# Space-time evolution of the Quark Gluon Plasma

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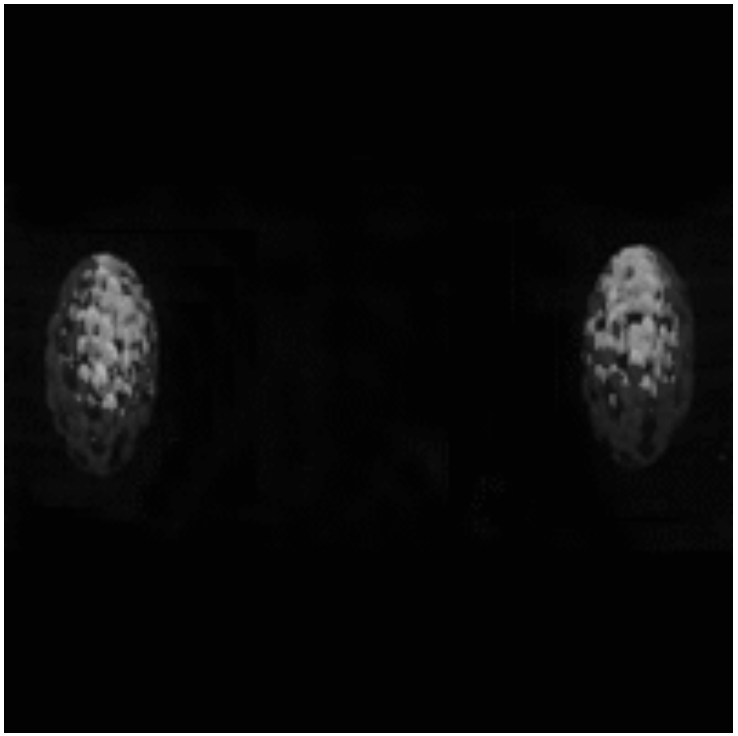
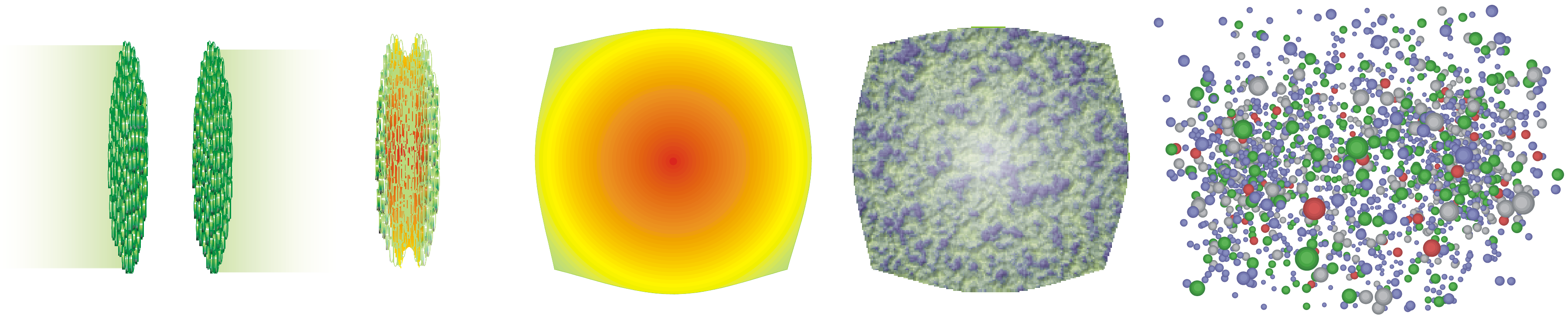
# High-energy nucleus-nucleus Collisions



# High-Energy Nuclear Collisions



Time →

Plot: Steffen A. Bass, Duke University

1) Initial condition:

-Baryon transfer

- $E_T$ production

-Partonic dof

2) System evolves:

- parton/hadron expansion

3) Bulk freeze-out

- hadronic dof

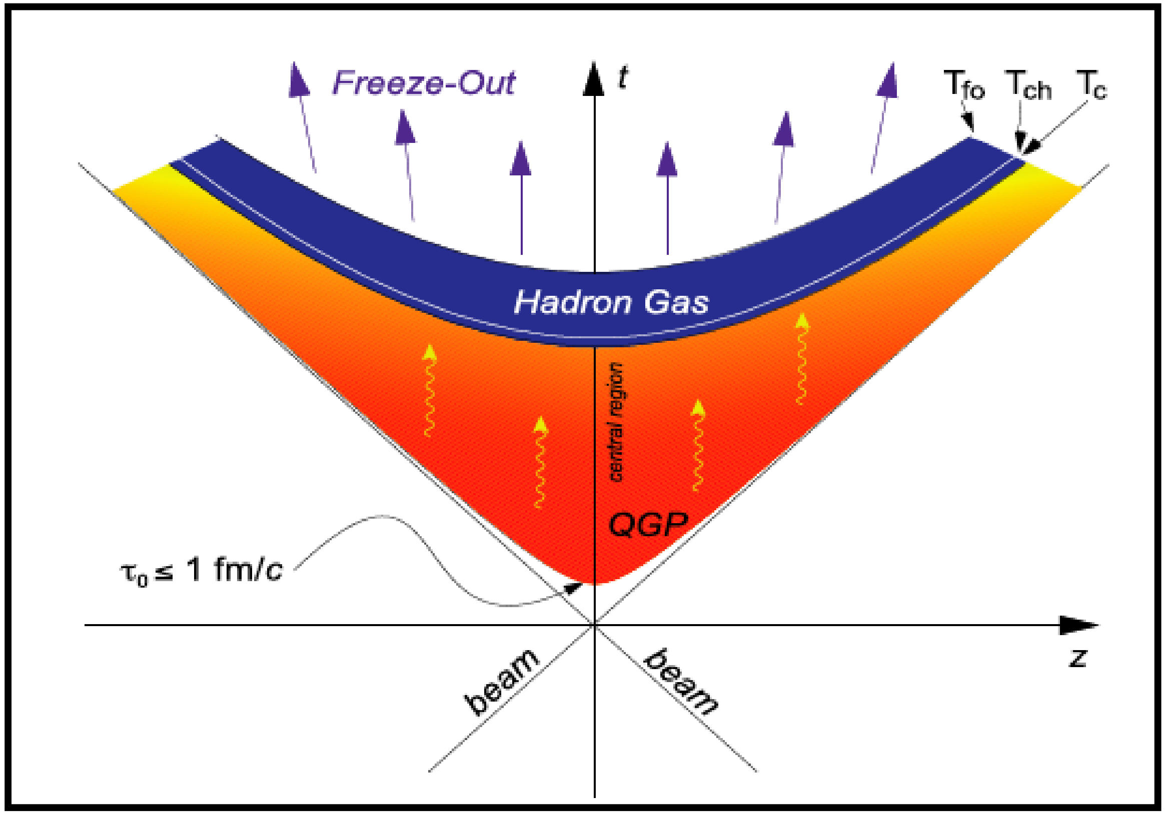
- inel. interactions cease:

particle ratios, $T_{ch}$, $\mu_B$

- elas. interactions cease

Particle spectra, $T_{th}$, $<\beta_T>$

# Space-time evolution




Plot: courtesy of R. Stock.

- **QGP life time**
  10 fm/c ≈ 3•10$^{-23}$ s

- **thermalization time**
  0.2 fm/c ≈ 7•10$^{-25}$ s
  → hydrodynamical expansion until freeze-out
  simplest model: only longitudinal expansion, 1d
  → Bjorken model

- **collision time**
  2R/γ = 0.005 fm/c
  ≈ 2•10$^{-26}$ s

# Outline

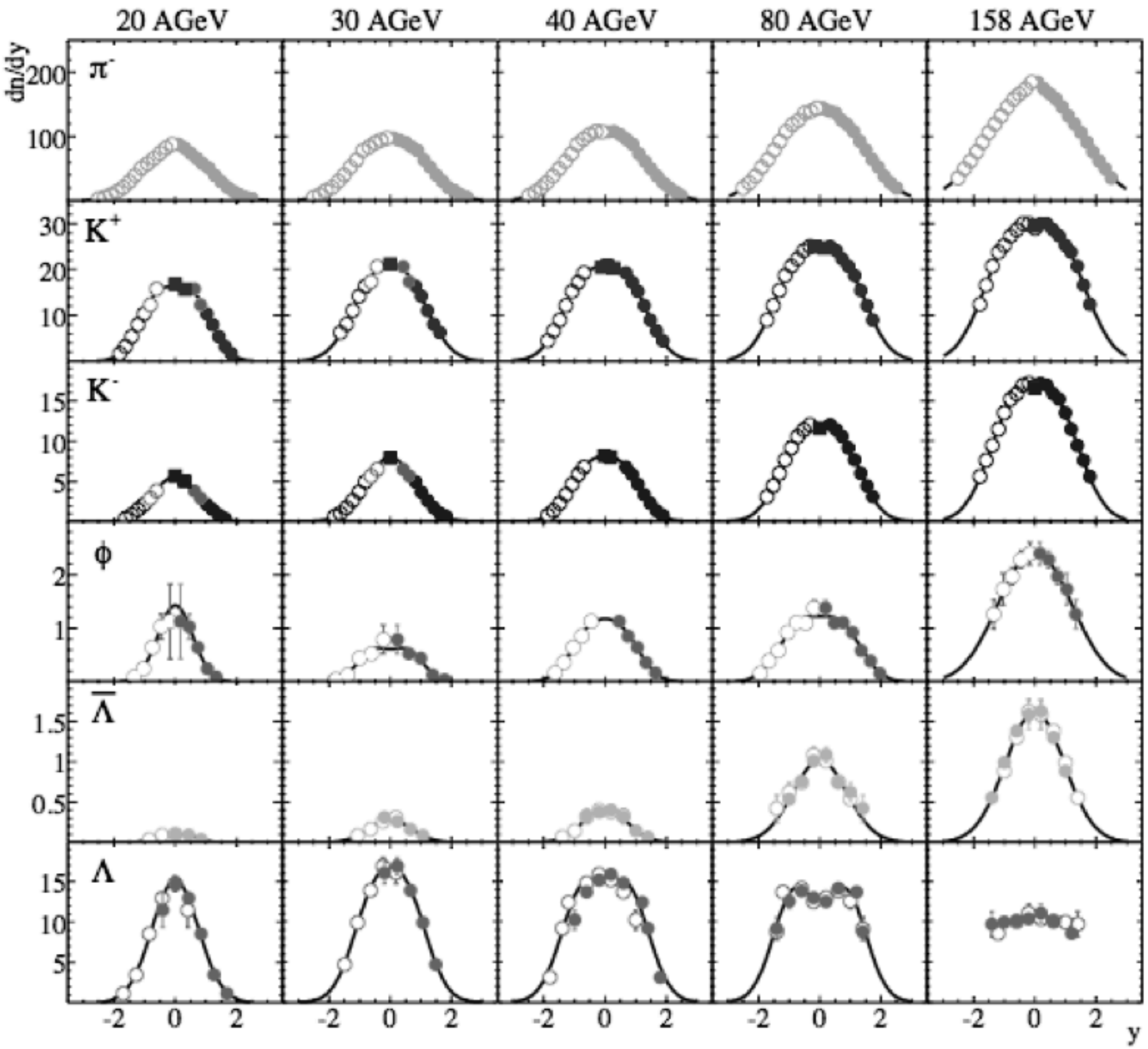
- Introduction
- Longitudinal expansion – Bjorken picture
- Transverse expansion

