Space-time evolution of the Quark Gluon Plasma

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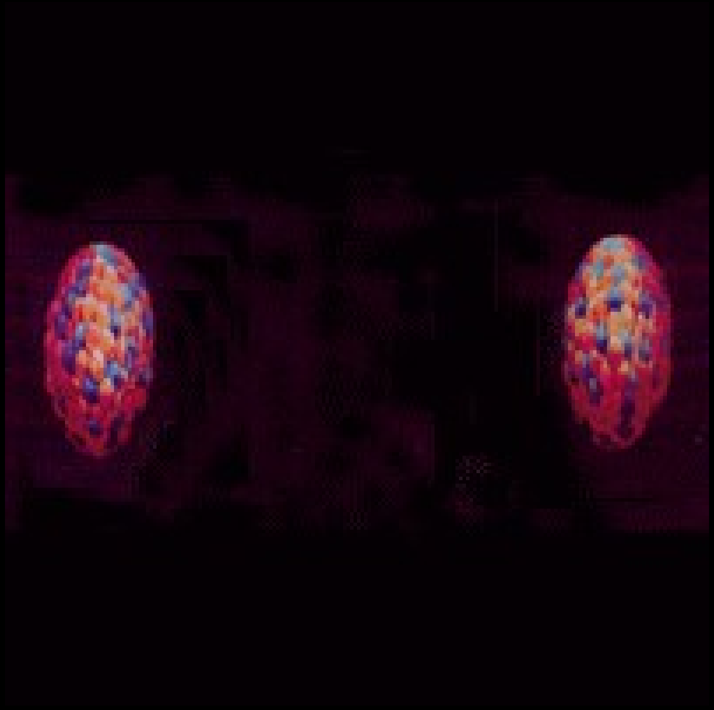
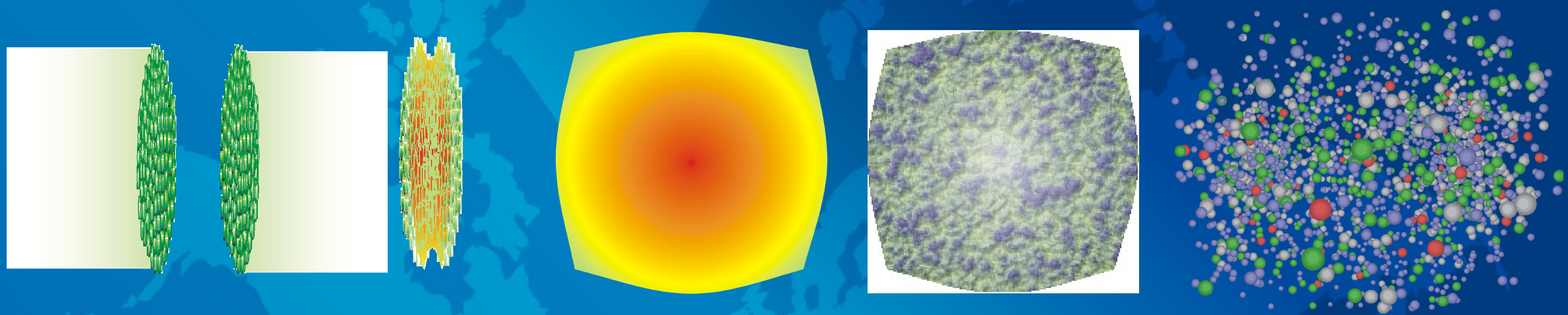
High-energy nucleus-nucleus Collisions



High-Energy Nuclear Collisions



Time →

Plot: Steffen A. Bass, Duke University

1) Initial condition:

-Baryon transfer

- E_T production

-Partonic dof

2) System evolves:

- parton/hadron expansion

3) Bulk freeze-out

- hadronic dof

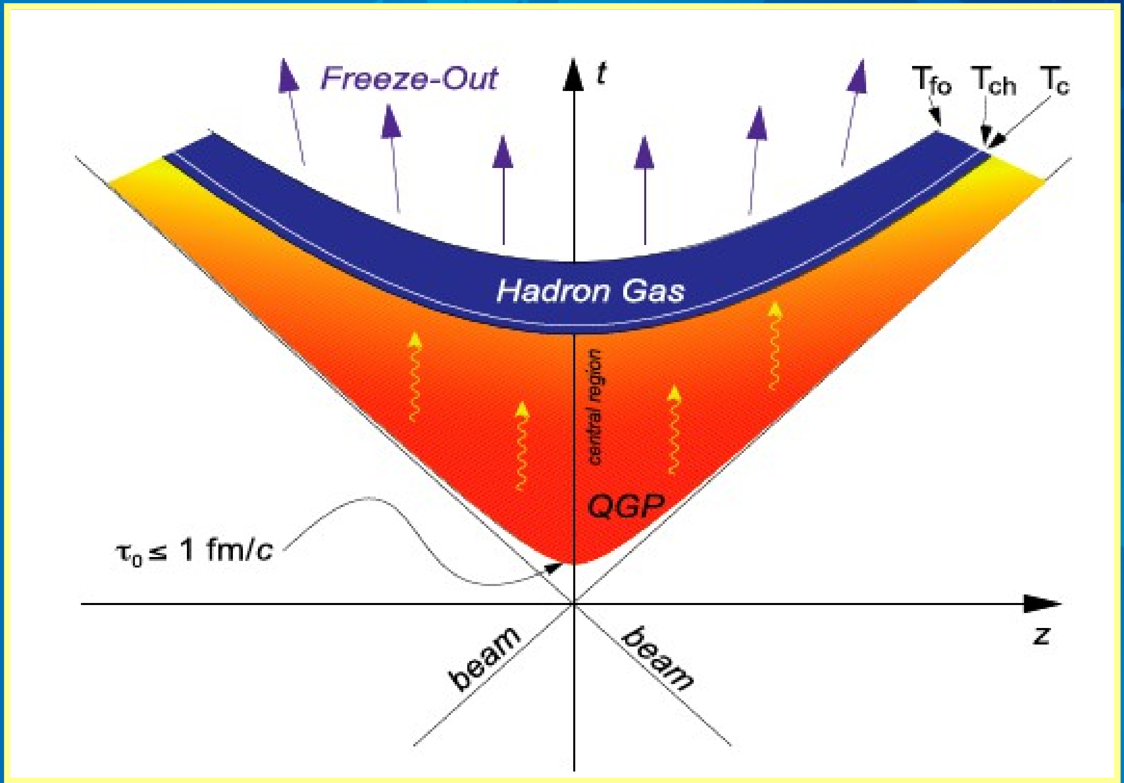
- inel. interactions cease:

particle ratios, T_{ch}, μ_B

- elas. interactions cease

Particle spectra, T_{th}, $\langle\beta_T\rangle$

Space-time evolution



Plot: courtesy of R. Stock.

- **QGP life time**
  10 fm/c ≈ $3 \cdot 10^{-23}$ s
- **thermalization time**
  0.2 fm/c ≈ $7 \cdot 10^{-25}$ s
  → hydrodynamical expansion until freeze-out
  simplest model: only longitudinal expansion, 1d
  → Bjorken model
- **collision time**
  $2R/\gamma$ = 0.005 fm/c
  ≈ $2 \cdot 10^{-26}$ s

Outline

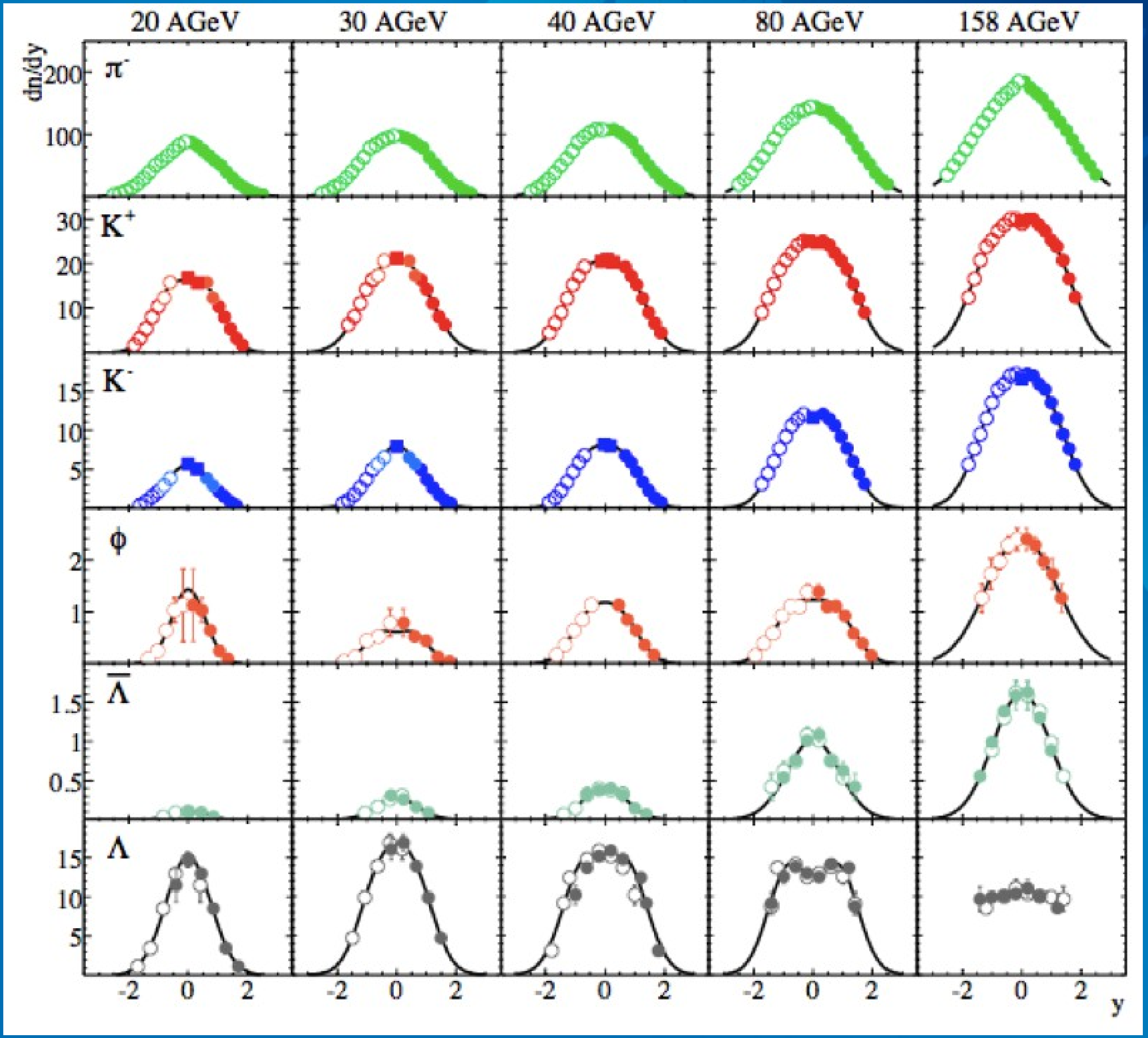
- Introduction
- Longitudinal expansion – Bjorken picture
- Transverse expansion

