QGP Physics – from Fixed Target to LHC

4. Basics of Nucleon-Nucleon and Nucleus-Nucleus Collisions

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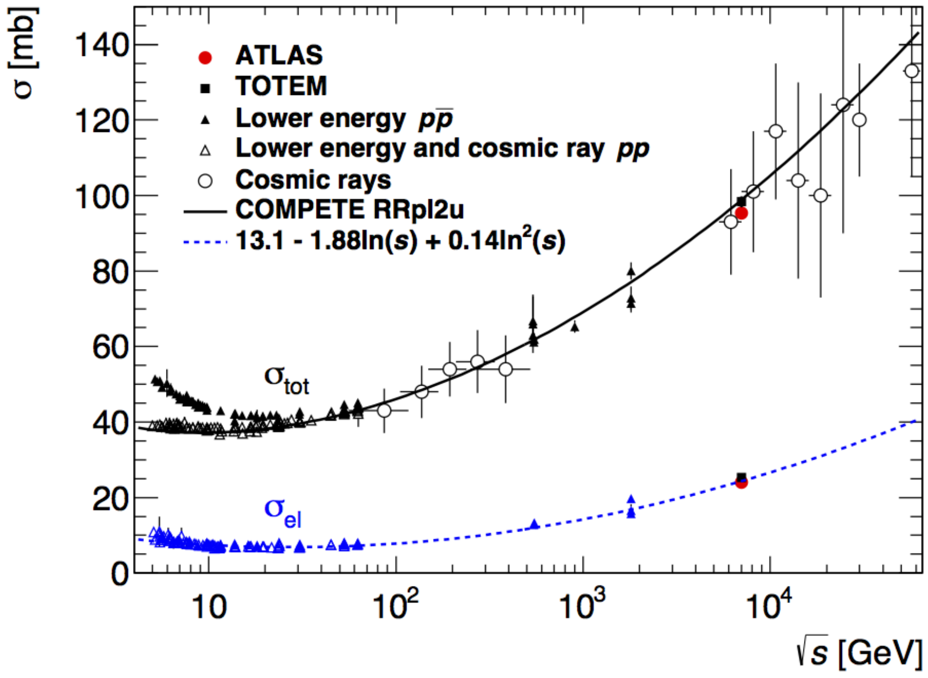
General Consideration

- QCD predictive power for dynamical processes only at high p_T (above a few GeV/c or so) where α_s is sufficiently small so that perturbative methods are applicable
- Need to work with (QCD inspired) models, and confront them with data

Total p+p(pbar) Cross Section



Picture: ATLAS,
arXiv:1408.5778 (→ Link)

Above ~ √s = 20 GeV all hadronic cross sections rise with increasing √s

Data are in agreement with Pomeranchuk's theorem which states that for hadronic collisions at asymptotic energies the following relation holds:

$$\sigma_{\mathrm{tot}}(h+X) = \sigma_{\mathrm{tot}}(\bar{h}+X)$$

Useful parameterization inspired by Regge theory:

$$\sigma_{\mathrm{tot}} = X s^{\epsilon} + Y s^{\epsilon'}$$

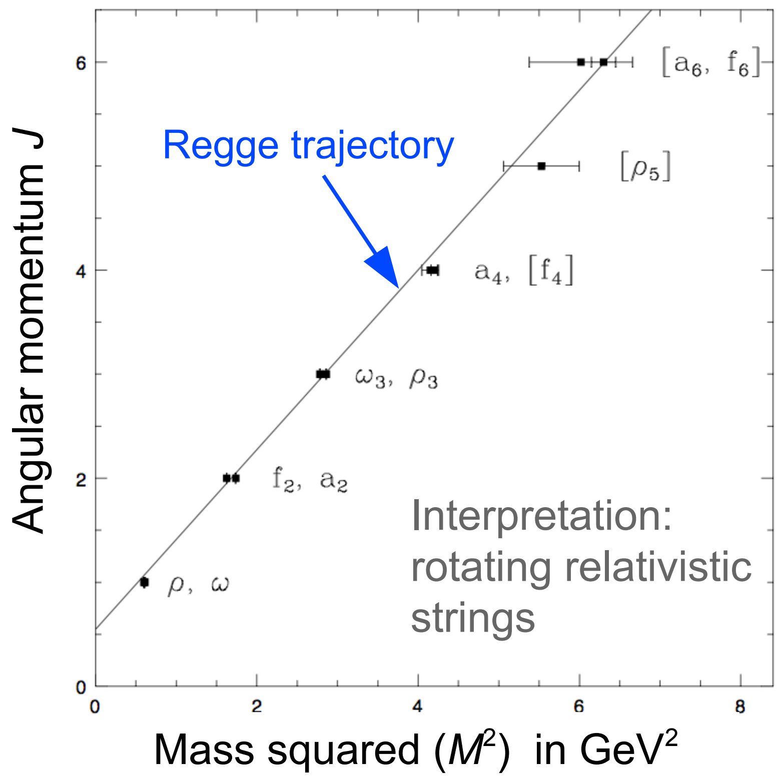
$$\epsilon = 0.08 - 0.1, \quad \epsilon' \approx -0.45$$

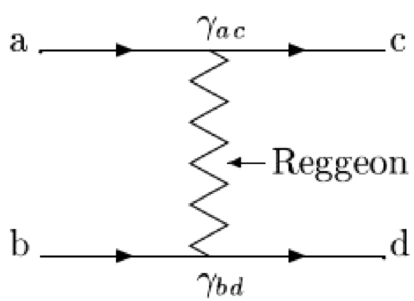
The first term corresponds to Pomeron exchange, the second to „normal“ Regge (Reggeon) exchange

[Regge Theory – A Candidate for the Fundamental Theory of the Strong Interaction in the 1960s]

[Tullio Regge, 1931-2014]

Chew-Frautschi-Plot





Regge theory is based on a generalization of angular momentum to non-integer (complex) values.

The interaction of hadrons is mediated by the exchange of "Regge trajectories". We can think of Regge exchange as the superposition of the exchange of many particles.

The intercepts $\alpha_i(t=0)$ of the Regge trajectories are related to the total cross section:

$$\sigma_{\text{total}} \sim \sum_{\text{Regge traj.}} A_i s^{\alpha_i(0)-1}$$

The rise of the total cross section with $\sqrt{s}$ is explained by the exchange of a trajectory with

$$\alpha(0) \geq 1$$

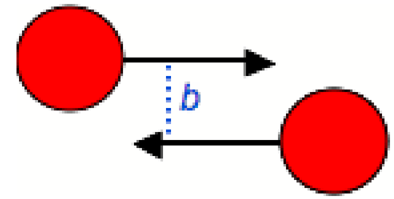
This is the pomeron. (→ link)

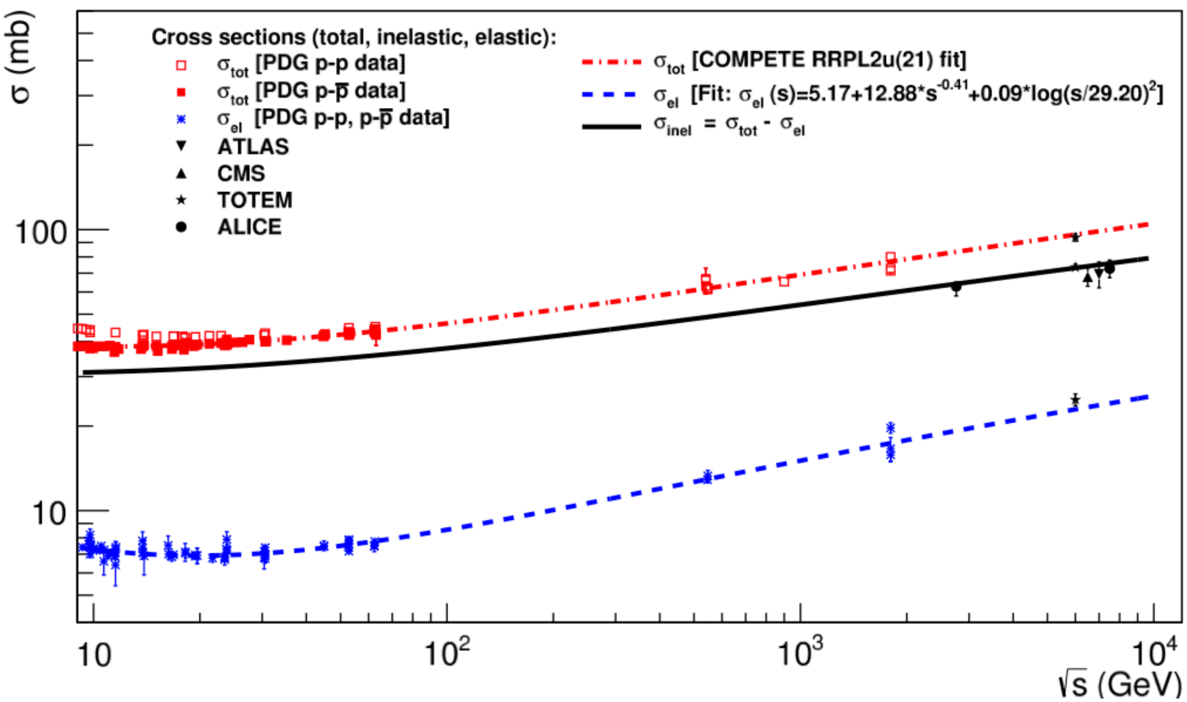
Total inelastic Nucleon-Nucleon Cross Section

Naïve expectation for the total inelastic p+p cross section:

$$\sigma_{\text{geo}} = \pi \cdot b_{\max}^2 = \pi \cdot (2 r_{\text{proton}})^2 = \pi \cdot (1.6\,\text{fm}^2) = 80\,\text{mb}$$

$$(1\text{b} = 10^{-28}\text{m}^2, 1\,\text{fm}^2 = 10^{-30}\,\text{m}^2 = 10\,\text{mb})$$





ALICE, arXiv:1303.0737 (→ link)

From data:

$$\sigma_{\text{inel}} = \sigma_{\text{total}} - \sigma_{\text{elastic}}$$

$\sqrt{s}$ (GeV)	σ_{inel} (p+p)
17.2	≈ 32 mb
200	≈ 42 mb
2760	≈ 63 mb

Total inelastic NN cross section is needed as input for Glauber calculations for A+A

Diffractive Collisions (I)

(Single) diffraction in p+p:
"Projectile" proton is excited to a hadronic state X with mass *M*

$$p_{\mathrm{proj}} + p_{\mathrm{targ}} \rightarrow X + p_{\mathrm{targ}}$$

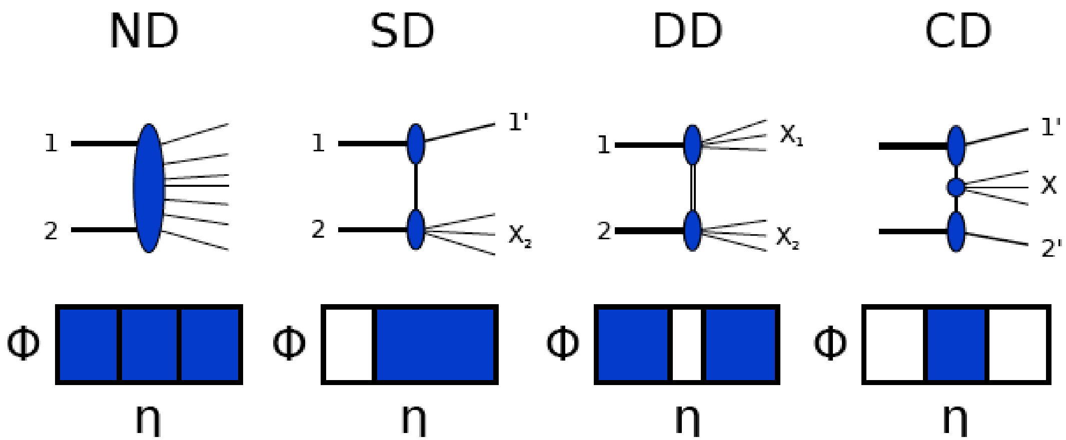
The excited state *X* fragments, giving rise to the production of (a small number) of particles in the forward direction

Theoretical view:

- Diffractive events correspond to the exchange of a Pomeron
- The Pomeron carries the quantum numbers of the vacuum ($J^{PC} = 0^{++}$)
- Thus, there is no exchange of quantum numbers like color or charge
- In a QCD picture the Pomeron can be considered as a two- or multi-gluon state, see, e.g., O. Nachtmann (→ link)

Diffractive Collisions (II)

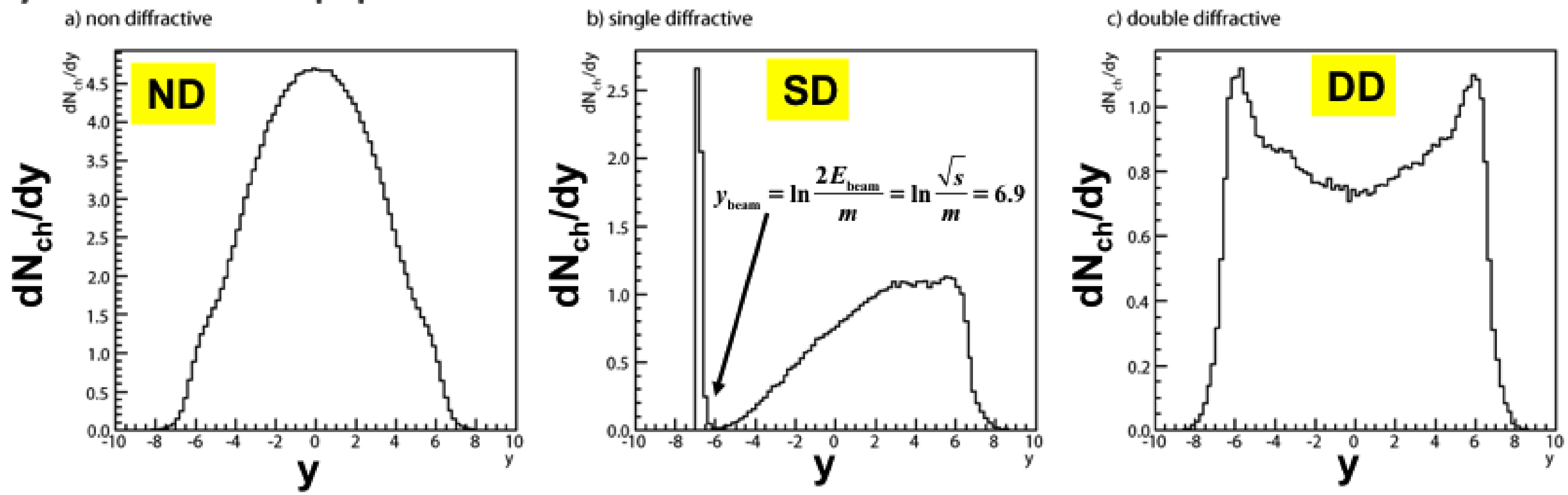
A characteristic feature of diffractive collisions are large regions in rapidity in which no particles are found ("rapidity gaps"):



Plot: F. Reidt, Bachelor thesis (→ Link)

ND: non-diffractive process
SD: single diffractive dissociation
DD: double diffractive dissociation
CD: central diffraction

Pythia simulation: p+p at $\sqrt{s}$ = 900 GeV:



Diffractive Collisions (III)

$$\sigma_{\text{tot}} = \sigma_{\text{elastic}} + \sigma_{\text{inel}} \qquad \sigma_{\text{inel}} = \sigma_{\text{ND}} + \sigma_{\text{SD}} + \sigma_{\text{DD}} + \sigma_{\text{CD}}$$

small, < 1 mb

Data from UA5:

UA5, Z. Phys. C33, 175, 1986 (→ Link)

$p + \bar{p}$	√s = 200 GeV	√s = 900 GeV
Total inelastic	**(41.8 ± 0.6) mb**	**(50.3 ± 0.4 ± 1.0) mb**
Single-diffractive	**(4.8 ± 0.5 ± 0.8) mb**	**(7.8 ± 0.5 ± 1.8) mb**
Double-diffractive	**(3.5 ± 2.2) mb**	**(4.0 ± 2.5) mb**
Non-diffractive	**≈ 33.5 mb**	**≈ 38.5 mb**

About 20-25% of the inelastic cross section is due to diffractive processes for √s = 200 - 900 GeV

Expectation for p+p at 14 TeV: σ_{tot} = 102 mb, σ_{ND} = 76 mb, σ_{SD} = 12 mb

(nucl-ex/0701067)

Average Charged Particle Multiplicity

- Total number of produced charged particles in a p+p collision
  - related to soft processes and hence difficult to calculate from first QCD principles
  - Thus, a large variety of models describing soft particles production exists
  - d*N*/dη measurements at the LHC help constrain models
- History
  - Feynman concluded in the 1970‘s that for asymptotically large energies the mean total number of produced particles increases as

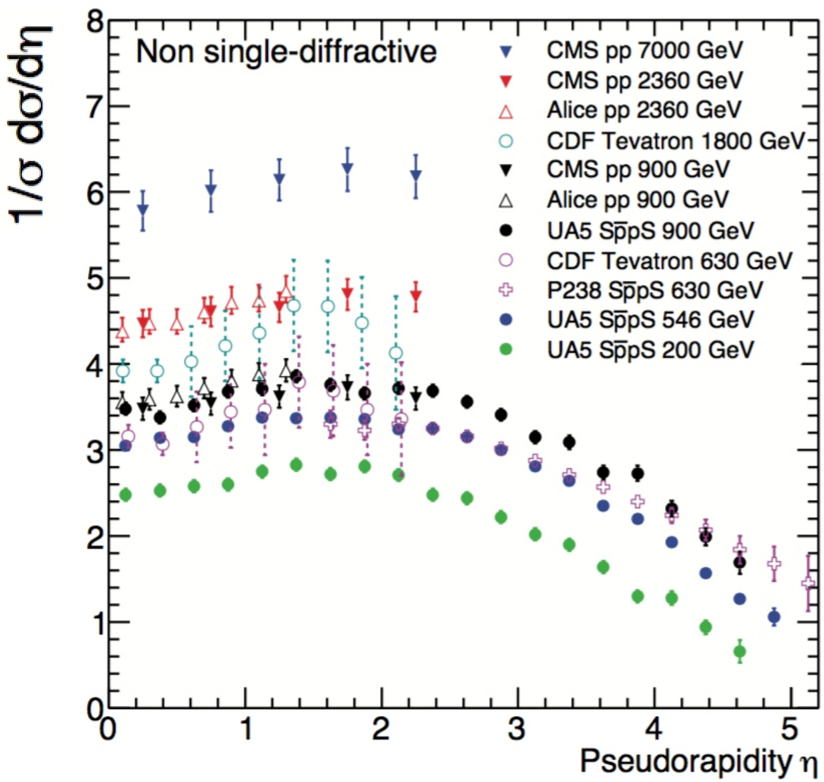
$$\langle N_{\mathrm{ch}} \rangle \propto \ln \sqrt{s} \quad \text{(follows from 'Feynman scaling')},$$

$$\text{i.e., from } E \frac{d^3\sigma}{d^3p} = F(x_F) \cdot F(p_T) \stackrel{!}{=} B \cdot F(p_T), x_F = \frac{p_L^*}{\sqrt{s}/2}$$

