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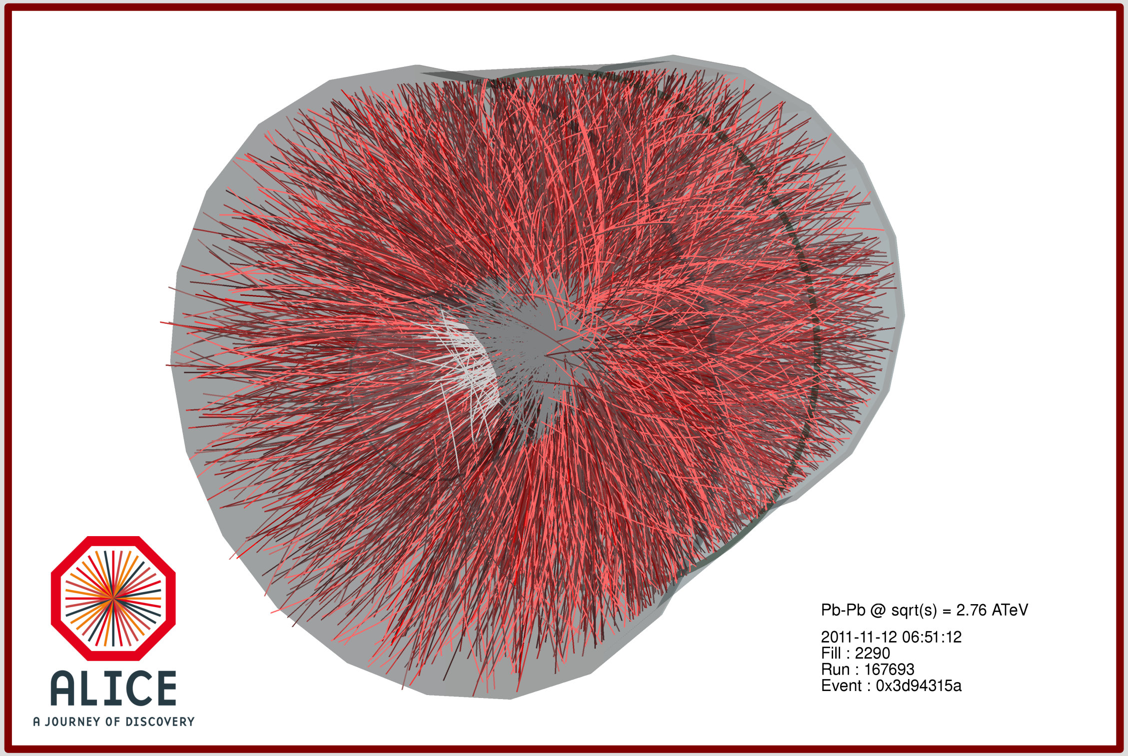
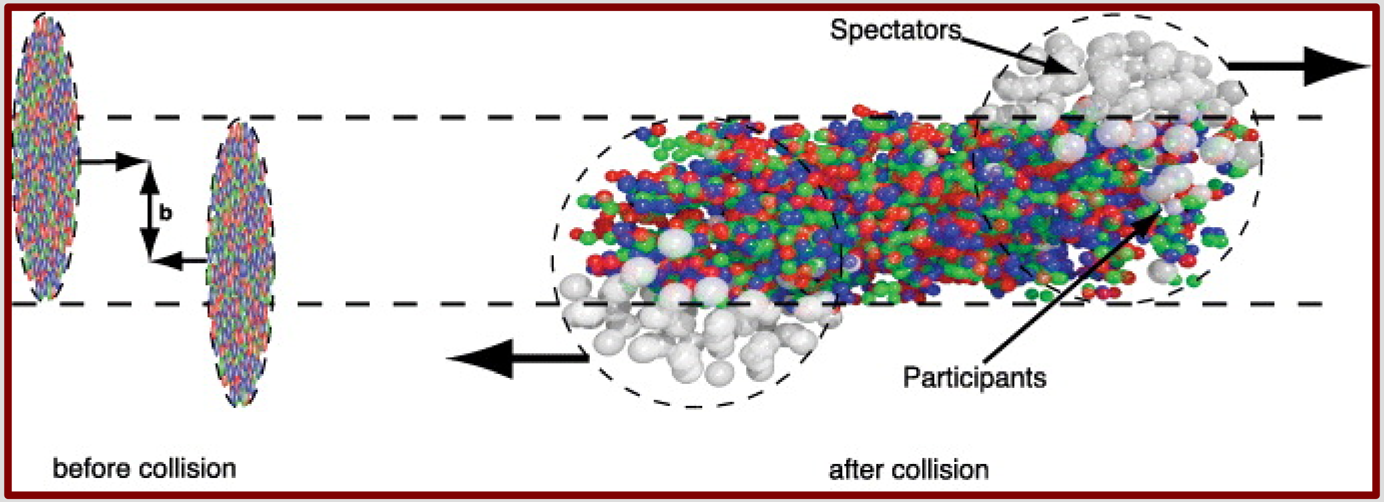


Glauber modelling in high-energy nuclear collisions

Jeremy Wilkinson

Introduction: Centrality in Pb-Pb collisions

- Proton-proton collisions: large multiplicities of charged particles produced through multi-parton interactions (fluctuations in gluon PDFs)
- Heavy-ion collisions: Larger system → much higher overall multiplicity. High-multiplicity events also occur due to nucleon-nucleon collisions



- Additional measurement parameter – centrality – defined in terms of N_{part} (number of participants, aka "wounded nucleons") and N_{coll} (number of binary collisions); characterises shape & size of overlap region

Introduction: Glauber model

- Problem: Impossible to directly measure centrality
  - → Impact parameter (*b*) on order of femtometres, N_{part} & N_{coll} can't be directly measured
  - → Theoretical models developed to estimate
- Leading technique: Glauber model, named after Roy Glauber (right)
- Basic assumptions ("optical limit"):
  - → Nucleons at high energy → undeflected due to large momentum (linear trajectory)
  - → Nucleus large compared to nucleon-nucleon force
  - → Motion of nucleons independent of nucleus
    - → overall cross-section described in terms of nucleon-nucleon cross section

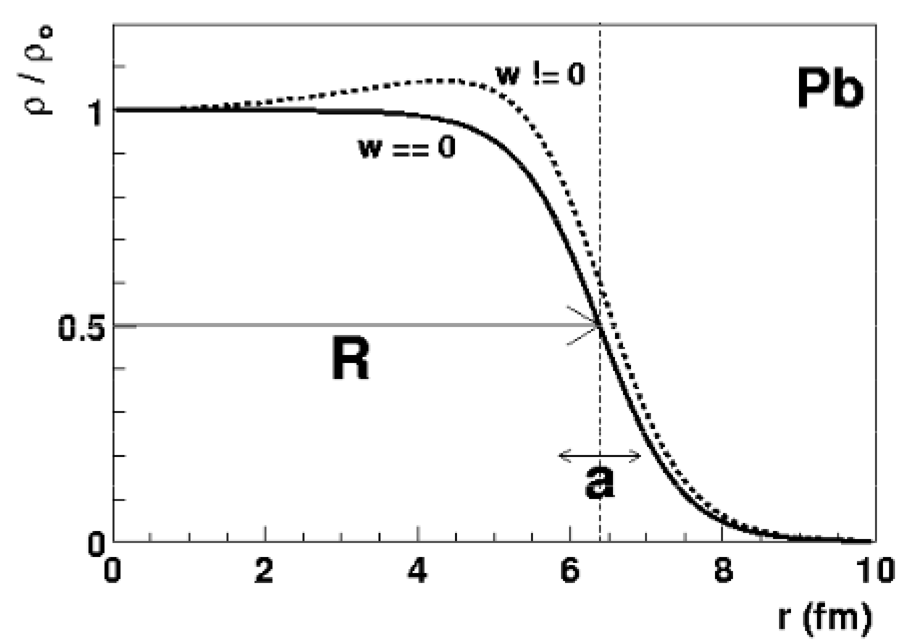
Roy Glauber (Nobel Prize, 2005)

Introduction: Woods-Saxon distribution

- Input for Glauber: inelastic nucleon-nucleon cross section,
  density profile of nucleus
- Woods-Saxon distribution describes nuclear density profile:

$$\rho(r) = \frac{\rho_0\left(1 + wr^2/R^2\right)}{1 + \exp((r-R)/a)}$$

- Parameters (see table) determined via e^--nucleus scattering (depends only on charge distribution of nucleus)
- Differences between distributions for protons and neutrons negligible

