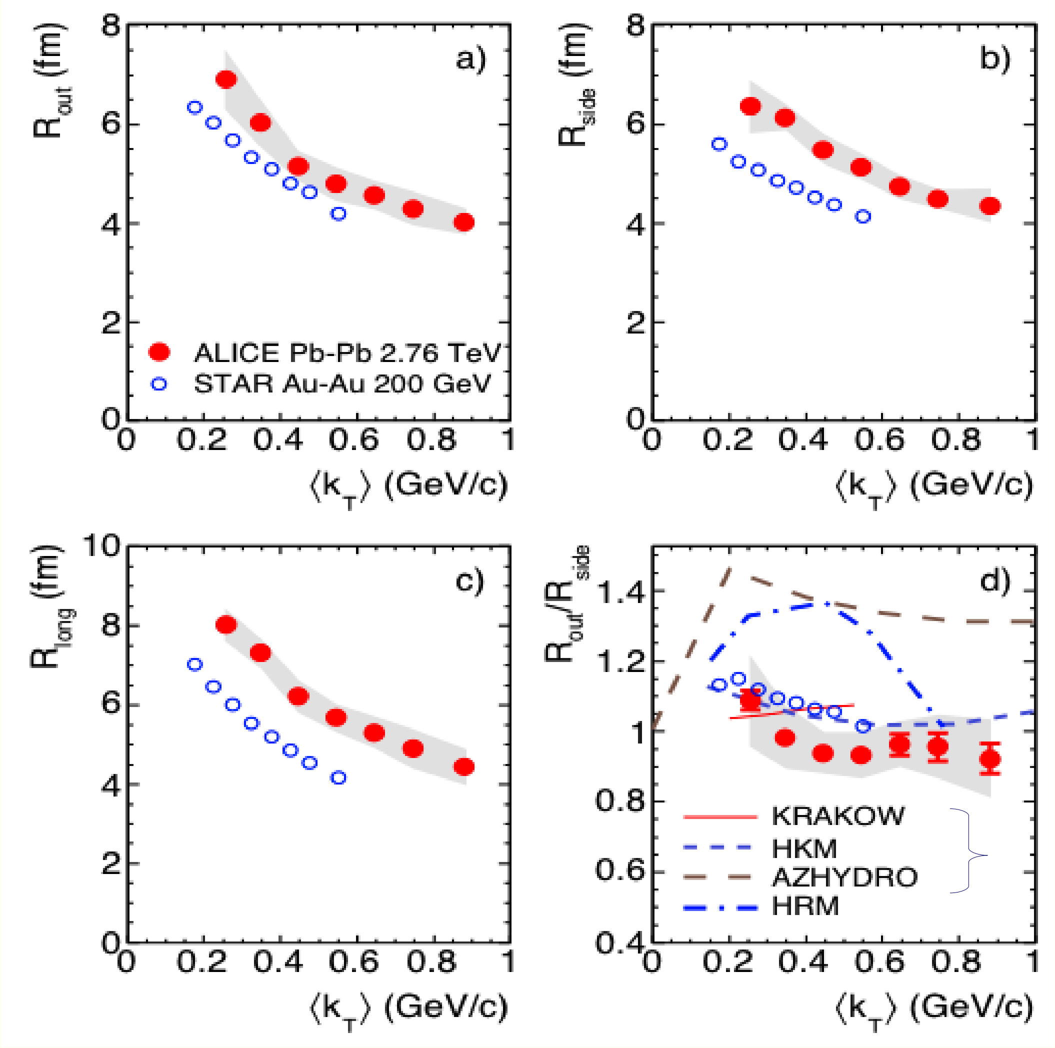
at 158 AGeV:
short but finite emission duration