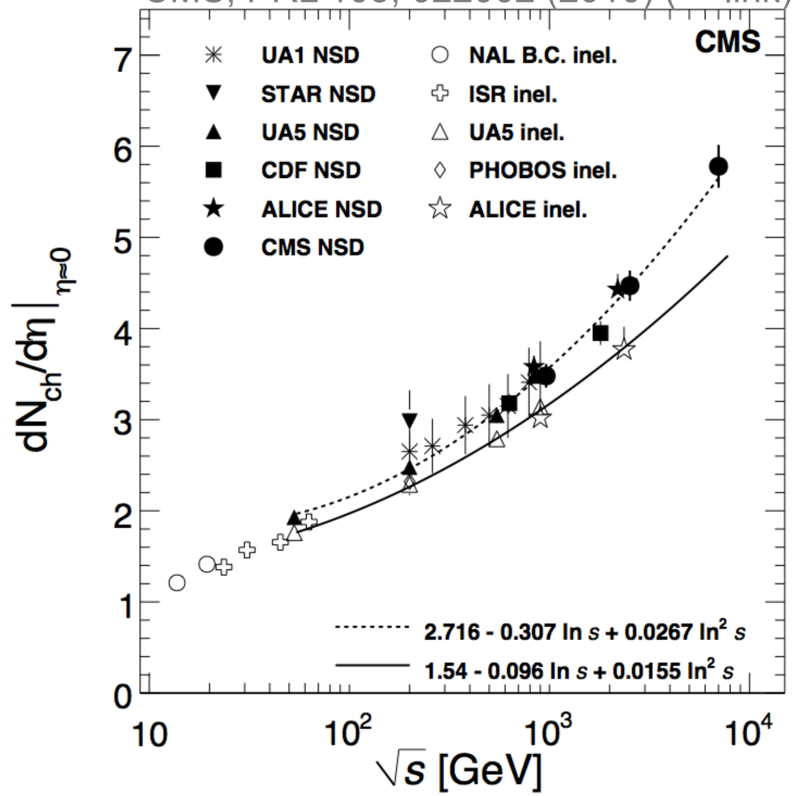
  - Maximum beam rapidity also scales as ln √s, thus Feynman scaling implies

$$dN/dy = \text{constant} \quad \text{(i.e., independent of } \sqrt{s})$$

√s Dependence of $dN_{ch}/d\eta$



CMS, PRL 105, 022002 (2010) (→ link)



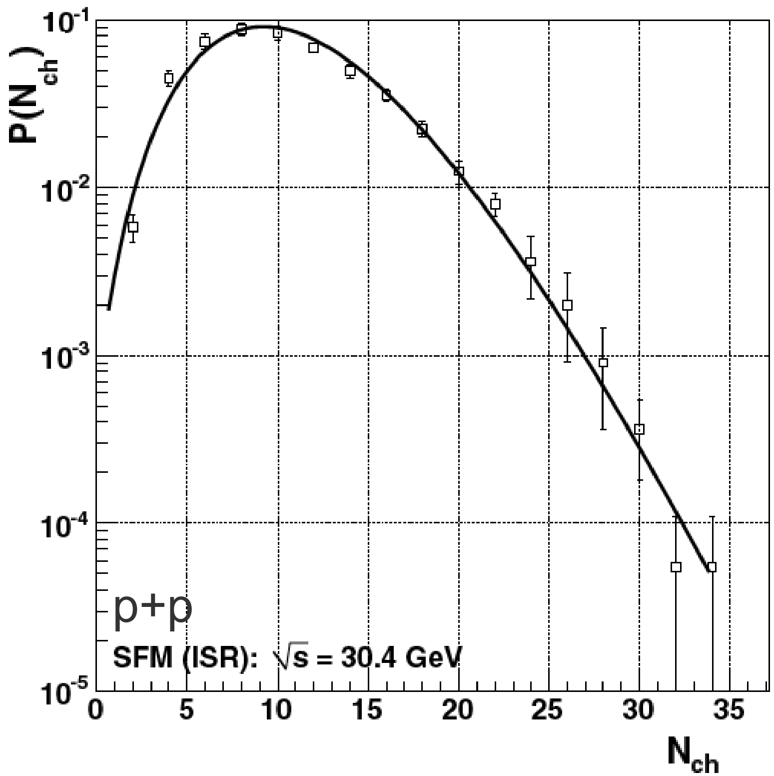
$dN_{ch}/d\eta$ rises with $\sqrt{s}$: This corresponds to a violation of Feynman scaling

CMS parameterization: $dN_{\mathrm{ch,NSD}}/d\eta|_{\eta=0} = 2.716 - 0.307 \ln s + 0.0267 \ln^2 s$

Rise with $\sqrt{s}$ also nicely described with: $dN_{\mathrm{ch}}/d\eta|_{\eta=0} \propto s^{0.11}$

J. F. Grosse-Oetringhaus, K.R., Charged Particle Multiplicity in Proton-Proton Collisions, 2010 (→ link)

Charged Particle Multiplicity Distributions



Multiplicity distributions in pp, e^+e^-, and lepton-hadron collisions well described by a Negative Binomial Distribution (NBD).

However, deviations from the NBD were discovered by UA5 at $\sqrt{s}$ = 900 GeV and later confirmed at the Tevatron at $\sqrt{s}$ = 1800 GeV (shoulder structure at $n \approx 2 \langle n \rangle$)

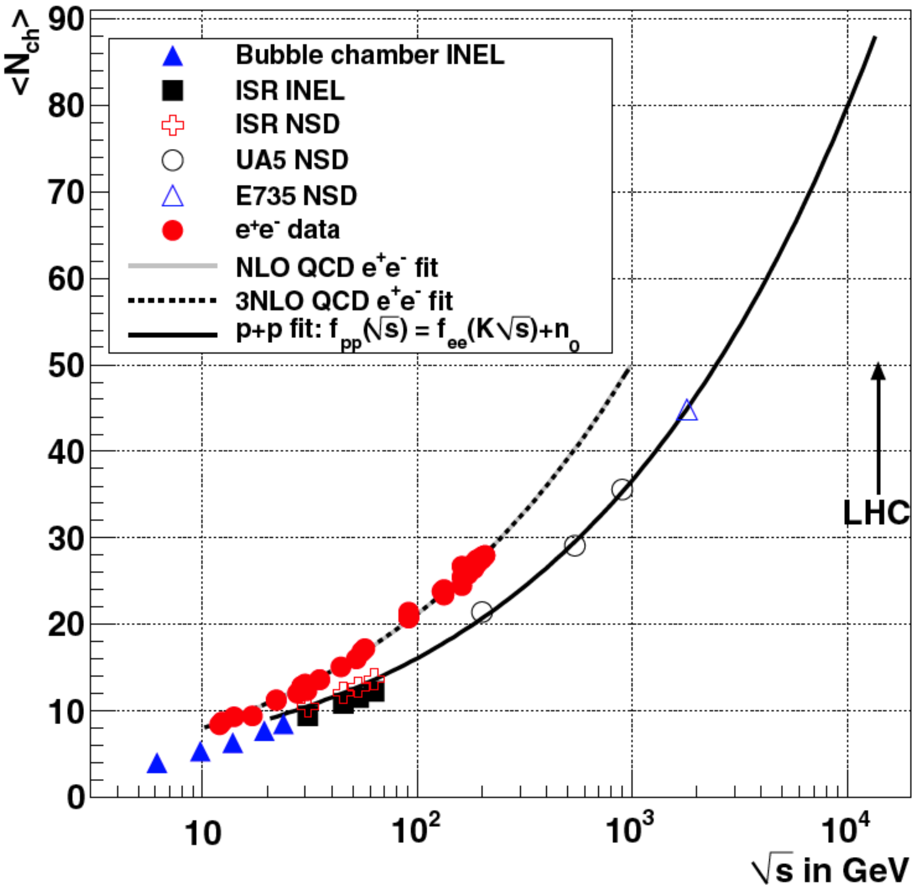
In limited η-intervals ($|\eta| < 0.5$) NBD describes the distributions up to 1.8 TeV

$$P^{\mathrm{NBD}}_{\mu,k}(n) = \frac{(n+k-1)\cdot(n+k-2)\cdot\ldots\cdot k}{\Gamma(n+1)} \left(\frac{\mu/k}{1+\mu/k}\right)^n \frac{1}{(1+\mu/k)^k}$$

$$\langle n \rangle = \mu,\ D := \sqrt{\langle n^2 \rangle - \langle n \rangle^2} = \sqrt{\mu\left(1+\frac{\mu}{k}\right)}$$

Limits of the NBD:
$k \to \infty$: Poisson distribution
integer k, k<0: Binomial distribution ($N = -k$, $p = -\langle n \rangle / k$)

Charged Particle Multiplicity in p+p and e+e-: An Interesting Similarity



The increase of N_{ch} with $\sqrt{s}$ looks rather similar in p+p and e^+e^-.

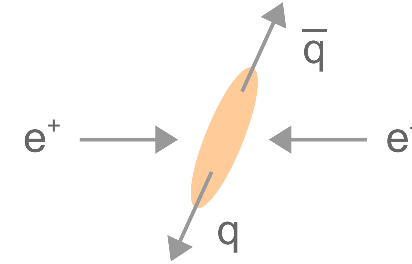
Roughly speaking, the energy available for particle production in p+p seems to be ~ 30 – 50%:

$$f(\sqrt{s}) := N_{ch}^{e+e-}(\sqrt{s})$$

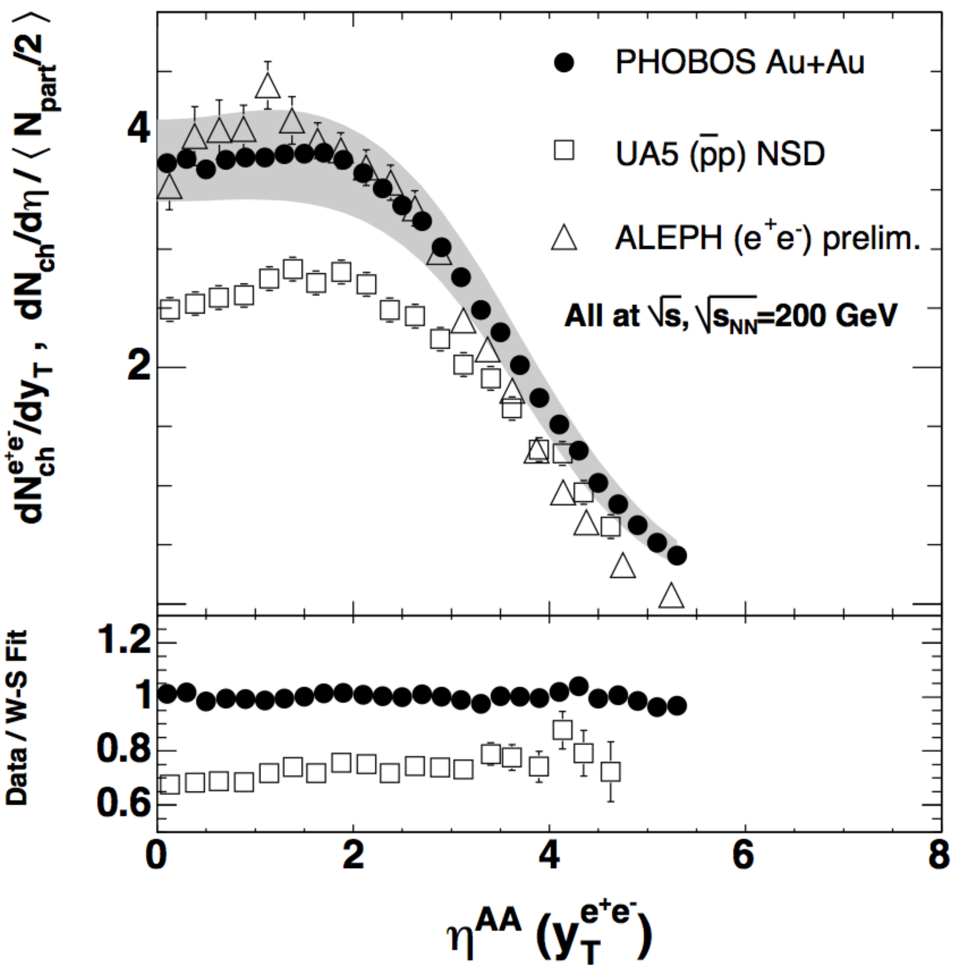
$$\rightarrow N_{ch}^{p+p} = f(K\sqrt{s_{pp}}) + n_0$$

A fit yields: $K \approx 0.35, \quad n_0 \approx 2.2$

Similarity of dN_{ch}/dy in e^+e^-, p+p, and A+A



PHOBOS, Nucl. Phys. A757, 28 (2005)



e^+e^-: Rapidity w.r.t. thrust axis:

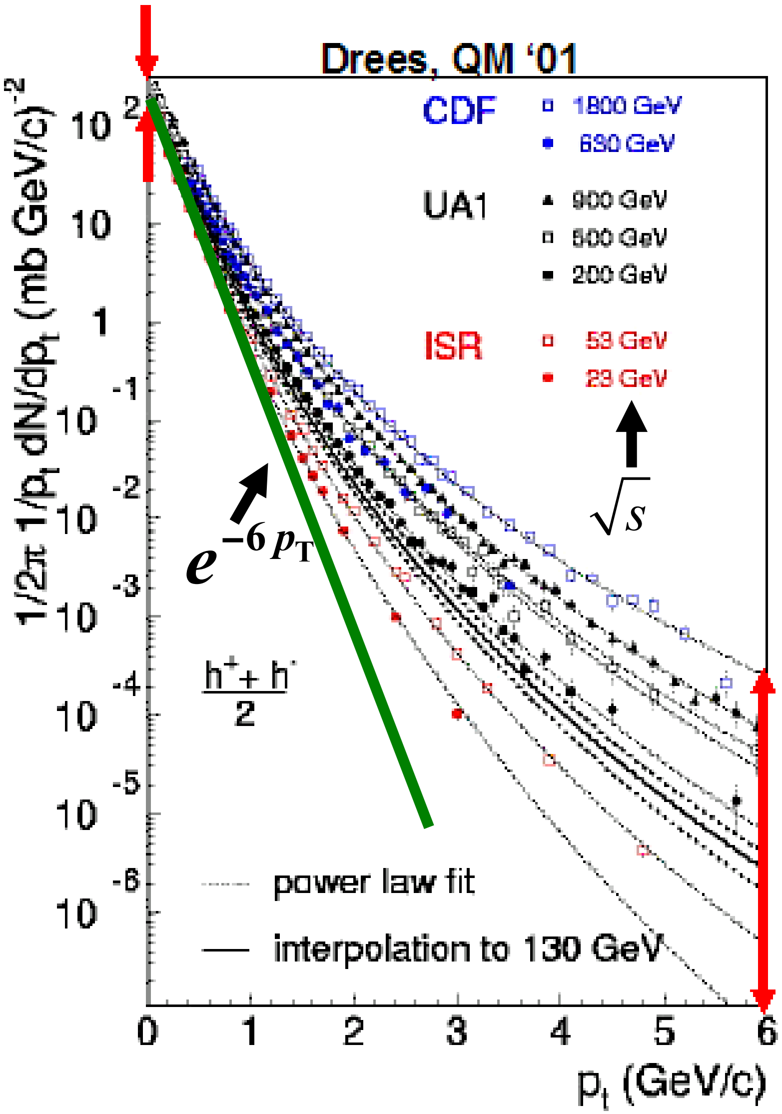
$$y_T^{e^+e^-} = \frac{1}{2}\ln\left(\frac{E + \vec{p}\cdot\vec{n}_{\text{thrust}}}{E - \vec{p}\cdot\vec{n}_{\text{thrust}}}\right)$$

Remarkable similarity between particle production in e^++e^-, p+p, **and** A+A

Effective energy fraction $K \approx 100\%$ in Au+Au

Hint at universal particle production mechanism?

Transverse Momentum Spectrum of Charged Particles



Transverse momentum spectra of charged particles for different $\sqrt{s}$:

Small p_T (roughly < 2 GeV/c):

$$\frac{1}{p_\mathrm{T}} \frac{\mathrm{d}N_x}{\mathrm{d}p_\mathrm{T}} \approx A(\sqrt{s}) \cdot e^{-6\,p_\mathrm{T}}$$

High p_T :

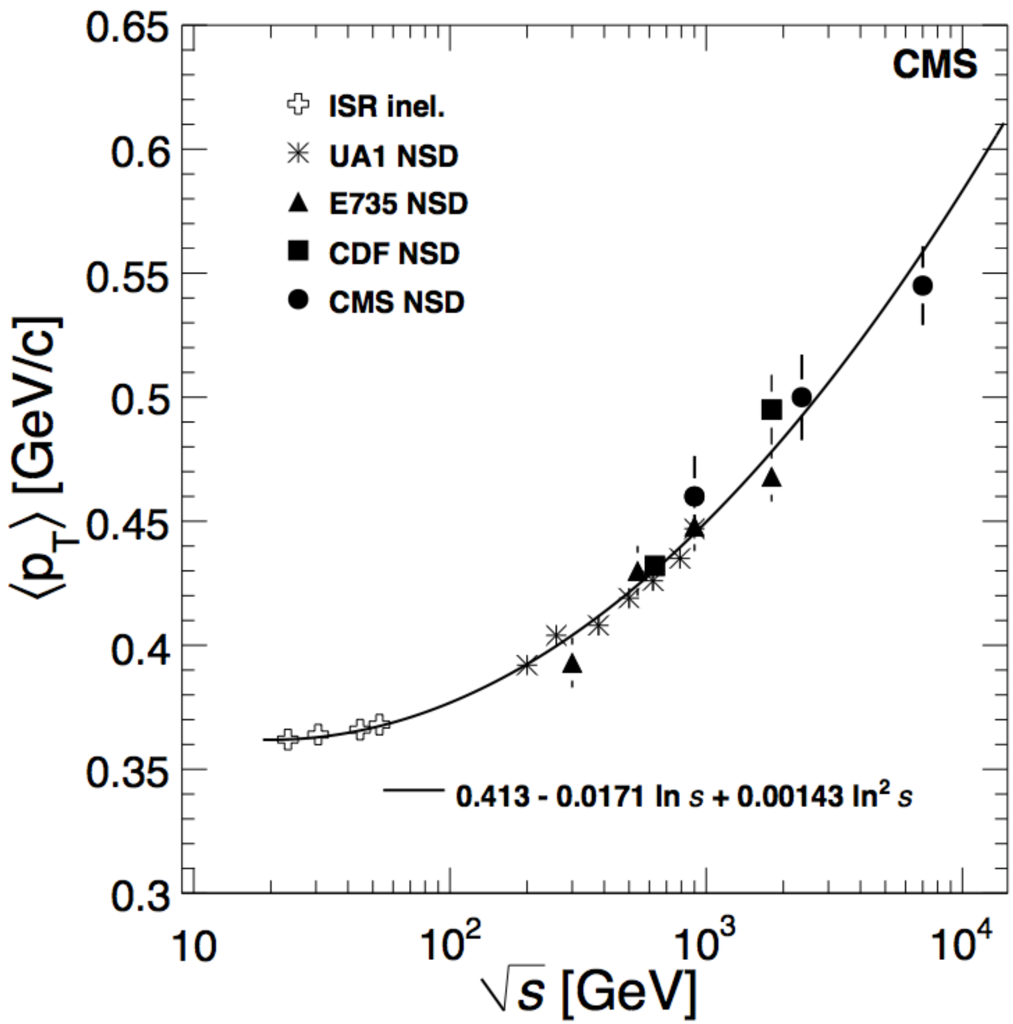
$$\frac{1}{p_\mathrm{T}} \frac{\mathrm{d}N_x}{\mathrm{d}p_\mathrm{T}} = B(\sqrt{s}) \cdot \frac{1}{p_\mathrm{T}^{n(\sqrt{s})}}$$