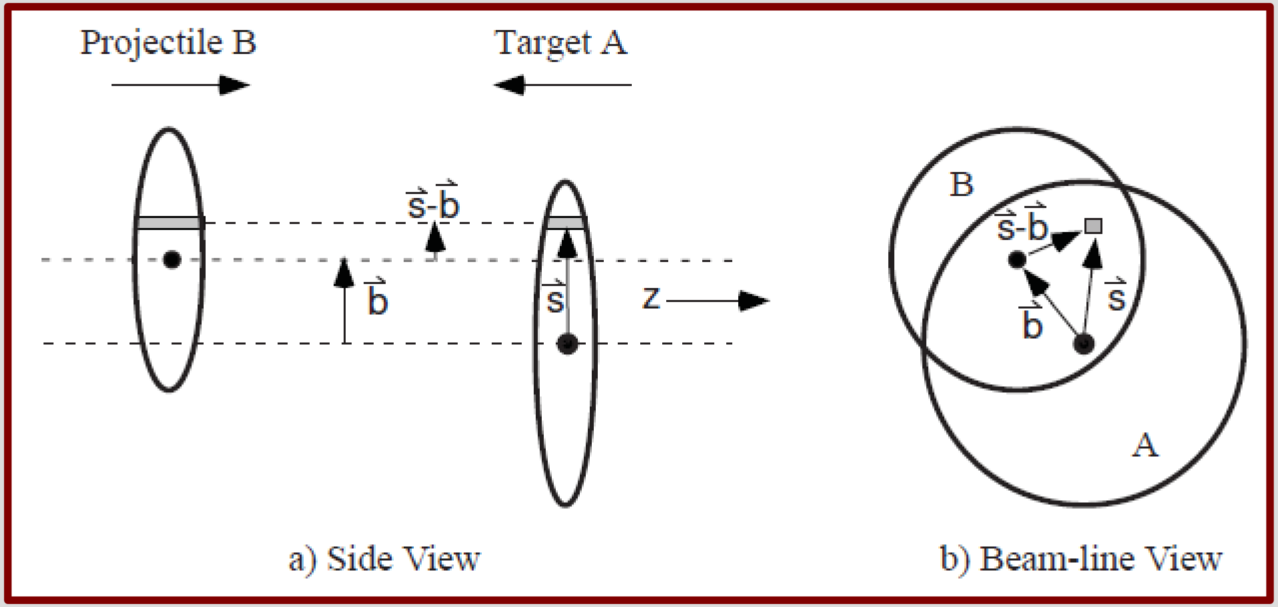


Nucleus	A	R (fm)	a (fm)	w
Au	197	6.38	0.535	0
Pb	208	6.68	0.546	0

H. DeVries, C.W. De Jager, C. DeVries, 1987

Collision schematics

- “Projectile” B colliding with “Target” A at relativistic speed
- Impact parameter **b**, flux tube of nucleon at **s** relative to nucleus centre



- Probability per unit transverse area of nucleon in flux tube:

$$\hat{T}_A\,(\mathrm{s}) = \int \hat{\rho}_A(\mathrm{s}, z_A) dz_A$$

- ρ_A = prob. of location per unit volume

Optical-limit approximation

- Product of T_A, T_B can be used to define nuclear "thickness function"

$$\hat{T}_{AB}(\mathbf{b}) = \int \hat{T}_A(\mathbf{s})\,\hat{T}_B(\mathbf{s}-\mathbf{b})\,d^2s.$$

- Units: inverse area → effective overlap area of specific nucleons in A and B
- T(**b**) σ_{inel}^{NN} = probability of interaction (σ_{inel}^{NN} = inelastic cross section; elastic processes have little energy loss)
- Probability of *n* interactions then given by binomial distribution

$$P(n,\mathrm{b}) = \binom{AB}{n} \left[\hat{T}_{AB}(\mathrm{b})\,\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^n \left[1-\hat{T}_{AB}(\mathrm{b})\,\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^{AB-n}$$

- Can be used to calculate N_{coll}, N_{part}, σ_{inel}^{tot}

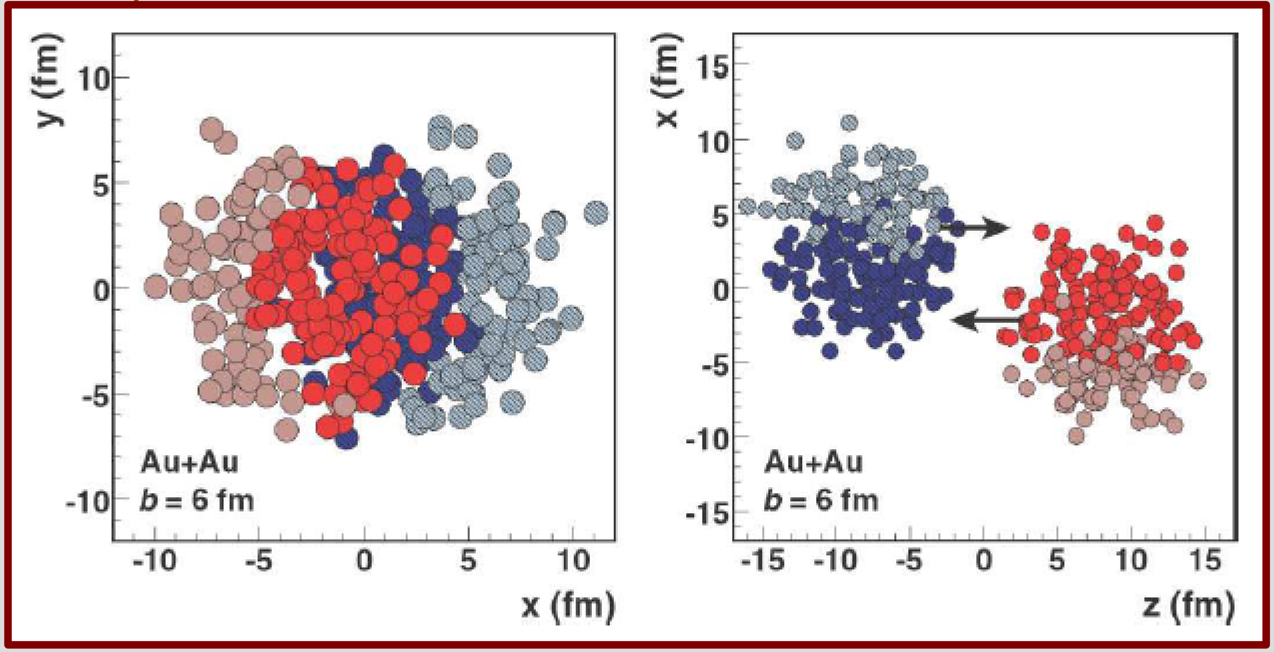
$$\sigma_{\mathrm{inel}}^{\mathrm{A+B}} = \int_0^\infty 2\pi b\,db \left\{1-\left[1-\hat{T}_{AB}(b)\,\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^{AB}\right\}$$

$$N_{\mathrm{coll}}(b) = \sum_{n=1}^{AB} nP(n,b) = AB\hat{T}_{AB}(b)\,\sigma_{\mathrm{inel}}^{\mathrm{NN}}$$

$$\begin{aligned} N_{\mathrm{part}}(\mathrm{b}) &= A\int \hat{T}_A(\mathrm{s})\left\{1-\left[1-\hat{T}_B(\mathrm{s}-\mathrm{b})\,\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^B\right\}d^2s + \\ &\quad B\int \hat{T}_B(\mathrm{s}-\mathrm{b})\left\{1-\left[1-\hat{T}_A(\mathrm{s})\,\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^A\right\}d^2s, \end{aligned}$$