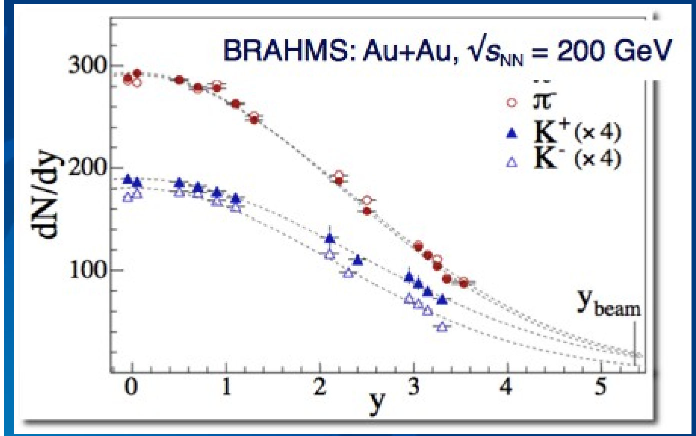
  - transverse radial flow

  - transverse elliptic flow v_2

  - higher harmonics, v_3, v_4, v_5, …
- Hydrodynamical model description
- Summary

Rapidity distribution in A-A

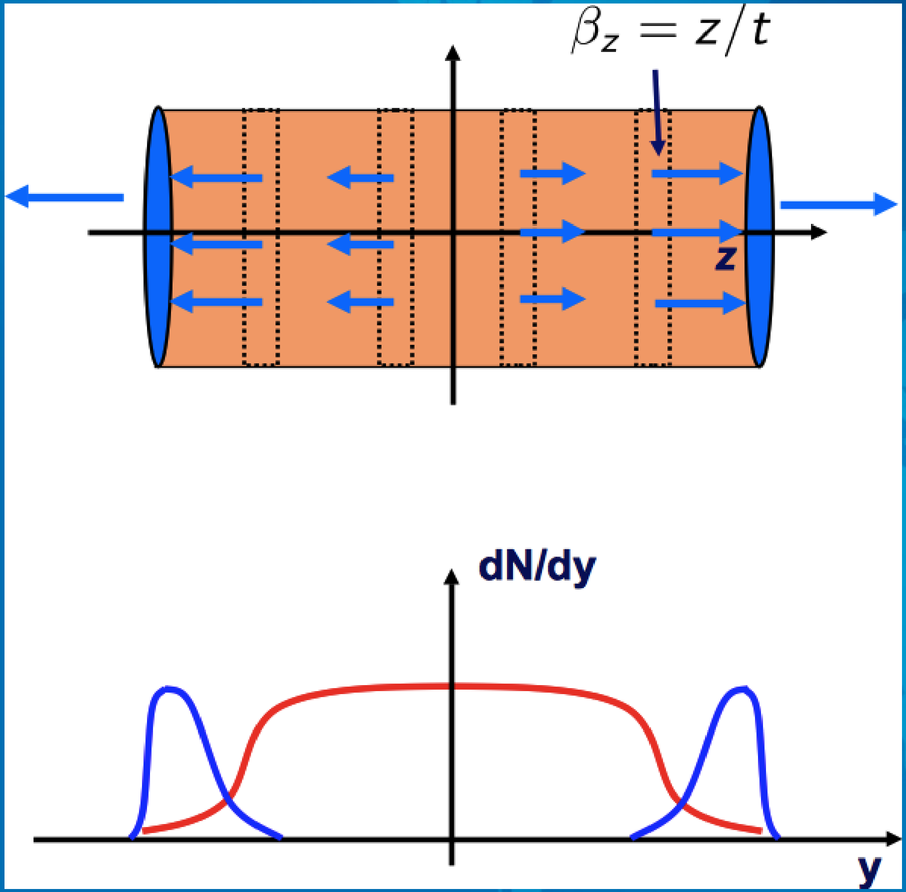




With increasing collision energy:

- wider distribution,
- becomes flatter around mid-rapidity

Bjorken model



Velocity of the local system at position z at time t:

$$\beta_z = z/t$$

Proper time τ in this system:

$$\tau = t/\gamma = t\sqrt{1-\beta^2}$$
$$= \sqrt{t^2 - z^2}$$

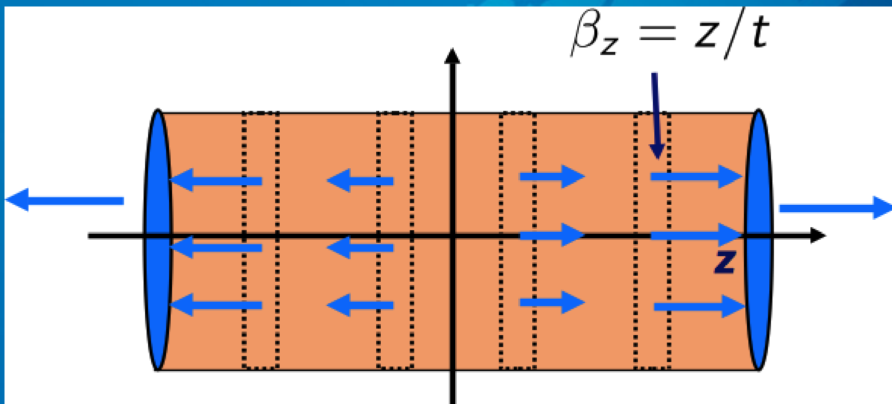
In the Bjorken model all thermodynamic quantities only depend on τ, e.g., the particle density:

$$n(t,z) = n(\tau)$$

This leads to a constant rapidity densityof the produced particles (at least atcentral rapidities):

$$\frac{dN_{ch}}{dy} = \text{const.}$$

1d - Bjorken model (I)



The 1D Bjorken model is based on the assumption that dNch/dy ist constant (around mid-rapidity).
This means that the central region is invariant under Lorentz transformation. This implies βz = z/t and that all thermodynamic quantities depend only on the proper time τ

Initial conditions in the Bjorken model:

$$\varepsilon(\tau_0) = \varepsilon_0, \quad u^\mu = \frac{1}{\tau_0}(t, 0, 0, z) = \frac{x^\mu}{\tau_0}$$

Initial energy density

In this case the equations of ideal hydrodynamics simplify to

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} + \frac{\varepsilon + p}{\tau} = 0$$

$\varepsilon = E/V$: energy density
p : pressure
$s = S/V$: entropy density

Bjorken model (II)

For an ideal gas of quarks and gluons, i.e., for

$$\varepsilon = 3p, \quad \varepsilon \propto T^4$$

This leads to

$$\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}, \quad T(\tau) = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}$$

The temperature drops to the critical temperature at the proper time

$$\tau_c = \tau_0 \left(\frac{T_0}{T_c}\right)^3$$

And thus the lifetime of the QGP in the Bjorken model is

$$\Delta\tau_{\mathrm{QGP}} = \tau_c - \tau_0 = \tau_0 \left[\left(\frac{T_0}{T_c}\right)^3 - 1\right]$$

QGP lifetime in 1d - Bjorken model

$$\varepsilon_0 = 11\,\mathrm{GeV/fm^3} = 11 \cdot 0.197^3\,\mathrm{GeV}^4 \quad \text{for } \tau_0 = 1\mathrm{fm}/c$$

$$1 = \hbar c = 0.197\,\mathrm{GeV} \cdot \mathrm{fm}$$

$$\varepsilon_0 = g_{\mathrm{QGP}} \frac{\pi^2}{30} T^4 \quad \rightarrow \quad T_0 = \left(\frac{30}{\pi^2} \frac{\varepsilon}{g} \right)^{1/4}$$

Parameters	$\Delta\tau_{QGP}$
$\varepsilon_0\, \tau$ = 3 GeV/fm2	0.84 fm/c
$\varepsilon_0\, \tau$ = 5 GeV/fm2	1.70 fm/c
$\varepsilon_0\, \tau$ = 11 GeV/fm2	3.9 fm/c

Fixed parameters: Nf = 2, Tc = 170 MeV, τ0 = 1 fm/c

Quick estimate for LHC

Bjorken formula: $$\varepsilon \cdot \tau_0 = \frac{\langle m_T \rangle}{A} \left.\frac{\mathrm{d}N}{\mathrm{d}y}\right|_{y=0}$$

Transverse area in collisions with $b \approx 0$:

$$A \approx \pi R_{\mathrm{Pb}}^2 = \pi (6.62\,\mathrm{fm})^2 \approx 140\,\mathrm{fm}^2$$

Estimate for the mean transverse momentum:

$$\langle p_T \rangle = 0.66\,\mathrm{GeV}/c \rightsquigarrow \langle m_T \rangle \approx \sqrt{(0.138\,\mathrm{GeV})^2 + (0.66\,\mathrm{GeV})^2} = 0.67\,\mathrm{GeV}$$

Measured charged particle multiplicity:

$$dN_{ch}/d\eta \approx 1601 \pm 60 \quad (5\%\ \text{most central})$$

$$\rightsquigarrow \quad \left.\frac{\mathrm{d}N}{\mathrm{d}y}\right|_{y=0} = \frac{3}{2} \cdot \underbrace{\left(1 - \frac{m^2}{\langle m_T \rangle}\right)^{-1/2}}_{1.02} \cdot \left.\frac{\mathrm{d}N_{ch}}{\mathrm{d}\eta}\right|_{\eta=0} = 2450 \pm 92$$