Average p_T :

$$\langle p_\mathrm{T} \rangle = \frac{\int_0^\infty p_\mathrm{T} \frac{\mathrm{d}N_x}{\mathrm{d}p_\mathrm{T}} \mathrm{d}p_\mathrm{T}}{\int_0^\infty \frac{\mathrm{d}N_x}{\mathrm{d}p_\mathrm{T}} \mathrm{d}p_\mathrm{T}} \approx 300 - 400 \mathrm{MeV}/c$$

pretty energy-independent for $\sqrt{s}$ < 100 GeV

$\langle p_T \rangle$ vs. $\sqrt{s}$

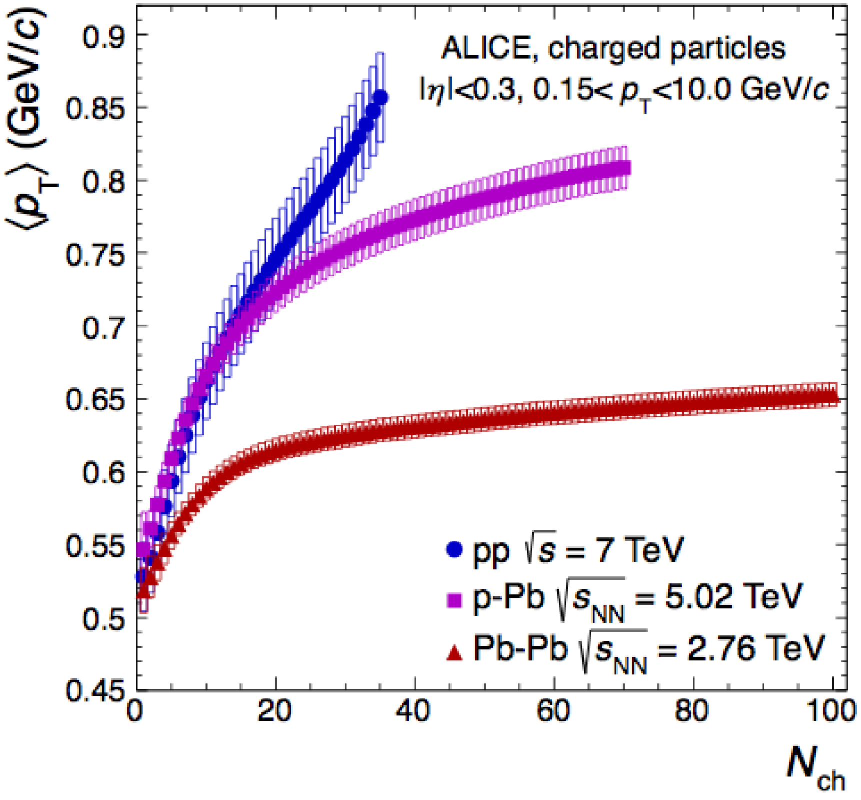


CMS, PRL 105, 022002 (2010)
(→ link)

CDF, PRL 61, 1819 (1988)

Increase of $\langle p_T \rangle$ with $\sqrt{s}$ (most likely) reflects increase in particle production from hard parton-parton scattering

$\langle p_T \rangle$ vs. N_{ch}



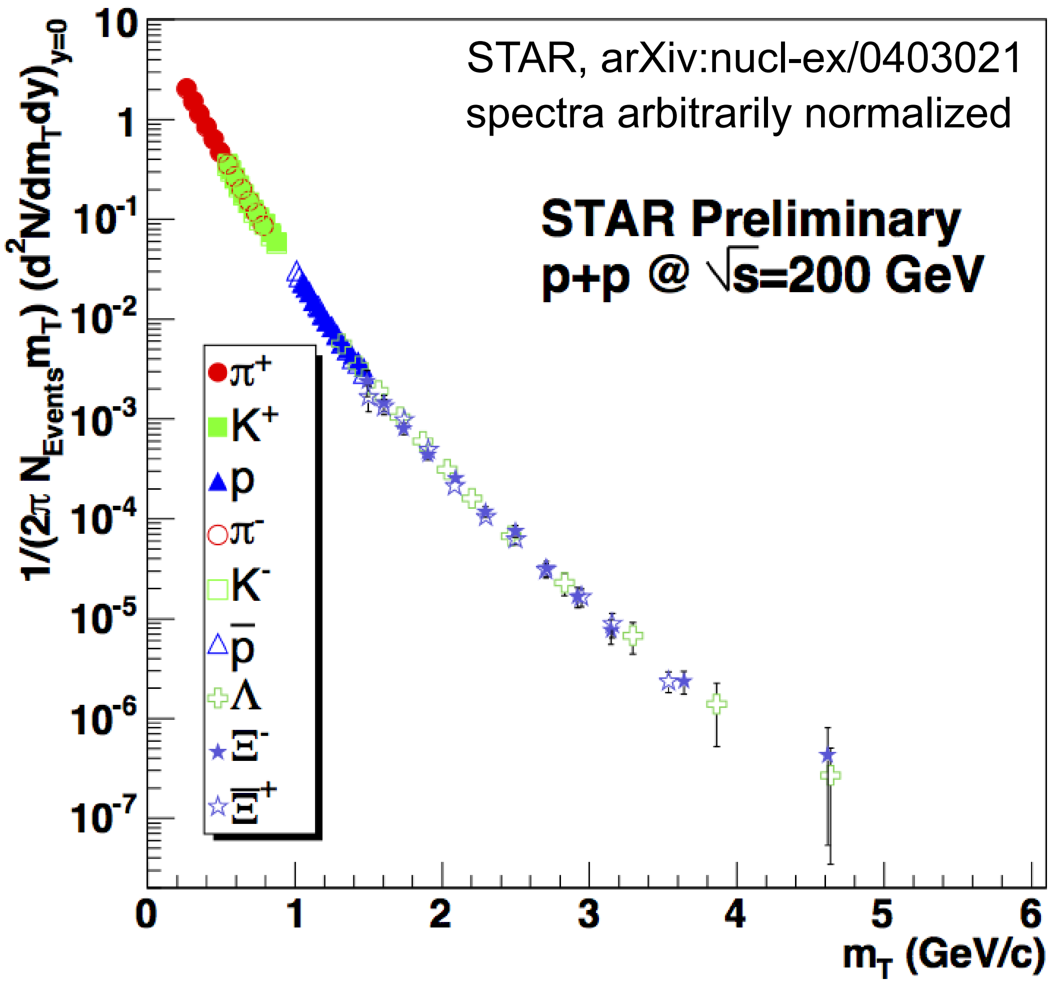
ALICE, arXiv:1307.1094 (→ Link)

For $\sqrt{s}$ > ~ 60 GeV the mean transverse momentum rises with N_{ch} in pp collisions.

The rise is still not fully understood. Multiple hard parton-parton scatterings in the same p+p collision are often used to explain it.

For it to work, however, each new interaction should add proportionately less to the total N_{ch} than to the total p_T.

m_T Scaling in pp Collisions



$$m_T = \sqrt{m^2 + p_T^2}$$

m_T scaling:

m_T spectra for different particle species (approximately) have the same shape

Example:

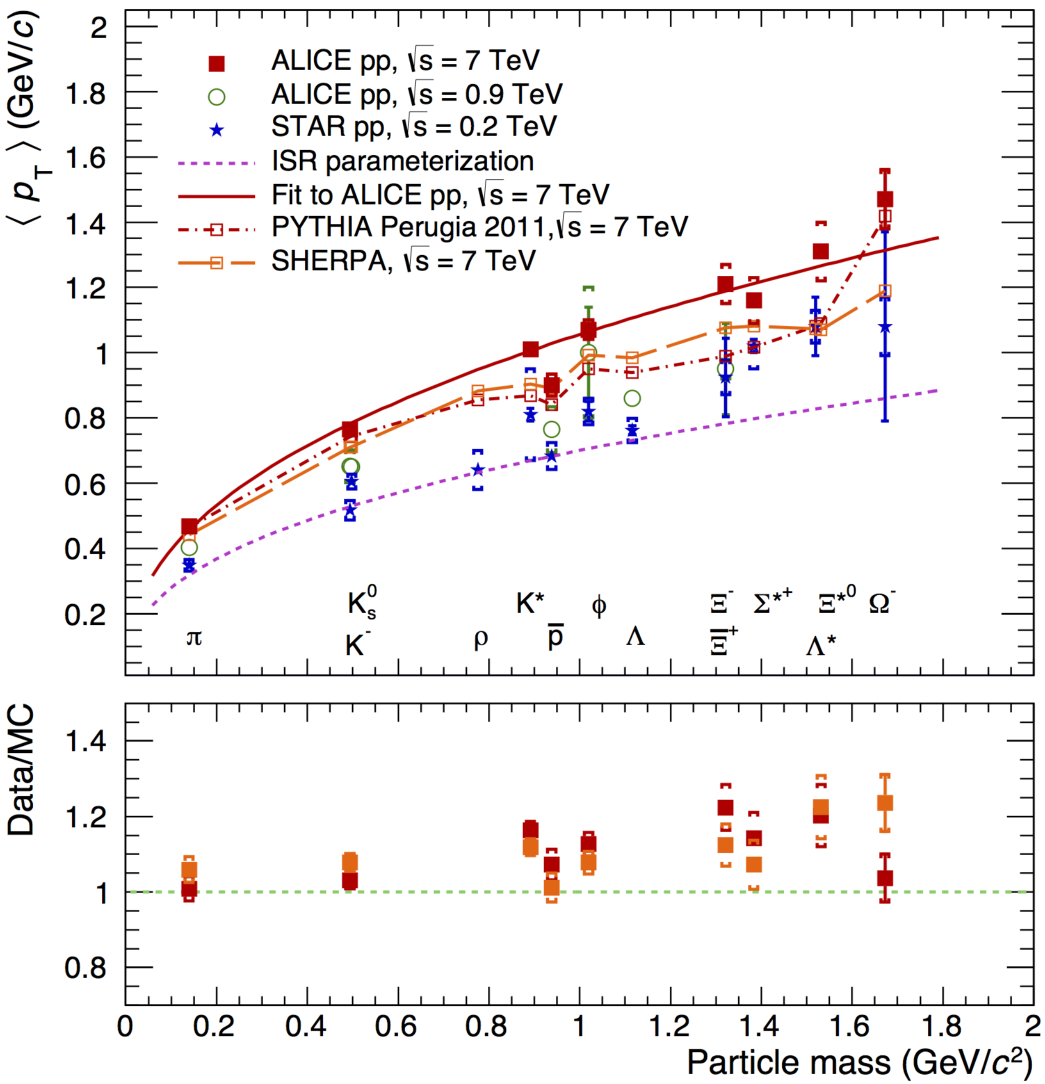
$$\frac{dN/dm_T|_\eta}{dN/dm_T|_{\pi^0}} \approx 0.45$$

Useful functional form (at low p_T):

$$E\frac{d^3\sigma}{d^3p} \propto \frac{1}{\exp(m_T/T) - 1}$$

Bartke et al.,
Nucl.Phys., B120, 14 (1976), → link

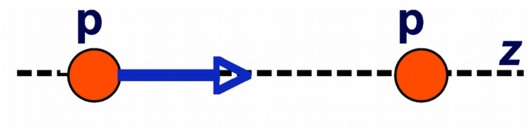
<p_T> for Different Particle Species



ALICE, arXiv:1406.3206

Average p_T increases with particle mass

Stopping in Nucleon-Nucleon Collisions

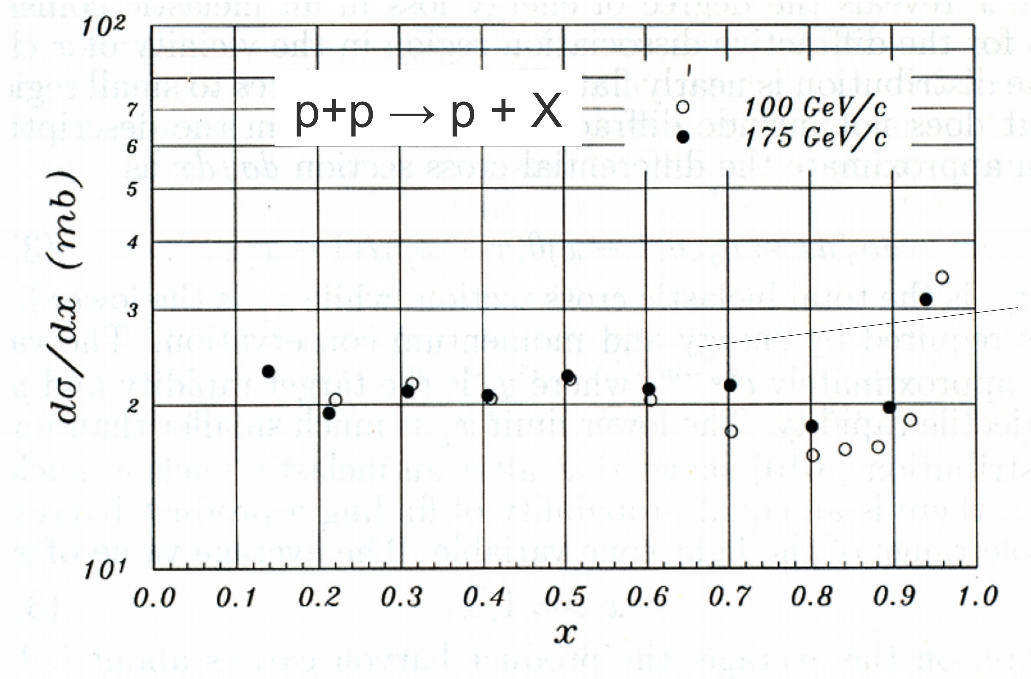


Feynman-*x* $x_{\mathrm{F}} := \frac{p_z}{p_{z,0}} \approx \frac{E}{E_0} \approx \frac{m_{\mathrm{T}}}{m} e^{y-y_0}$

Longitudinal momentum before collisions: $p_{z,0}$

Longitudinal momentum after collisions: p_z

$$E \approx \frac{m_T}{2} e^y$$



$$\frac{\mathrm{d}n_p}{\mathrm{d}y} = \underbrace{\frac{\mathrm{d}n_p}{\mathrm{d}x_{\mathrm{F}}}}_{\approx \, \text{constant}} \cdot \frac{\mathrm{d}x_{\mathrm{F}}}{\mathrm{d}y} \propto e^{y-y_0}$$

Feynman-*x* distribution of the leading proton is approximately constant.

$$\langle y \rangle \approx \frac{\int_{-\infty}^{y_0} y \, e^{y-y_0} \, \mathrm{d}y}{\int_{-\infty}^{y_0} e^{y-y_0} \, \mathrm{d}y} = y_0 - 1$$

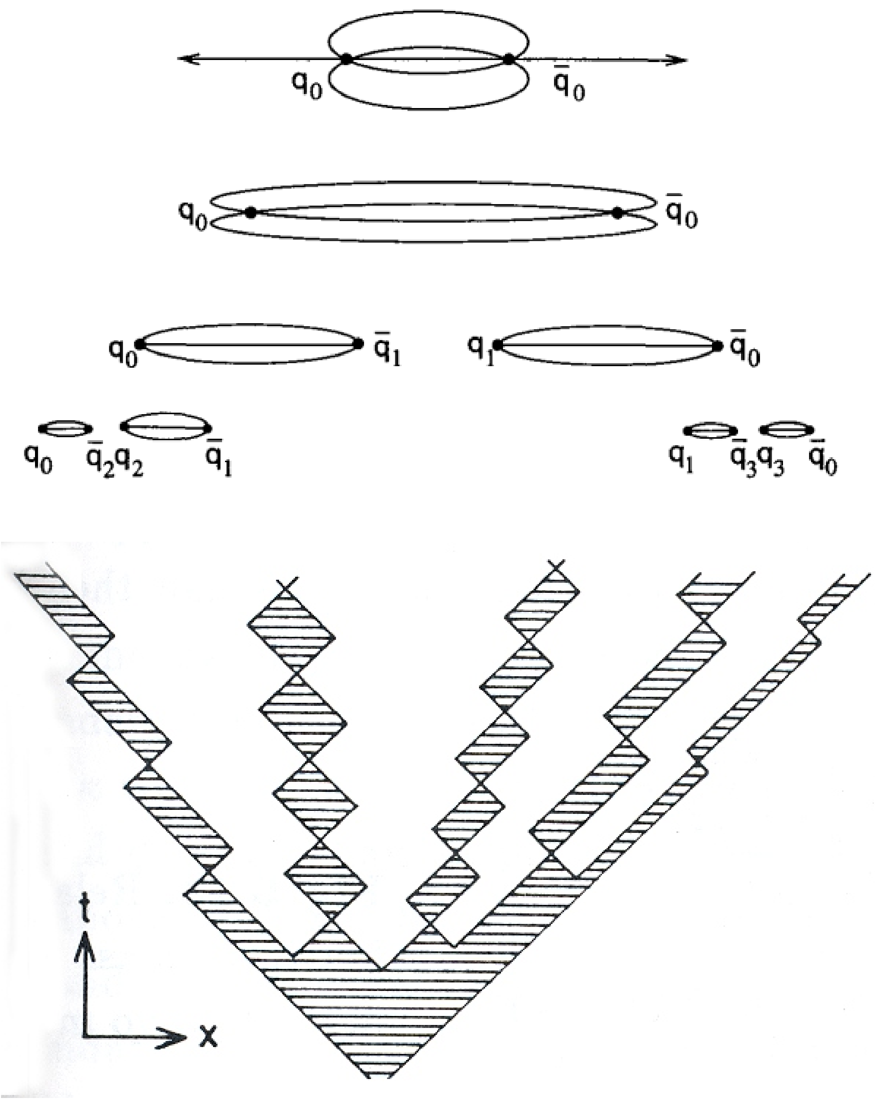
On average, a proton loses roughly one unit of rapidity ($\Delta y \approx 1$) in an inelastic p+p collision (approximately independent of the initial energy (?))