$$\Delta\tau^2 = \frac{1}{\beta_t^2}(R_{out}^2 - R_{side}^2)$$

Δτ ≈ 2 fm/c i.e. short, consistent with small density change



ALICE PbPb collisions at the LHC at $\sqrt{s_{nn}}$ = 2.76 TeV



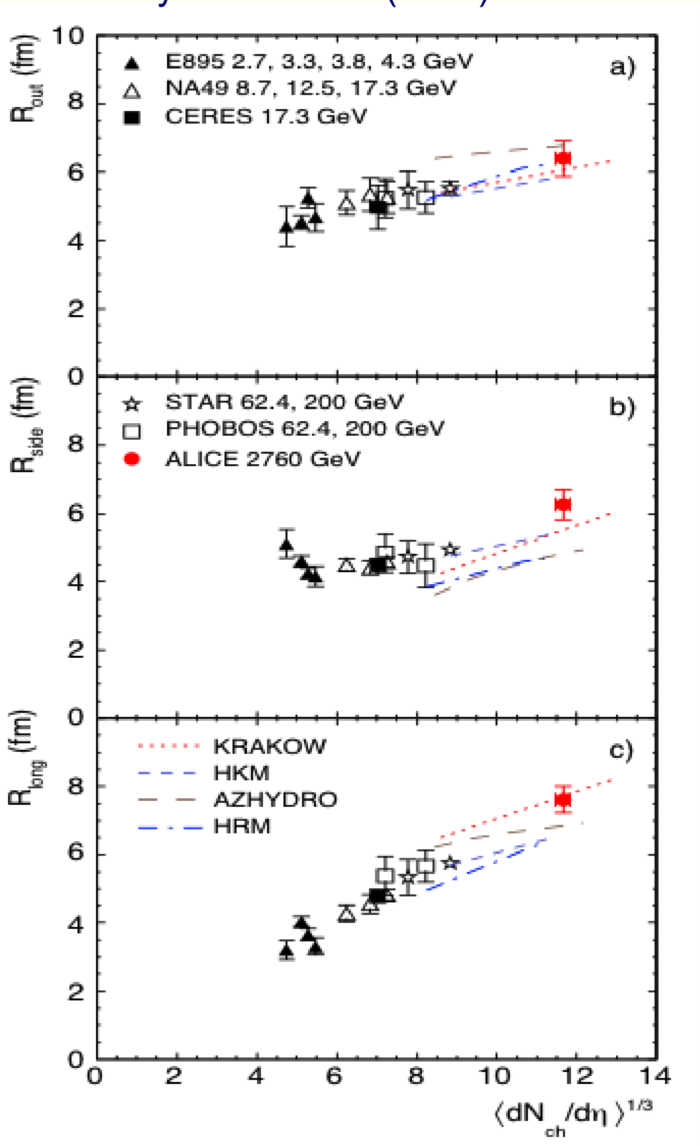
hydro models

k_t dependence shows shape typical for hydrodynamically expanding source
rather similar to RHIC data
but radii are larger
R_{out}/R_{side} dropping below 1
reproduced reasonably well by Krakow and Kiew hydro models