Larger (up to ≈ 1.2) if p and K are taken into account

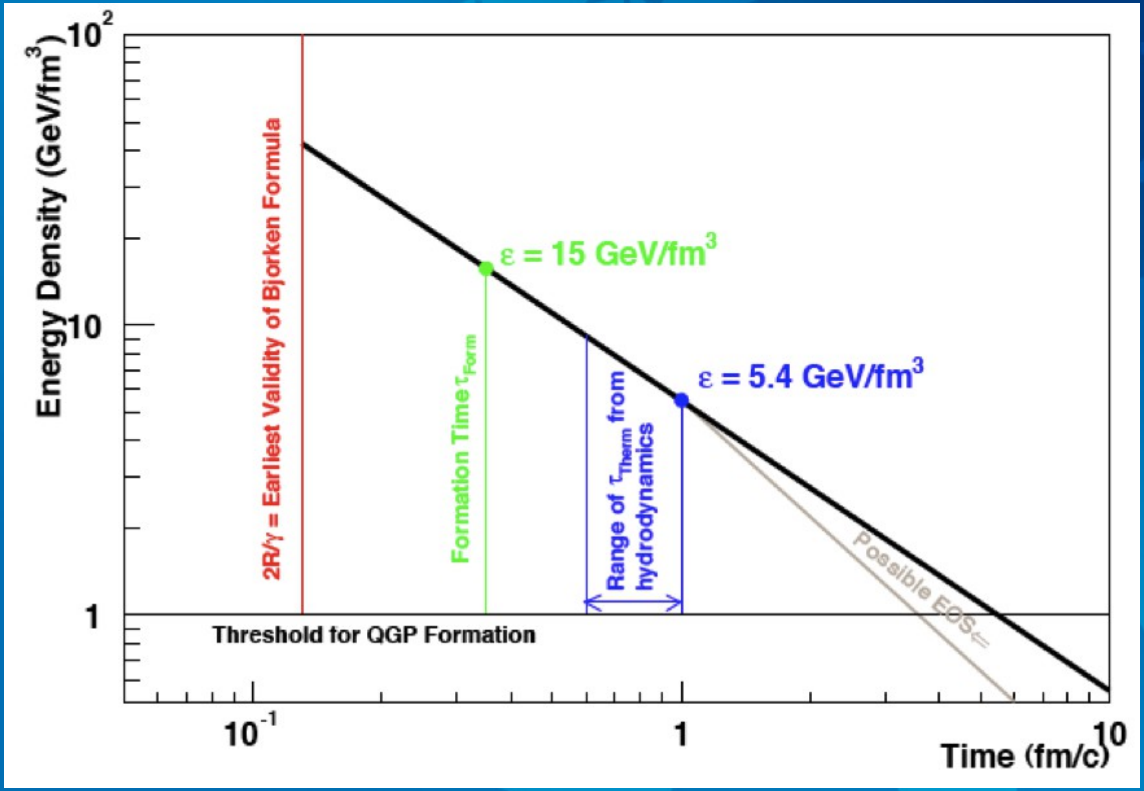
LHC:

$$\varepsilon \cdot \tau_0 = (11.7 \pm 0.43)\,\mathrm{GeV}/\mathrm{fm}^2 \quad (\mathrm{Pb{+}Pb@}\sqrt{s_{NN}} = 2.76\,\mathrm{TeV})$$

RHIC:

$$\varepsilon \cdot \tau_0 \approx 5\,\mathrm{GeV}/\mathrm{fm}^2 \quad (\mathrm{Au{+}Au@}\sqrt{s_{NN}} = 0.2\,\mathrm{TeV})$$

Energy density evolution in 1d-Bjorken

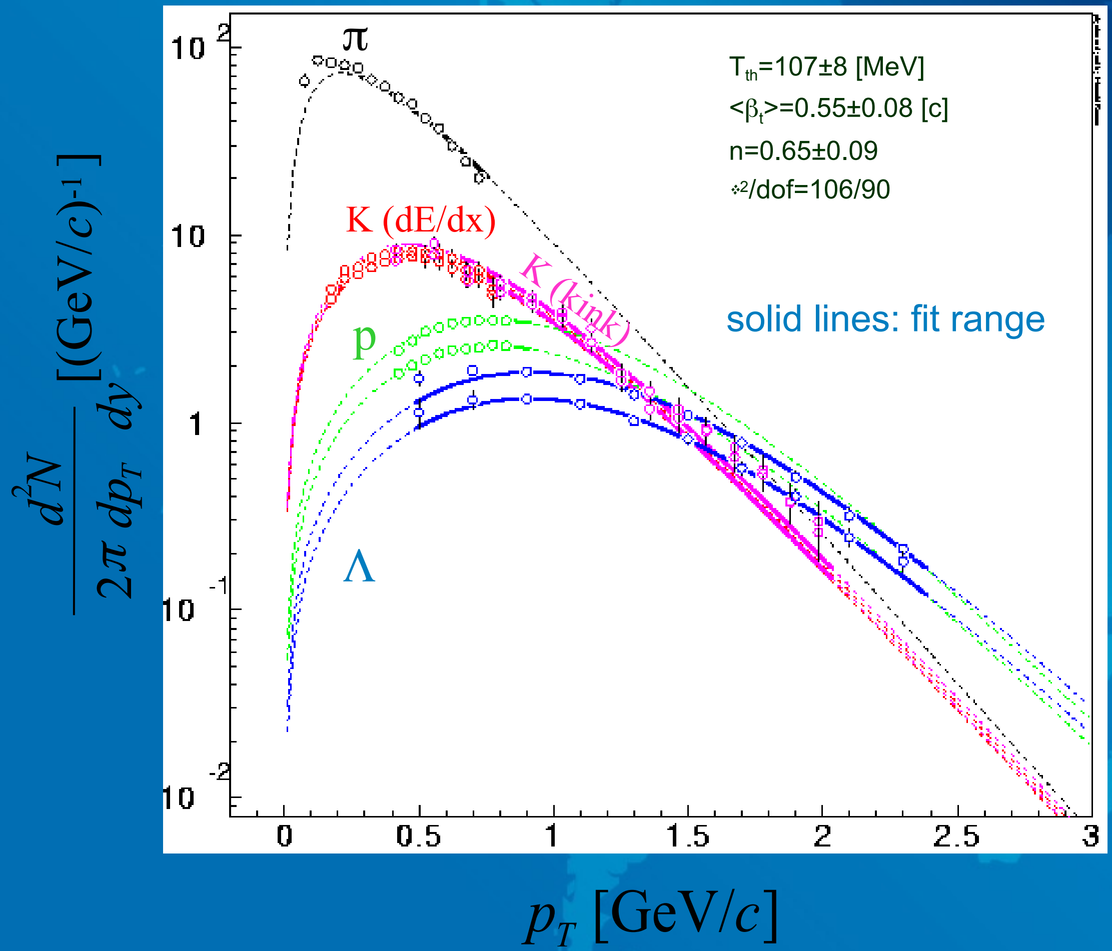


τ_0 = 1 fm/*c* is generally considered as a conservative estimate for the use in the Bjorken formula.
Other estimates yields shorter times (e.g. τ_0 = 0.35 fm/*c*) resulting in initial energy densities at RHIC of up to 15 GeV/fm^3

Transverse Expansion

Transverse radial flow: particle spectra

Particle Spectra*



*Au+Au @130 GeV, STAR

- Typical **mass ordering** in inverse slope from **light** π to **heavier** Λ
- Two-parameter fit describes yields of π, K, p, Λ
- T_{th} = 90 ± 10 MeV
- <β_t> = 0.55 ± 0.08 c

⇒ Disentangle **collective motion** from thermal random walk

Thermal Model + Radial Flow Fit

Source: each volume element is assumed to be

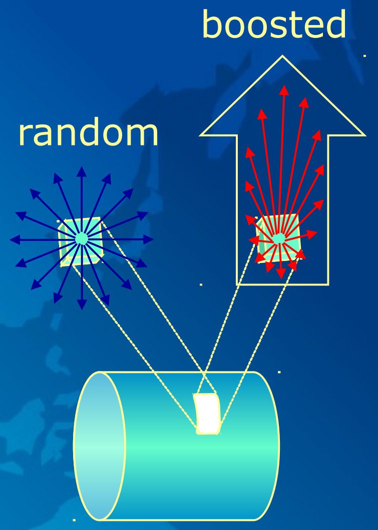
- in local **thermal equilibrium**: T_{fo}
- **boosted** in transverse radial direction: $\rho = f(\beta_s)$

E.Schnedermann, J.Sollfrank, and U.Heinz, Phys. Rev. C48, 2462(1993).

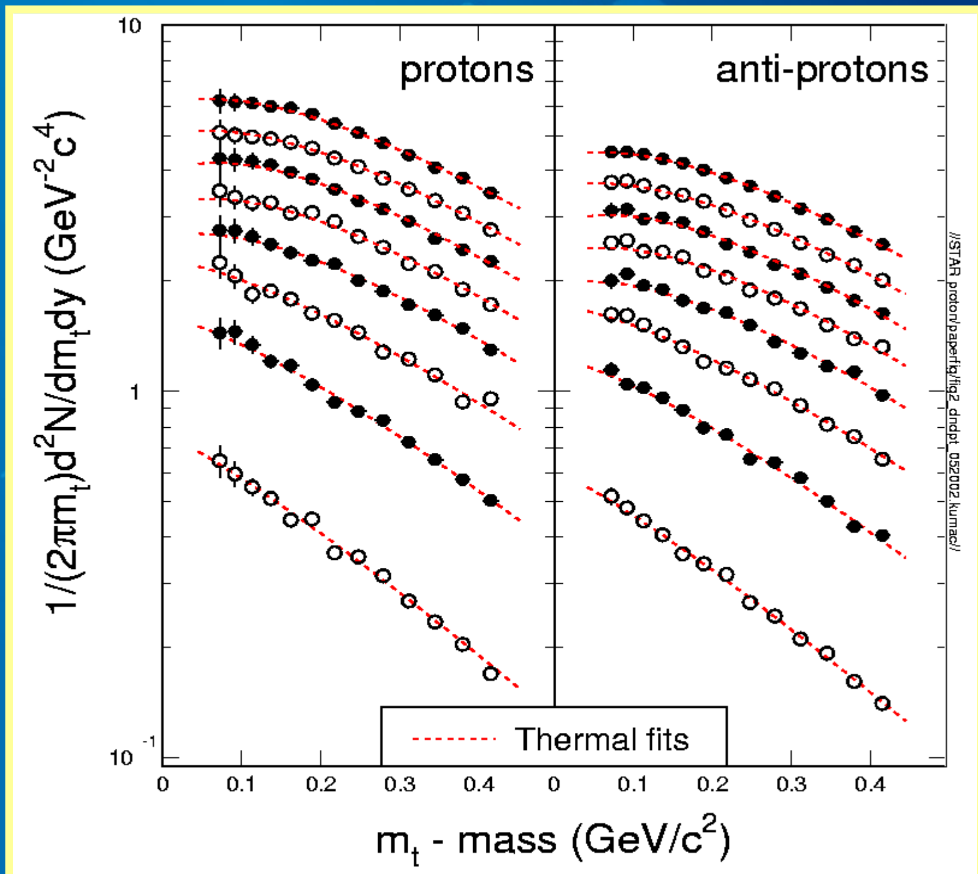
$$E\frac{d^3N}{dp^3} \propto \int_\sigma e^{-(u^\mu p_\mu)/T_{fo}} p d\sigma_\mu \Rightarrow$$

$$\frac{dN}{m_T dm_T} \propto \int_0^R r dr m_T K_1\left(\frac{m_T \cosh\rho}{T_{fo}}\right) I_0\left(\frac{p_T \sinh\rho}{T_{fo}}\right)$$

$$\rho = \tanh^{-1}\beta_T \qquad \beta_T = \beta_S\left(\frac{r}{R}\right)^\alpha \qquad \alpha = 0.5,\ 1,\ 2$$