Classical String Model: History

- Idea: Hadrons = Quarks connected via a color flux tube (“string”), i.e., a tube which contains the color field lines
- Initially conceived as a fundamental theory of the strong interaction (end of the 1960's), but substantial problems (unobserved massless states, inconsistent in 3+1 dimensions, ...)
- In the beginning of the 1970's QCD became the accepted theory of the strong interaction
- Today: string model for hadrons is a phenomenological model for (soft) particle production
- Interestingly, the mathematical framework of the hadronic string theory developed into today's supersymmetric string theory
  - Elementary particles (quarks and leptons) = vibrating strings
  - Dimensions of the string ~ 10^{-35} m (Planck length)

History of string theory:
- http://www.damtp.cam.ac.uk/user/mbg15/superstrings/superstrings.html (→ link)
- workshop 'The birth of string theory' (→ link)

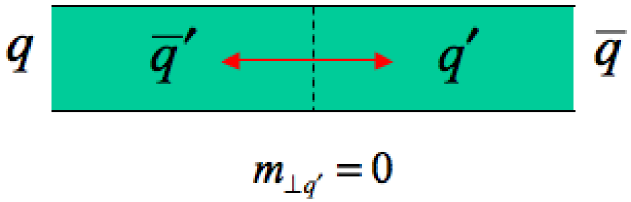
Classical String Model: Particle Production via String Breaking (I)

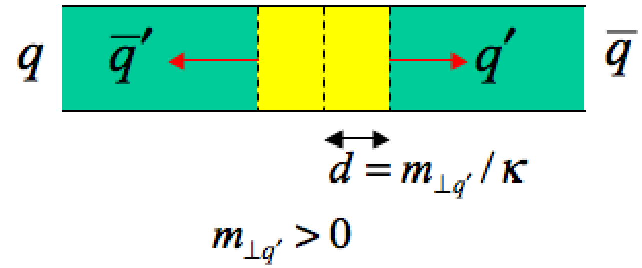


- We consider a string formed by a quark-antiquark pair
- The string can break by producing quark-antiquark pairs in the intense color field
- The basic assumption of the symmetric Lund model is that the vertices at which the quark and the antiquark are produced lie approximately on a curve on constant proper time
- This characteristics leads to a flat rapidity distribution of the produced particles

Classical String Breaking:
String Breaking via Tunneling (I)

In the Lund scheme, quantum mechanical tunneling leads to the q-qbar break-ups:





In terms of the transverse mass of q' the probability that the break-up will occur is:

$$P \propto \exp\left(-\frac{\pi m_{\perp q'}^2}{k}\right) = \exp\left(-\frac{\pi p_{\perp q'}^2}{k}\right)\exp\left(-\frac{\pi m_{q'}^2}{k}\right)$$

This leads to a transverse momentum distribution for the quarks of the form:

$$\frac{\mathrm{d}N_\mathrm{quark}}{\mathrm{d}p_T} = \mathrm{const.} \cdot \exp\left(-\pi p_T^2/k\right) \quad \rightsquigarrow \quad \sqrt{\langle p_T^2 \rangle_\mathrm{quark}} = \sqrt{k/\pi}$$

For pions (two quarks) one obtains: $\sqrt{\langle p_T^2 \rangle_\mathrm{pion}} = \sqrt{2k/\pi}$

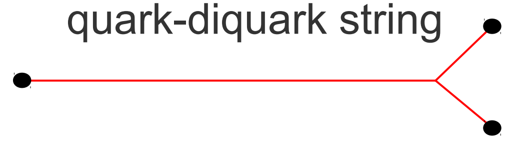
With a string tension of 1 GeV/fm this yields $\langle p_T \rangle_\mathrm{pion} \approx 0.37$ GeV/c, in agreement with data

Classical String Breaking:
String Breaking via Tunneling (II)

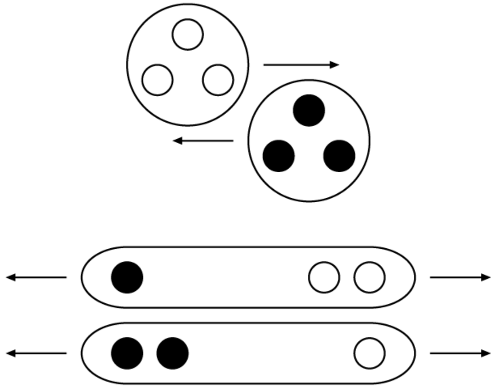
The tunneling process implies heavy-quark suppression:

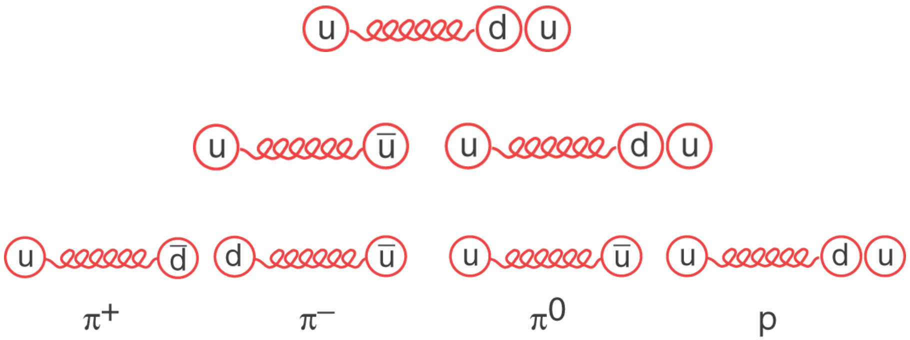
$$u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$$

The production of baryons can be modeled by replacing the q-qbar pair by an quark-diquark pair



Collisions of hadrons described as excitation of quark-diquarks strings:





Classical String Model: Summary

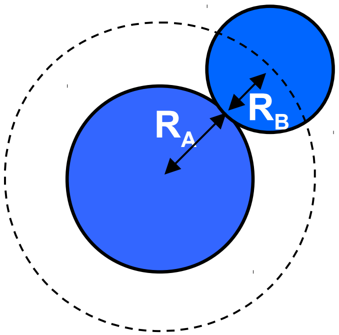
- The string model is strongly physically motivated and intuitively compelling
- It describes many general features of particle production in hadronic collisions
  - Average transverse momentum
  - $\sqrt{s}$ independence of $\langle p_T \rangle$ (string breaking is a local process)
  - Shape of the rapidity distribution of the produced particles
- Universal, after fitting to e^+e^- data little freedom elsewhere
- But: It has many free parameters, particularly for the flavor sector

See also:
P. Richardson, Lecture at CTEQ school, 2006 (→ link)
Torbjörn Sjöstrand (→ link)

Part II: Nucleus-Nucleus Collisions

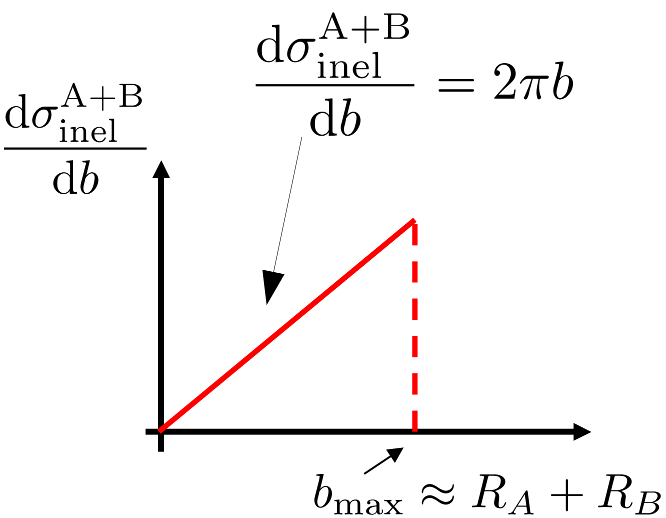
Ultra-Relativistic Nucleus-Nucleus Collisions: Many Aspects Controlled by Nuclear Geometry

- Ultra-relativistic energies
  - De Broglie wave length much smaller than size of the nucleon
  - Wave character of the nucleon can be neglected for the estimation of the total cross section
- Nucleus-Nucleus collision can be considered as a collision of two black disks

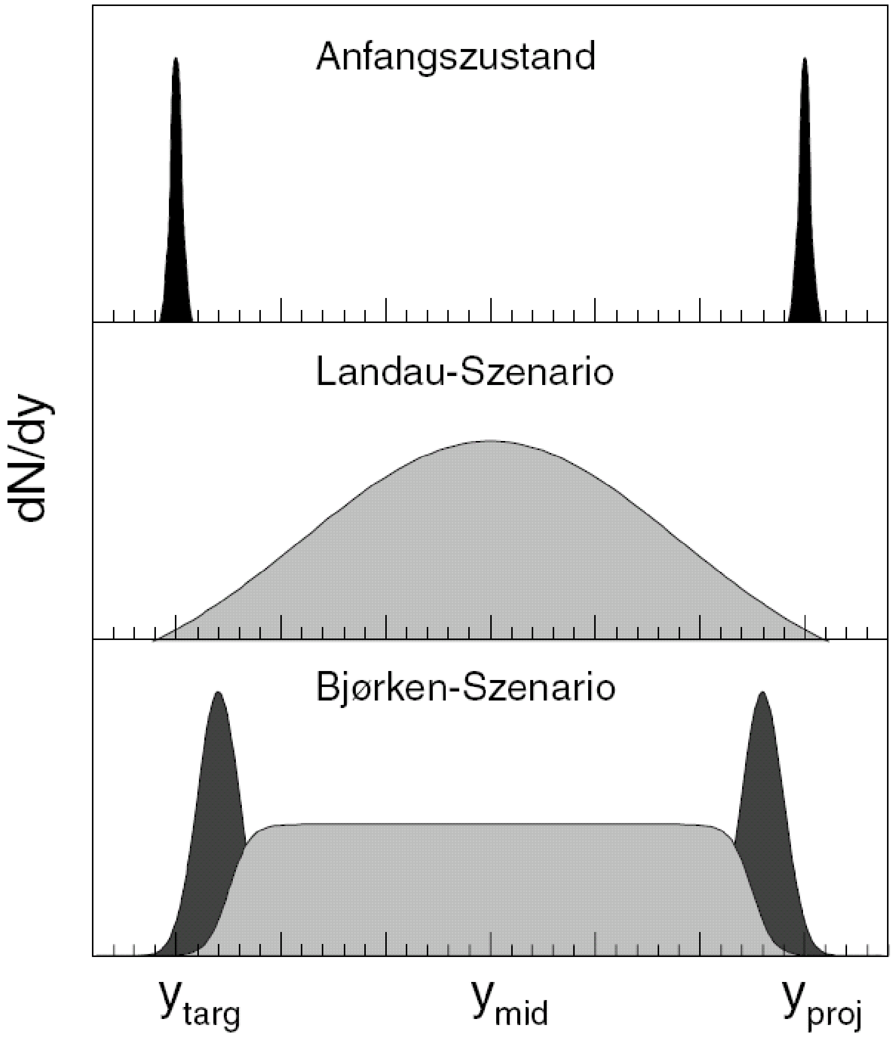


$$R_A \approx r_0 \cdot A^{1/3}, \; r_0 = 1,2\,\mathrm{fm}$$

$$\sigma_{\mathrm{inel}}^{\mathrm{A+B}} \approx \sigma_{\mathrm{geo}} \approx \pi r_0^2 (A^{1/3} + B^{1/3})^2$$



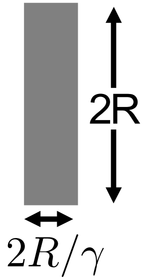
Nucleus-Nucleus Collisions: Landau and Bjorken Scenario



- Landau scenario
  - Complete stopping of the nuclei
  - Initial condition for hydrodynamic expansion

$$V_0 = V_{\mathrm{nucleus}}^{\mathrm{rest}} / \gamma_{\mathrm{CMS}}$$

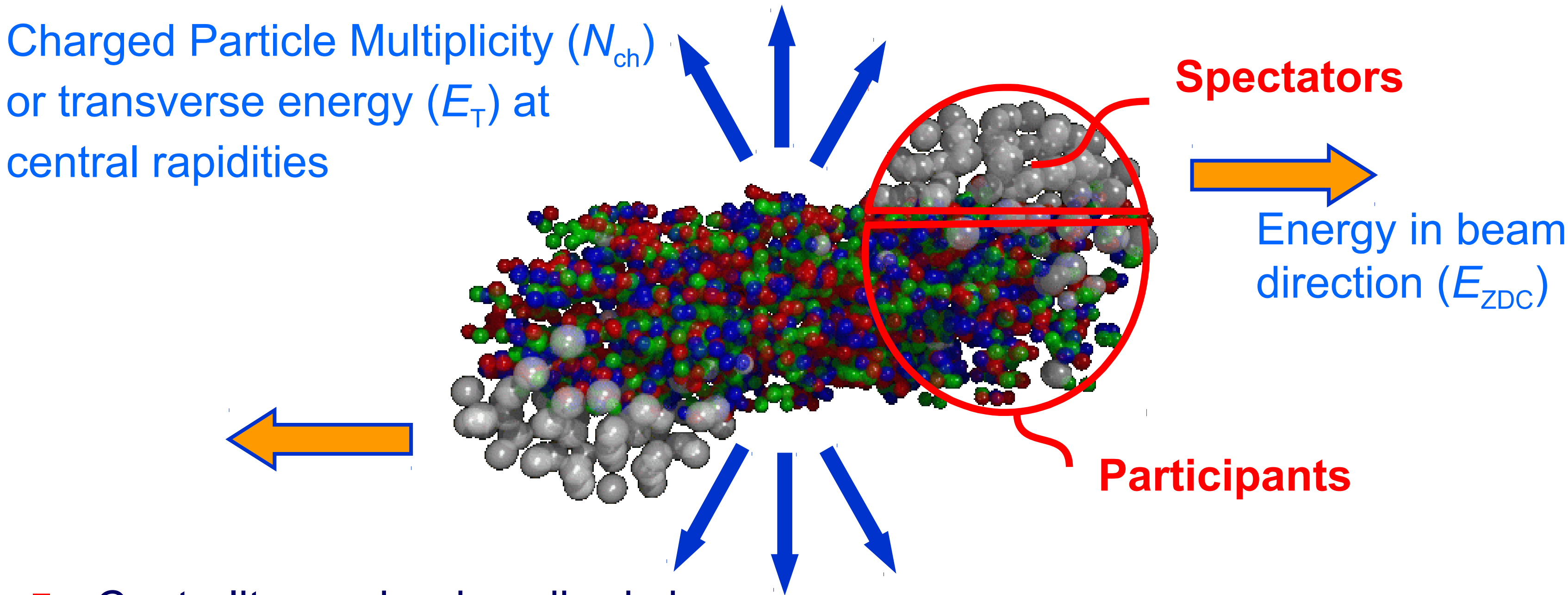
$$\varepsilon_0 = \sqrt{s}/V$$



- Bjorken scenario
  - transparency
  - flat rapidity distribution

Complete stopping of the nuclei in central collisions up to $\sqrt{s_{NN}}$ ~ 5 - 10 GeV, transparency (baryon-free QGP at central rapidities) for $\sqrt{s_{NN}}$ > ~ 100 GeV

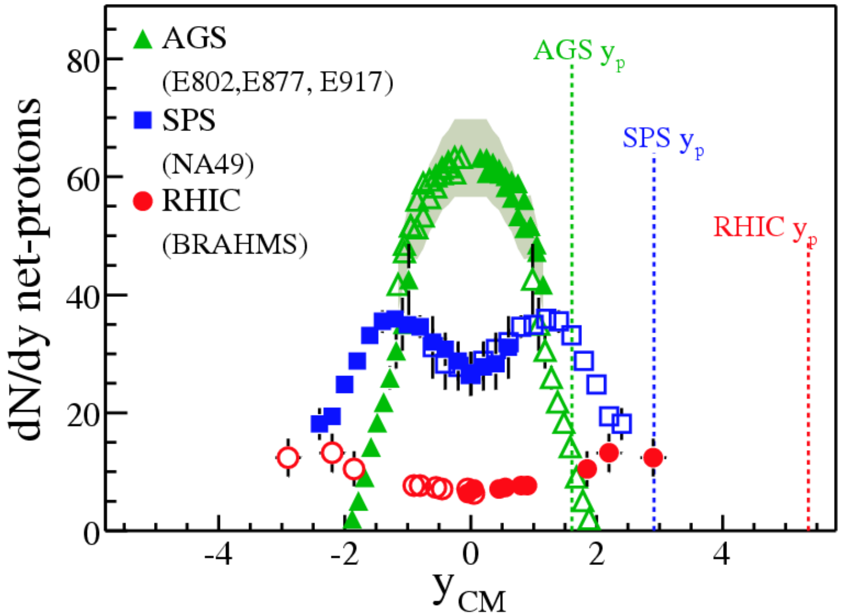
N_{part} and N_{coll} in Nucleus-Nucleus-Collisions

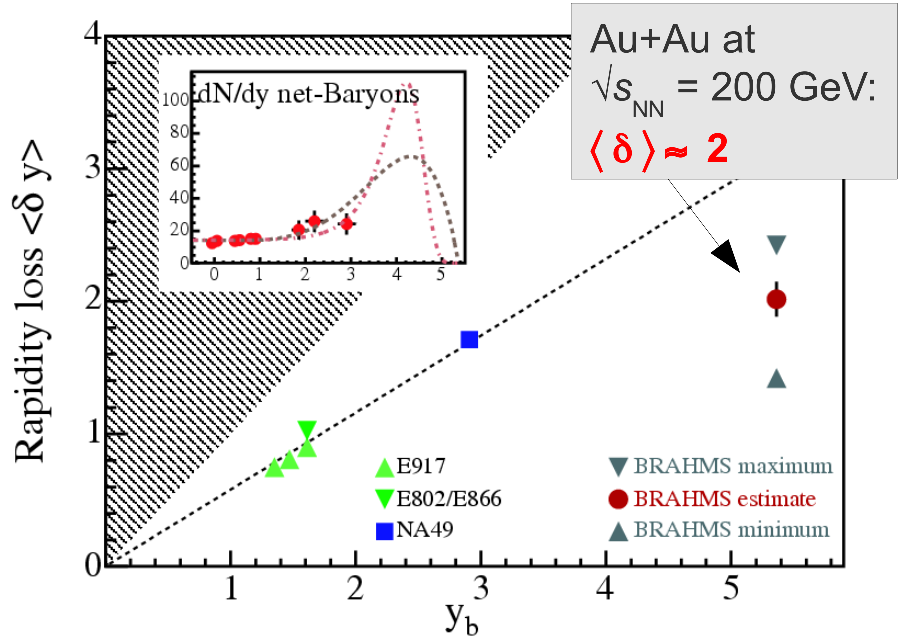


- Centrality can be described via
  - N_{coll}: number of inelastic nucleon-nucleon collisions
  - N_{part}: number of nucleons which underwent at least one inelastic nucleon-nucleon collisions
- This simplifies the comparison between theory and experiment and between different experiments
- Typically not directly measured but determined from Glauber calculations

Stopping in A+A Collisions

Brahms, PRL 93:102301, 2004 (→ Link)





Stopping inferred from rapidity distribution of net-baryons (baryons-antibaryons)

$$\langle \delta y \rangle = y_p - \langle y \rangle \qquad \langle y \rangle = \frac{2}{N_\mathrm{part}} \int_0^{y_p} y \frac{dN_{B-\bar{B}}}{dy}\, dy$$

MC generator used to go from the measured net-protons to net-baryons

Average energy per net baryon:

$$E = \frac{1}{N_\mathrm{part}} \int_{-y_p}^{y_p} \langle m_T \rangle \cosh y \frac{\mathrm{d}N_{B-\bar{B}}}{\mathrm{d}y}\, \mathrm{d}y \approx 27 \pm 6\,\mathrm{GeV}$$

Thus, the average energy loss of a nucleon in central Au+Au@200GeV is 73 ± 6 GeV

Particle Multiplicities in p+A (d+A) Collisions

- Proton-nucleon collision
  - Example:

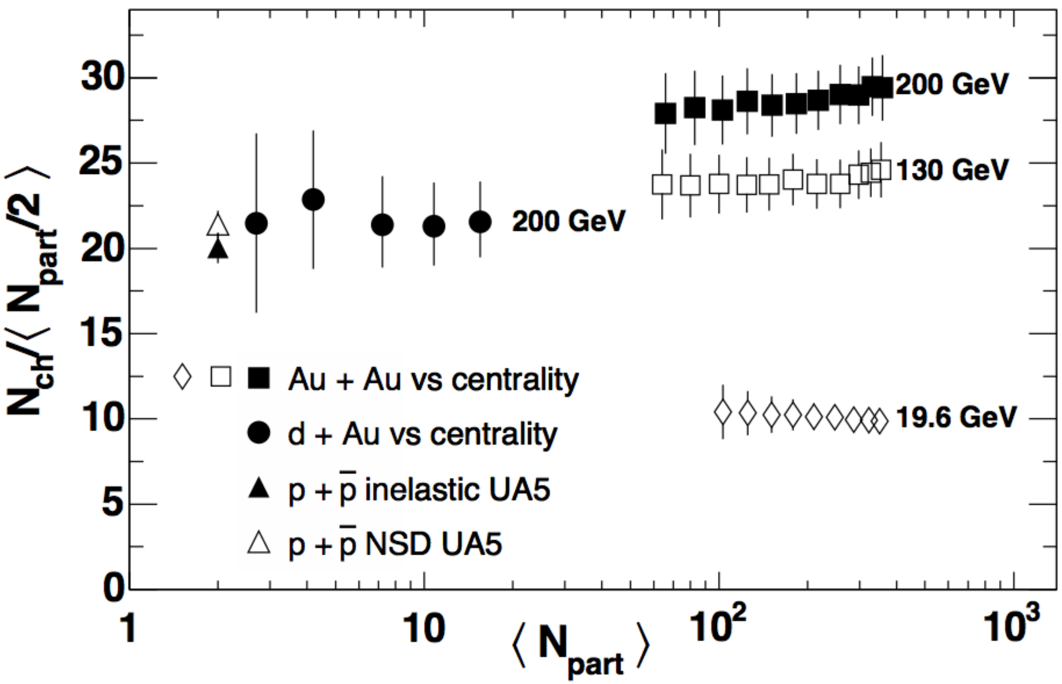
$N_{coll} = 3$, $N_{part} = 4$

- How do particle multiplicities scale? With N_{part} or N_{coll}?
- Observation: Particle multiplicities scale with N_{part}

$$\left\langle N_{ch}^{p+A} \right\rangle \approx \frac{N_{\text{part}}}{2} \left\langle N_{ch}^{p+p} \right\rangle$$

(Wounded Nucleon Model)

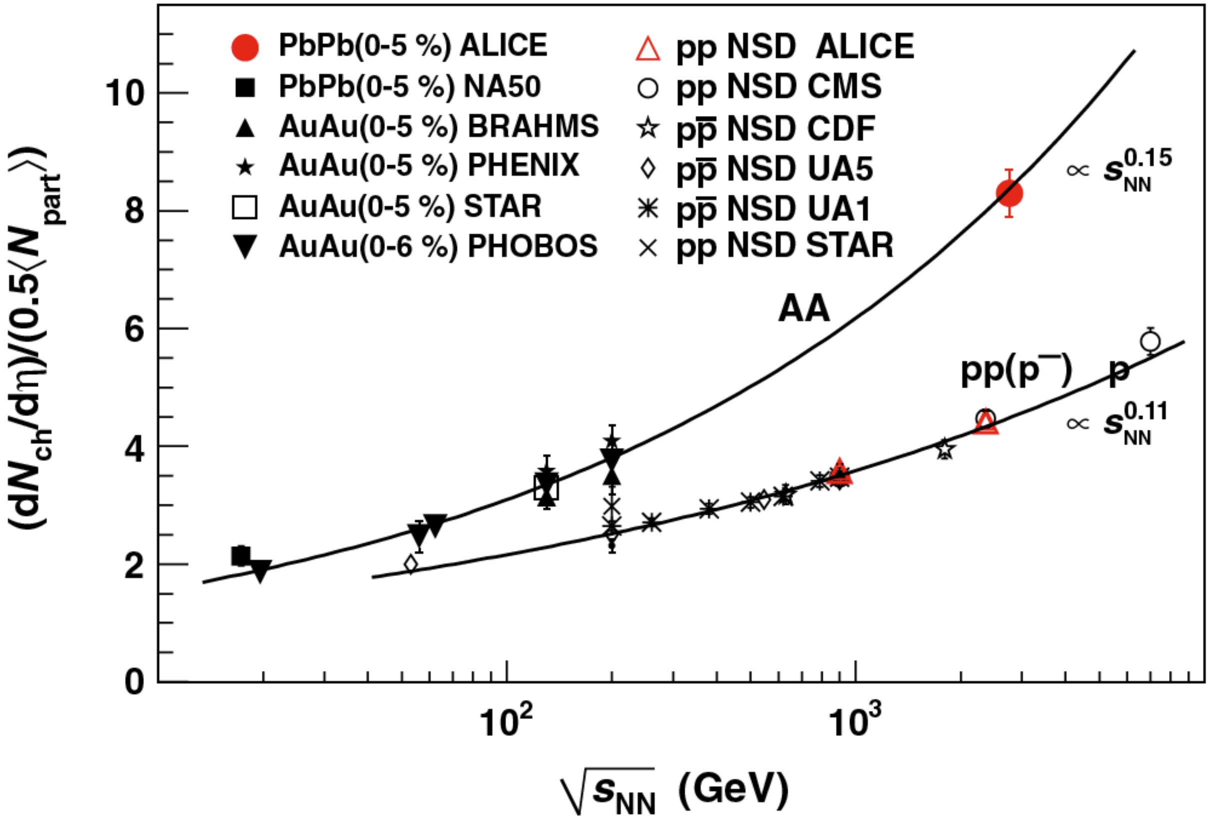
PHOBOS, Nucl.Phys. A757 (2005) 28-101 (→ link)



However, from d+Au to Au+Au, there is a jump in N_{ch}/N_{part}!

$\sqrt{s_{NN}}$ Dependence of the Charged Particle Multiplicity in Central A+A Collisions

ALICE: http://link.aps.org/doi/10.1103/PhysRevLett.105.252301 (→ link)



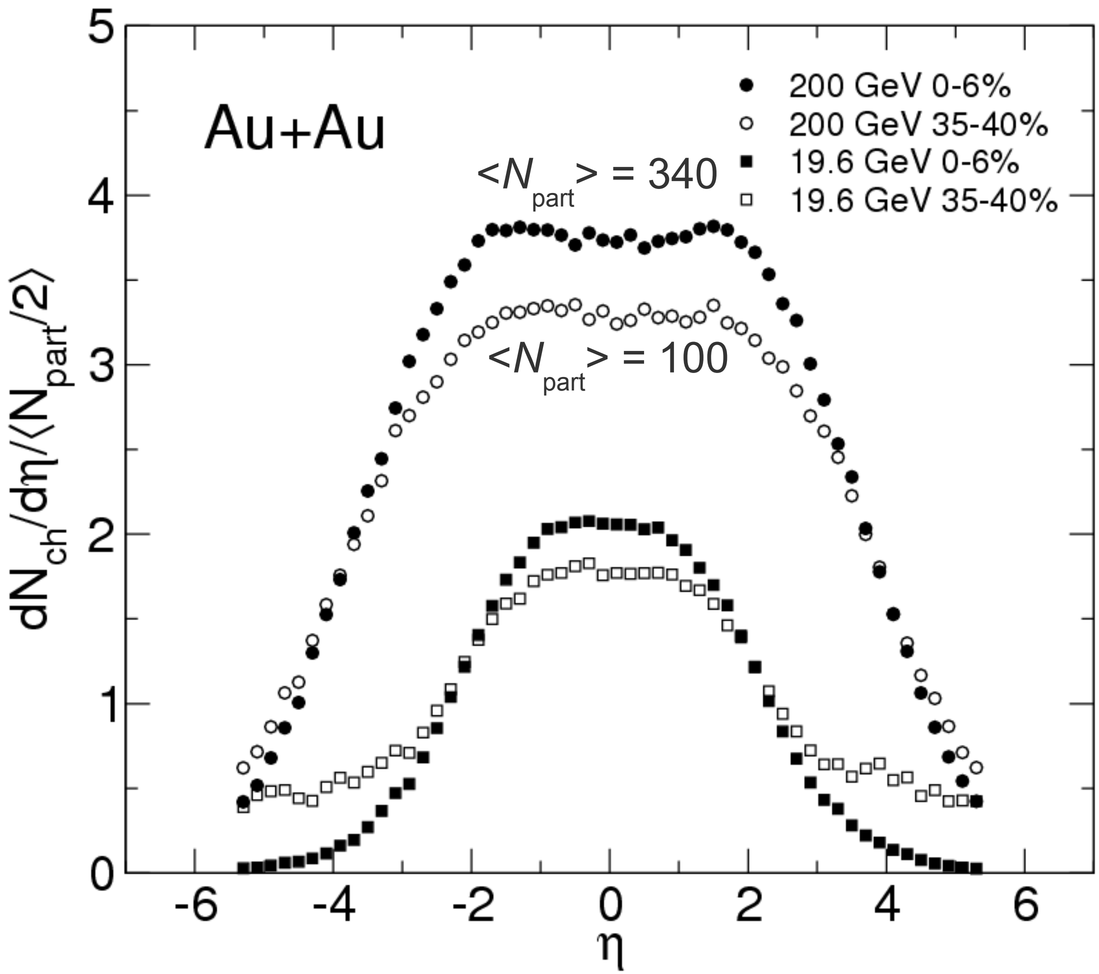
From $\sqrt{s_{NN}}$ = 200 GeV (Au+Au, RHIC) to $\sqrt{s_{NN}}$ = 2760 GeV (Pb+Pb, LHC) the charged particle multiplicity increases by about a factor 2.2.

Stronger increase with $\sqrt{s}$ in central A+A than in p+p

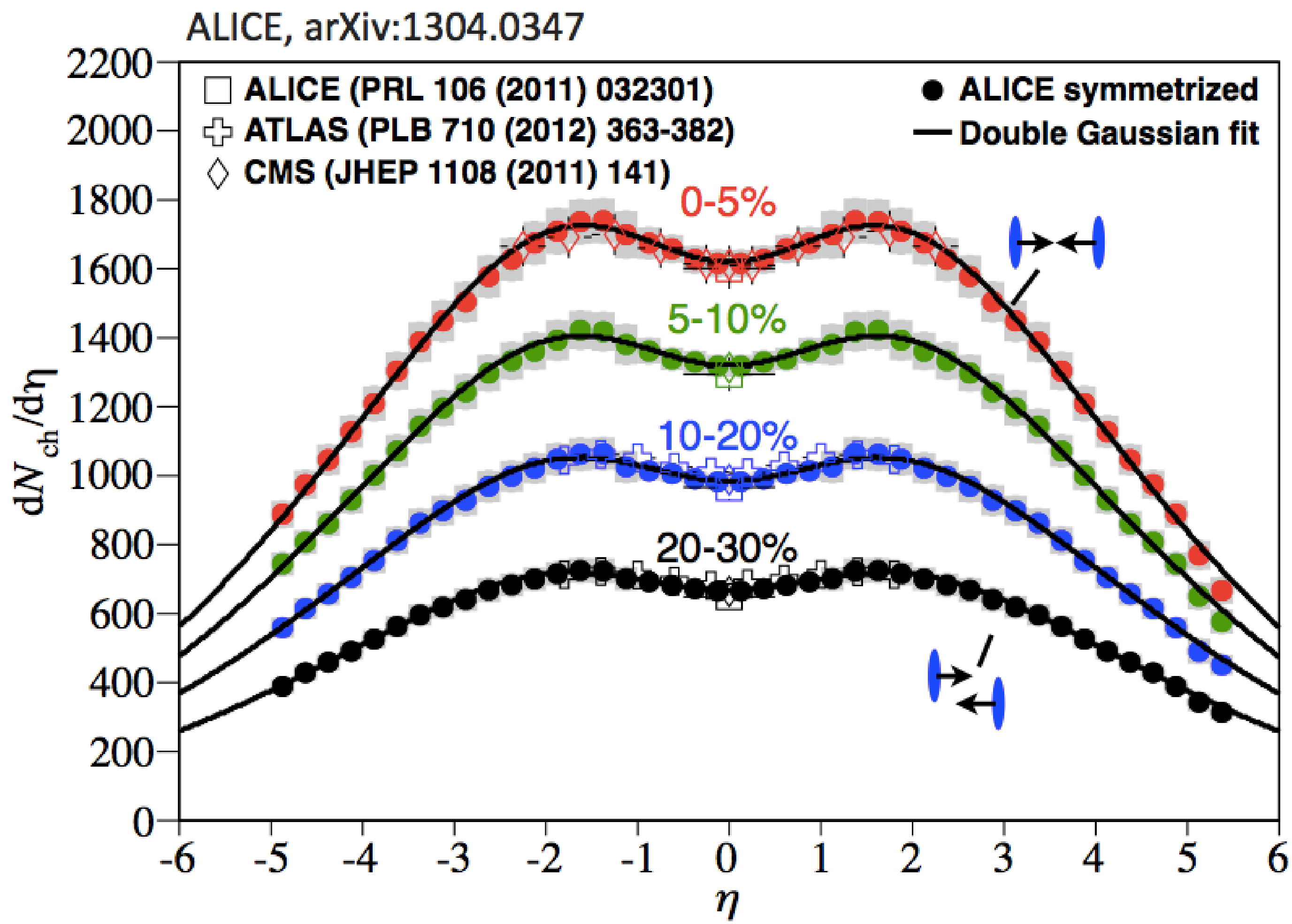
Charged Particle Pseudorapidity Distributions in Au+Au Collisions at 19.4 and 200 GeV

- Multiplicity increases with centrality
- N_{part} scaling only approximately satisfied
- Total charged particles multiplicity in central Au+Au at 200 GeV:

  ≈ 5000

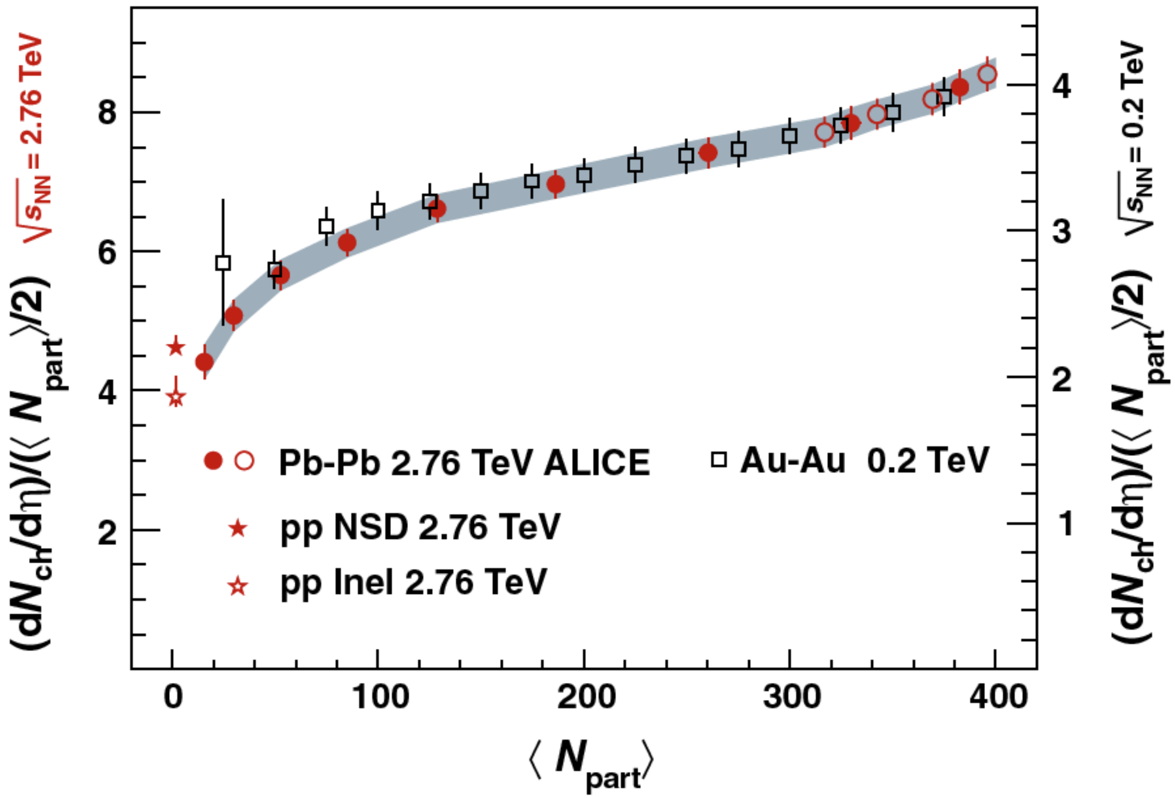


Charged Particle Pseudorapidity Distributions in Pb+Pb Collisions at 2760 GeV



~ 25 000 produces particles in total in central Pb-Pb
(full phase space, charged + neutrals)

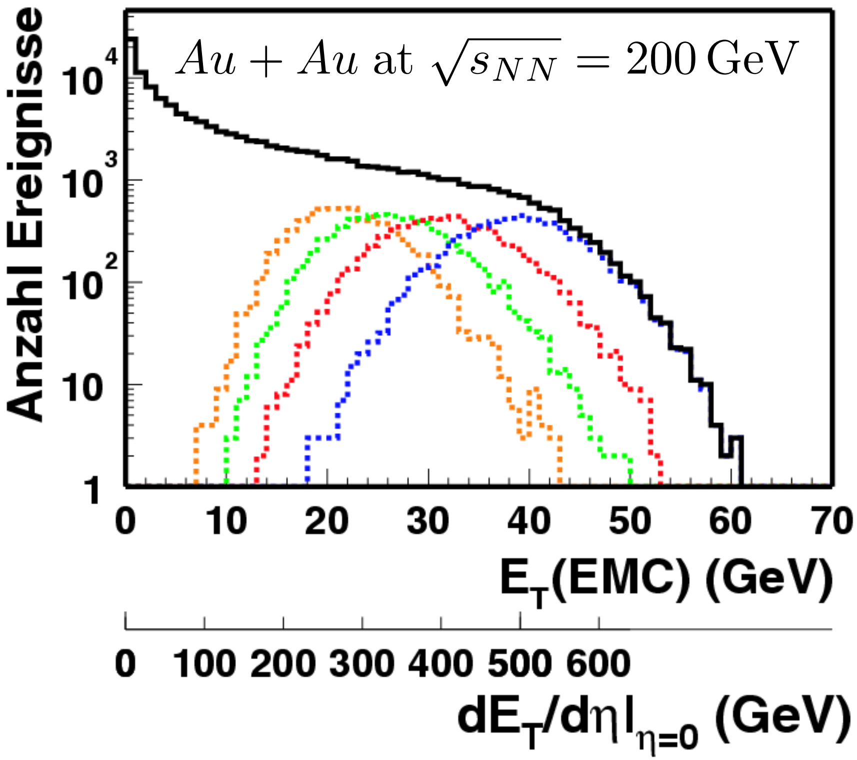
N_{part} Dependence of $dN_{ch}/d\eta$



ALICE: http://link.aps.org/doi/10.1103/PhysRevLett.106.032301 (→ link)

Relative increase of N_{ch} with centrality independent of $\sqrt{s_{NN}}$

Transverse Energy



E_T : Total energy in transverse direction

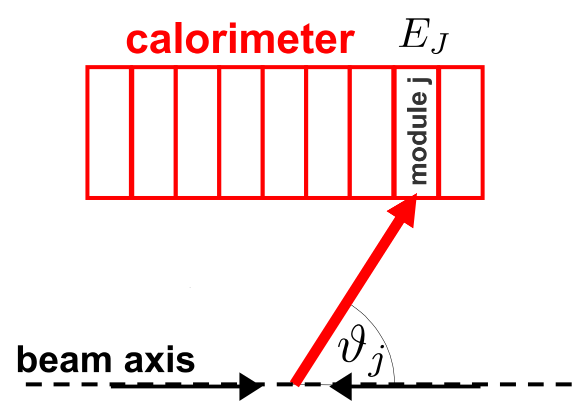
$$E_T = \sum_{i=1}^{N_{\mathrm{particles}}} m_{T,i}$$