Alternative approach: Glauber Monte Carlo

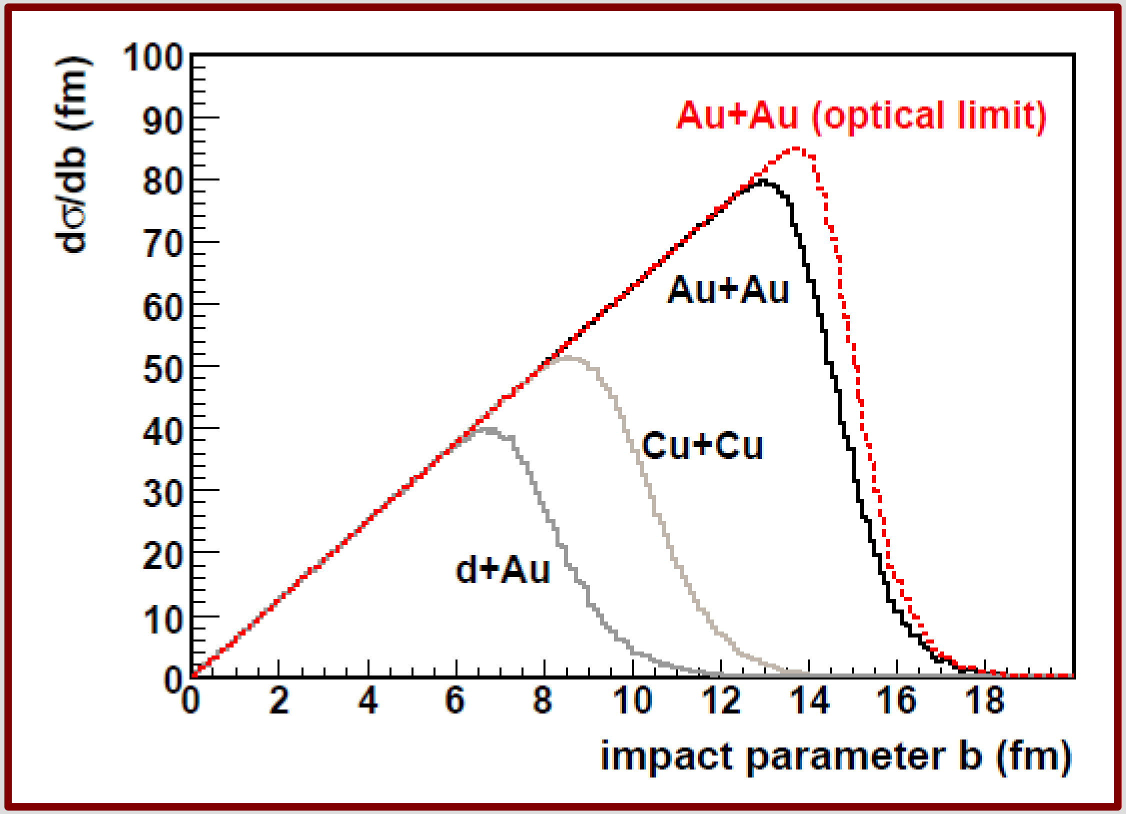
- Simple approach to Glauber calculations
- Nucleons have straight-line trajectories, σ independent of prev. interactions
- Nucleons distributed in 3D space according to Woods-Saxon (e.g. Au+Au, $\sqrt{s_{NN}}$ = 200 GeV)



- Impact parameter drawn at random from $d\sigma/db = 2\pi b$, collision happens if distance between nucleons < $\sqrt{(\sigma_{inel}^{NN}/\pi)}$

Sample Glauber MC calculation

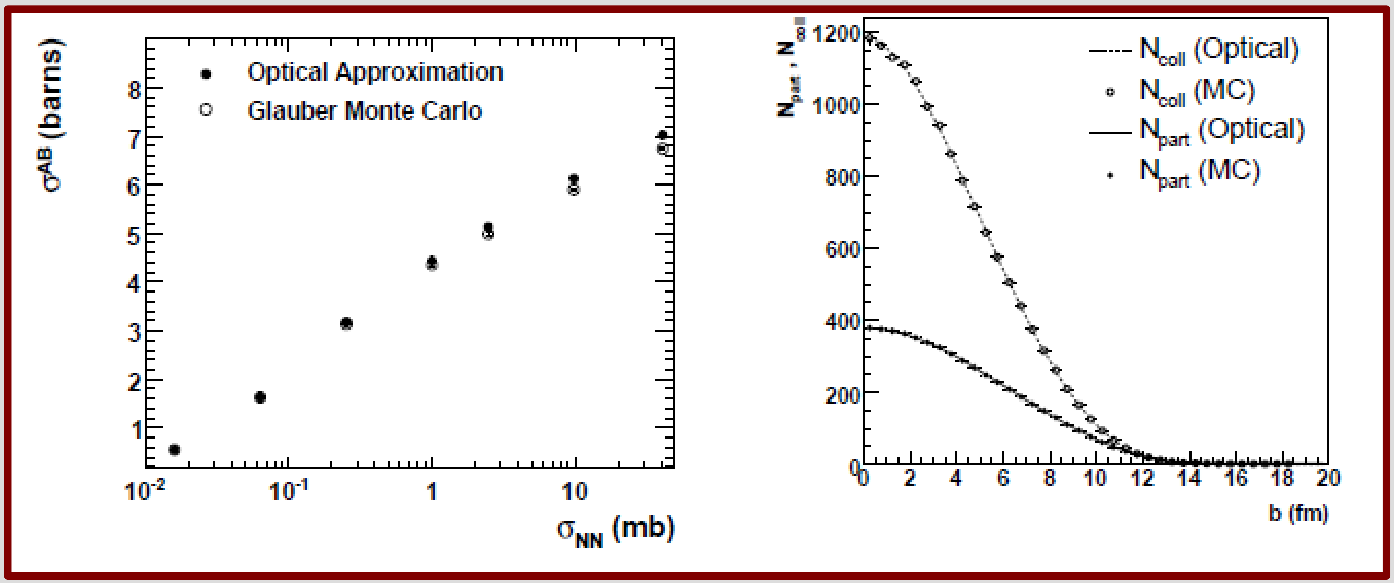
- Impact parameter distribution shown for Au+Au, Cu+Cu and d+Au collisions



- Optical approach in Au+Au leads to larger cross section – perturbation seems small, but is significant (will come back to this later)

Optical Glauber vs. Glauber Monte Carlo

- Optical approach doesn't consider spatial coordinates of nucleons
- Nucleons see target as having smooth density (eikonal approach)
  → Doesn't account for full physics of collision
  → Distortions between approaches in calculation of calculated N_{part} & N_{coll}, esp. at large σ, or for small A / B



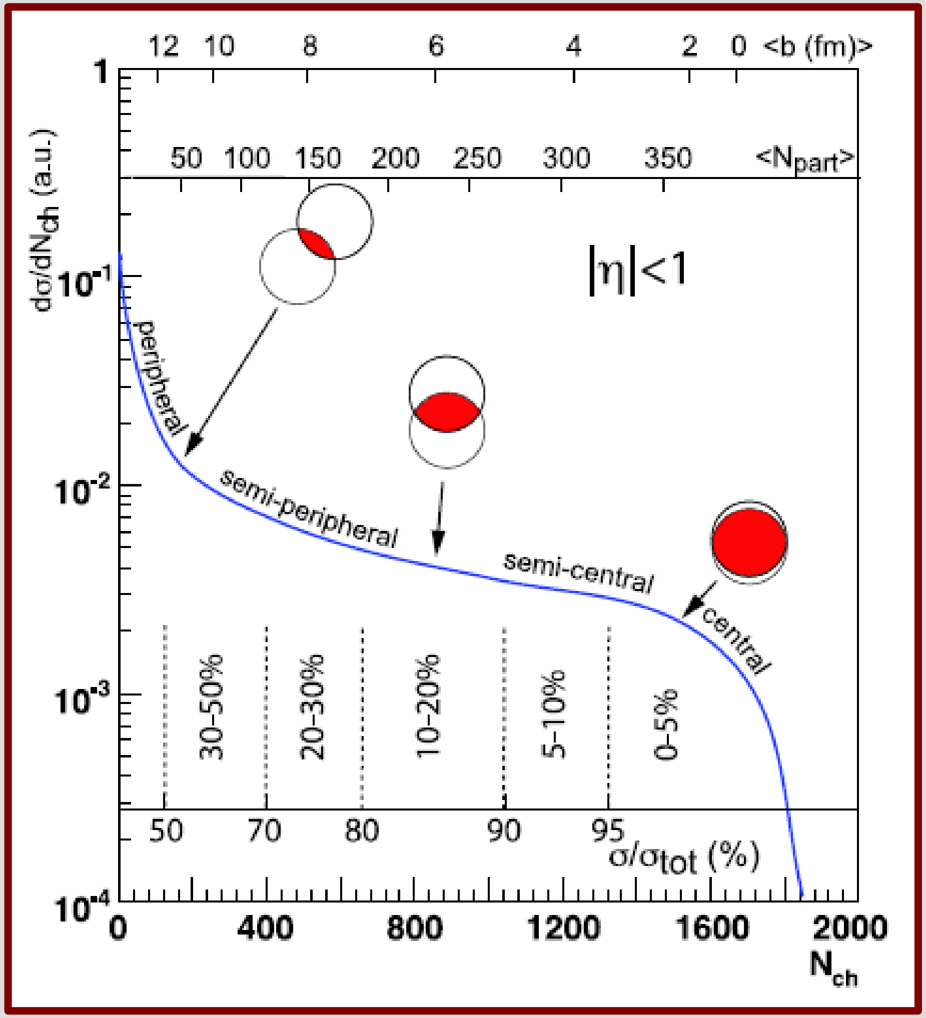
- σ^{AB} converges between approaches for pointlike σ_{NN} (left); little difference for geometric quantities (right)