(anti-)Protons From RHIC Au+Au@130GeV



$$m_T = \sqrt{p_T^2 + mass^2}$$

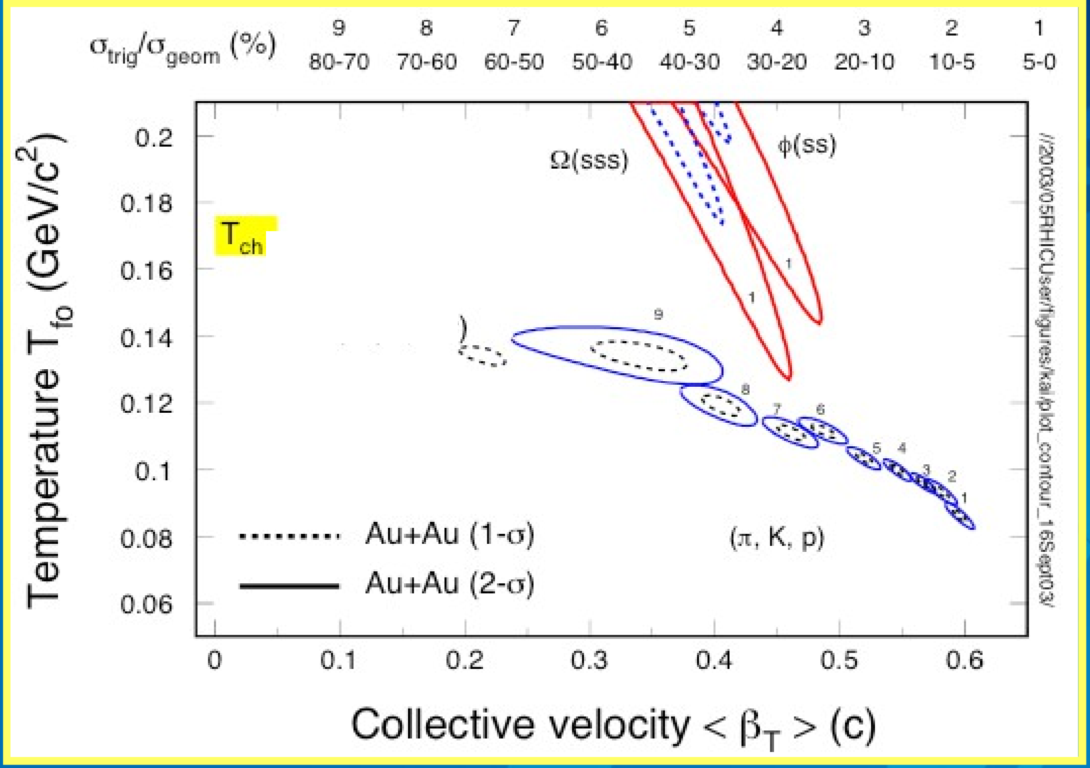
Centrality dependence:

- **spectra** at low momentum de-populated, become **flatter** at larger momentum

ü stronger **collective flow** in more central collisions, $\langle \beta_t \rangle$ = 0.55 ± 0.08

STAR: Phys. Rev. C70, 041901(R).

Kinetic Freeze-out at RHIC



STAR Data: Nucl. Phys. A757, (2005 102),

*A. Baran, W. Broniowski and W. Florkowski, Acta. Phys. Polon. B 35 (2004) 779.

0) T and β_T are anti-correlated

1) Multi-**strange hadrons** φ and Ω **freeze-out earlier** than (π, K, p)

ü Collectivity prior to hadronization

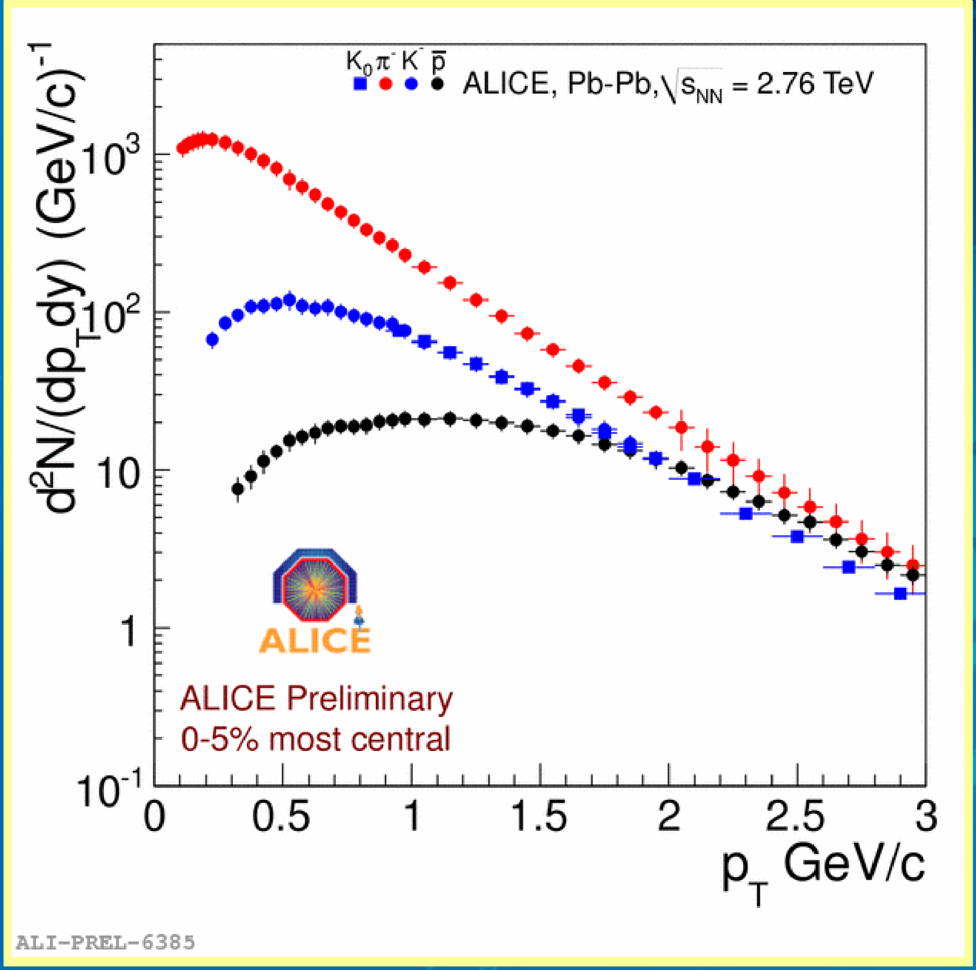
2) Sudden single freeze-out*:

Resonance decays lower T_{fo} for (π, K, p)