Pragmatic definition:

$$E_T = \sum_{i=1}^{N_{\mathrm{particles}}} E_i \cdot \sin\vartheta_i$$

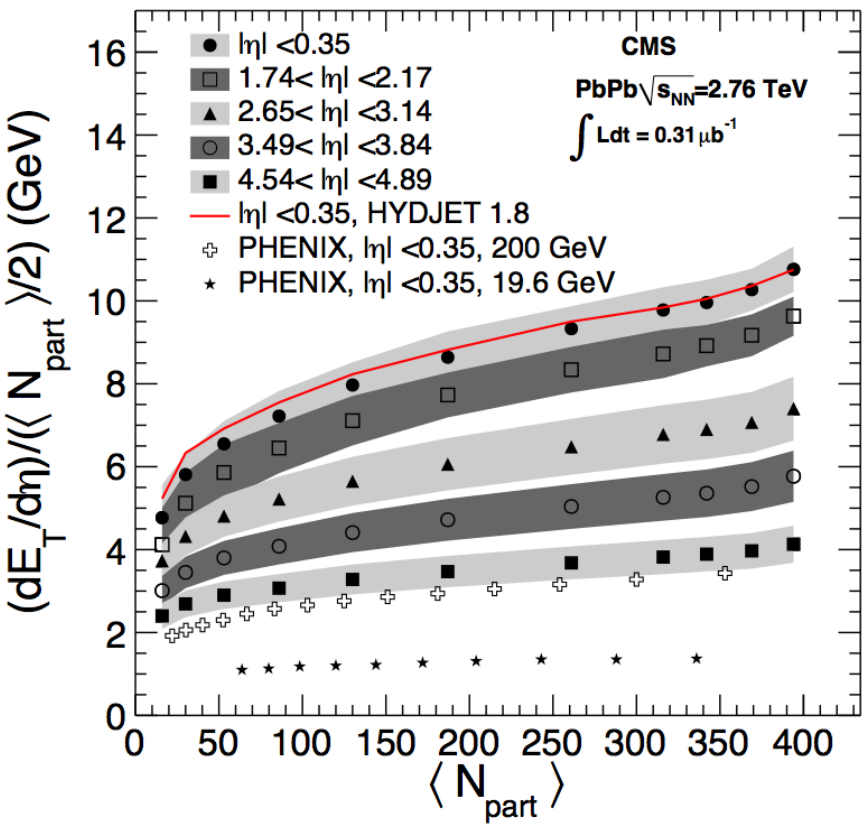
$$\simeq \sum_{j=1}^{N_{\mathrm{modules}}} E_j \cdot \sin\vartheta_j$$

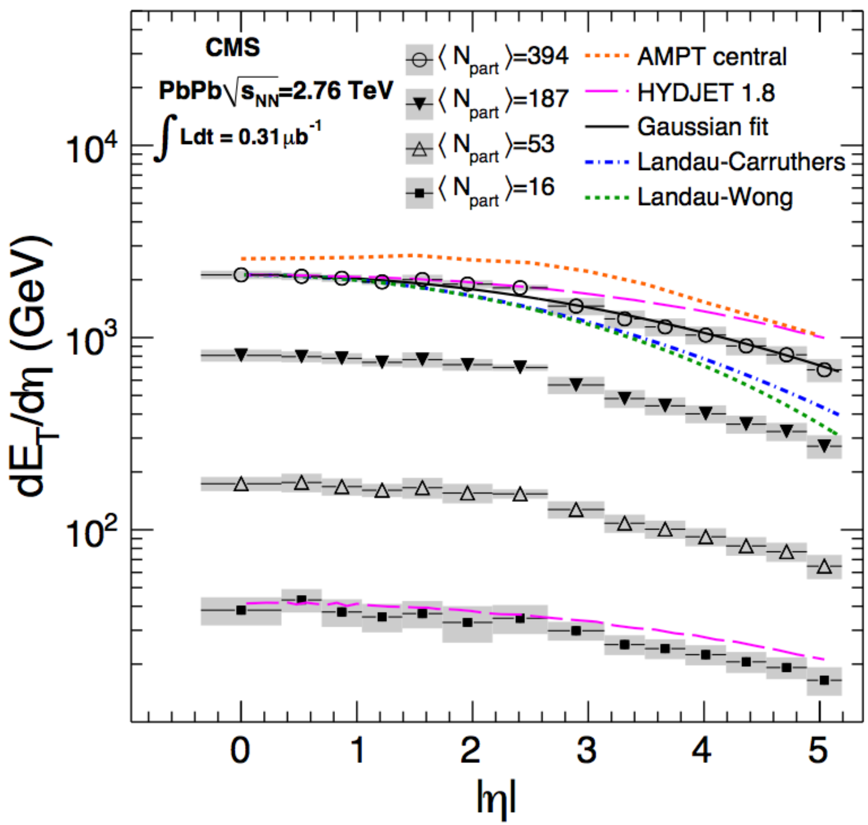
E_T and N_{ch} provide similar information



Transverse Energy (II)

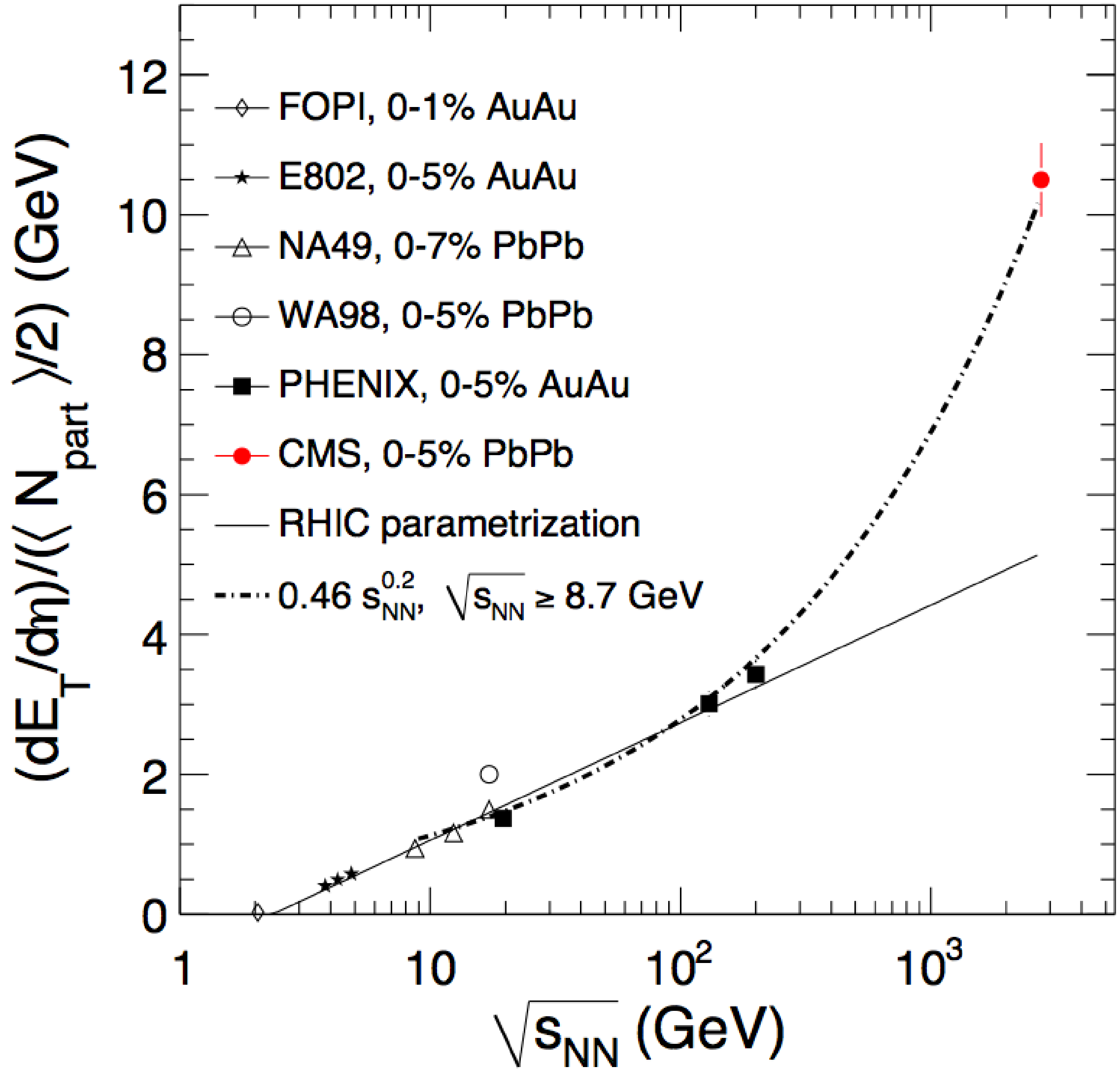
CMS, Phys. Rev. Lett. 109 (2012) 152303 (→ link)





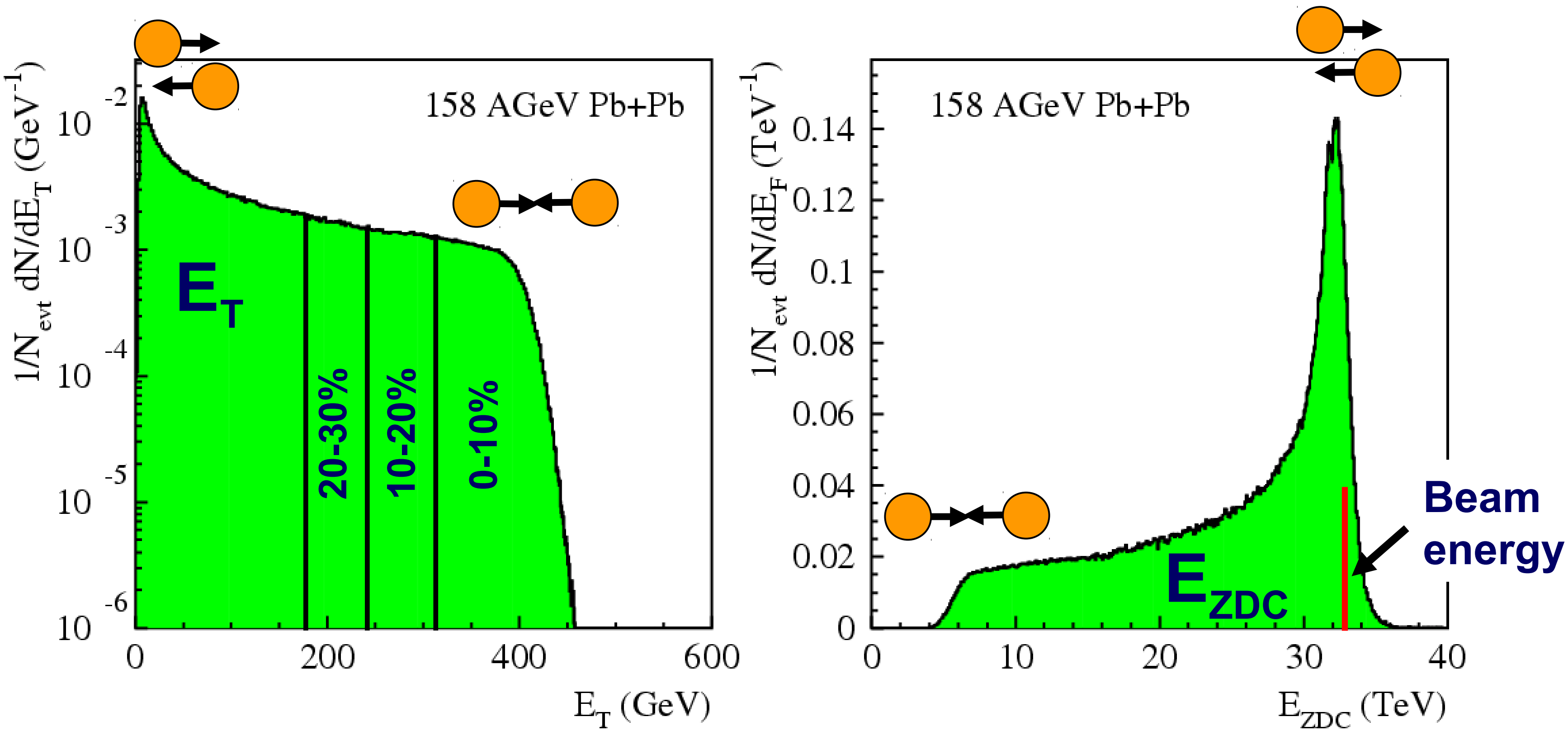
Transverse Energy (III)

CMS, Phys. Rev. Lett. 109 (2012) 152303 (→ link)



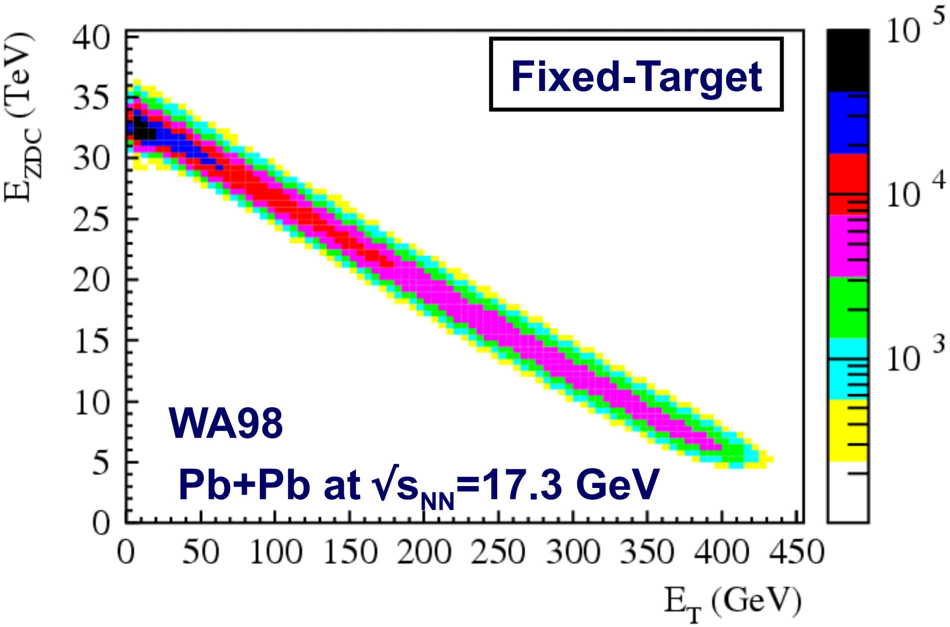
$$\left.\frac{dE_T}{d\eta}\right|_{\eta=0}^{0-5\%} \approx 2000\,\text{GeV}$$

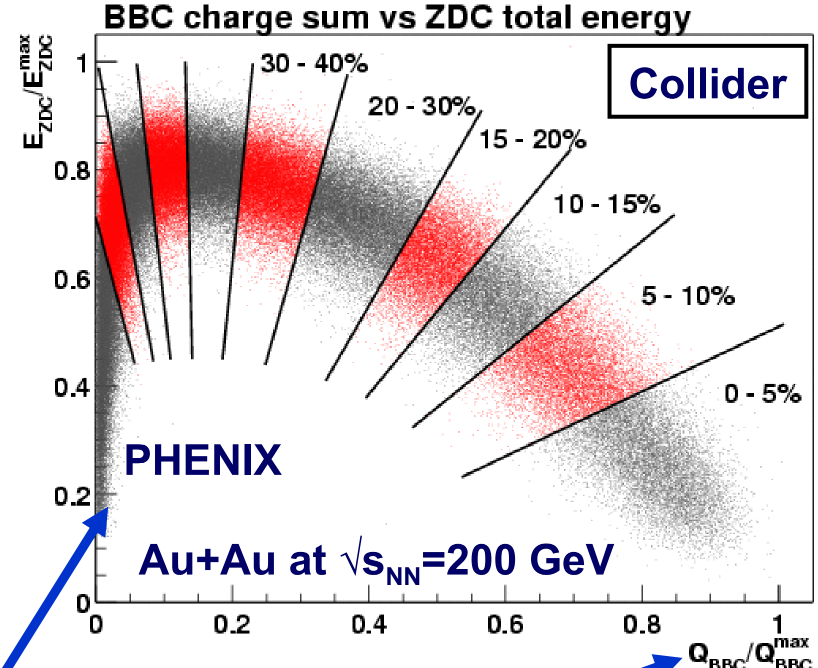
Centrality Selection: Fixed-Target Experiment



- Shape of E_T and E_{ZDC} follows from nuclear geometry
- Centrality selection: Cuts on E_T, E_{ZDC} (or charged particle multiplicity)

Centrality: Correlation between E_T and E_{ZDC}



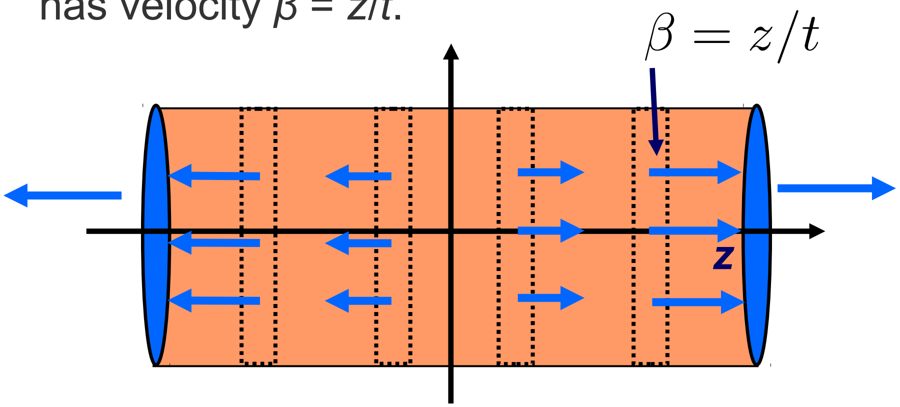


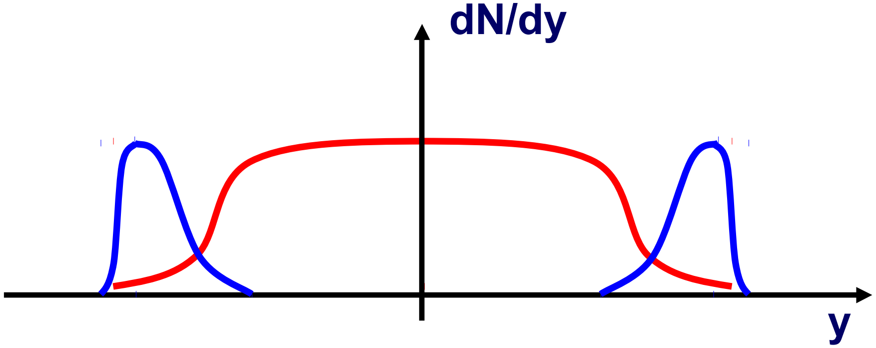
Q_{BBC}: measure of N_{ch} in $3.0 < \eta < 3.9$

Forward energy at colliders typically only due to neutrons (charged fragments deflected by beam magnets). Incomplete fragmentation of the nuclei leads to decrease of forward energy in very peripheral collisions (in contrast to fixed-target experiments)

Space-Time Evolution: Bjorken Model

Particle at position *z* and time *t* streaming from the origin (0,0) has velocity *β* = *z*/*t*.