Relating Glauber to real collisions

- As mentioned, N_{part} & N_{coll} not measured directly
  - → Observables (e.g. dN_{Evt}/dN_{ch}) must be mapped to these quantities via Glauber calculations
  - → "Centrality classes": percentiles (fraction of total integral) of centrality distribution.

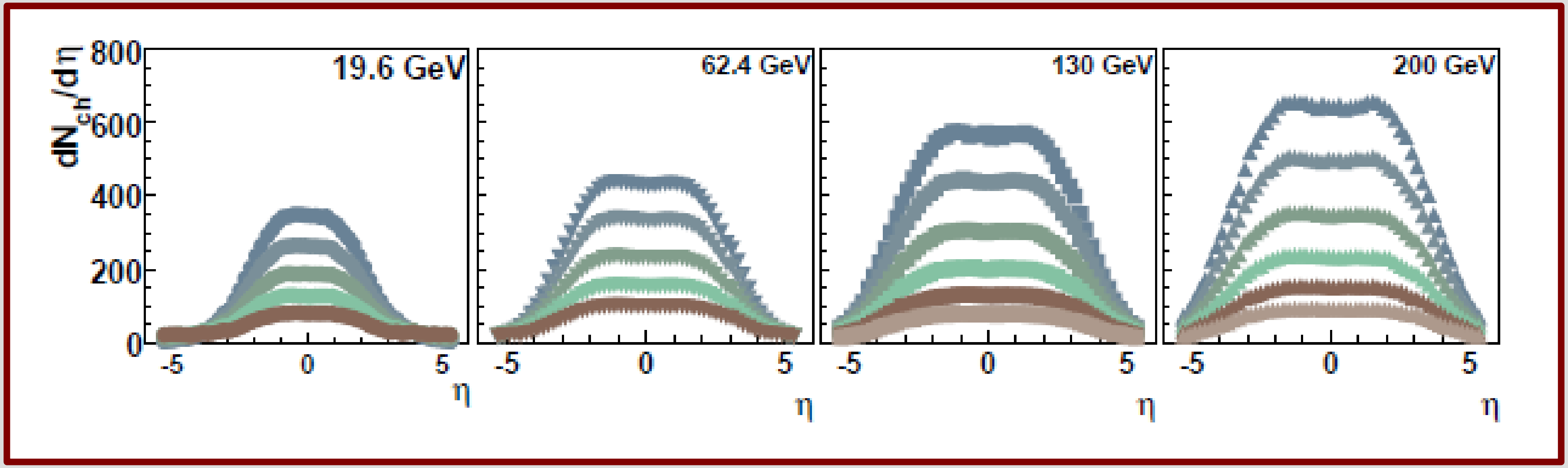
- Convention: 0% = most central, 100% = most peripheral

- Classes justifiable as we expect monotonic relation between b and N_{ch}; peripheral → low mult, central → high mult



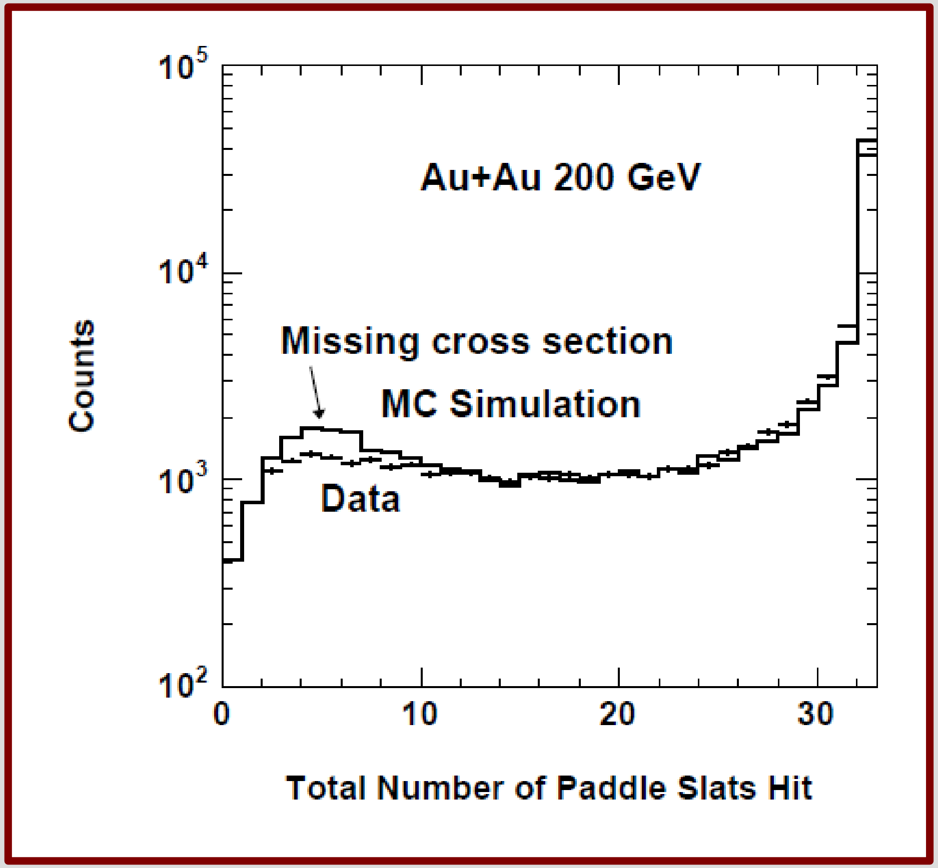
Glauber observables

- N_{ch} scales with q^2 (hardness) of collision; jet events have higher mult than minimum-bias collisions
- Assume majority of nucleon-nucleon collisions analogous to MB pp events
- Can estimate N_{ch} online via (energy deposited)/(<E> per charged particle) (e.g. PHOBOS paddles), or offline by counting charged tracks (e.g. STAR Time Projection Chamber, ALICE Silicon Pixel Detector)
- Below: $dN_{ch}/d\eta$ in PHOBOS for Au+Au collisions