ü **Collectivity prior** to **hadronization**

ü **Partonic Collectivity ?**

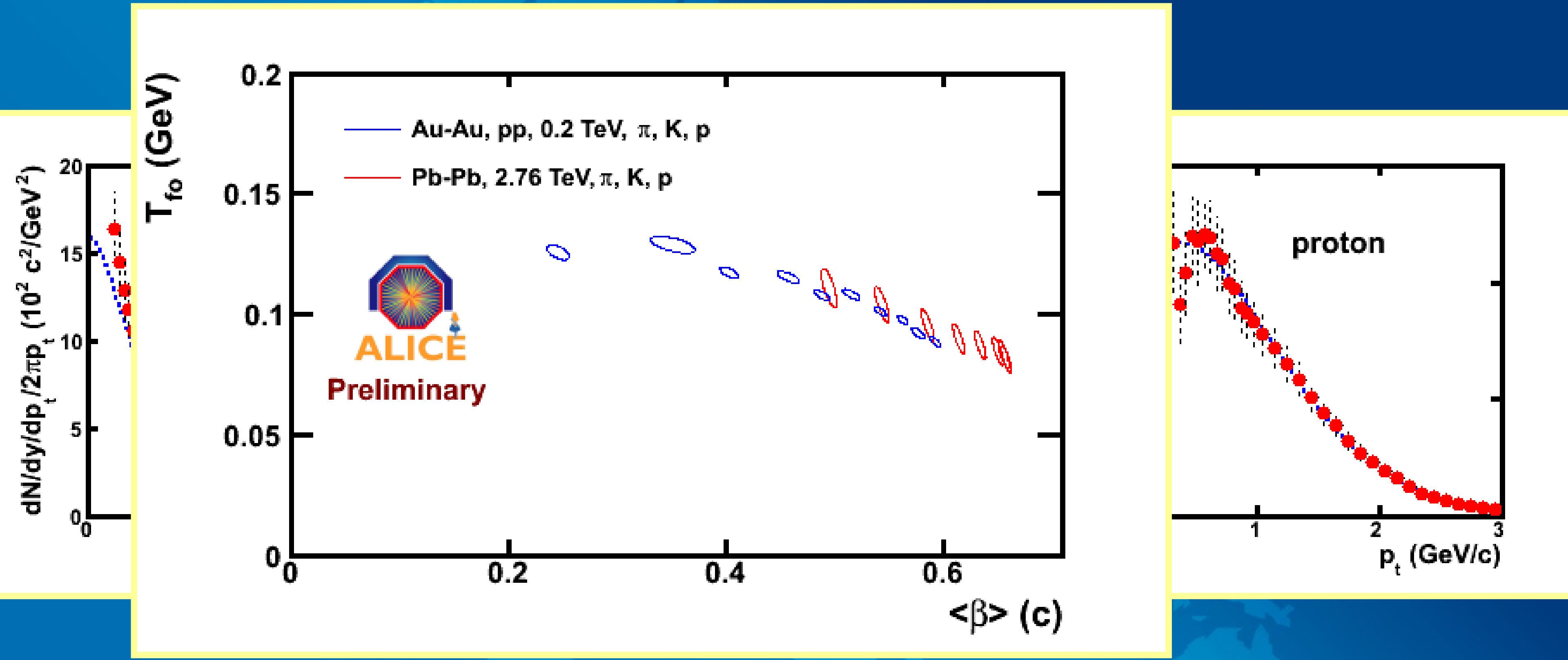
LHC: Identified particle spectra



Spectra harder at LHC

⇒**stronger** collective **flow** at LHC than at RHIC

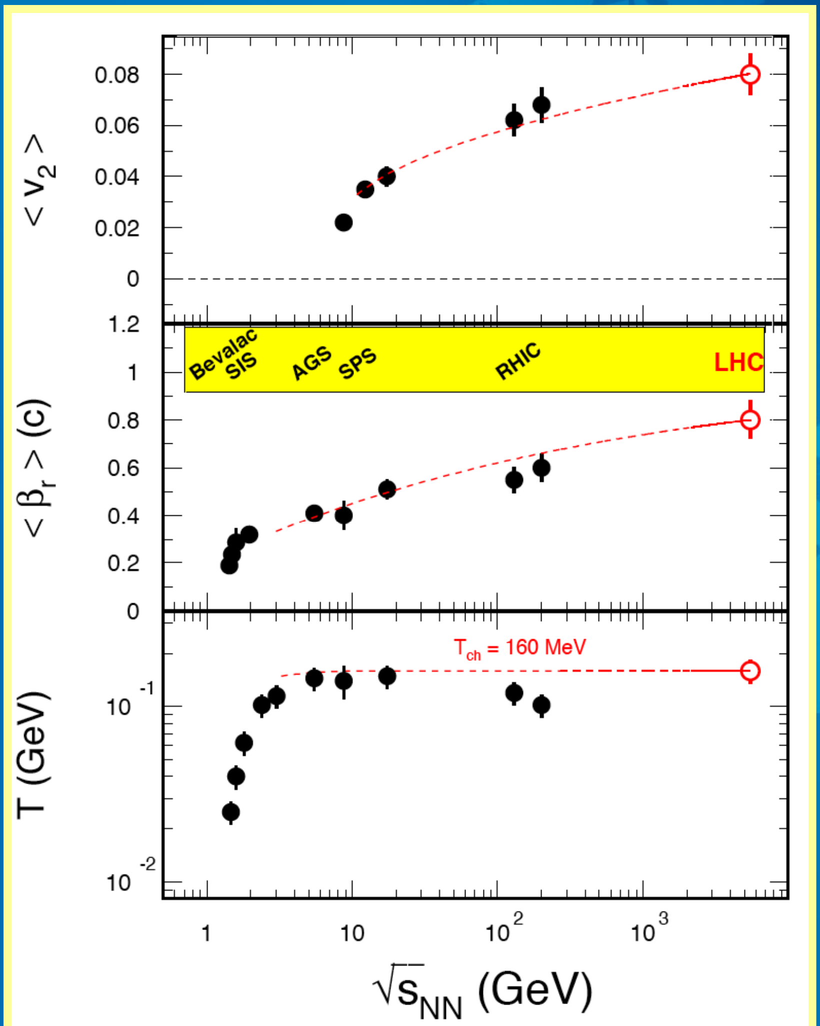
Collective expansion



Blast wave parametrization **describes spectra** at **10% level**

Collective **flow velocity increases** from RHIC to LHC by **10%**

Collective Flow - Energy Dependence



K.S., ISMD07, arXiv:0801.1436 [nucl-ex].

Collectivity parameters $<\beta_T>$ and $<v_2>$ **increase** with **collision energy**

strong collective **expansion** at RHIC !

$<\beta_T>$RHIC » 0.6

expected **strong partonic expansion** at **LHC**,

$<\beta_T>$LHC ü 0.8, T_{fo} ü T_{ch}

Lesson I

- At LHC, Initial energy density approx. 50 GeV/fm^3

- much larger than critical energy density ε_c = 0.7 GeV/fm^3

- Strong **collective expansion**, $\langle\beta_T\rangle$ = 0.6 – 0.7

  at **highest collider energies**

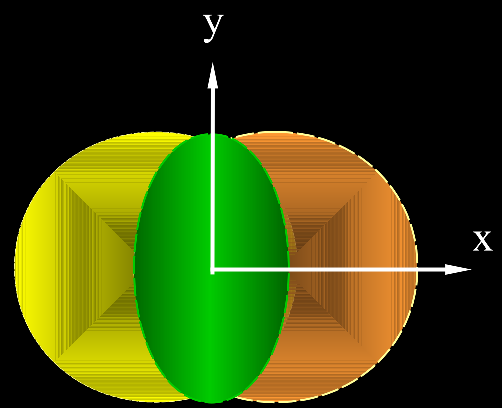
- Particles carrying **strange quarks** show that **collective expansion** develops **before hadronization**, $\langle\beta_T\rangle$ = 0.3 - 0.4 – among quarks and gluons (?)

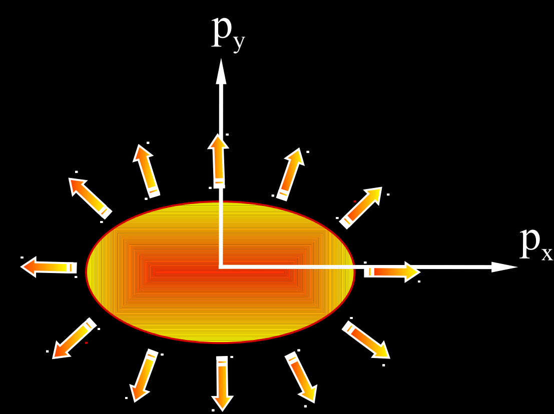
Transverse Expansion

Transverse elliptic flow: event anisotropy

Anisotropy Parameter v_2

coordinate-space-anisotropy ü momentum-space-anisotropy

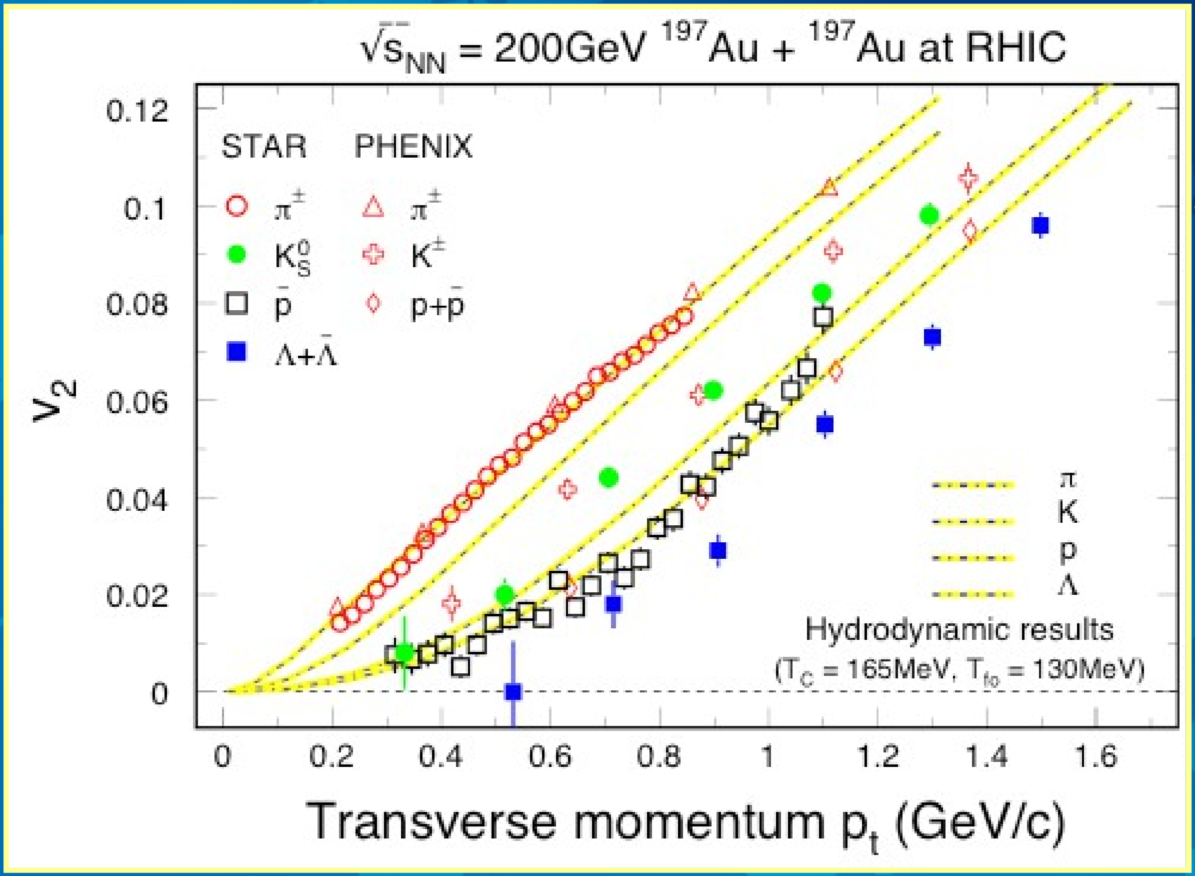




$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \qquad v_2 = \langle \cos 2\varphi \rangle, \;\; \varphi = \tan^{-1}(\frac{p_y}{p_x})$$

Initial/final conditions, EoS, degrees of freedom

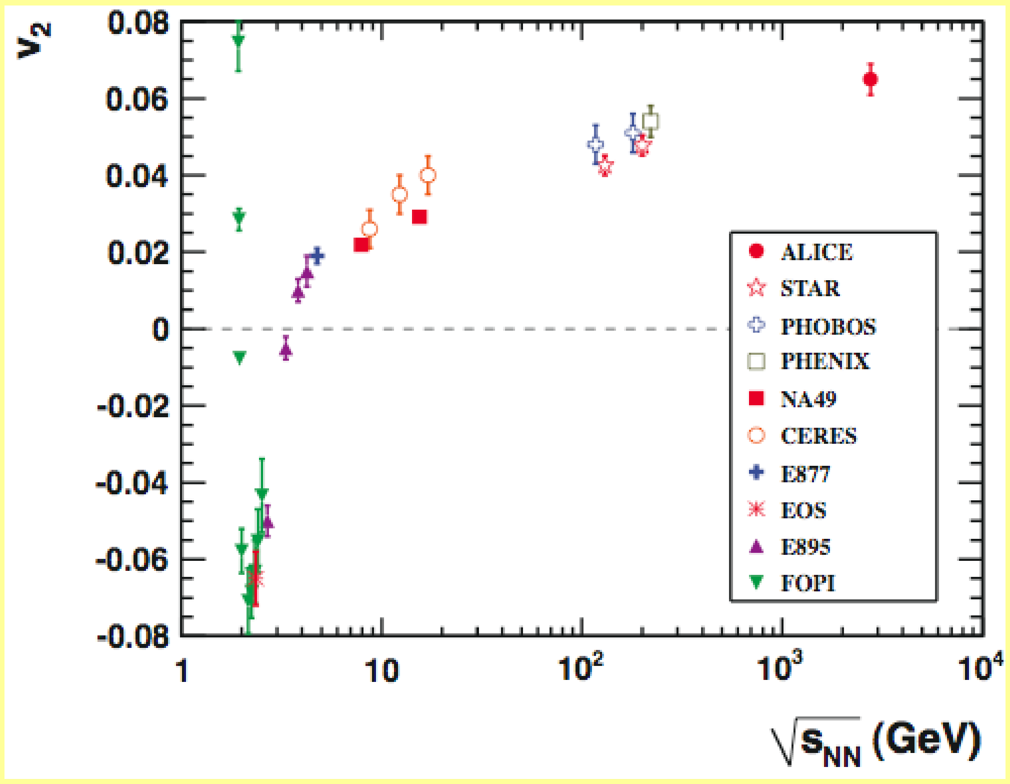
v_2 in the low-p_T Region



P. Huovinen, private communications, 2004

- v_2 approx. linear in p_T, mass ordering from light π to heavier Λ
- characteristic of hydrodynamic flow, sensitive to EOS !