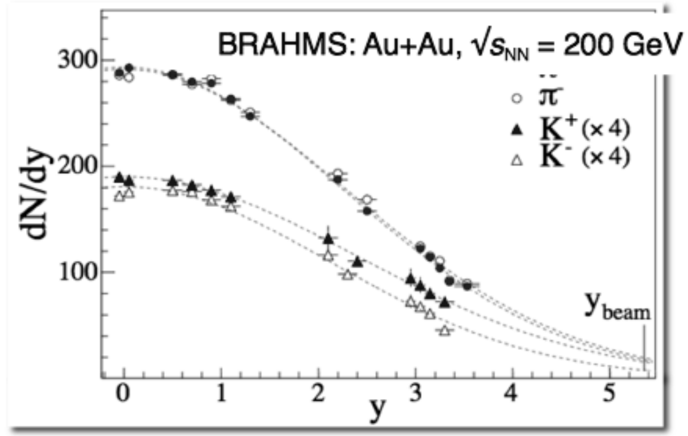
  - transverse radial flow

  - transverse elliptic flow $v_2$

  - higher harmonics, $v_3$, $v_4$, $v_5$, ...
- Hydrodynamical model description
- Summary

# Rapidity distribution in A-A

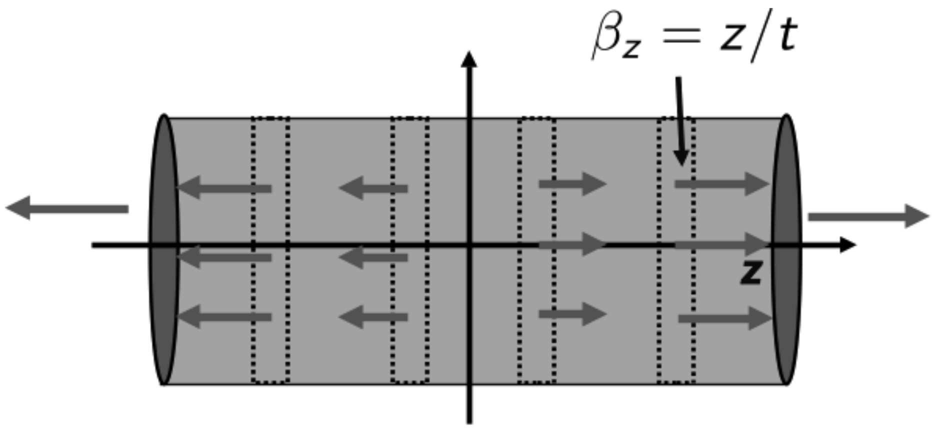


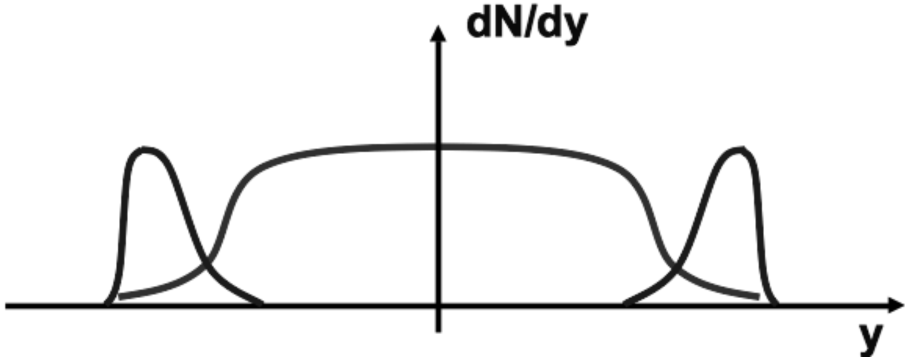




With increasing collision energy:

- wider distribution,
- becomes flatter around mid-rapidity

# Bjorken model




Velocity of the local system at position z at time t:

$$\beta_z = z/t$$

Proper time τ in this system:

$$\tau = t/\gamma = t\sqrt{1-\beta^2}$$
$$= \sqrt{t^2 - z^2}$$

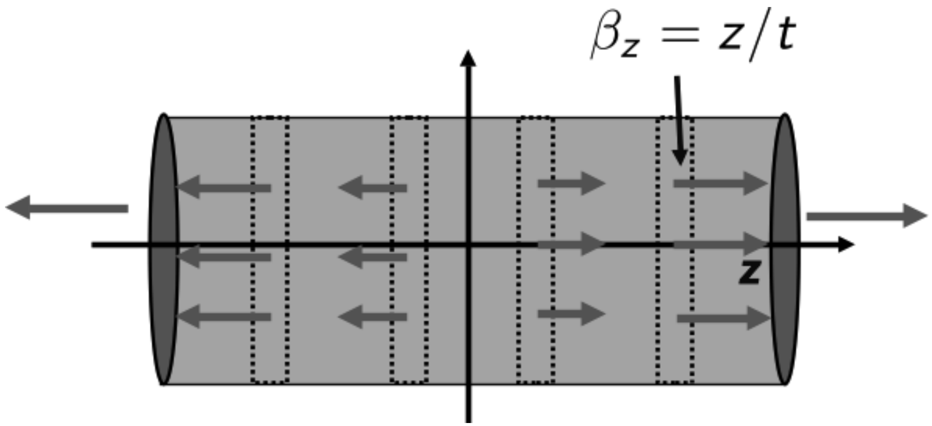
In the Bjorken model all thermodynamic quantities only depend on τ, e.g., the particle density:

$$n(t,z) = n(\tau)$$

This leads to a constant rapidity densityof the produced particles (at least atcentral rapidities):

$$\frac{dN_{ch}}{dy} = \text{const.}$$

# 1d - Bjorken model (I)




The 1D Bjorken model is based on the assumption that dNch/dy ist constant (around mid-rapidity).
This means that the central region is invariant under Lorentz transformation.This implies βz = z/t and that all thermodynamic quantities depend only on the proper time τ

Initial conditions in the Bjorken model:

$$\varepsilon(\tau_0) = \varepsilon_0, \quad u^\mu = \frac{1}{\tau_0}(t,0,0,z) = \frac{x^\mu}{\tau_0}$$

Initial energy density

In this case the equations of ideal hydrodynamics simplify to

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + p}{\tau} = 0$$

| | |
|---|---|
| $\varepsilon = E/V$ : | energy density |
| $p$ : | pressure |
| $s = S/V$ : | entropy density |

# Bjorken model (II)

For an ideal gas of quarks and gluons, i.e., for

$$\varepsilon = 3p, \quad \varepsilon \propto T^4$$

This leads to

$$\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}, \quad T(\tau) = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}$$

The temperature drops to the critical temperature at the proper time

$$\tau_c = \tau_0 \left(\frac{T_0}{T_c}\right)^3$$

And thus the lifetime of the QGP in the Bjorken model is

$$\Delta\tau_{\mathrm{QGP}} = \tau_c - \tau_0 = \tau_0 \left[\left(\frac{T_0}{T_c}\right)^3 - 1\right]$$

# QGP lifetime in 1d - Bjorken model

$$\varepsilon_0 = 11\,\mathrm{GeV/fm^3} = 11 \cdot 0.197^3\,\mathrm{GeV}^4 \quad \mathrm{for}\ \tau_0 = 1\mathrm{fm}/c$$

$$1 = \hbar c = 0.197\,\mathrm{GeV}\cdot\mathrm{fm}$$

$$\varepsilon_0 = g_{\mathrm{QGP}}\frac{\pi^2}{30}T^4 \quad \rightarrow \quad T_0 = \left(\frac{30}{\pi^2}\frac{\varepsilon}{g}\right)^{1/4}$$

| Parameters | $\Delta\tau_{\mathrm{QGP}}$ |
|---|---|
| $\varepsilon_0\,\tau$ = 3 GeV/fm2 | 0.84 fm/c |
| $\varepsilon_0\,\tau$ = 5 GeV/fm2 | 1.70 fm/c |
| $\varepsilon_0\,\tau$ = 11 GeV/fm2 | 3.9 fm/c |

Fixed parameters: Nf = 2, Tc = 170 MeV, τ0 = 1 fm/c

# Quick estimate for LHC

Bjorken formula: $$\varepsilon \cdot \tau_0 = \frac{\langle m_T \rangle}{A} \left.\frac{\mathrm{d}N}{\mathrm{d}y}\right|_{y=0}$$

Transverse area in collisions with $b \approx 0$:

$$A \approx \pi R_{\mathrm{Pb}}^2 = \pi (6.62\,\mathrm{fm})^2 \approx 140\,\mathrm{fm}^2$$

Estimate for the mean transverse momentum:

$$\langle p_T \rangle = 0.66\,\mathrm{GeV}/c \rightsquigarrow \langle m_T \rangle \approx \sqrt{(0.138\,\mathrm{GeV})^2 + (0.66\,\mathrm{GeV})^2} = 0.67\,\mathrm{GeV}$$

Measured charged particle multiplicity:

$$dN_{ch}/d\eta \approx 1601 \pm 60 \quad (5\%\ \text{most central})$$

$$\rightsquigarrow \quad \left.\frac{\mathrm{d}N}{\mathrm{d}y}\right|_{y=0} = \frac{3}{2} \cdot \underbrace{\left(1 - \frac{m^2}{\langle m_T \rangle}\right)^{-1/2}}_{1.02} \cdot \left.\frac{\mathrm{d}N_{ch}}{\mathrm{d}\eta}\right|_{\eta=0} = 2450 \pm 92$$

Larger (up to ≈ 1.2) if p and K are taken into account

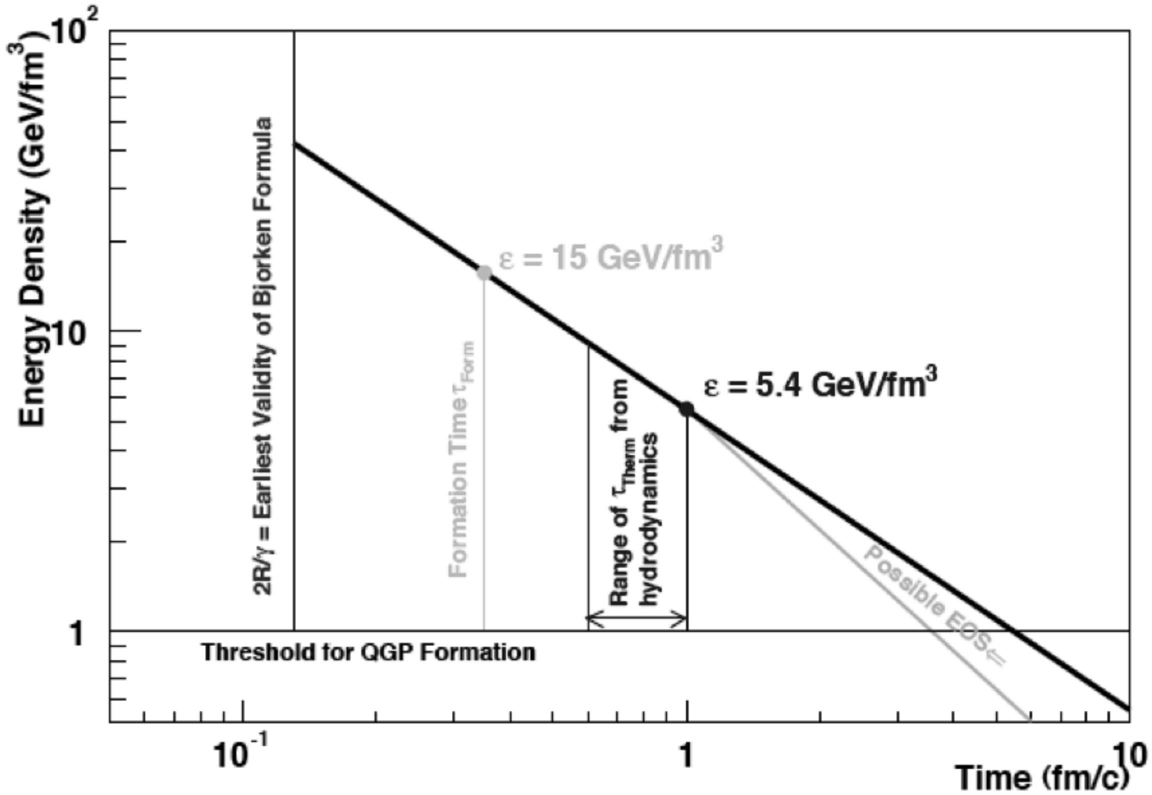
LHC:

$$\varepsilon \cdot \tau_0 = (11.7 \pm 0.43)\,\mathrm{GeV}/\mathrm{fm}^2 \quad (\mathrm{Pb{+}Pb}@\sqrt{s_{NN}} = 2.76\,\mathrm{TeV})$$