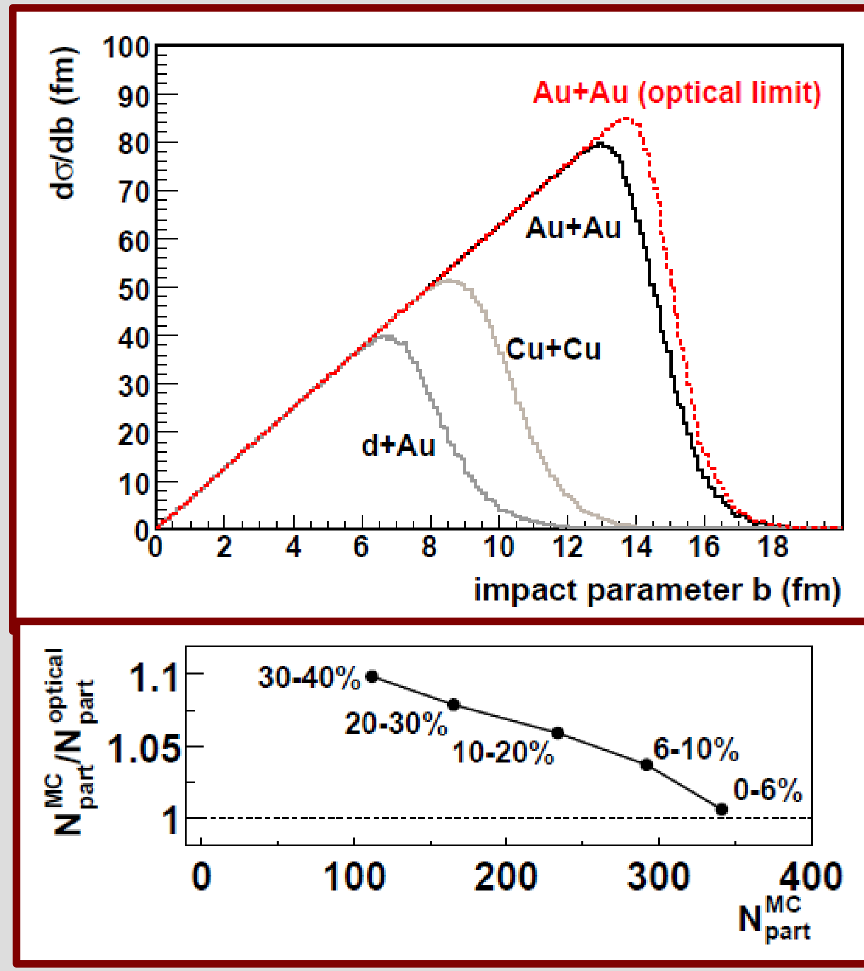
Relating Glauber to real collisions

- Monte Carlo approach can be adapted to include detector effects
  - → Detectors have finite resolution; no perfect 1-to-1 relation between *b* and measured N_{ch}
  - → Detector effects in simulation allow direct comparison between calculated + real distributions of N_{ch}
- Allows e.g. trigger inefficiencies to be accounted for



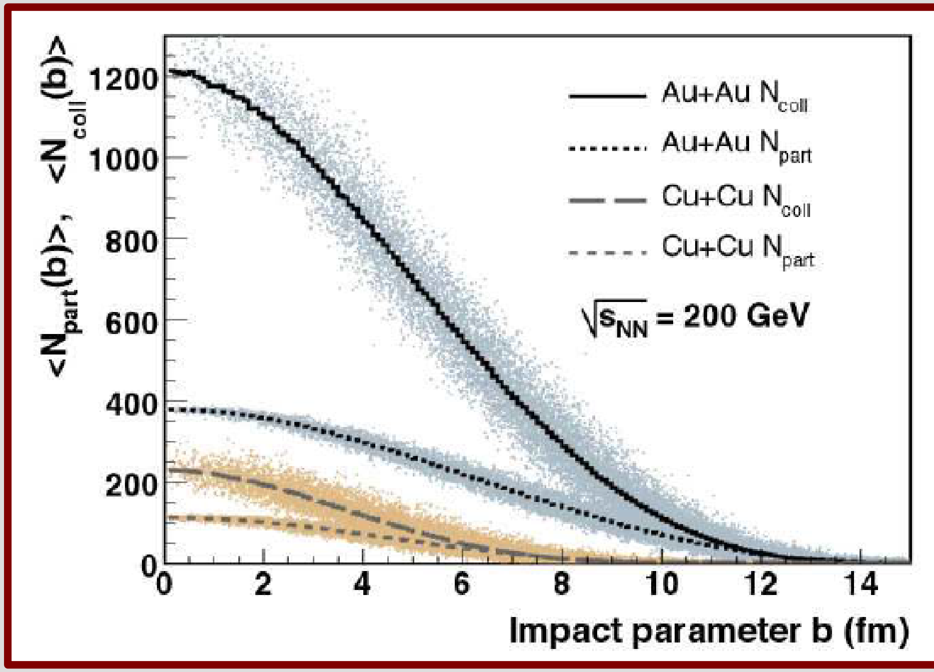
Estimating geometric quantities: σ

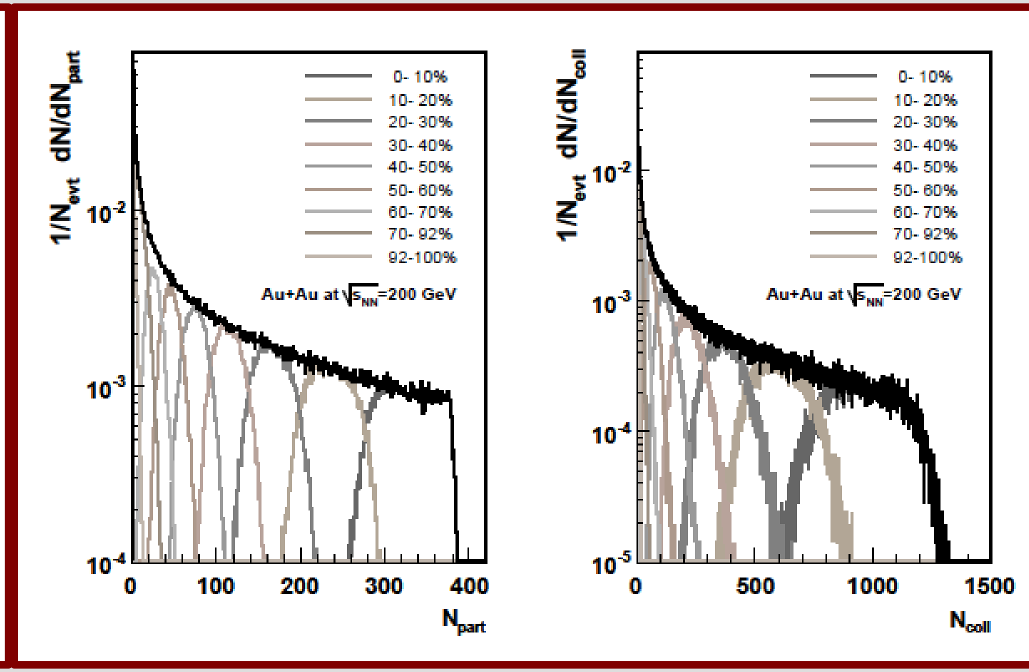
- Total geometric cross section (integral of distribution to right) simple in Glauber MC approach
- de Broglie wavelength small → quantum effects small → $\sigma_{geo} \sim \sigma_{inel}$
- Systematic uncertainty ~10%, mostly due to nuclear density profile
- Differences between optical + MC approaches lead to systematic differences in centrality binning for events



Estimating geometric quantities: N_{part}, N_{coll}

- Definition: participant (or “wounded”) nucleon takes part in at least one collision.



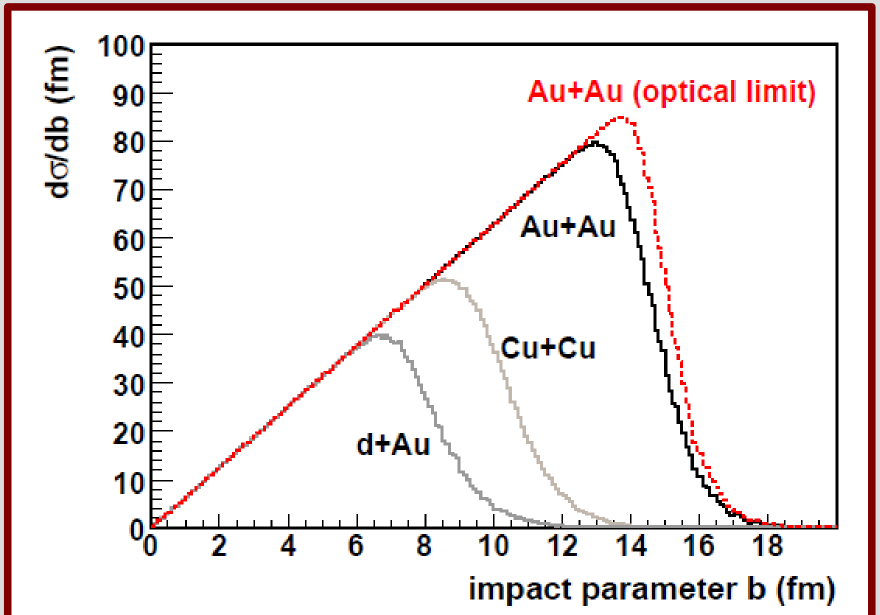


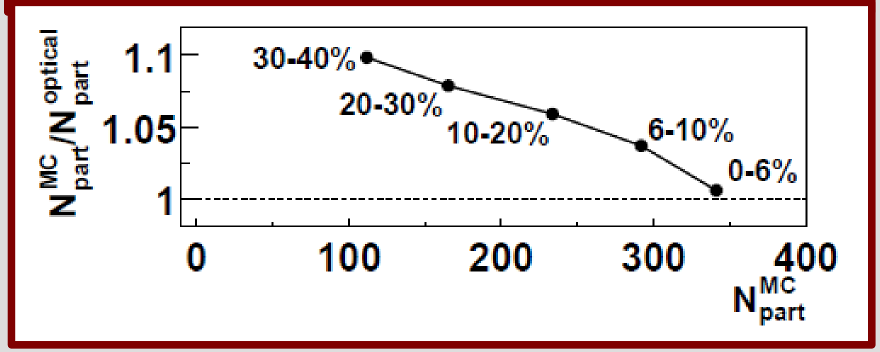
- Smearing accounted for by fluctuations in random distribution
- Shape of N_{part}, N_{coll} distributions due to peripheral events being more likely