Elliptic flow in ALICE



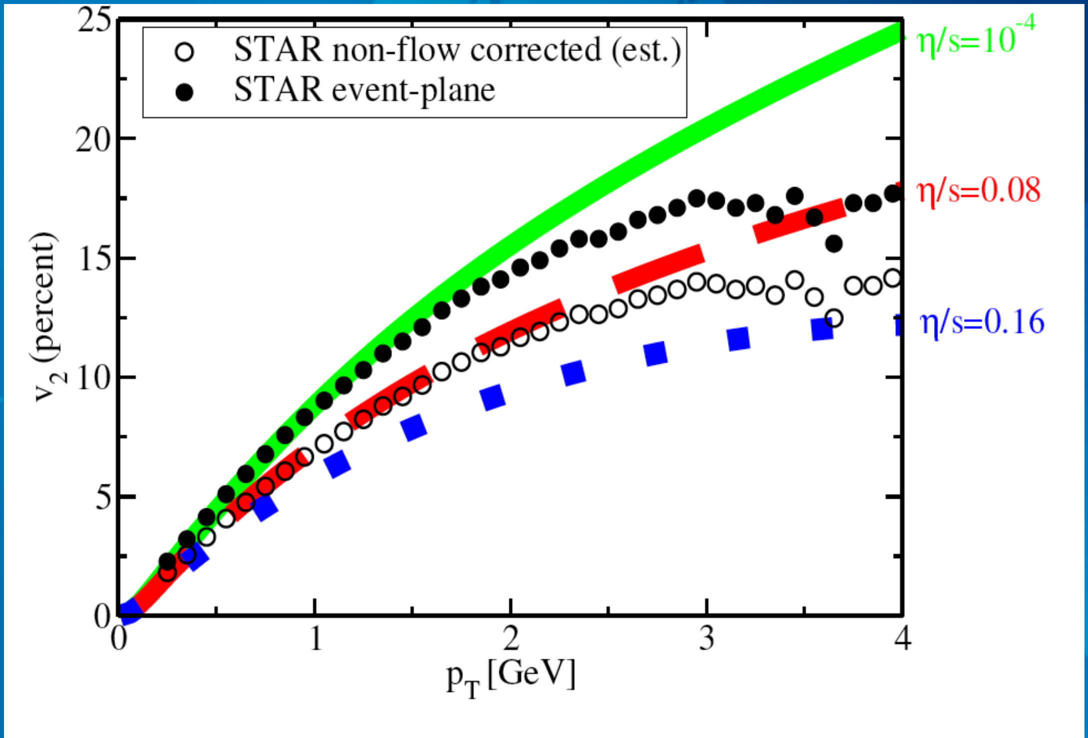
ALICE, submitted for publication, arXiv:1011.3914 [nucl-ex].

$\sqrt{s_{NN}}$ > ~ 4 GeV:initial excentricity leads to pressure gradients that cause positive v_2

2 < $\sqrt{s_{NN}}$ < 4 GeV:velocity of the nuclei is small so that presence of spectator matter inhibits in-plane particle emission (“squeeze-out”)

$\sqrt{s_{NN}}$ < 2 GeV:rotation of the collision system leads to fragments being emitted in-plane

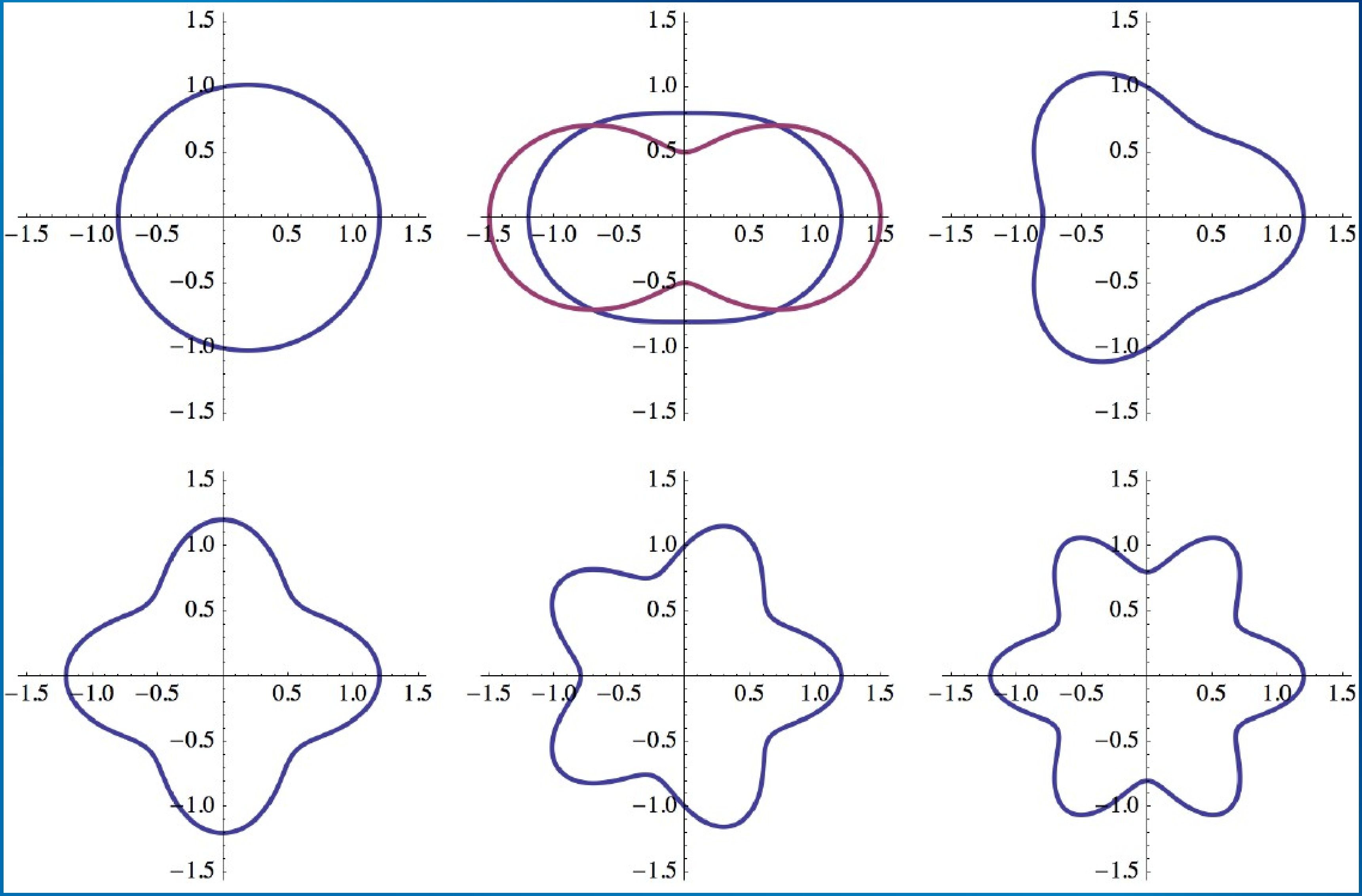
Non-ideal Hydro-dynamics



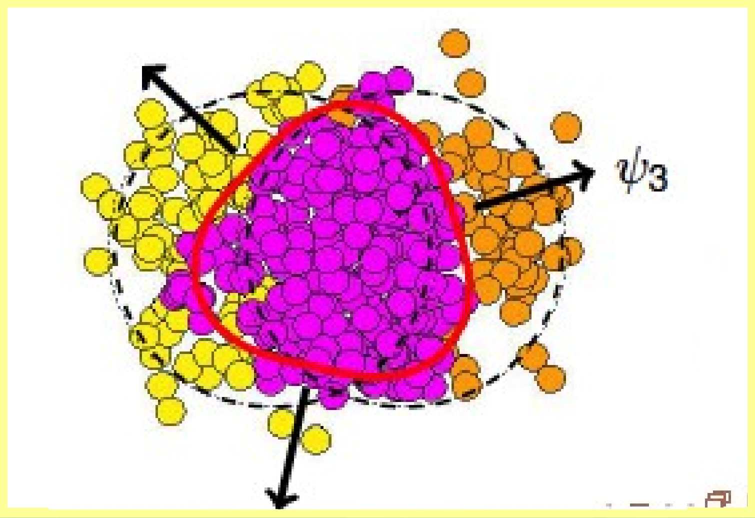
M.Luzum and R. Romatschke, PRC 78 034915 (2008); P. Romatschke, arXiv:0902.3663.

- Spectra and flow reproduced by ideal hydrodynamics calcs.
- Shear viscosity to entropy density ratio close to AdS/CFT bound
- viscosity leads to decrease in v_2, ultra-low viscosity sufficient to describe data
- Hydro-limit exceeded at LHC ?

$$f(\varphi) = 1 + 2v_n \cos(n\varphi)$$



Can there be v_3 ?



figs.: courtesy of M. Luzum.

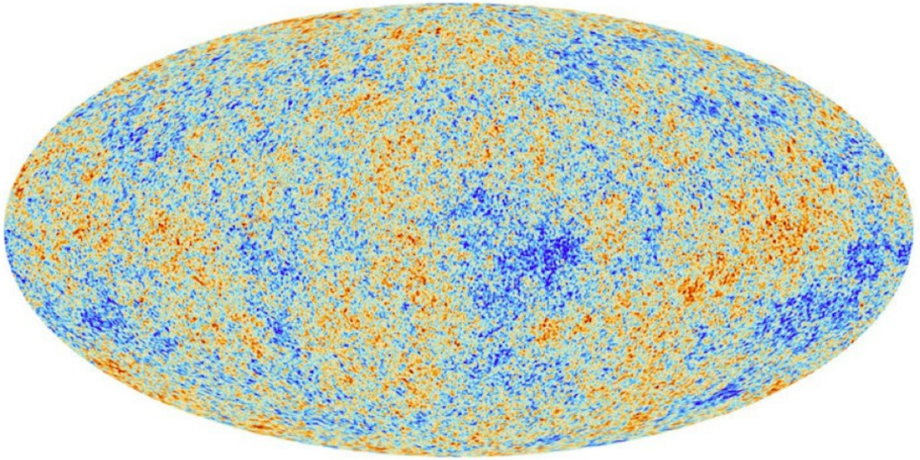
- reaction plane ü participant plane
- fluctuating initial state is seed for v_3