RHIC:

$$\varepsilon \cdot \tau_0 \approx 5\,\mathrm{GeV}/\mathrm{fm}^2 \quad (\mathrm{Au{+}Au}@\sqrt{s_{NN}} = 0.2\,\mathrm{TeV})$$

# Energy density evolution in 1d-Bjorken

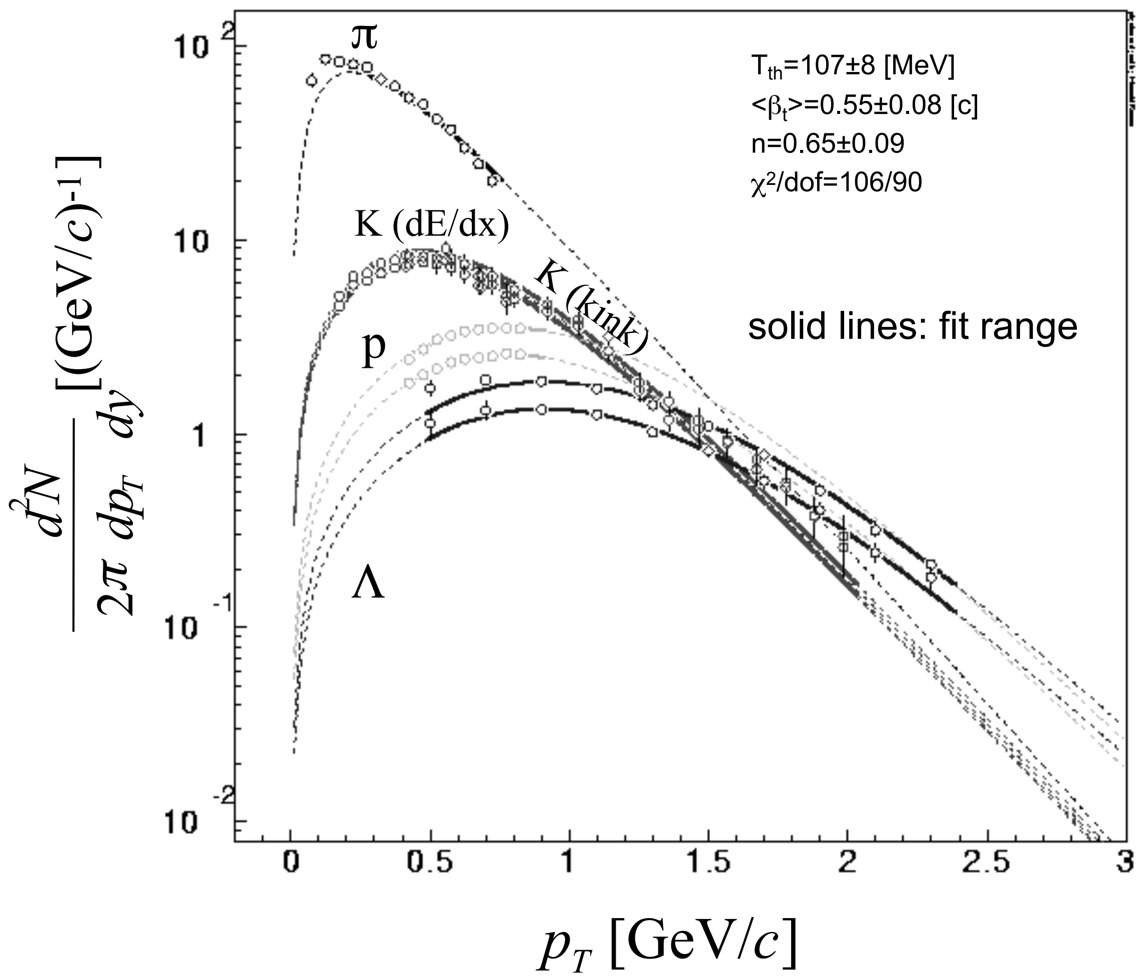



$\tau_0$ = 1 fm/*c* is generally considered as a conservative estimate for the use in the Bjorken formula.
Other estimates yields shorter times (e.g. $\tau_0$ = 0.35 fm/*c*) resulting in initial energy densities at RHIC of up to 15 GeV/fm$^3$

# Transverse Expansion

Transverse radial flow: particle spectra

# Particle Spectra*




*Au+Au @130 GeV, STAR

- Typical mass ordering in inverse slope from light π to heavier Λ
- Two-parameter fit describes yields of π, K, p, Λ
- $T_{th}$ = 90 ± 10 MeV
- <$\beta_t$> = 0.55 ± 0.08 c

⇒ Disentangle **collective motion** from thermal random walk

# Thermal Model + Radial Flow Fit

Source: each volume element is assumed to be

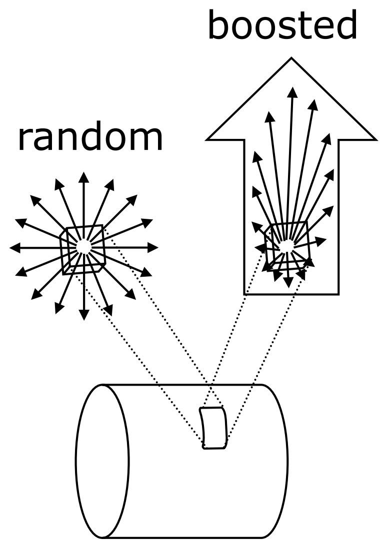
- in local **thermal equilibrium**: $T_{fo}$
- **boosted** in transverse radial direction: $\rho = f(\beta_s)$

E.Schnedermann, J.Sollfrank, and U.Heinz, Phys. Rev. C48, 2462(1993).

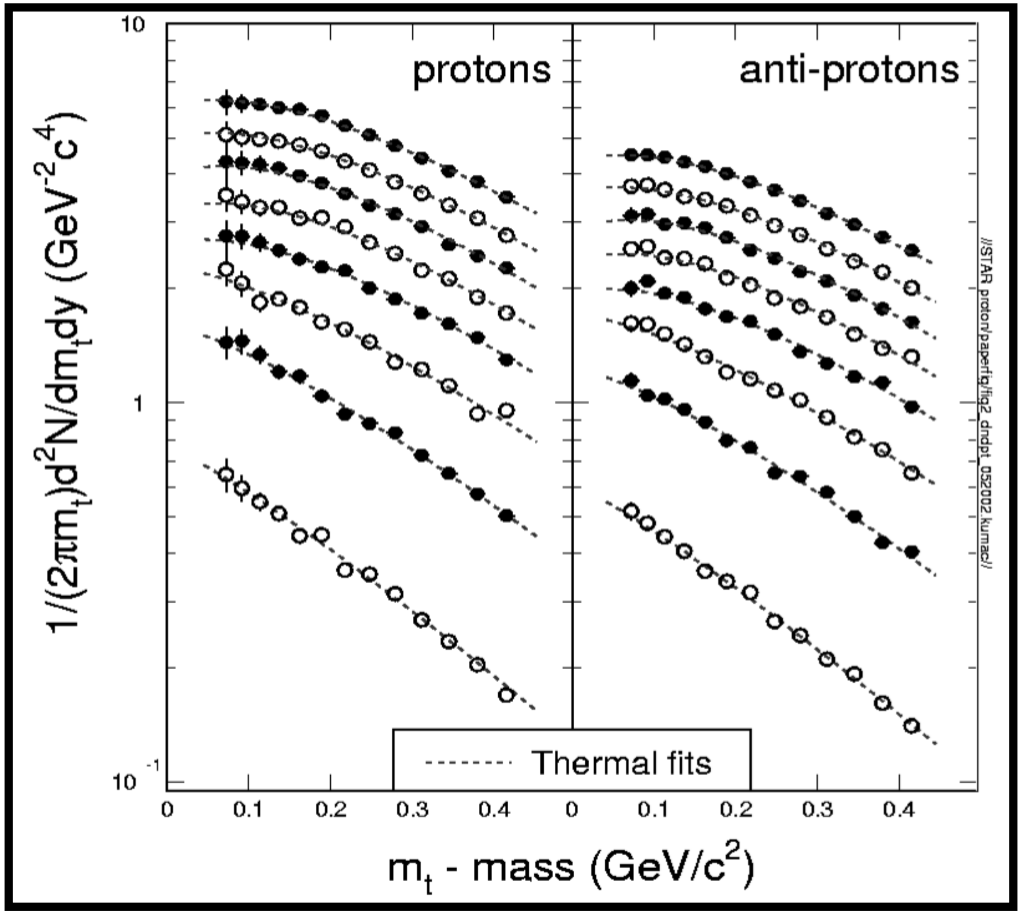
$$E\frac{d^3N}{dp^3} \propto \int_\sigma e^{-(u^\mu p_\mu)/T_{fo}} p\, d\sigma_\mu \Rightarrow$$

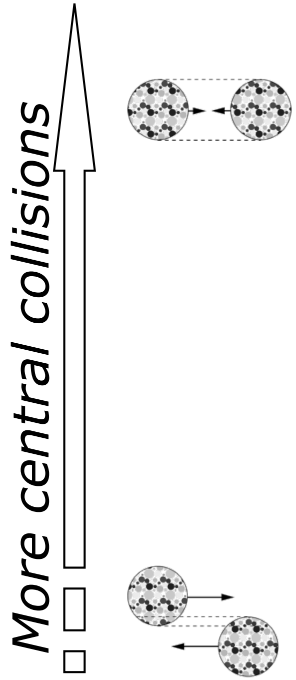
$$\frac{dN}{m_T dm_T} \propto \int_0^R r\,dr\,m_T K_1\left(\frac{m_T \cosh\rho}{T_{fo}}\right) I_0\left(\frac{p_T \sinh\rho}{T_{fo}}\right)$$

$$\rho = \tanh^{-1}\beta_T \qquad \beta_T = \beta_S\left(\frac{r}{R}\right)^\alpha \qquad \alpha = 0.5,\ 1,\ 2$$