ALICE, Phys. Lett. B696 (2011) 328

Updated energy dependence of HBT radii

Phys. Lett. B696(2011)328



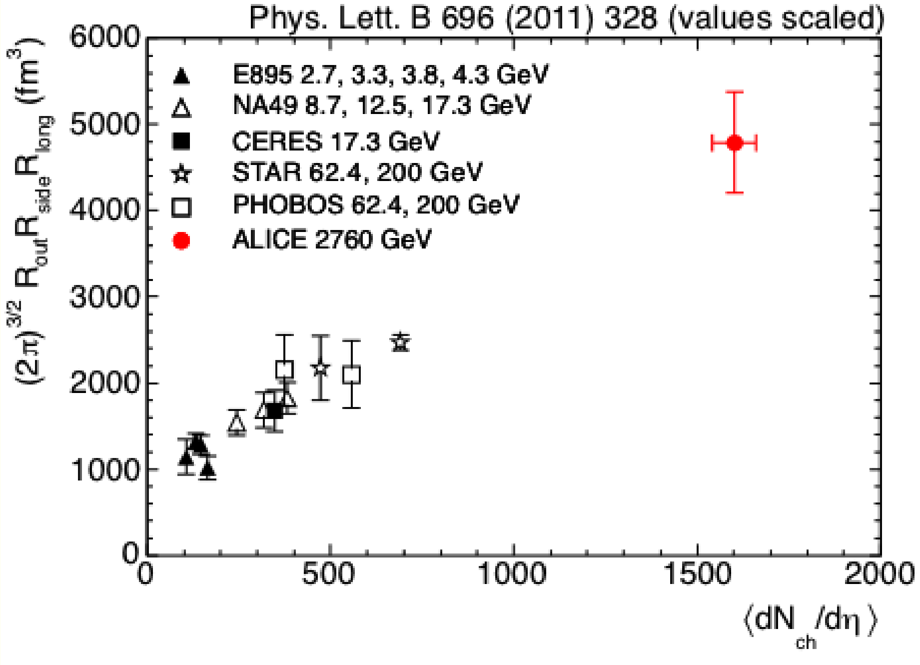
$\langle k_t \rangle$ = 0.3 GeV/c

- significant extension of reach of world data by ALICE
  at higher energies radii grow as cube root of multiplicity indicating freeze-out at constant density
- hydro models reproduce growth reasonably well

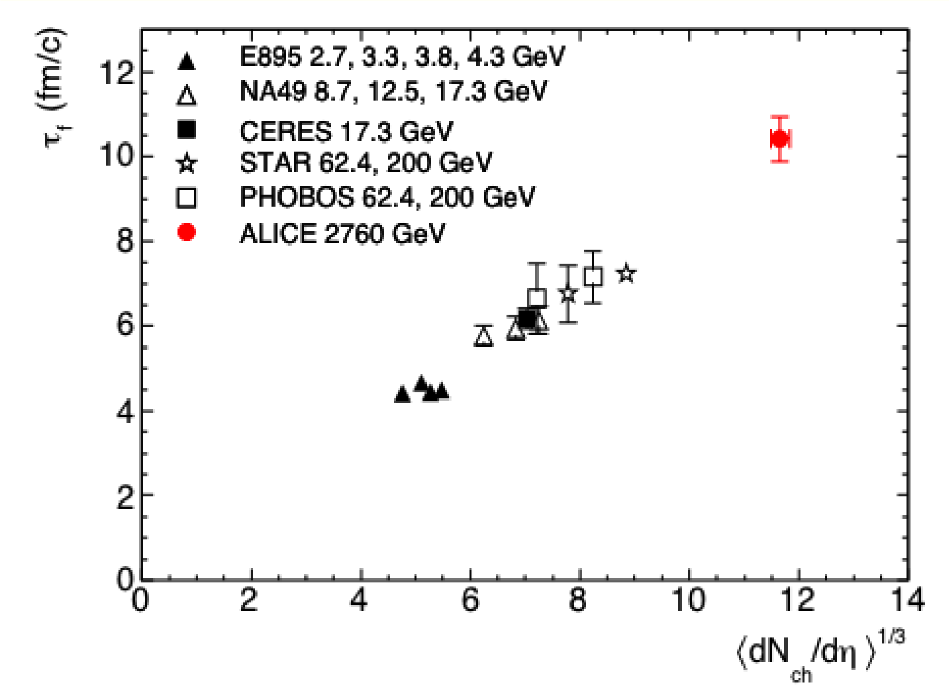
freeze-out volume and duration of expansion

coherence volume $V = (2\pi)^{3/2} R_{side}^2 R_{long}$

from R_{long}: expansion at LHC 10 fm/c



$$R_{long} = \tau_f \sqrt{T/m_t}$$



huge growth at LHC

ALICE, Phys. Lett. B696(2011)328

Volume in pp and heavy ion collisions different