Estimating geometric quantities: systematic uncertainties

- Systematics can be estimated by varying model parameters:
  - → Value of nucleon-nucleon cross section
  - → Woods-Saxon parameters
  - → Detector resolution parameters
  - → Gaussian, instead of "black disc", overlap function
  - → Centrality cuts in experiment
  - → Trigger efficiencies

- Lower plot: Systematic difference of N_{part} when considering optical and MC approaches





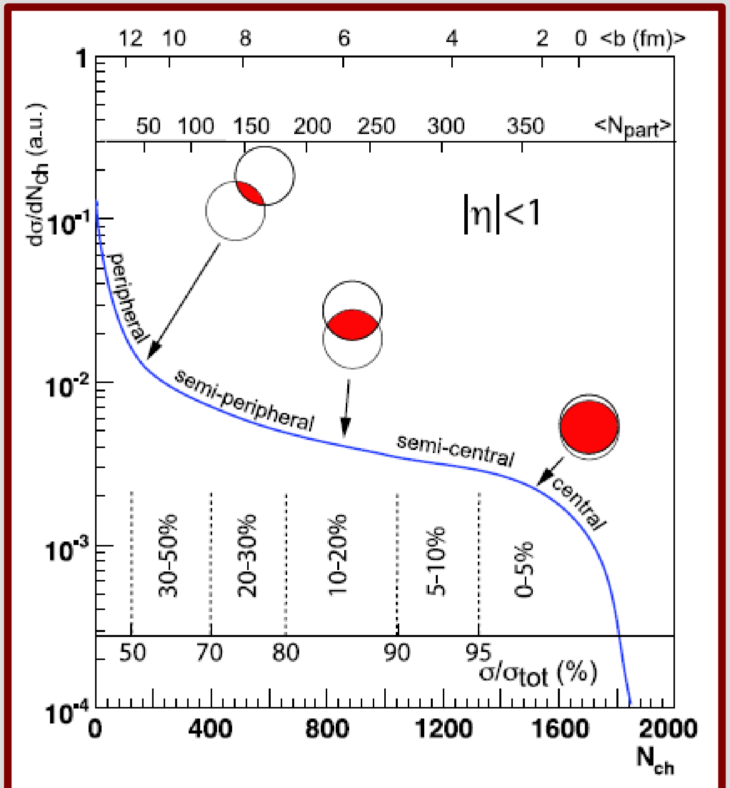
Estimating geometric quantities: eccentricity

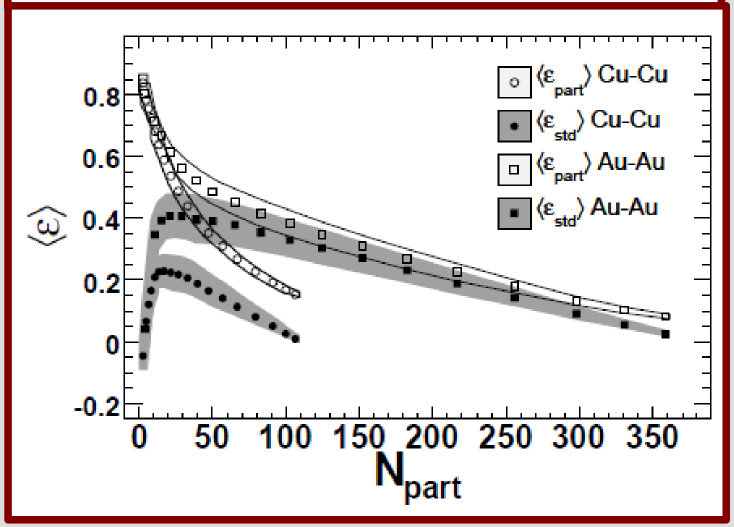
- Overlap region of nuclei is not spherically symmetric; more "almond-shaped" → hydrodynamic evolution leads to momentum anisotropy → "elliptic flow"
- Eccentricity:

$$\epsilon = \frac{\langle Y^2 - X^2 \rangle}{\langle Y^2 + X^2 \rangle}$$

- Can be calculated in Glauber model in "standard" or "participant" method
  → measuring eccentricity along reaction plane or principal axis of participant distribution

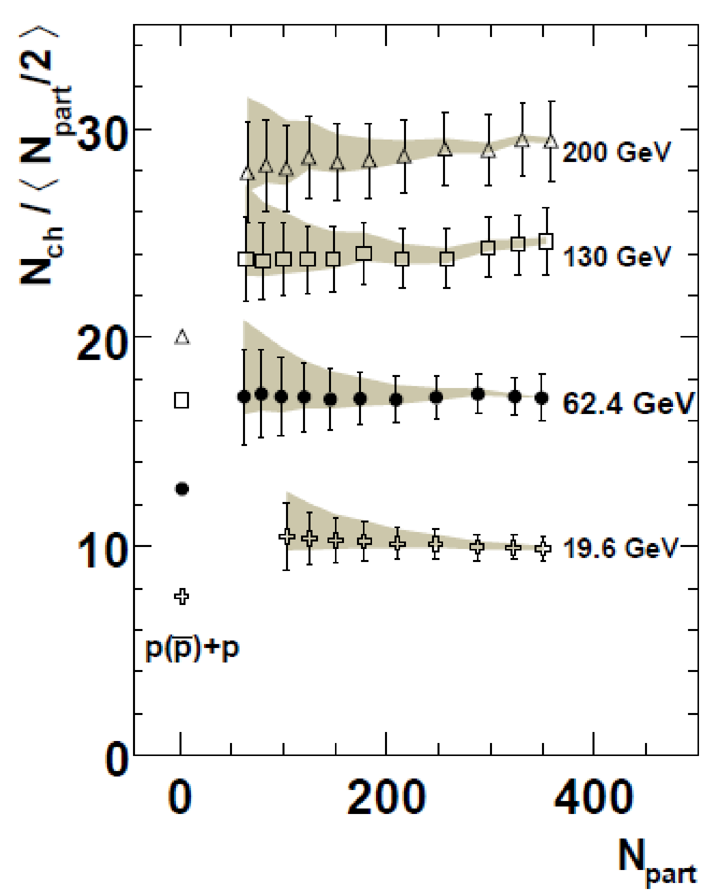
- Limiting behaviour for two methods very different

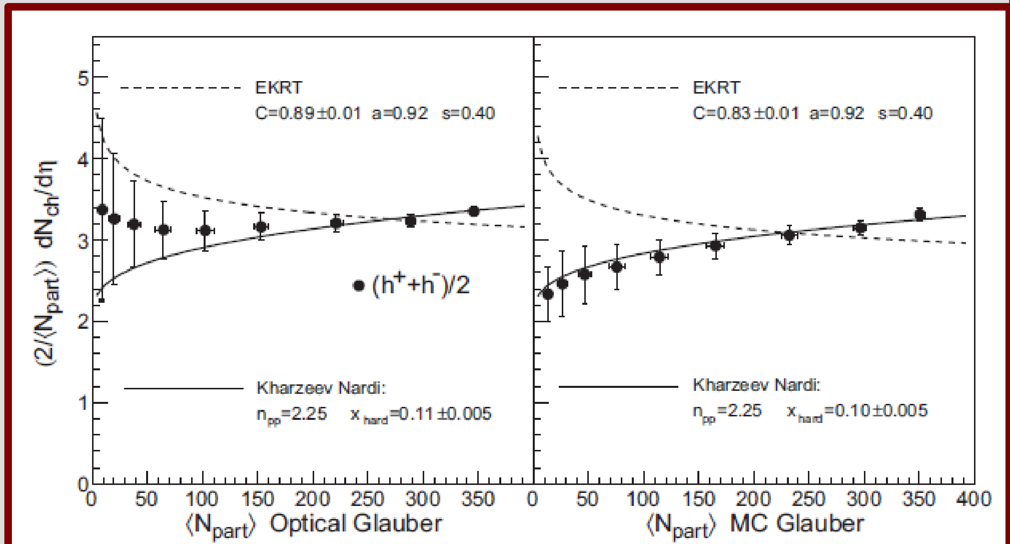




Phenomena in p-A and A-A: charged-particle yields

- Multiplicity determined in 1970s to be proportional to N_{part}
- Found to be roughly true for Au+Au collisions at varying RHIC energies (right, PHOBOS)
- But particle density does not scale linearly with N_{part} (below, STAR)
- Model agreements vary depending on approximation used for N_{part} (optical or GMC)