# (anti-)Protons From RHIC Au+Au@130GeV




$$m_T = \sqrt{p_T^2 + mass^2}$$

Centrality dependence:

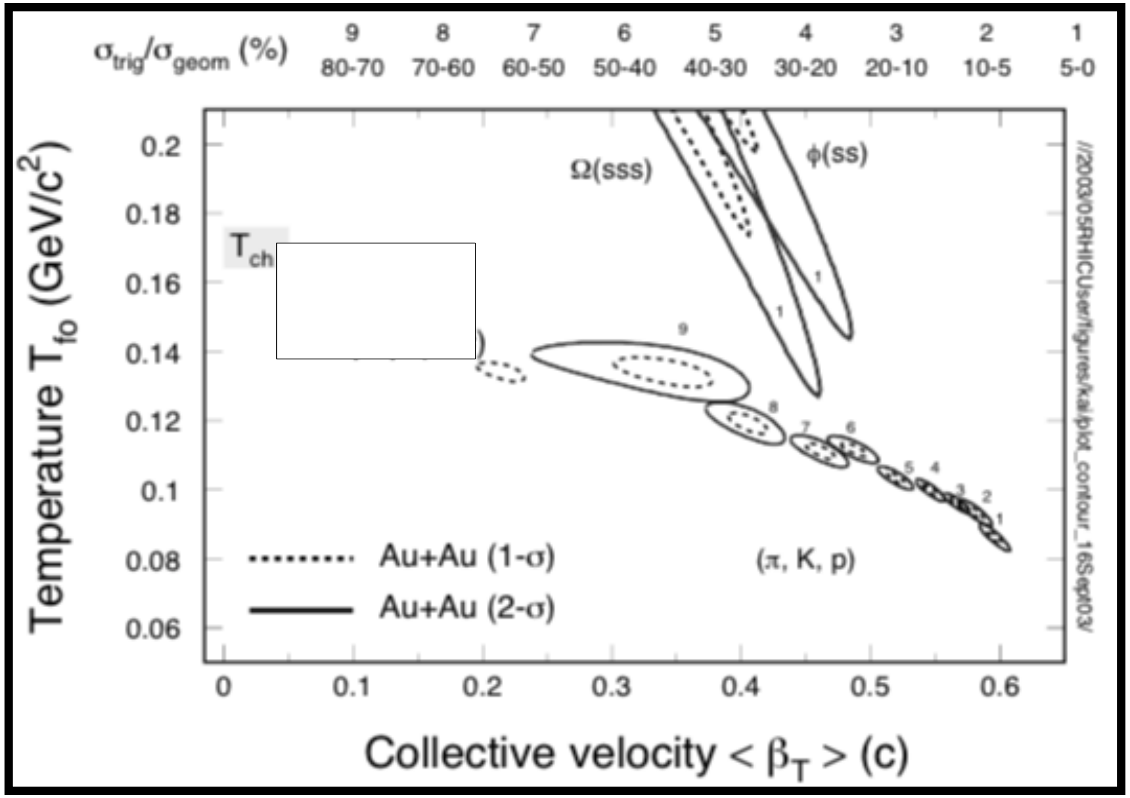
- spectra at low momentum de-populated, become flatter at larger momentum

⇒ stronger collective flow in more central collisions, $\langle\beta_t\rangle = 0.55 \pm 0.08$



STAR: Phys. Rev. C70, 041901(R).

# Kinetic Freeze-out at RHIC




STAR Data: Nucl. Phys. A757, (2005 102),

*A. Baran, W. Broniowski and W. Florkowski, Acta. Phys. Polon. B 35 (2004) 779.

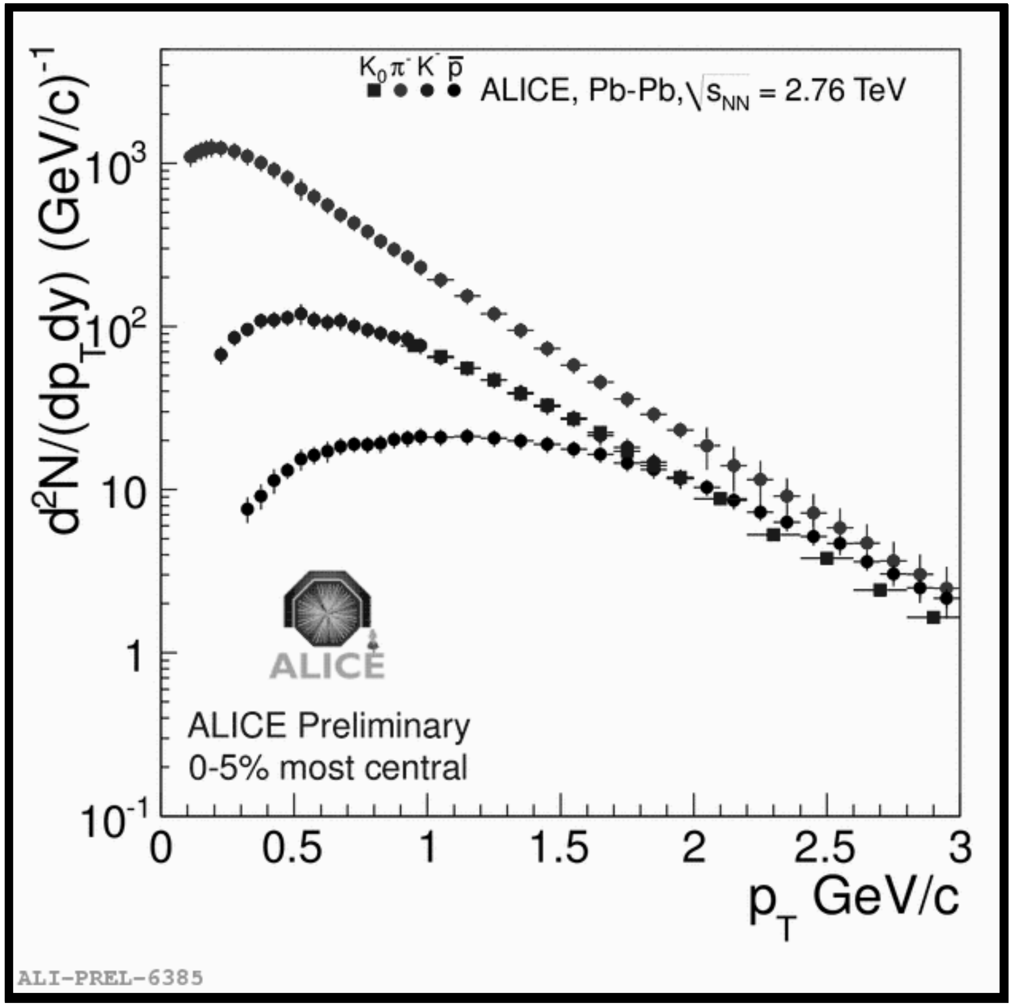
0) T and $\beta_T$ are anti-correlated

1) Multi-strange hadrons $\phi$ and $\Omega$ freeze-out **earlier** than ($\pi$, K, p)
$\Rightarrow$ Collectivity prior to hadronization

2) Sudden single freeze-out*:
Resonance decays lower $T_{fo}$ for ($\pi$, K, p)
$\Rightarrow$ **Collectivity prior** to **hadronization**