Velocity profile of the local rest frames in the Bjorken model:

$$\beta = z/t$$

Proper time τ in the frame at (*z*, *t*):

$$\begin{aligned} \tau &= t/\gamma = t\sqrt{1-\beta^2} \\ &= \sqrt{t^2 - z^2} \end{aligned}$$

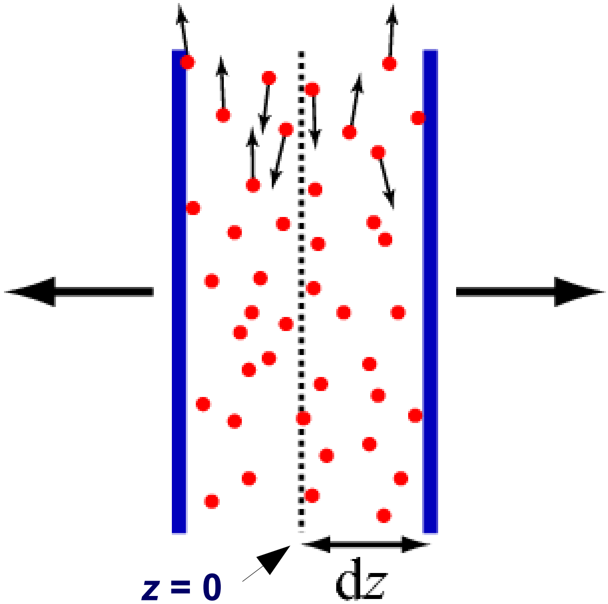
In the Bjorken model all thermodynamic quantities only depend on τ, e.g., the particle density:

$$n(t,z) = n(\tau)$$

This leads to a constant rapidity density of the produced particles (at least at central rapidities):

$$\frac{dN_{ch}}{dy} = \text{const.}$$

Bjorken's Estimate of the Energy Density



J. D. Bjorken,
Phys. Rev. D, 27, 140 (1983) (→ link)

Total energy in a central slice [0,dz] at time $\tau = \tau_0$:

$$E = N \cdot \langle m_T \cosh y \rangle |_{y=0} = N \cdot \langle m_T \rangle$$

Energy density:

$$\varepsilon = \frac{\langle m_T \rangle}{A} \left. \frac{\mathrm{d}N}{\mathrm{d}z} \right|_{z=0} = \frac{\langle m_T \rangle}{A} \left. \frac{\mathrm{d}N}{\mathrm{d}y} \right|_{y=0} \left. \frac{\mathrm{d}y}{\mathrm{d}z} \right|_{z=0}$$

transverse area

1D Bjorken flow: relation between *z* position of a slice and rapidity *y*

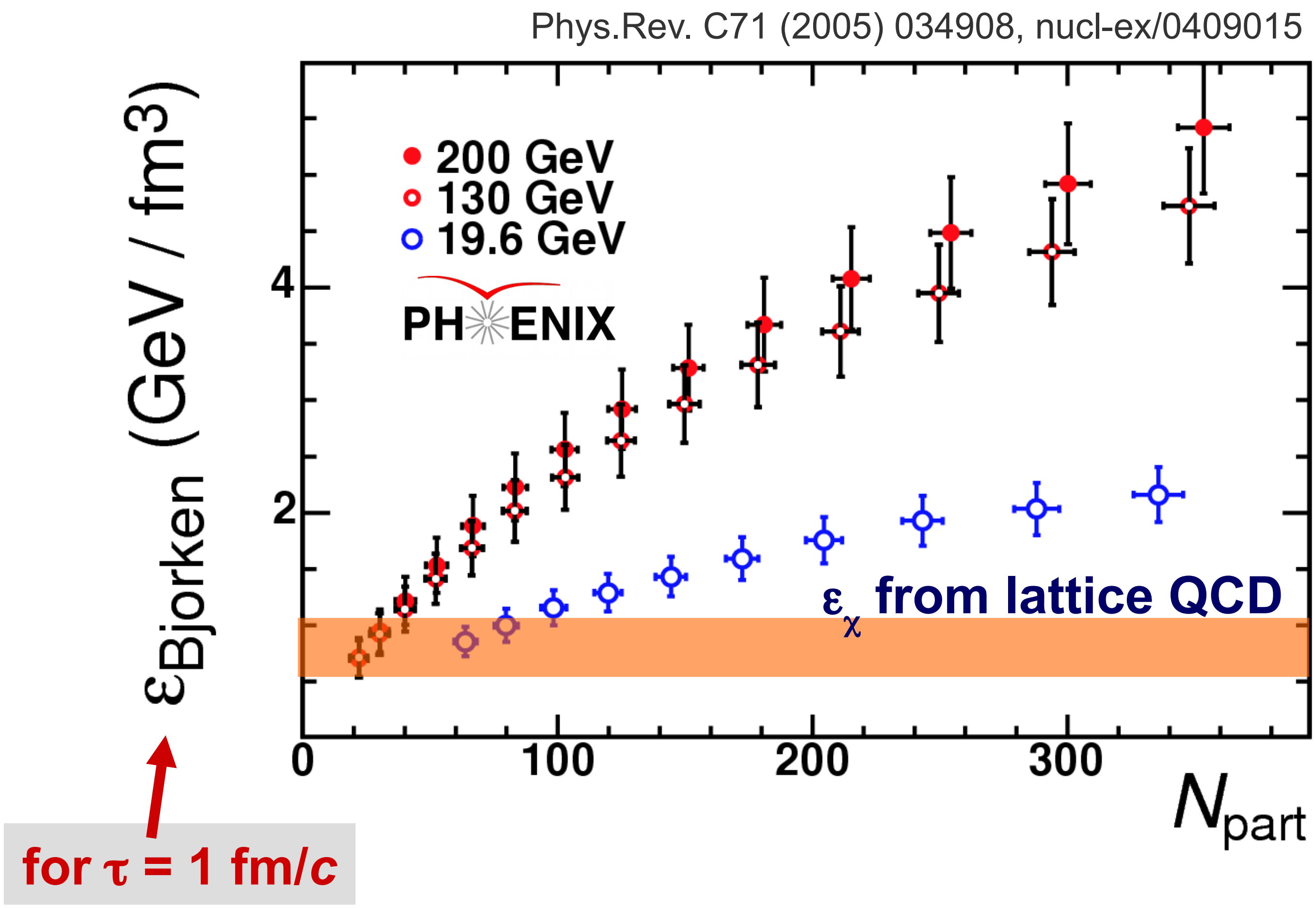
$$z = \tau \sinh y \quad \Rightarrow \quad \left. \frac{dy}{dz} \right|_{z=0} = \left. \frac{1}{\tau \cosh y(z)} \right|_{z=0} = \frac{1}{\tau}$$

Bjorken formula for the initial energy density:

$$\varepsilon = \frac{\langle m_T \rangle}{A \cdot \tau_0} \left. \frac{\mathrm{d}N}{\mathrm{d}y} \right|_{y=0} = \frac{1}{A \cdot \tau_0} \left. \frac{\mathrm{d}E_\mathrm{T}}{\mathrm{d}y} \right|_{y=0}$$

Thermalization time τ_0 = 1 fm/*c* (with large uncertainties)

Bjorken Energy Density from Data



Estimated energy densities in central A+A collision for CERN SPS and RHIC energies above critical value of $\approx 0.7\ \mathrm{GeV/fm^3}$ from lattice QCD

Bjorken Energy density in central Pb+Pb Collisions at the LHC

$$\varepsilon = \frac{1}{A \cdot \tau_0} \left. \frac{\mathrm{d}E_\mathrm{T}}{\mathrm{d}y} \right|_{y=0} = \frac{1}{A \cdot \tau_0} J(y,\eta) \left. \frac{\mathrm{d}E_\mathrm{T}}{\mathrm{d}\eta} \right|_{\eta=0} \qquad \text{with } J(y,\eta) \approx 1.09$$

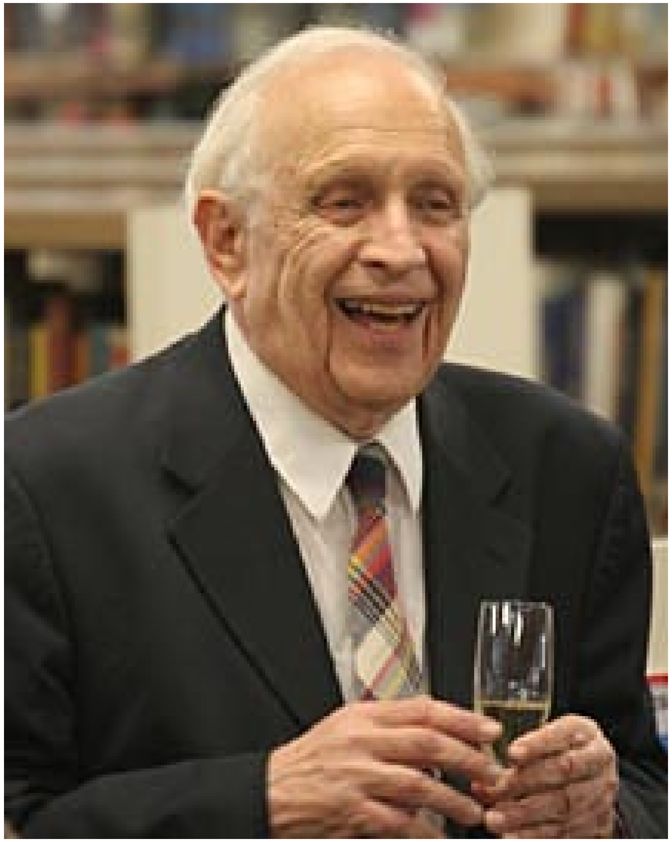
Central Pb+Pb collisions ($b \approx 0$):

$$A = \pi R_\mathrm{Pb}^2 \quad \text{with } R_\mathrm{Pb} \approx 7\,\mathrm{fm}$$

$$dE_T/d\eta = 2000\,\mathrm{GeV}$$

$$\rightsquigarrow \quad \varepsilon_\mathrm{LHC} = 14\,\mathrm{GeV/fm^3} \approx 2.6 \times \varepsilon_\mathrm{RHIC} \quad \text{for} \quad \tau_0 = 1\,\mathrm{fm}/c$$

Glauber Model: Basic Assumptions



Nobel prize in physics 2005 for
his contributions to quantum optics

Glauber model for nucleus-nucleus collisions

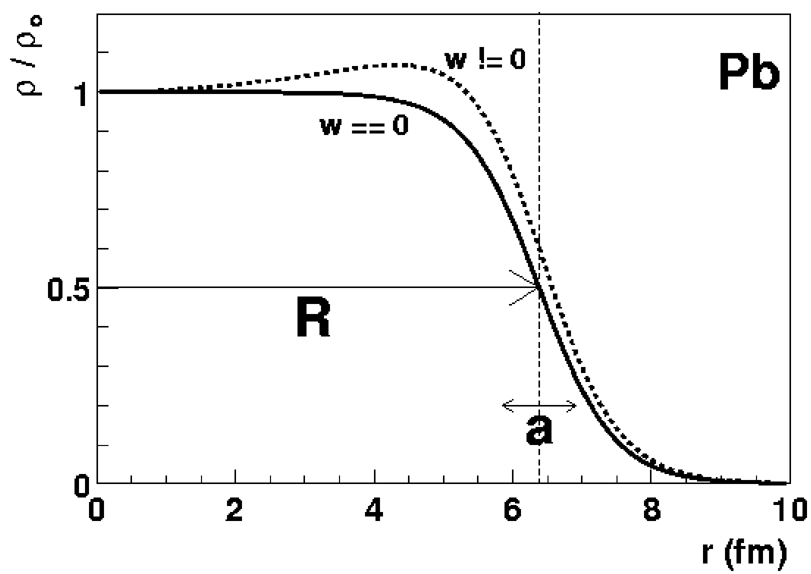
- Nucleons travel on straight trajectories (after a nucleon-nucleon collisions)
- Nucleon-nucleon cross section is independent of the number of collisions a nucleon underwent before
- Input: density profile of the nucleus and inelastic nucleon-nucleon cross section

Review article:
Glauber modeling in high energy nuclear collisions, 2007 (→ link)

Glauber Model: Nuclear Geometry

Woods-Saxon nuclear density profile:

$$\rho(r) = \frac{\rho_0 \left(1 + wr^2/R^2\right)}{1 + \exp((r-R)/a)}$$



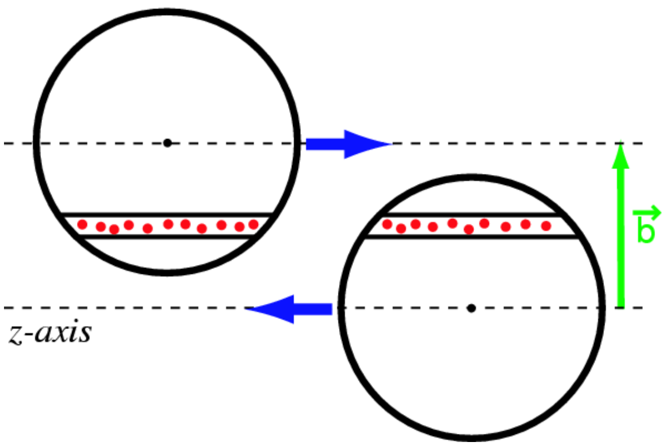
Nucleus	A	R (fm)	a (fm)	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	**197**	**6.38**	**0.535**	**0**
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

H. DeVries, C.W. De Jager, C. DeVries, 1987

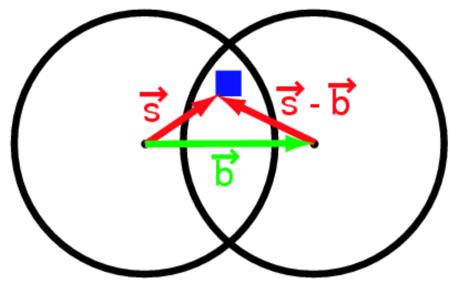
- Woods-Saxon parameters typically from e^--nucleus scattering (sensitive to charge distribution only)
- Difference between neutron and proton distribution small and typically neglected

Optical Glauber Model: Number of Nucleon-Nucleon Collisions

side view:



transverse plane:



Nuclear thickness function:

$$T_A(\vec{s}) := \int \rho_A(\vec{s}, z)\, dz$$

Normalization:

$$\int T_A(\vec{s})\, d^2 s = A$$

Nucleon "luminosity" at $\vec{s}$:

$$dT_{AB}(\vec{s}) = T_A(\vec{s}) \cdot T_B(\vec{s} - \vec{b})\, d^2 s$$

Nuclear overlap function:

$$T_{AB}(b) := \int T_A(\vec{s}) \cdot T_B(\vec{s} - \vec{b})\, d^2 s$$

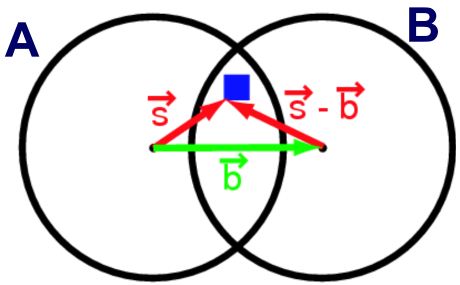
$$\langle N_{coll}(b) \rangle = T_{AB}(b) \cdot \sigma_{inel}^{p+p}$$

and, more generally

$$\langle N_{int}(b) \rangle = T_{AB}(b) \cdot \sigma_{int}^{p+p}$$

typically used for hard parton-parton interactions

Optical Glauber Model: Number of Participants



definition:

$$\hat{T}_{\mathrm{B}}(\vec{x}) := T_{\mathrm{B}}(\vec{x})/B$$

Probability that a "test nucleon" from nucleus A collides with a certain nucleon from nucleus B:

$$p_{\mathrm{int}} = \hat{T}_{\mathrm{B}}(\vec{s}-\vec{b}) \cdot \sigma_{\mathrm{inel}}^{\mathrm{p+p}}$$

Probability that a "test nucleon" from nucleus A collides with none of the B nucleons of nucleus B:

$$(1-p_{\mathrm{int}})^B = (1-\hat{T}_{\mathrm{B}}(\vec{s}-\vec{b}) \cdot \sigma_{\mathrm{inel}}^{\mathrm{p+p}})^B$$

Probability that a "test nucleon" undergoes at least one inelastic nucleon-nucleon collision:

$$1-(1-\hat{T}_{\mathrm{B}}(\vec{s}-\vec{b}) \cdot \sigma_{\mathrm{inel}}^{\mathrm{p+p}})^B$$

Number of participants in nucleus A:

$$\langle N_{\mathrm{part}}^{\mathrm{A}}(b) \rangle = A \int \hat{T}_{\mathrm{A}}(\vec{s}) \cdot \left(1-(1-\hat{T}_{\mathrm{B}}(\vec{s}-\vec{b}) \cdot \sigma_{\mathrm{inel}}^{\mathrm{p+p}})^B\right) \mathrm{d}^2 s$$

Total mean number of participants for A+B collisions with impact parameter *b*:

$$\langle N_{\mathrm{part}}(b) \rangle = \langle N_{\mathrm{part}}^{\mathrm{A}}(b) \rangle + \langle N_{\mathrm{part}}^{\mathrm{B}}(b) \rangle$$

[Exercise: Optical Glauber Calculation with Mathematica]

[notebook (→ link)]

Parameters

User input

```
SigmaNN := 6.4 (* Nucleon-nucleon cross section in fm^2: Pb+Pb at LHC: 6.4, Au+Au at RHIC 4.2 *)
A := 208 (* Pb:208, Au: 197 *)
R := 6.62 (* Radius in fm: Pb: 6.62, Au: 6.38 *)
a := 0.546 (* diffuseness parameter in fm *)
bmax := 20 (* maximum impact parameter in fm *)
bstep := 0.2(* stepsize in fm in calculation of Npart(b), etc *)
```

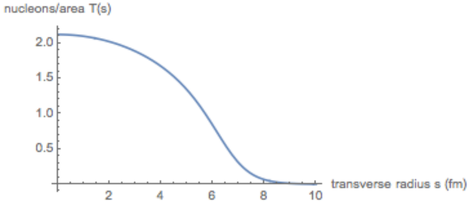
Define volume that fully subtends the nucleus (and thus also the overlap region)

```
xmax = R + 5 a (* transverse area that fully includes the larger of the two nuclei *)
9.35
ymax = xmax; (* transverse area that fully includes the larger of the two nuclei *)
zmax = xmax;
```

Calculation of thickness function T

Define nuclear geometry

```
rho := rho0 / (1 + Exp[(r - Rp) / ap]) (* nuclear density distribution as a function of the radius r *)
norm = Integrate[4 Pi r^2 rho /. rho0 → 1, {r, 0, Infinity}, Assumptions → {ap > 0, Rp > 0}] /. {Rp → R, ap → a};
(* normalization *)
rhonorm[s_?NumericQ, z_?NumericQ] = rho /. {r → Sqrt[s^2 + z^2], Rp → R, ap → a, rho0 → A / norm};
(*3D nucleon density distribution with correct normalization *)
t[s_?NumericQ] := NIntegrate[rhonorm[s, z], {z, -zmax, zmax}, AccuracyGoal → 4, Method → "LocalAdaptive"]
 (* nuclear thickness function*)
Monitor[tint = Interpolation[Table[{s, t[s]}, {s, 0., 20, bstep}]] , s];
Plot[tint[s], {s, 0, 10.}, AxesLabel → {"transverse radius s (fm)", "nucleons/area T(s)"}]
```



… and so on ...

output:

b (fm)	Ncoll	Npart	σ/σ_{tot} (%)
0	1951.1	405.9	0.
1	1906.8	401.1	0.401
2	1783.7	386.0	1.6
3	1603.8	360.6	3.61
4	1389.5	327.3	6.41
5	1159.8	288.6	10.
6	929.8	247.2	14.4
7	711.5	204.9	19.6
8	514.7	163.6	25.6
9	346.8	124.6	32.5
10	213.2	89.3	40.1
11	116.0	58.9	48.5
12	53.8	34.6	57.7
13	20.6	17.2	67.7
14	6.5	6.9	78.5
15	2.1	2.7	89.5
16	1.2	1.8	96.7
17	1.0	1.6	99.2
18	1.0	1.7	99.8
19	1.0	1.7	100.