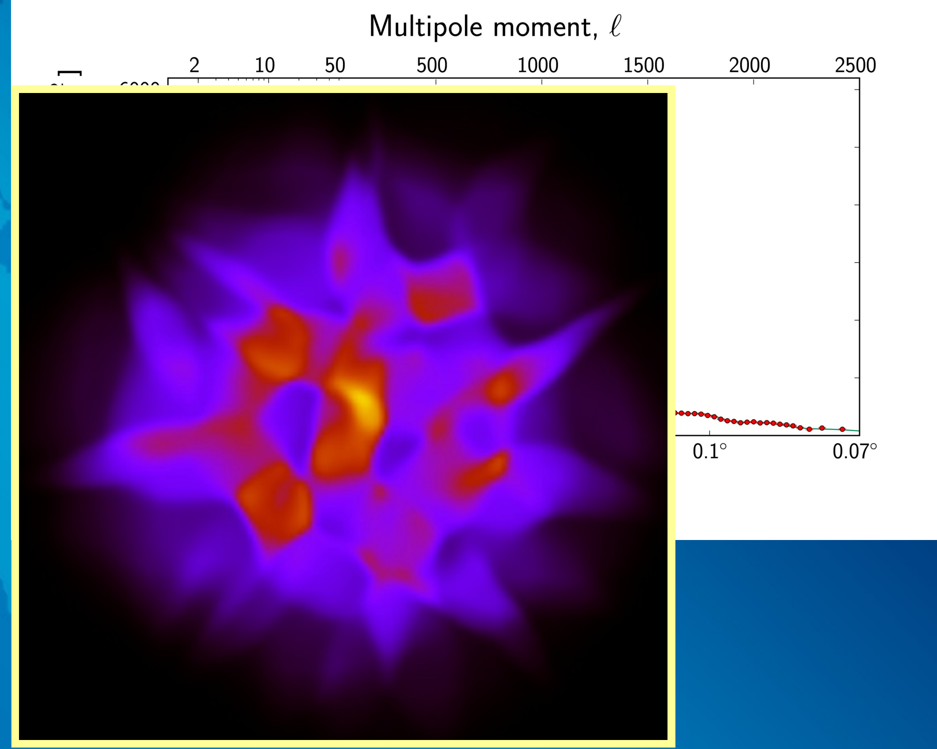
ü can CGC be challenged ?

higher harmonics

- extract power spectrum of v_n, like Planck*
- higher harmonics
- odd harmonics important
- v_3: access η/s
- Higher harmonics strongly damped ($v_{n,\, n>10} = 0$)

STAR, arXiv:1301.2187 [nucl-ex];
STAR, PRL 92 (2004) 062301;
A.Mocsy and P. Sorensen, NPA 855 (2011) 241;
B. Alver and G. Roland, PRC 81 (2010) 054904;
Planck data: EAS and the Planck collaboration
QGP plot: B. Schenke, S. Jeon, and C. Gale, arXiV:1109.6289.





Lesson II

- Geometrical anisotropy is seed for elliptic flow v_2
- Elliptic flow v_2 sensitive to QGP equation of state
- Triangular flow v_3 due to fluctuations, e.g. in initial energy density
- Triangular flow v_3 especially sensitive to shear viscosity /entropy density ratio
- Higher harmonics v_4, v_5, ... strongly damped

Hydrodynamical model description

Some basic concepts

Relativistic Hydrodynamics (I)

The energy-momentum tensor $T^{\mu\upsilon}$ is the four-momentum component in the μ direction per three-dimensional surface area perpendicular to the ν direction.

$$\Delta \mathbf{p} = (\Delta E, \Delta p_x, \Delta p_y, \Delta p_z)$$
$$\Delta \mathbf{x} = (\Delta t, \Delta x, \Delta y, \Delta z)$$

$$\mu = \nu = 0 : \quad T_R^{00} = \frac{\Delta E}{\Delta x \Delta y \Delta z} = \frac{\Delta E}{\Delta V} = \varepsilon$$

$$\mu = \nu = 1 : \quad T_R^{11} = \frac{\Delta p_x}{\Delta t \Delta y \Delta z}$$

force in *x* direction acting on an surface Δ*y* Δ*z* perpendicular to the force → pressure

$$T^{\mu\nu} = \begin{pmatrix} \text{energy density} & \text{energy flux density} \\ \text{momentum density} & \text{momentum flux density} \end{pmatrix} = \begin{pmatrix} \varepsilon & \vec{j}_\varepsilon \\ \vec{g} & \Pi \end{pmatrix}$$

Relativistic Hydrodynamics (II)

Isotropy in the fluid rest implies that

the energy flux T^{0j} and the momentum density T^{j0} vanish

and that $\Pi^{ij} = P\,\delta_{ij}$

$$T_R^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Off-diagonal elements ≠ 0 in case of viscous hydrodynamics, not considered here

→ ideal (perfect) fluid.

See also Ollitrault, arXiv:0708.2433.

Relativistic Hydrodynamics (III)

Energy-momentum tensor (in case of local thermalization) after Lorentz transformationto the lab frame:

$$T^{\mu\nu} = (\varepsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$$ metric tensor diag(1,-1,-1,-1)

Energy density and pressure in the co-moving system

4-velocity: $u^\mu = \mathrm{d}x^\mu / \mathrm{d}\tau = \gamma(1, \vec{v})$

Energy and momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0, \quad \nu = 0, \ldots, 3$$

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

in components:

$$\frac{\partial}{\partial t}\varepsilon + \vec{\nabla}\vec{j}_\varepsilon = 0 \;\; \text{(energy conservation)}$$

$$\frac{\partial}{\partial t} g_i + \nabla_j \Pi_{ij} = 0 \;\; \text{(momentum conservation)}$$

Conserved quantities, e.g., baryon number:

$$j^\mu_B(x) = n_B(x)\, u^\mu(x), \qquad \partial_\mu j^\mu_B(x) = 0 \;\; \Leftrightarrow \;\; \frac{\partial}{\partial t} N_\mathrm{B} + \vec{\nabla}(N_\mathrm{B}\vec{v}) = 0$$

continuity equation

$$N_\mathrm{B} = \gamma n_\mathrm{B}$$

Ingredients of Hydro - models

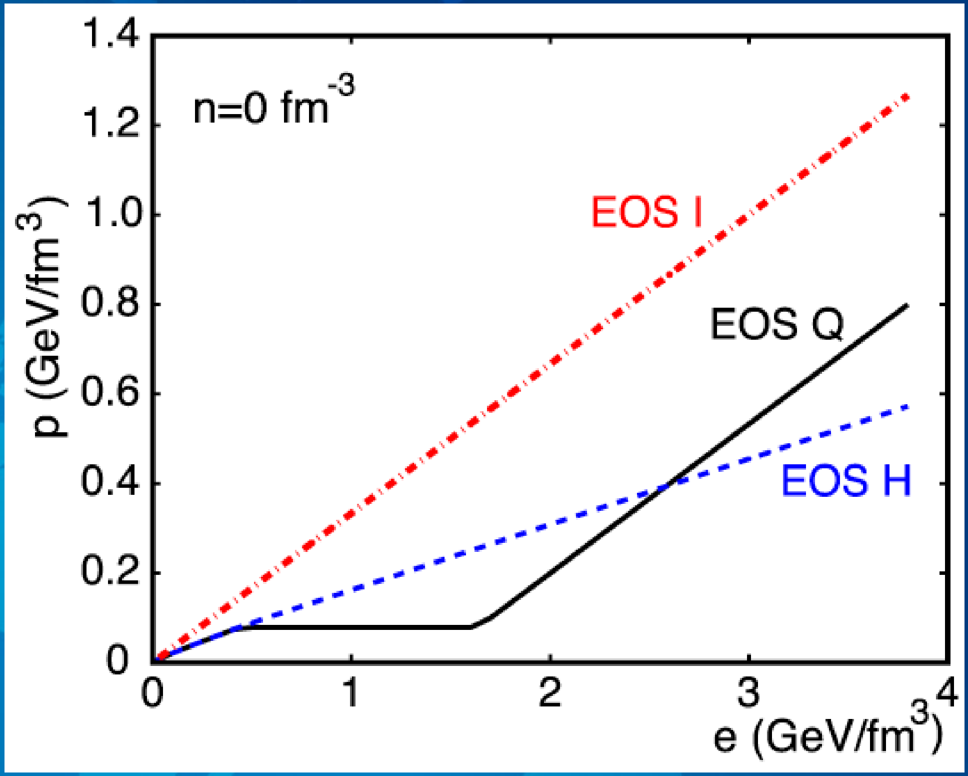
- Equation of motion and baryon number conservation:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu_{\rm B}(x) = 0$$

- 5 equations for 6 unknowns:

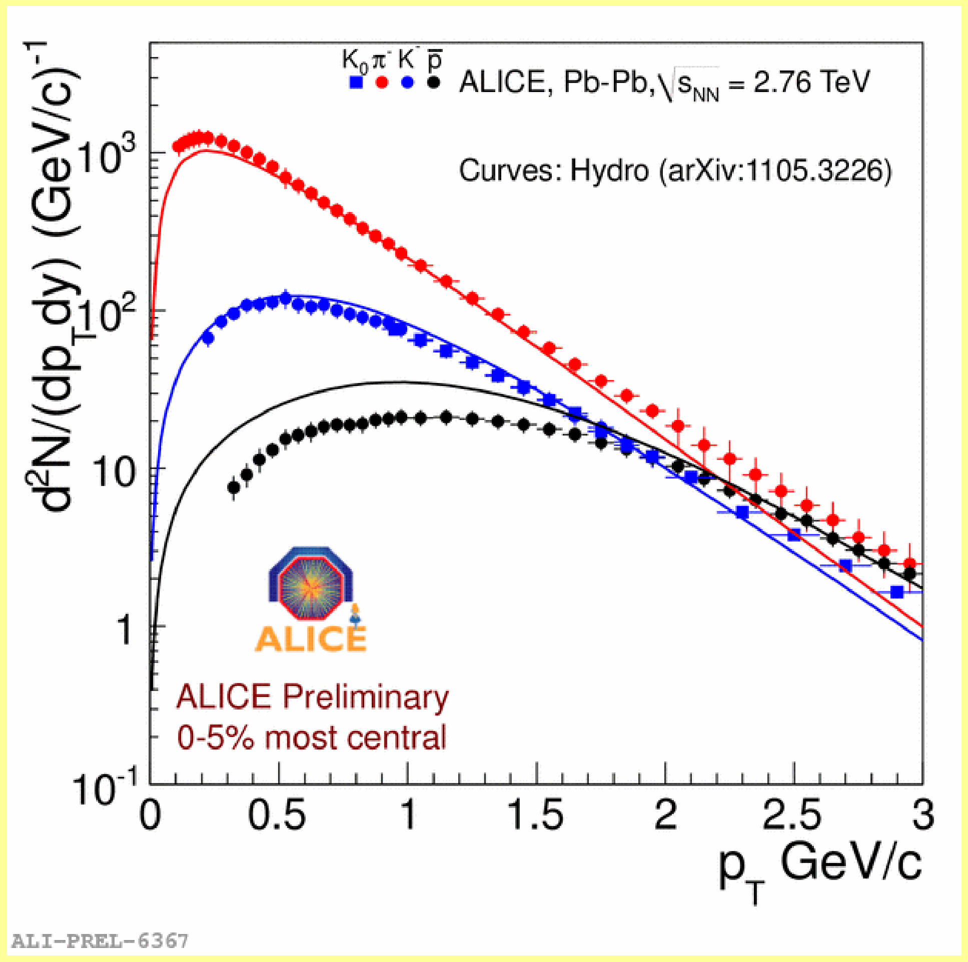
$$(u_x, u_y, u_z, \varepsilon, P, n_{\rm B})$$