$\Rightarrow$ **Partonic Collectivity ?**

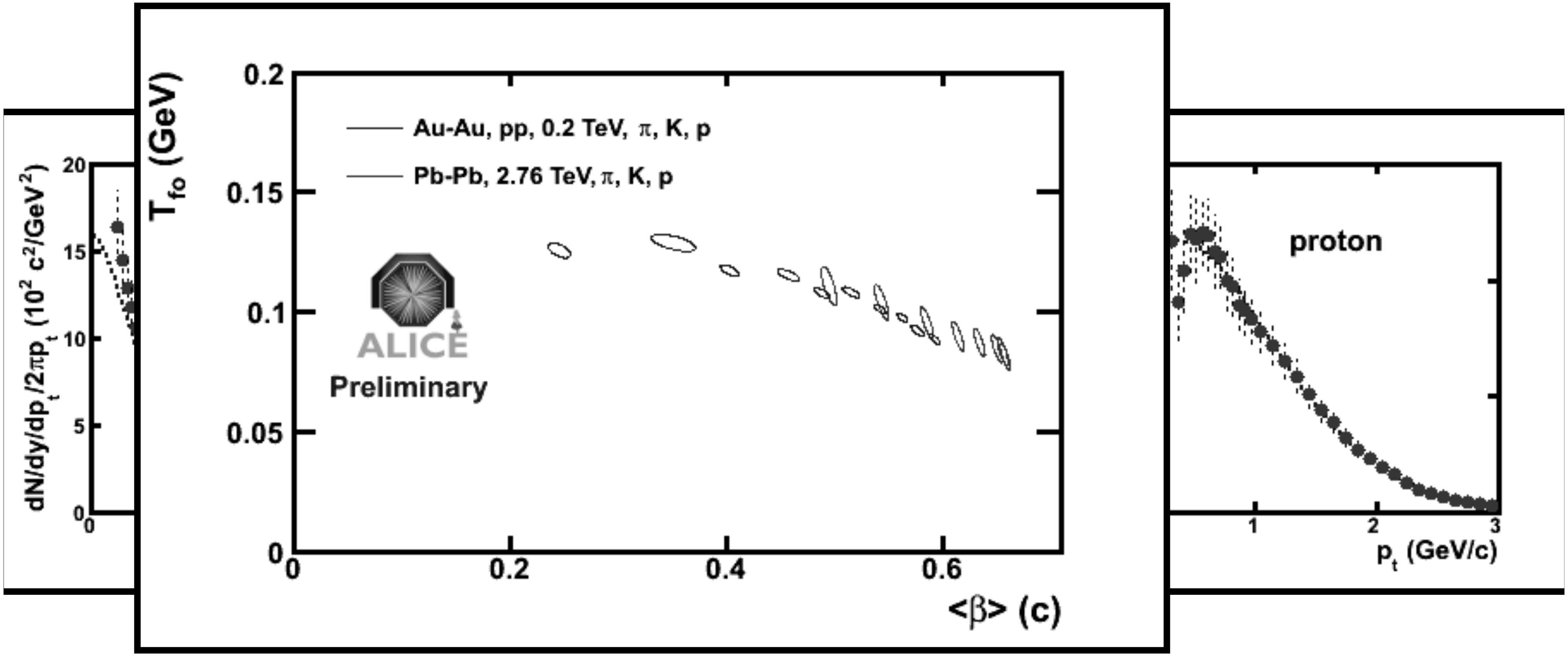
# LHC: Identified particle spectra




**Spectra harder** at LHC

$\Rightarrow$ **stronger** collective **flow** at LHC than at RHIC

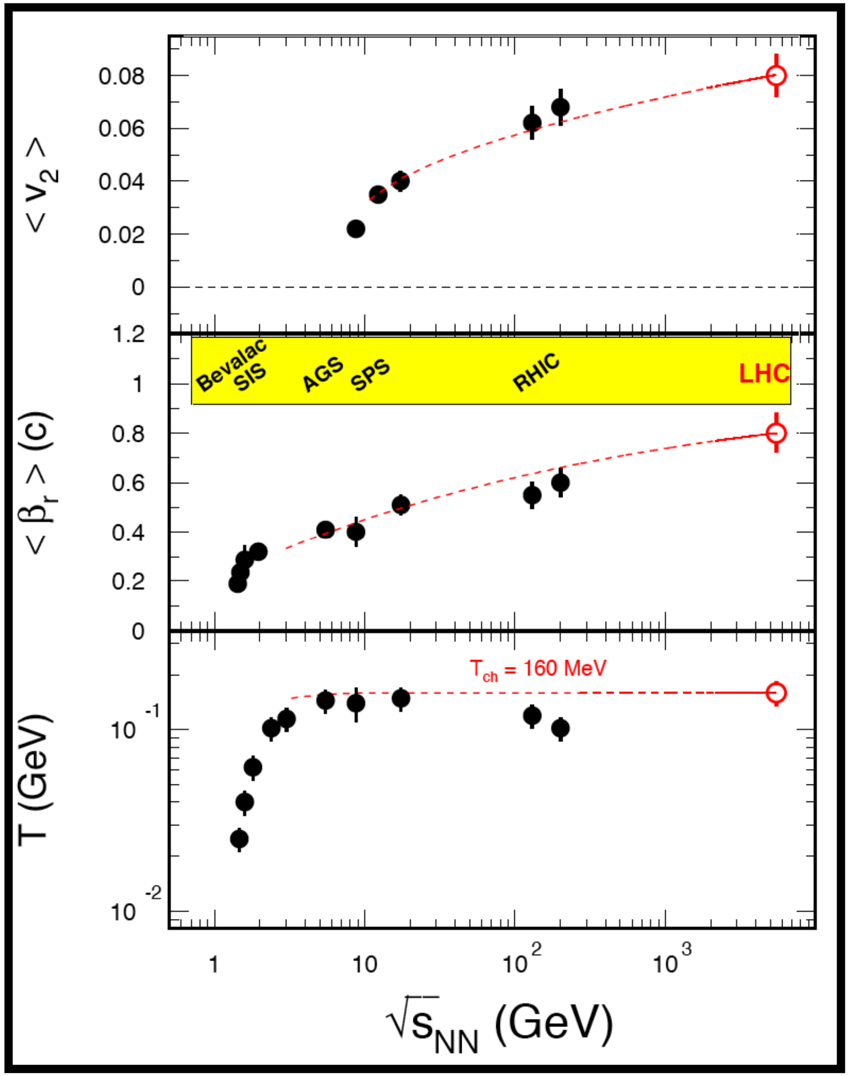
# Collective expansion




Blast wave parametrization **describes spectra** at **10% level**

Collective **flow velocity increases** from RHIC to LHC by **10%**

# Collective Flow - Energy Dependence




K.S., ISMD07, arXiv:0801.1436 [nucl-ex].

**Collectivity** parameters $<\beta_T>$ and $<v_2>$ **increase** with **collision energy**

**strong** collective **expansion** at RHIC !

$<\beta_T>$RHIC » 0.6

expected **strong partonic expansion** at **LHC**,

$<\beta_T>$LHC ≈ 0.8, $T_{fo}$ ≈ $T_{ch}$

# Lesson I

- At LHC, Initial energy density approx. 50 GeV/fm$^3$
- much larger than critical energy density $\varepsilon_c$ = 0.7 GeV/fm$^3$

- Strong **collective expansion**, $\langle\beta_T\rangle$ = 0.6 – 0.7
 at **highest collider energies**

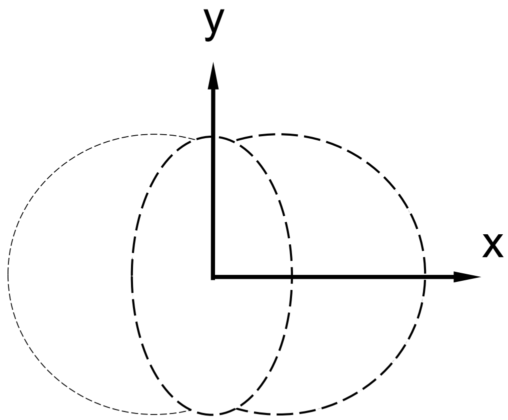
- Particles carrying **strange quarks** show that **collective expansion** develops **before hadronization**, $\langle\beta_T\rangle$ = 0.3 - 0.4
– among quarks and gluons (?)

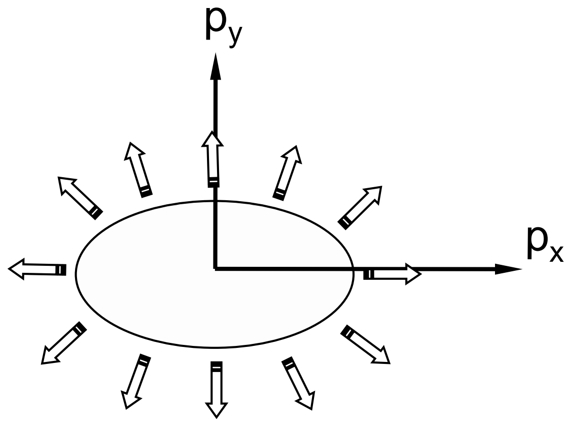
# Transverse Expansion

Transverse elliptic flow: event anisotropy

# Anisotropy Parameter $v_2$

coordinate-space-anisotropy ⇔ momentum-space-anisotropy

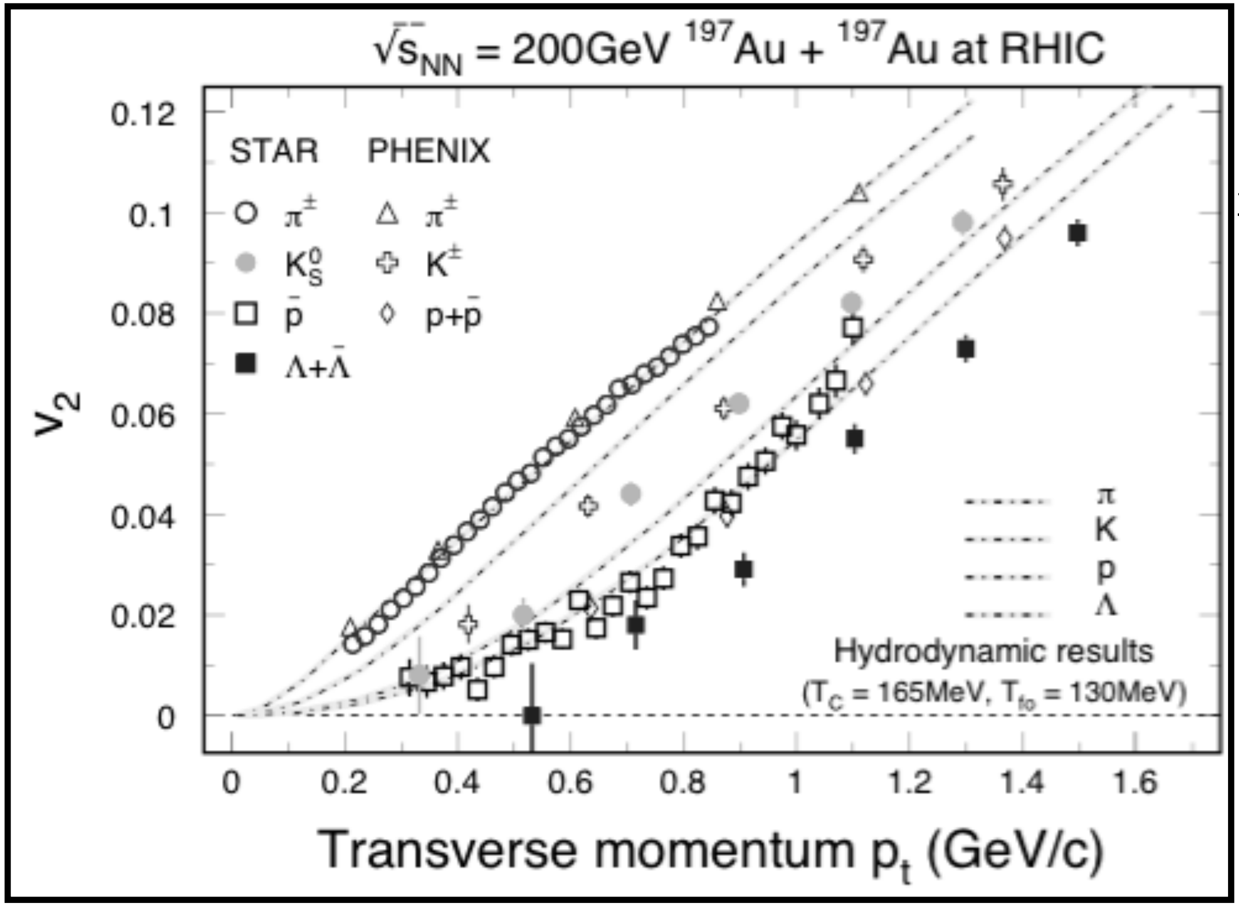



$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \qquad v_2 = \langle \cos 2\varphi \rangle, \;\; \varphi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

**Initial/final conditions, EoS, degrees of freedom**

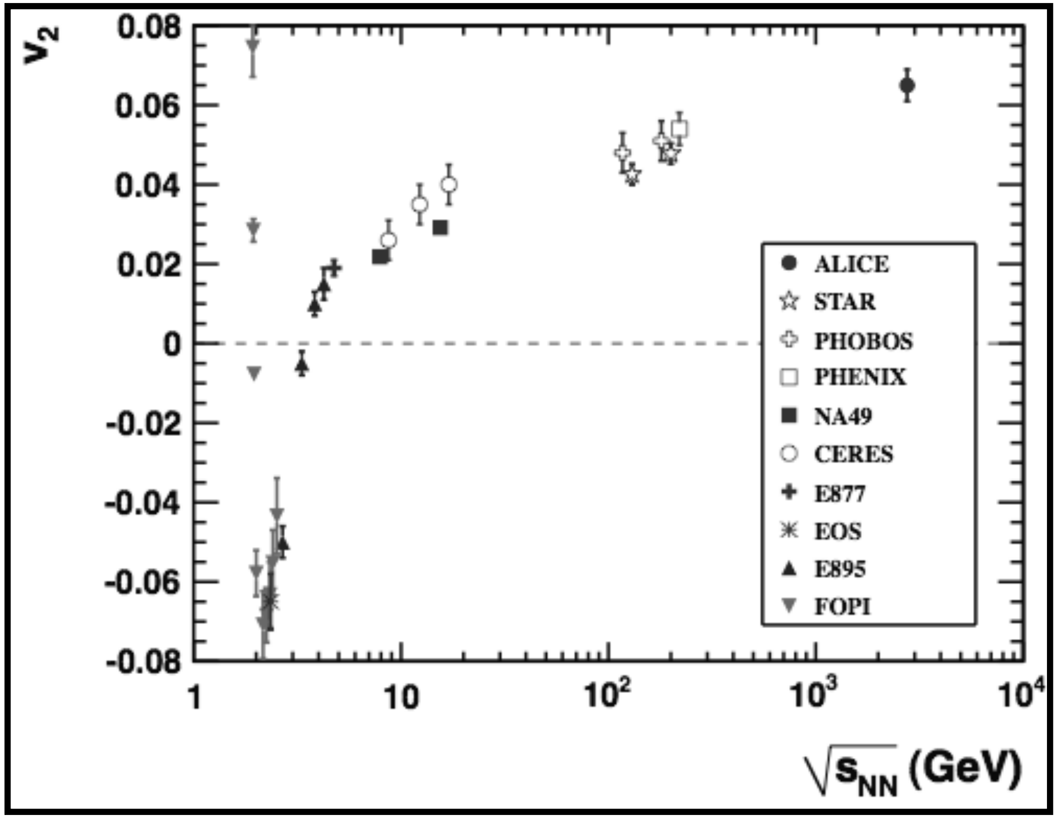
# $v_2$ in the low-$p_T$ Region




P. Huovinen, private communications, 2004

- $v_2$ approx. linear in $p_T$, mass ordering from light $\pi$ to heavier $\Lambda$
- characteristic of hydrodynamic flow, sensitive to EOS !