Phenomena in p-A and A-A: hard scattering

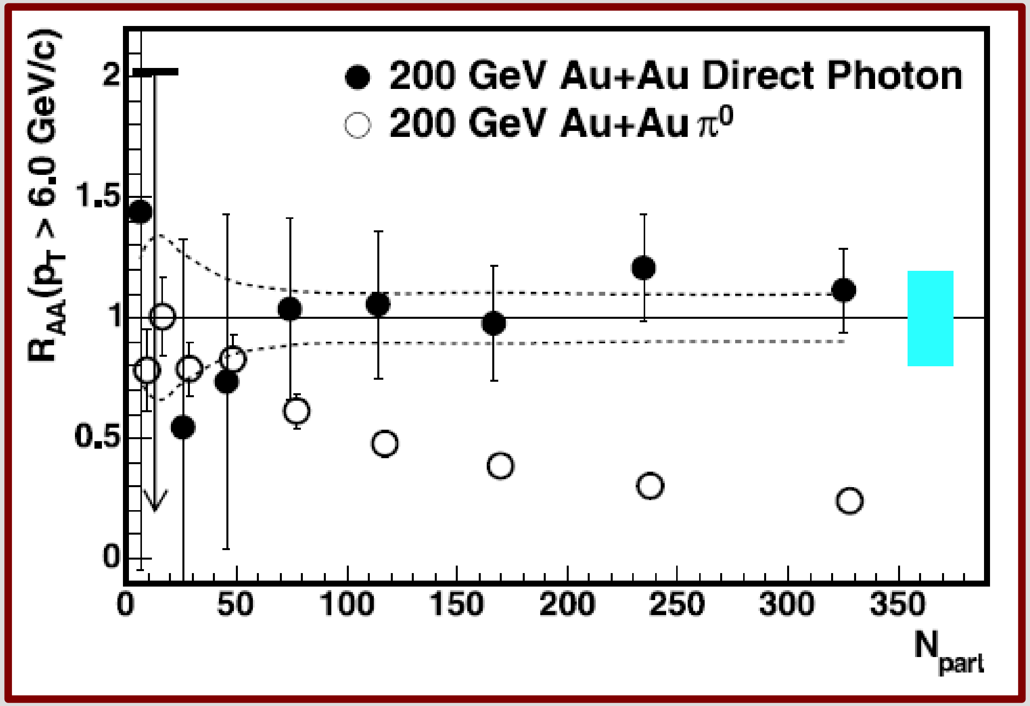
- Number of hard-scattering events proportional to T_{AB}

$$N_{\rm hard}^{\rm A+B,enc}(b) = T_{\rm AB}(b)\,\sigma_{\rm hard}^{\rm pp}$$

- Particle yields at RHIC + LHC usually measured vs. transverse momentum p_T
- Can define nuclear modification factor R_{AB} → effectively ratio of spectrum to that from proton-proton collisions

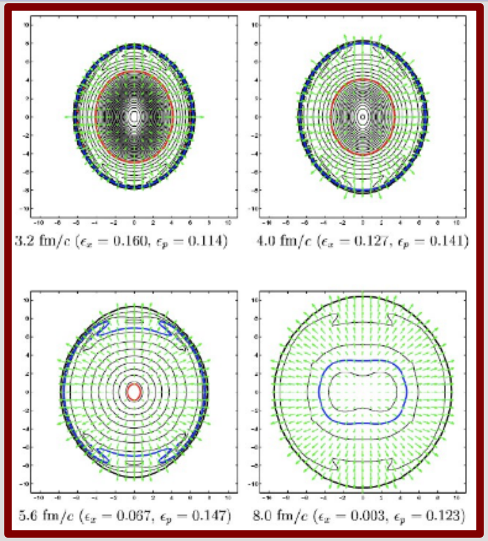
$$R_{\rm AB}(p_{\rm T}) = \frac{(N_{\rm inel}^{\rm AB})^{-1}\,{\rm d}N_x^{\rm A+B}/{\rm d}p_{\rm T}}{\langle T_{\rm AB}\rangle_{\rm f}\,{\rm d}\sigma_x^{\rm pp}/{\rm d}p_{\rm T}}$$

- Can be used to study energy loss in high-density medium

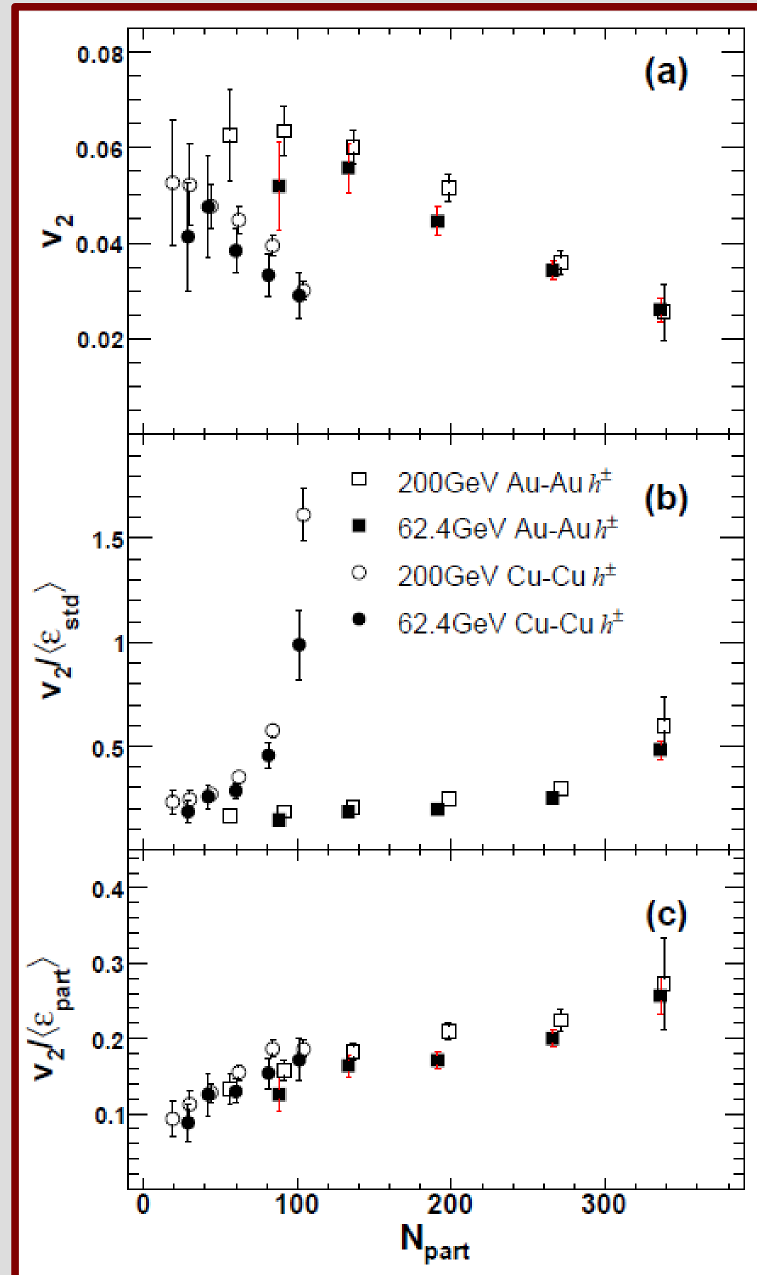


- Direct photons follow T_{AB} scaling; pions suppressed
  → energy loss of partons due to hard scattering in QGP (“jet quenching”)