- Equation of state: $P(\varepsilon, n_{\rm B})$
- (needed to close the system)
- Initial conditions, e.g., from Glauber calculation
- Freeze-out condition



EOS I: ultra-relativistic gas $P = \varepsilon/3$
EOS H: resonance gas, $P \approx 0.15\ \varepsilon$
EOS Q: phase transition, QGP resonance gas

LHC: Identified particle spectra

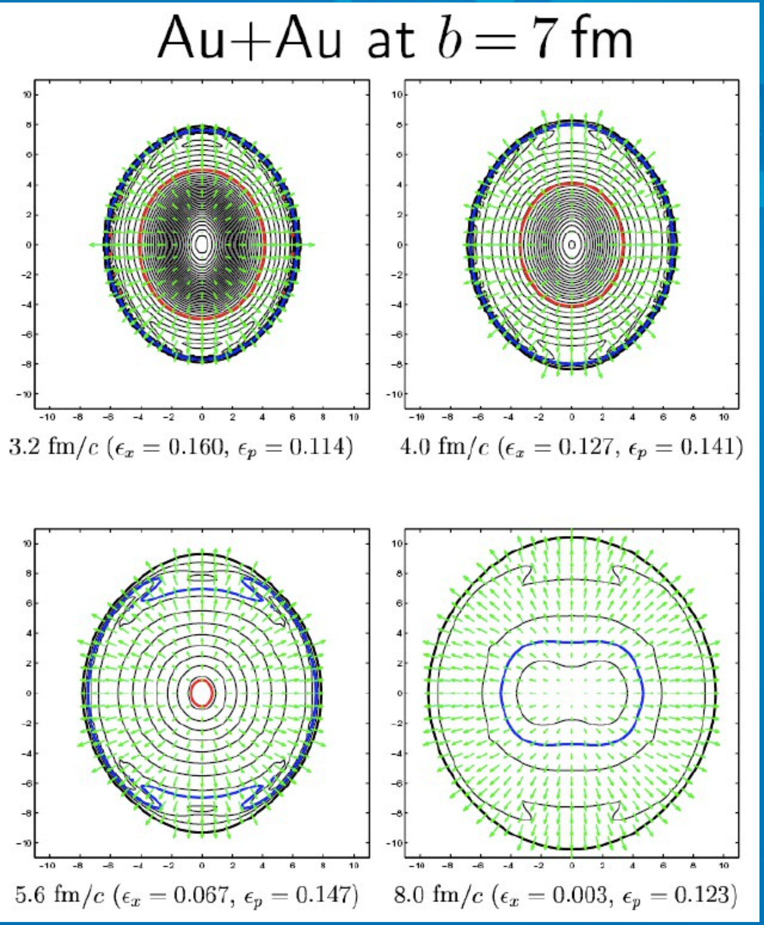


Initial conditions fixed by pion abundance

Protons overestimated

Annihilation of protons and anti-protons in the hadron phase ?

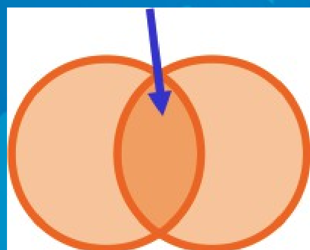
Elliptic flow in Hydro - models



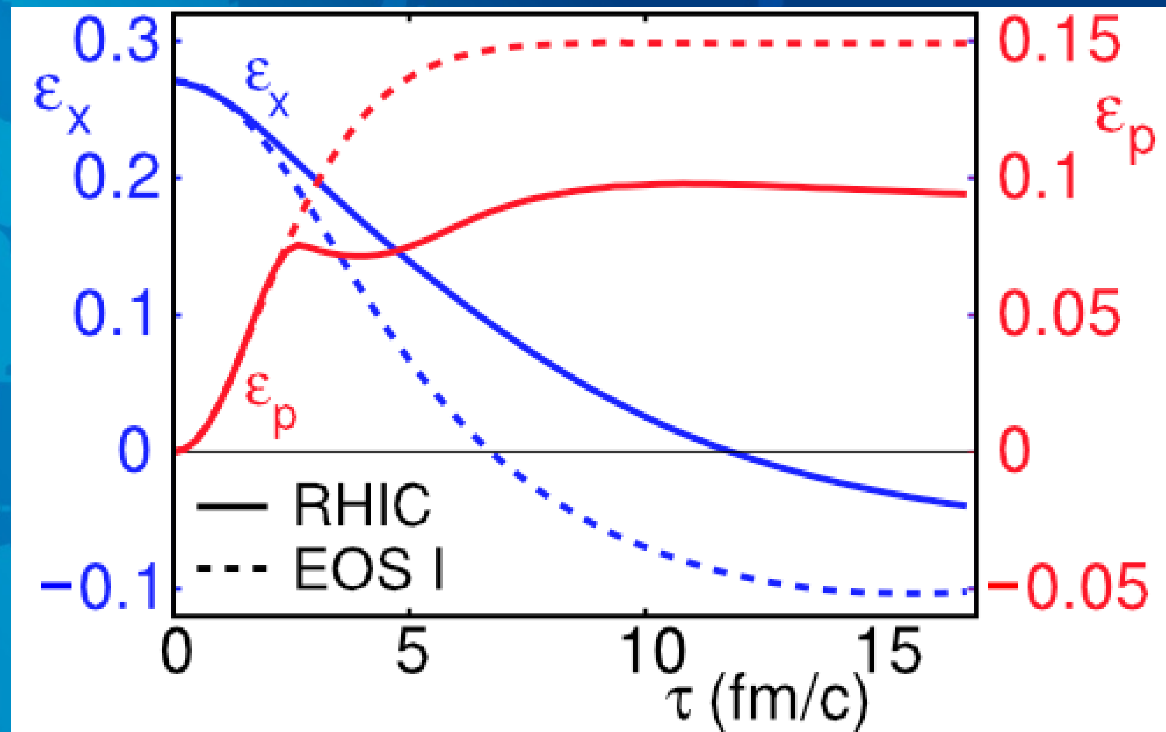
Elliptic flow is “self-quenching”:The cause of elliptic flow, the initial spacial anisotropy, decreases as the momentum anisotropy increases

Anisotropy in momentum space

Anisotropy in coordinate space



Anisotropy in momentum space



Ulrich Heinz, Peter Kolb, arXiv:nucl-th/0305084

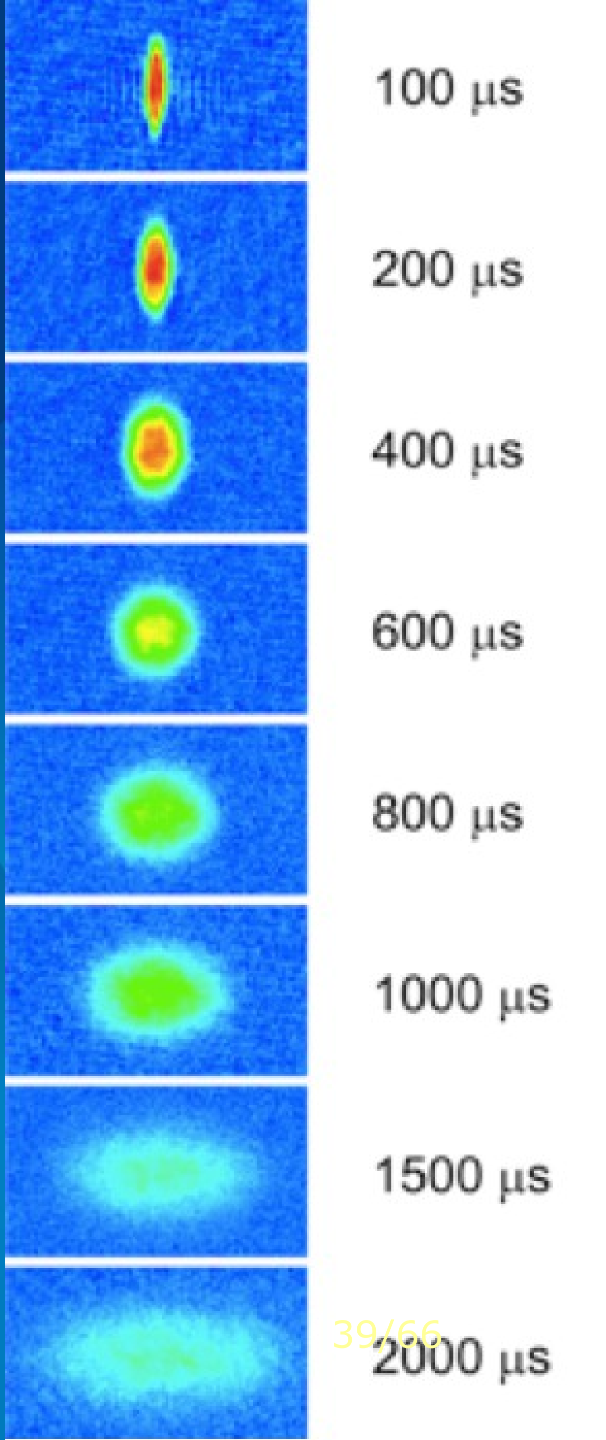
In hydrodynamic models the momentum anisotropy develops in the early (QGP)phase of the collision. Thermalization times of less then 1 fm/c are neededto describe the data.

Cold atomic gases

200 000 Li-6 atoms in an highly anisotropic trap (aspect ratio 29:1)

Very strong interactions between atoms (Feshbach resonance)

Once the atoms are released the one observed a flow pattern similar to elliptic flow in heavy-ion collisions



Lesson III

- **First results** from **ALICE** show large **increase** in **energy density** (**factor 2-3** compared to RHIC)
- **longer life-time** of qgp
- **larger collective flow** effects
- **anisotropic flow** comparable to **ultra-low viscosity**
- triangular flow sensitive to initial energy density fluctuations and viscosity/entropy ratio
- Hydrodynamical model provides framework to characeterize QGP, i.e. equation of state, viscosity/entropy ratio