# Elliptic flow in ALICE



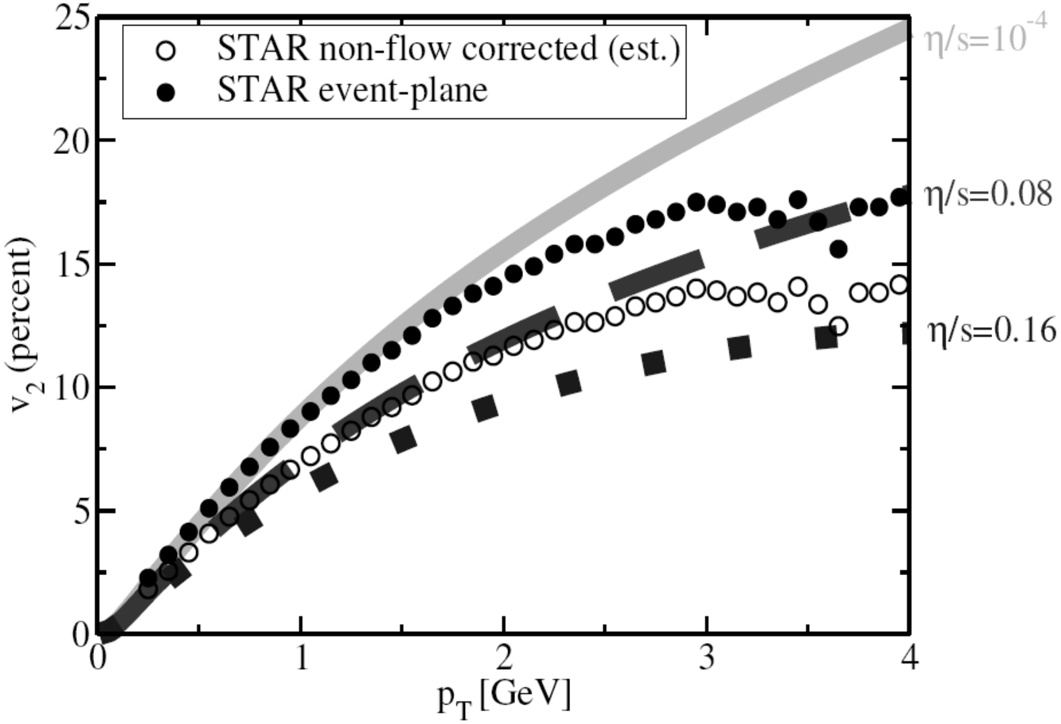

ALICE, submitted for publication, arXiv:1011.3914 [nucl-ex].

$\sqrt{s_{NN}}$ > ~ 4 GeV: initial excentricity leads to pressure gradients that cause positive $v_2$

2 < $\sqrt{s_{NN}}$ < 4 GeV: velocity of the nuclei is small so that presence of spectator matter inhibits in-plane particle emission ("squeeze-out")

$\sqrt{s_{NN}}$ < 2 GeV: rotation of the collision system leads to fragments being emitted in-plane

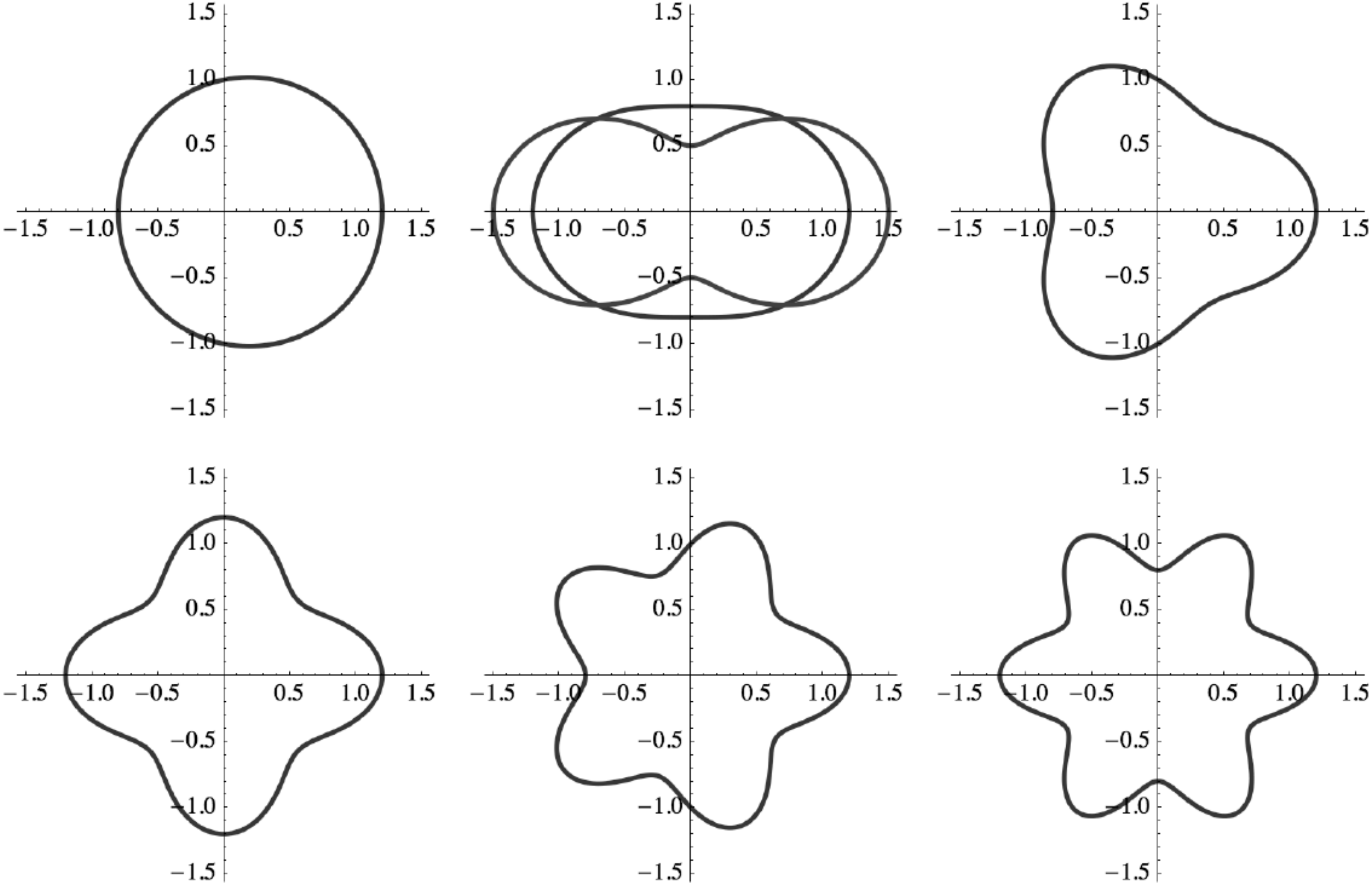
# Non-ideal Hydro-dynamics



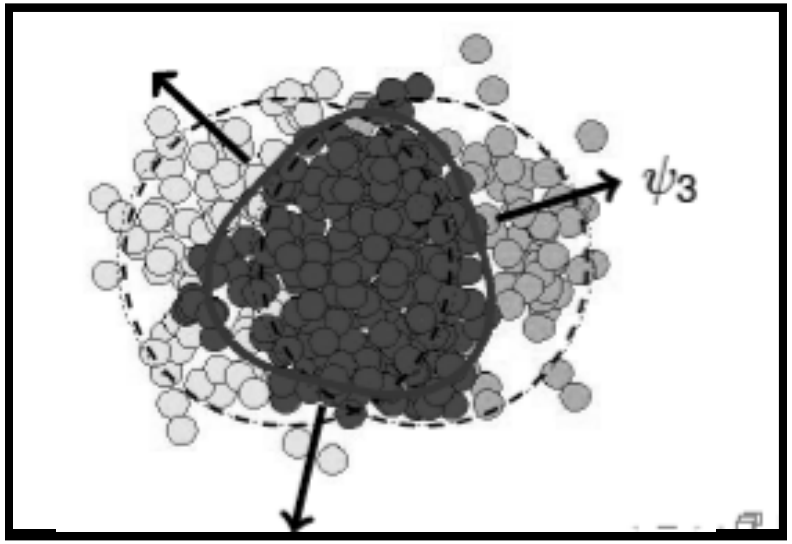

M.Luzum and R. Romatschke, PRC 78 034915 (2008); P. Romatschke, arXiv:0902.3663.

- Spectra and flow reproduced by ideal hydrodynamics calcs.
- Shear viscosity to entropy density ratio close to AdS/CFT bound
- viscosity leads to decrease in $v_2$, ultra-low viscosity sufficient to describe data
- Hydro-limit exceeded at LHC ?

$$f(\varphi) = 1 + 2v_n \cos(n\varphi)$$



# Can there be $v_3$ ?

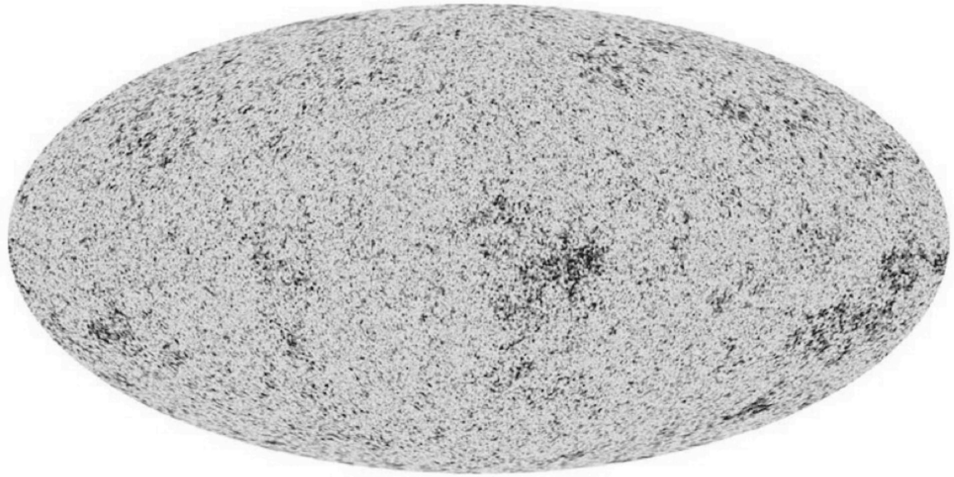



figs.: courtesy of M. Luzum.