Phenomena in p-A and A-A: eccentricity

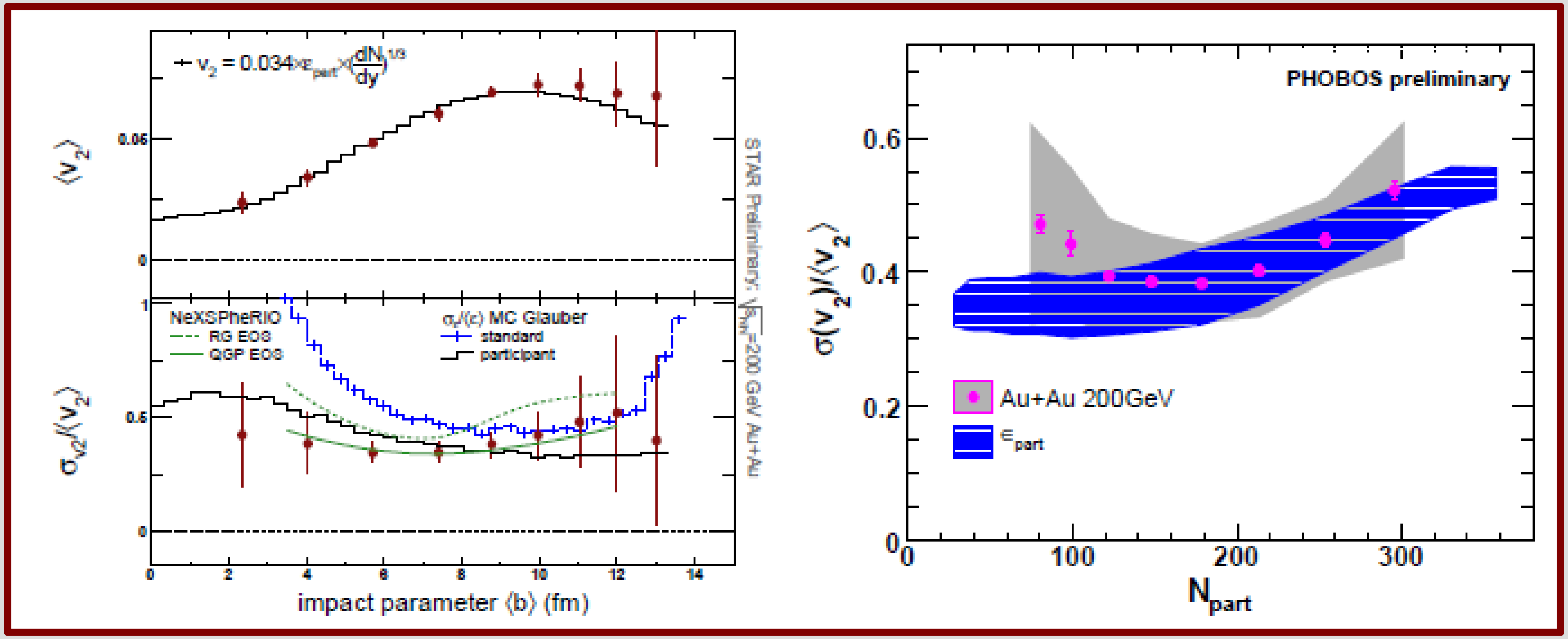


- Hydrodynamics: initial-state spatial anisotropy → final-state momentum anisotropy
- Second term in Fourier expansion of $dN/d\phi$: $2v_2 \cos[2(\phi - \Psi_R)]$; Ψ_R = angle of reaction plane
- Assumption: v_2 scales linearly with ϵ
- Dividing measured v_2 by ϵ: ϵ_{part} drives hydrodynamic evolution of system



Phenomena in p-A and A-A: eccentricity fluctuations

- RMS width of v_2 also measured; L: STAR, R: PHOBOS



- Agreement with ϵ_{part} implies fluctuations accounted for by fluctuations in initial-state geometry, meaning other sources such as Colour-Glass Condensate unnecessary for description

Phenomena in p-A and A-A: J/Ψ absorption in nuclear matter

- Due to large mass (> Λ_{QCD}), charm quarks produced in early stages of collision, not through thermal processes
  → Can use pQCD calculations to determine production rate

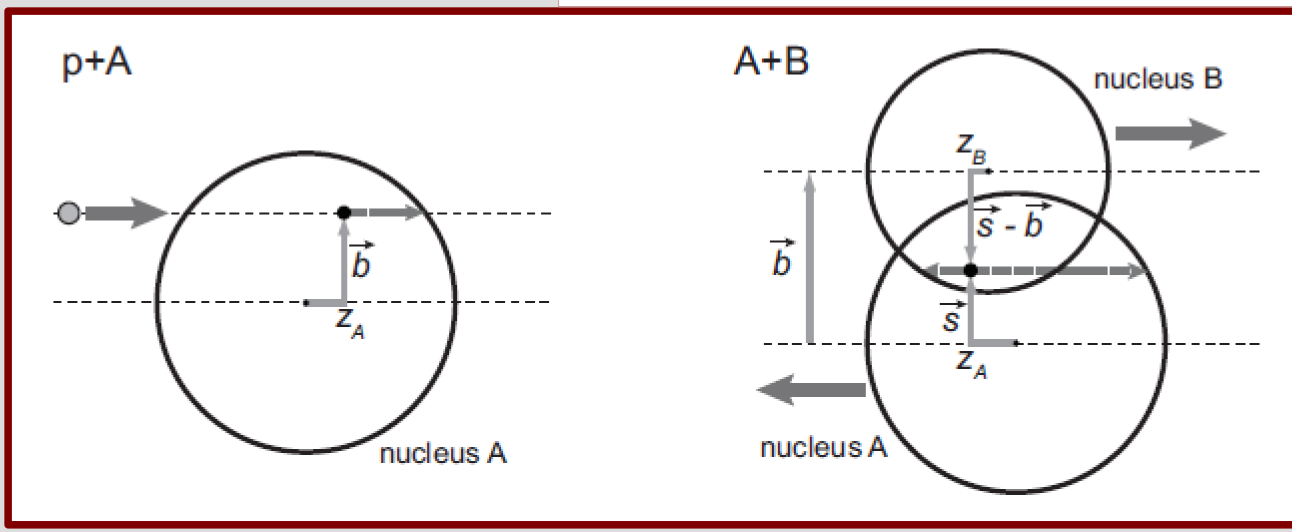
- J/Ψ suppression in heavy-ion collisions considered signature of QGP (due to screening of $c\bar{c}$ binding by free colour charges
  → but suppression also noticed in p-A collisions
  → must be quantified before concluding on suppression in A-A collisions (p-A system size considered too small to create QGP)

- Possible "cold nuclear matter" effects: modification of PDFs in nucleus (shadowing); absorption of pre-resonant $c\bar{c}$ pairs
  → Glauber model can be used for latter

Phenomena in p-A and A-A: J/Ψ absorption in nuclear matter

- Processes inhibiting formation can be parametrised with constant absorption cross section σ_{abs}.
- Break-up probability p_{abs}:

$$p_{\mathrm{abs}}(\mathrm{b}, z_{\mathrm{A}}) = \sigma_{\mathrm{abs}}\, \hat{T}_{\mathrm{A}>}(\mathrm{b}, z_{\mathrm{A}}) \quad \text{with} \quad \hat{T}_{\mathrm{A}>}(\mathrm{b}, z_{\mathrm{A}}) = \int_{z_{\mathrm{A}}}^{\infty} \hat{\rho}_{\mathrm{A}}(\mathrm{b}, z)\, \mathrm{d}z$$



- "Normal" suppression level classified as: $S_{\mathrm{A+B}} = \exp(-L\, \rho_0\, \sigma_{\mathrm{abs}})$
- "Anomalous" suppression beyond this (as seen at SPS energies at CERN) seen as possible signal for QGP formation

Summary

- Glauber model in nuclear physics depends only on nuclear geometry
- Gives access to quantities that are otherwise unmeasurable (N_{coll}, N_{part})
- N_{part} allows centrality-dependent measurements to be made and compared between different experiments
  - → Calculation simple, implemented in very similar way
  - → Theoretical bias small
- Many heavy-ion phenomena explicable through geometry
  - → Multiplicity scaling with N_{part}
  - → Role of anisotropy fluctuations in understanding elliptic flow

- Glauber model plays major role in understanding nuclear geometry in experiments at RHIC & LHC