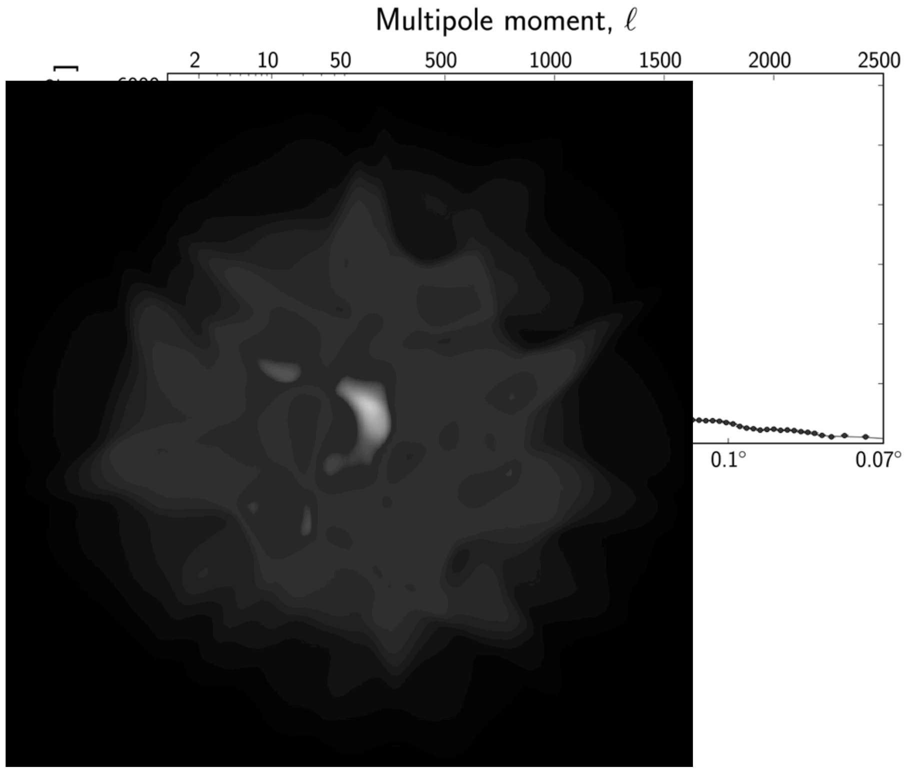
- reaction plane ≠ participant plane
- fluctuating initial state is seed for $v_3$

⇒ can CGC be challenged ?

# higher harmonics



- extract power spectrum of $v_n$, like Planck*
- higher harmonics
- odd harmonics important
- $v_3$: access $\eta/s$
- Higher harmonics strongly damped ($v_{n,\, n>10} = 0$)




STAR, arXiv:1301.2187 [nucl-ex];
STAR, PRL 92 (2004) 062301;
A. Mocsy and P. Sorensen, NPA 855 (2011) 241;
B. Alver and G. Roland, PRC 81 (2010) 054904;
Planck data: EAS and the Planck collaboration
QGP plot: B. Schenke, S. Jeon, and C. Gale, arXiV:1109.6289.

# Lesson II

- Geometrical anisotropy is seed for elliptic flow $v_2$
- Elliptic flow $v_2$ sensitive to QGP equation of state
- Triangular flow $v_3$ due to fluctuations, e.g. in initial energy density
- Triangular flow $v_3$ especially sensitive to shear viscosity / entropy density ratio
- Higher harmonics $v_4$, $v_5$, ... strongly damped

# Hydrodynamical model description

Some basic concepts

# Relativistic Hydrodynamics (I)

The energy-momentum tensor $T^{\mu\upsilon}$ is the four-momentum component in the μ direction per three-dimensional surface area perpendicular to the v direction.

$$\Delta\mathbf{p} = (\Delta E, \Delta p_x, \Delta p_y, \Delta p_z)$$
$$\Delta\mathbf{x} = (\Delta t, \Delta x, \Delta y, \Delta z)$$

$$\mu = \nu = 0: \quad T_R^{00} = \frac{\Delta E}{\Delta x \Delta y \Delta z} = \frac{\Delta E}{\Delta V} = \varepsilon$$

$$\mu = \nu = 1: \quad T_R^{11} = \frac{\Delta p_x}{\Delta t \Delta y \Delta z}$$

force in *x* direction acting on an surface Δ*y* Δ*z* perpendicular to the force → pressure

$$T^{\mu\nu} = \begin{pmatrix} \text{energy density} & \text{energy flux density} \\ \text{momentum density} & \text{momentum flux density} \end{pmatrix} \equiv \begin{pmatrix} \varepsilon & \vec{j}_\varepsilon \\ \vec{g} & \Pi \end{pmatrix}$$

# Relativistic Hydrodynamics (II)

Isotropy in the fluid rest implies that

the energy flux $T^{0j}$ and the momentum density $T^{j0}$ vanish

and that $\Pi^{ij} = P\,\delta_{ij}$

$$T_R^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Off-diagonal elements ≠ 0 in case of viscous

hydrodynamics, not considered here

→ ideal (perfect) fluid.

See also Ollitrault, arXiv:0708.2433.

# Relativistic Hydrodynamics (III)

Energy-momentum tensor (in case of local thermalization)
after Lorentz transformationto the lab frame:

$$T^{\mu\nu} = (\varepsilon + P)\, u^{\mu} u^{\nu} - P\, g^{\mu\nu}$$

metric tensor diag(1,-1,-1,-1)

Energy density
and pressure in
the co-moving system

4-velocity: $u^{\mu} = \mathrm{d}x^{\mu}/\mathrm{d}\tau = \gamma(1, \vec{v})$

Energy and momentum conservation:

$$\partial_{\mu} T^{\mu\nu} = 0, \quad \nu = 0, \ldots, 3$$

$$\partial_{\mu} = (\frac{\partial}{\partial t}, \vec{\nabla})$$

in components:

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t}\varepsilon + \vec{\nabla}\vec{j}_{\varepsilon} = 0 \;\; \text{(energy conservation)} \\ \frac{\partial}{\partial t} g_i + \nabla_j \Pi_{ij} = 0 \;\; \text{(momentum conservation)} \end{array} \right.$$

Conserved quantities, e.g., baryon number:

$$j^{\mu}_{B}(x) = n_{B}(x)\, u^{\mu}(x), \qquad \partial_{\mu} j^{\mu}_{B}(x) = 0 \;\Leftrightarrow\; \frac{\partial}{\partial t} N_{\mathrm{B}} + \vec{\nabla}(N_{\mathrm{B}} \vec{v}) = 0$$

continuity equation

$$N_{\mathrm{B}} = \gamma n_{\mathrm{B}}$$

# Ingredients of Hydro - models

- Equation of motion and baryon number conservation:

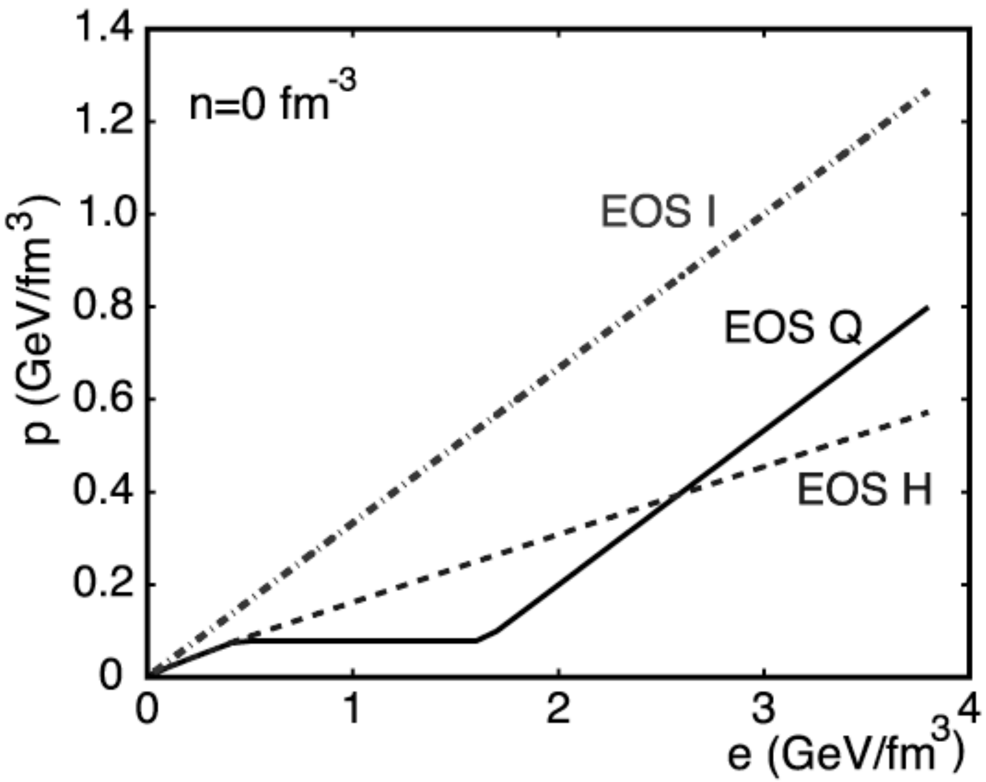
$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu_{\mathrm{B}}(x) = 0$$

- 5 equations for 6 unknowns:

$$(u_x, u_y, u_z, \varepsilon, P, n_{\mathrm{B}})$$

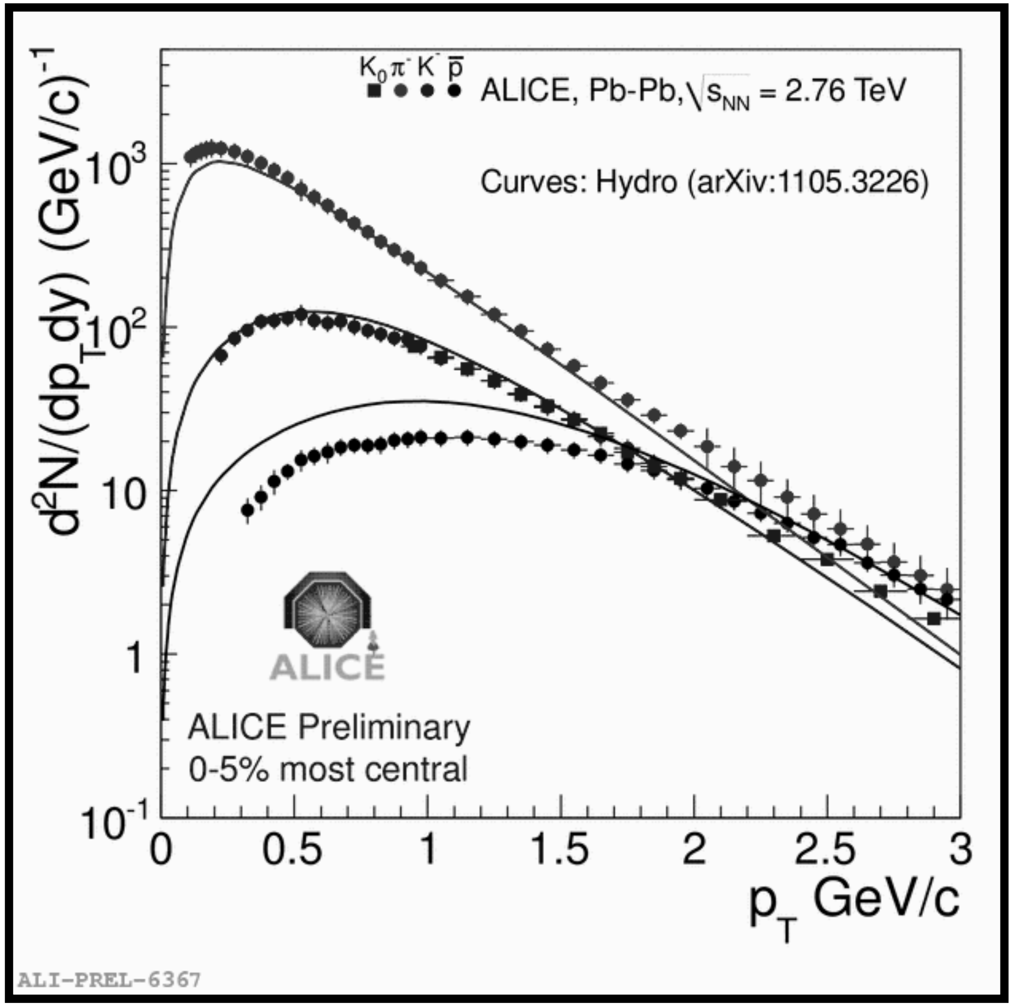
- Equation of state: $P(\varepsilon, n_{\mathrm{B}})$
- (needed to close the system)
- Initial conditions,

  e.g., from Glauber calculation
- Freeze-out condition




EOS I: ultra-relativistic gas $P = \varepsilon/3$
EOS H: resonance gas, $P \approx 0.15\ \varepsilon$
EOS Q: phase transition,
QGP ↔ resonance gas

# LHC: Identified particle spectra

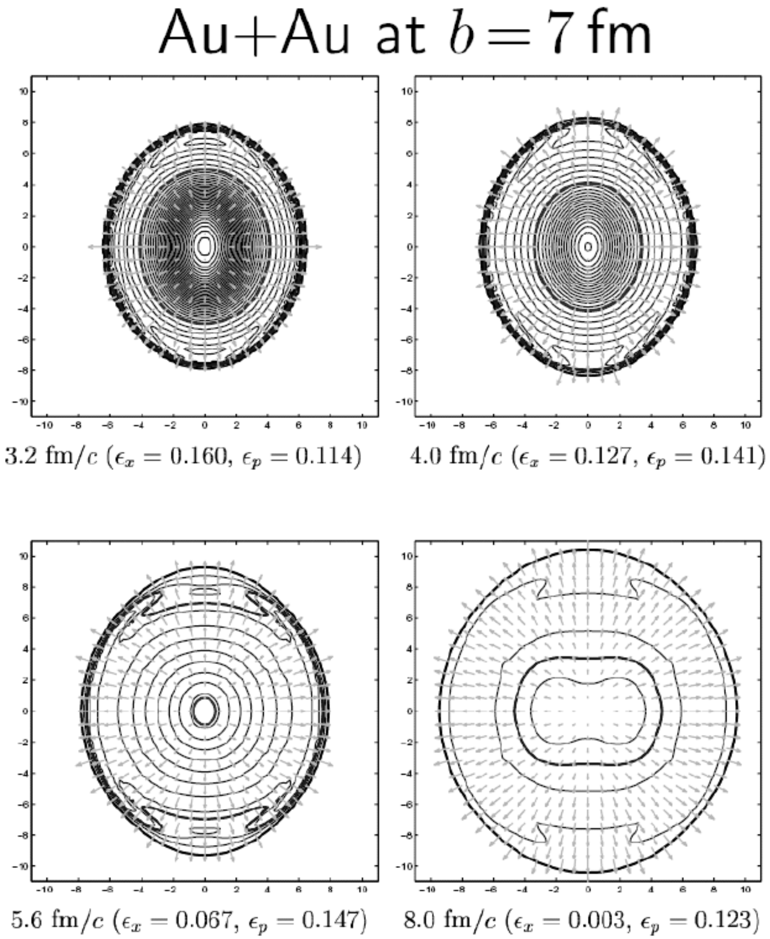



Initial conditions fixed by pion abundance

Protons overestimated

Annihilation of protons and anti-protons in the hadron phase ?

# Elliptic flow in Hydro - models




3.2 fm/$c$ ($\epsilon_x = 0.160$, $\epsilon_p = 0.114$) 4.0 fm/$c$ ($\epsilon_x = 0.127$, $\epsilon_p = 0.141$)

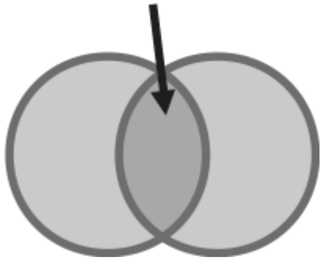
5.6 fm/$c$ ($\epsilon_x = 0.067$, $\epsilon_p = 0.147$) 8.0 fm/$c$ ($\epsilon_x = 0.003$, $\epsilon_p = 0.123$)

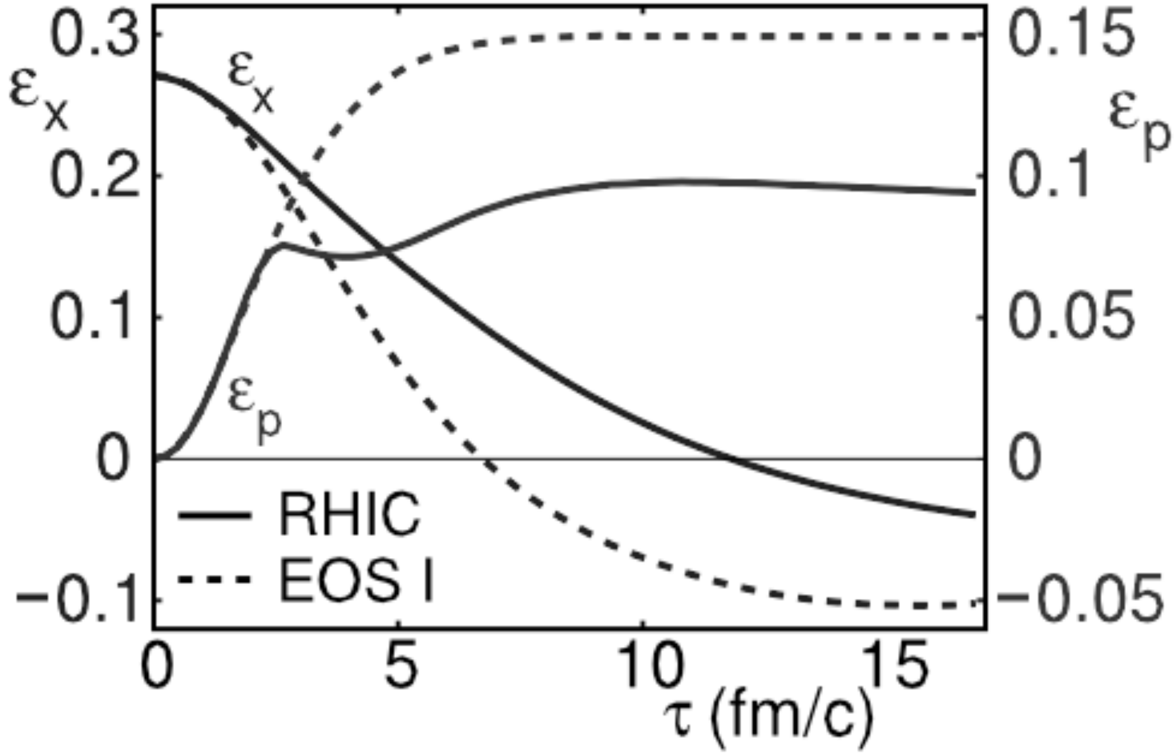
Elliptic flow is “self-quenching”:The cause of elliptic flow, the initial spacial anisotropy, decreases as the momentum anisotropy increases

# Anisotropy in momentum space

Anisotropy in coordinate space

Anisotropy in momentum space






Ulrich Heinz, Peter Kolb, arXiv:nucl-th/0305084

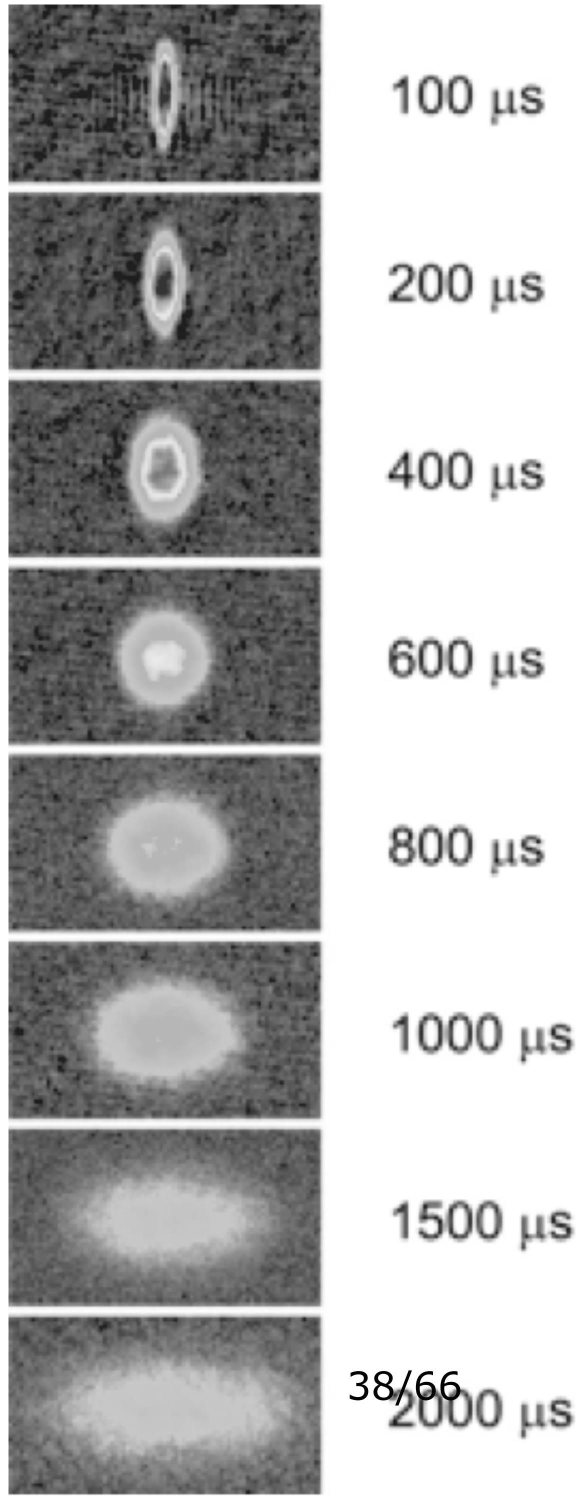
In hydrodynamic models the momentum anisotropy develops in the early (QGP)phase of the collision. Thermalization times of less then 1 fm/c are neededto describe the data.

# Cold atomic gases

200 000 Li-6 atoms in an highly anisotropic trap (aspect ratio 29:1)

Very strong interactions between atoms (Feshbach resonance)

Once the atoms are released the one observed a flow pattern similar to elliptic flow in heavy-ion collisions



# Lesson III

- **First results** from **ALICE** at LHC show large **increase** in **energy density** (**factor 2-3** compared to RHIC)

- **longer life-time** of qgp

- **larger collective flow** effects

- **anisotropic flow** comparable to **ultra-low viscosity**

- triangular flow sensitive to initial energy density fluctuations and viscosity/entropy ratio

- Hydrodynamical model provides framework to characeterize QGP, i.e. equation of state, viscosity/entropy ratio