[Exercise: Optical Glauber Calculation with Mathematica II]

Calculate the average number of ccbar and bbbar quark pairs per unit rapidity at midrapidity for the 10% most central events assuming

$$\frac{d\sigma_{c\bar{c}}}{dy} = 200\,\mu\mathrm{b}, \qquad \frac{d\sigma_{b\bar{b}}}{dy} = 20\,\mu\mathrm{b}$$

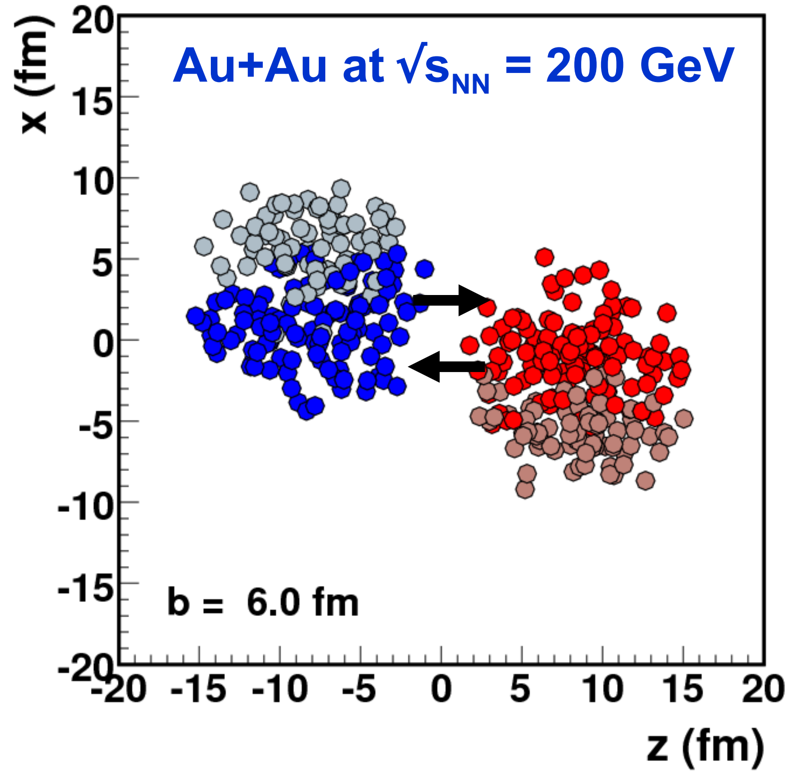
What are the corresponding numbers for the 10% most central Cu+Cu collisions?

Glauber Model: Monte Carlo Approach (I)

- In practice, most experiments use Glauber Monte Carlo models to determine N_{part} and N_{coll}
- Nucleons distributed according to Woods-Saxon distribution
- Impact parameter randomly drawn from $d\sigma/db = 2\pi b$
- A collision between two nucleons takes place if their distance *d* in the transverse plane satisfies

$$d \leq \sqrt{\sigma_{\text{inel}}^{\text{NN}}/\pi}$$

Glauber Monte Carlo event



Glauber Model: C++ Code Snippets (Glauber MC)

Glauber MC: Main loop

```
// produce n_events Glauber MC collisions
for (int i=0; i< n_events; i++) {

  // sample impact parameter distribution
  float b = fImpact->GetRandom();

  // Distribute nucleons according to Woods-Saxon dsitribution
  // and displace them by -b/2 and b/2 on the x axis.
  // Moreover, set collision counter of each nucleon to zero
  Target->DistributeNucleons(-b/2);
  Projectile->DistributeNucleons(+b/2);

  int Npart = 0;
  int Ncoll = 0;

  for (int ip=0; ip<Projectile->GetMassNumber(); ip++) {
    for (int it=0; it<Target->GetMassNumber(); it++) {

      // squared transverse distance of the nucleons
      float dx   = Projectile->nucleon[ip].x - Target->nucleon[it].x;
      float dy   = Projectile->nucleon[ip].y - Target->nucleon[it].y;
      float dxy2 = dx*dx + dy*dy;

      // check if there is a nn collision
      if (dxy2 < sigma_nn_inel_fm2/pi) {
        Ncoll++;
        if (Projectile->nucleon[ip].ncoll++ == 0) Npart++;
        if (Target->nucleon[it].ncoll++ == 0) Npart++;
      }

    }
  }
  cout << "Event " << i << ": Npart = " << Npart
       << ", Ncoll = " << Ncoll << endl;

}  // event loop
```

Function that distributes nucleons

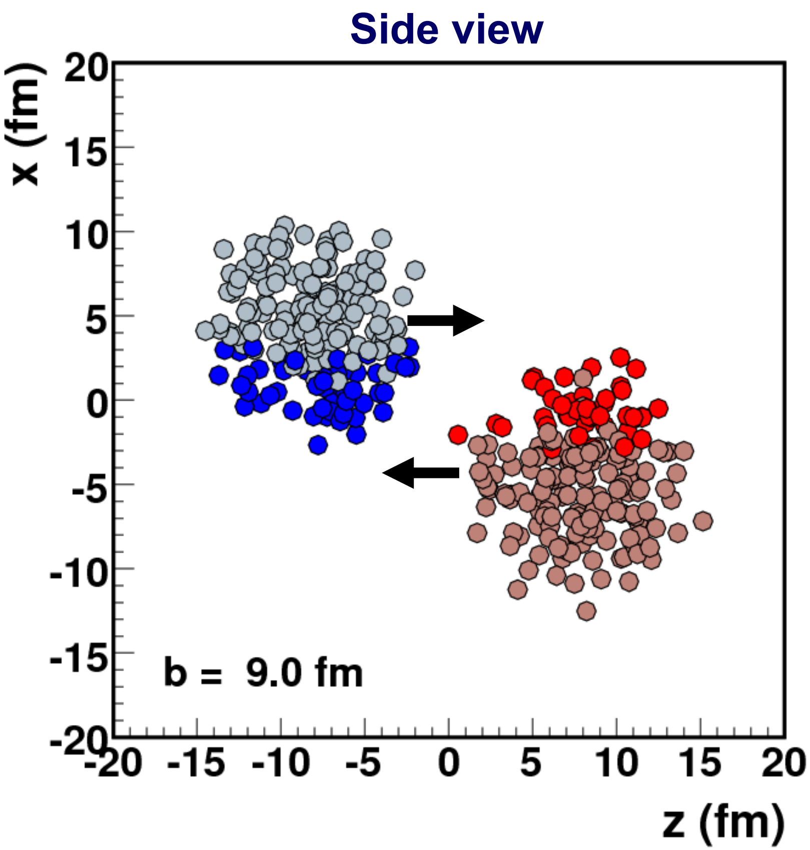
```
void nucleus::DistributeNucleons(const float& x_offset) {

  // loop over all nucleons
  for(int i=0; i<A; i++) {

    float r     = ws->GetRandom(); // radius from Woods-Saxon
    float theta = th->GetRandom();
    float phi   = 2.* pi * gRandom->Rndm();

    // coordinates in local coordinate system
    nucl[i].x = r * sin(theta) * cos(phi) + x_offset;
    nucl[i].y = r * sin(theta) * sin(phi);
    nucl[i].z = r * cos(theta);

    // set collision counter to zero
    nucl[i].ncoll = 0;

  }
}
```

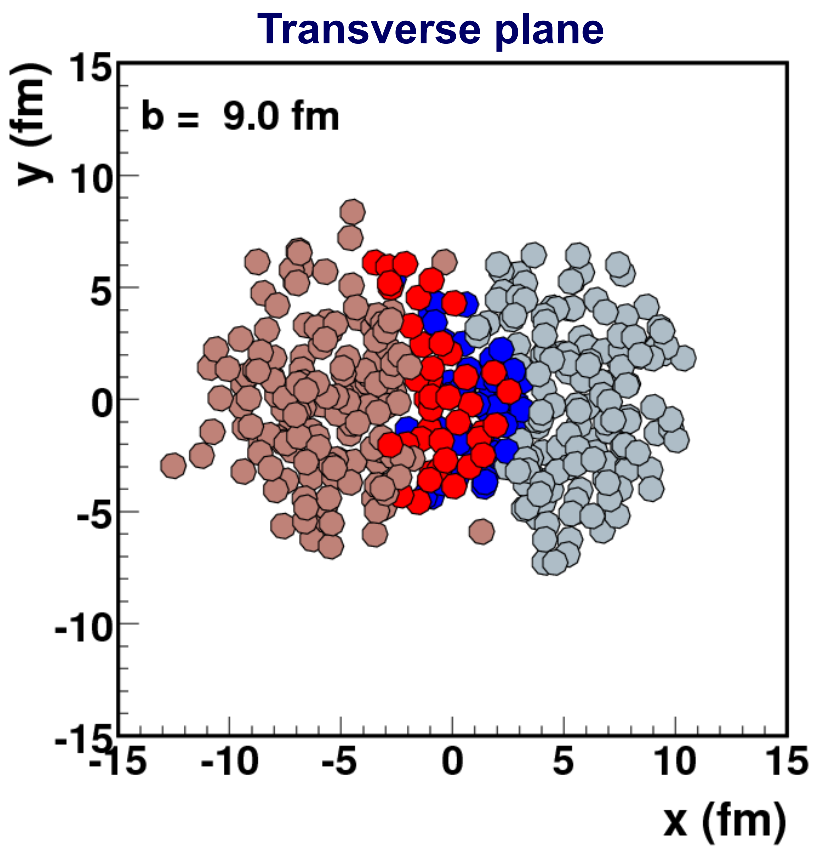
Woods-Saxon Distribution

```
//! Defines Woods-Saxon distribution
void nucleus::DefineShape() {

  // probability distribution for the radius
  ws = new TF1("Woods-Saxon","x*x/(1+exp((x-[0])/[1]))",0.,20.);
  ws->SetParameter(0, ws_radius);
  ws->SetParameter(1, ws_diffuseness);

}
```

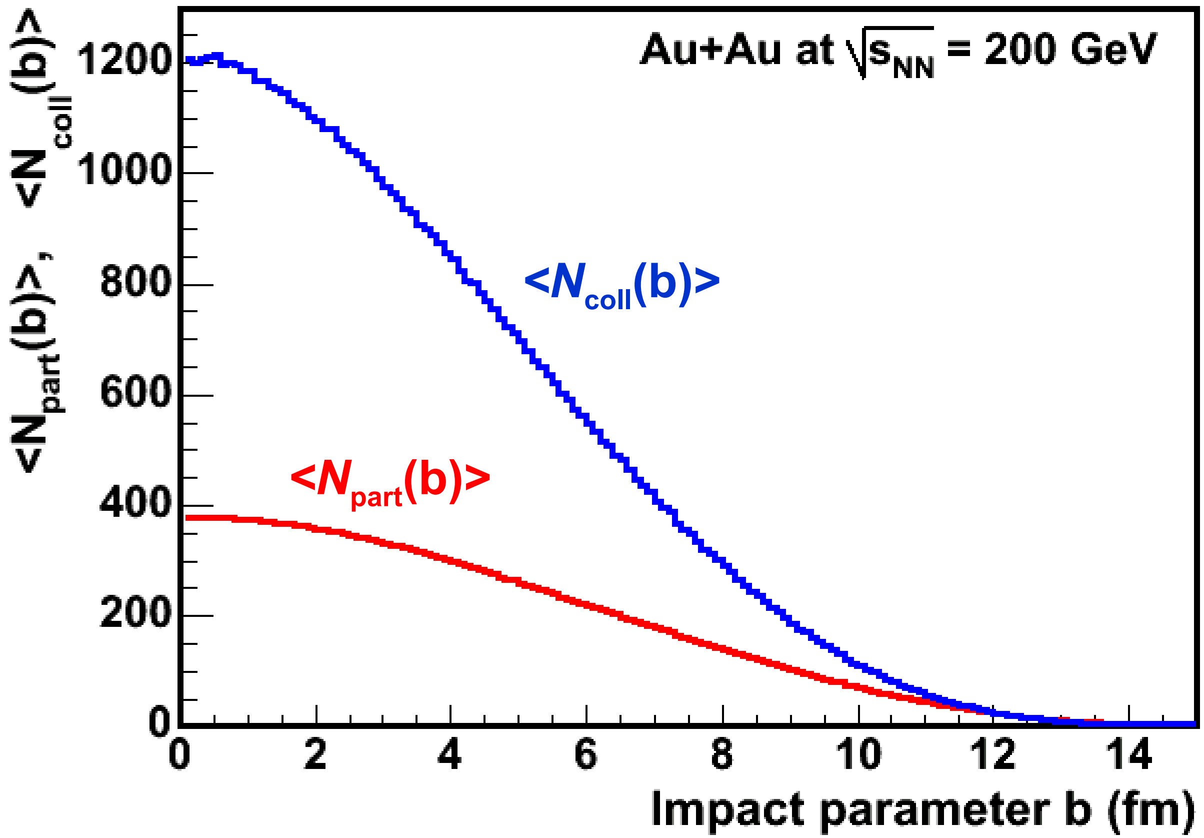
Glauber Model: Monte Carlo Approach (II)

Au+Au at $\sqrt{s_{NN}}$ = 200 GeV





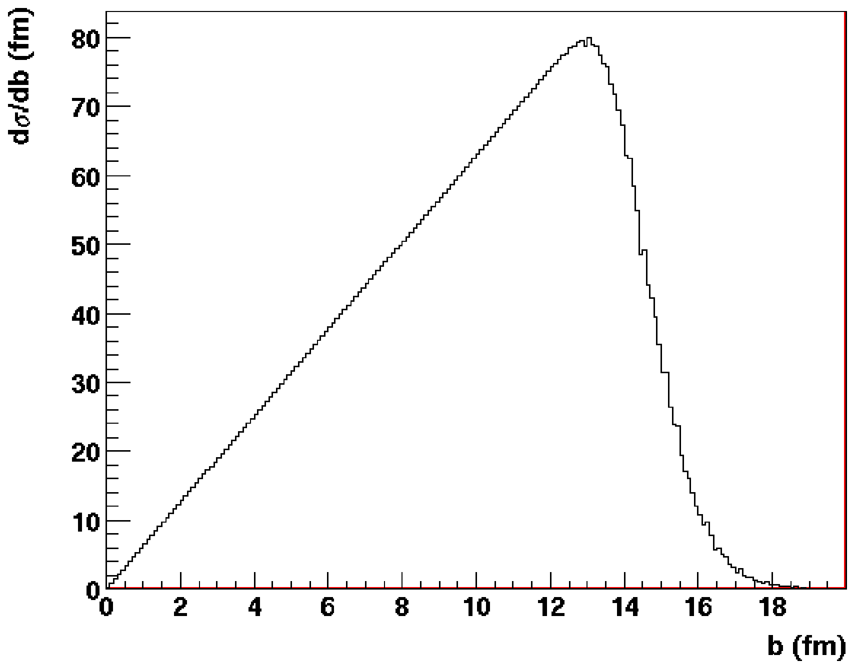
<N_{part}(b)> and <N_{coll}(b)> from Glauber MC



Approximate relation: $N_{\mathrm{coll}} \propto N_{\mathrm{part}}^{4/3}$

Glauber Model: Impact Parameter Distribution and Total Inelastic Cross Section

Glauber MC:



Analytic ("optical Glauber"):

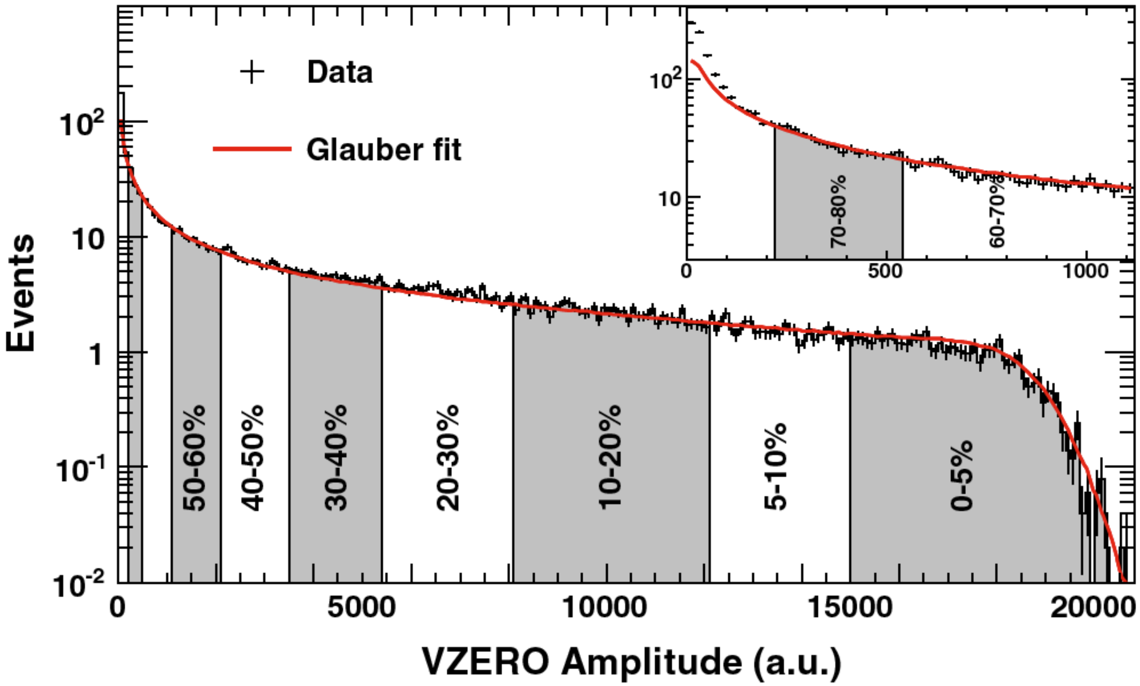
$$p_{\mathrm{inel}}^{\mathrm{A+B}}(b) = 1 - \exp(-T_{AB}(b) \cdot \sigma_{\mathrm{inel}}^{\mathrm{NN}})$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}b} = 2\pi\, b\, p_{\mathrm{inel}}^{\mathrm{A+B}}(b)$$

$$\sigma_{\mathrm{inel}}^{A+B} = \int_0^\infty \frac{\mathrm{d}\sigma}{\mathrm{d}b}\, \mathrm{d}b$$

Result: $\sigma_{\mathrm{inel}}^{\mathrm{Au+Au@200GeV}} \approx 6.9\mathrm{b}$

<N_{part}> and <N_{coll}> for Experimentally Defined Centrality Classes



Centrality	$dN_{ch}/d\eta$	$\langle N_{part} \rangle$
0%–5%	1601 ± 60	382.8 ± 3.1
5%–10%	1294 ± 49	329.7 ± 4.6
10%–20%	966 ± 37	260.5 ± 4.4
20%–30%	649 ± 23	186.4 ± 3.9
30%–40%	426 ± 15	128.9 ± 3.3
40%–50%	261 ± 9	85.0 ± 2.6
50%–60%	149 ± 6	52.8 ± 2.0
60%–70%	76 ± 4	30.0 ± 1.3
70%–80%	35 ± 2	15.8 ± 0.6

Measured multiplicity distribution described within the Glauber model by assuming a certain centrality dependence for the number of ancestor particles, e.g.

$$N_{\mathrm{ancestors}} = f \cdot N_{\mathrm{part}} + (1 - f) \cdot N_{\mathrm{coll}}$$

Each ancestor than “produces” charged particles according to a Negative Binomial Distribution (NBD). The same centrality cuts as used for real data are then applied to the simulated multiplicity in order to obtain <N_{part}> and <N_{coll}> for a given centrality class.

Points to Take Home

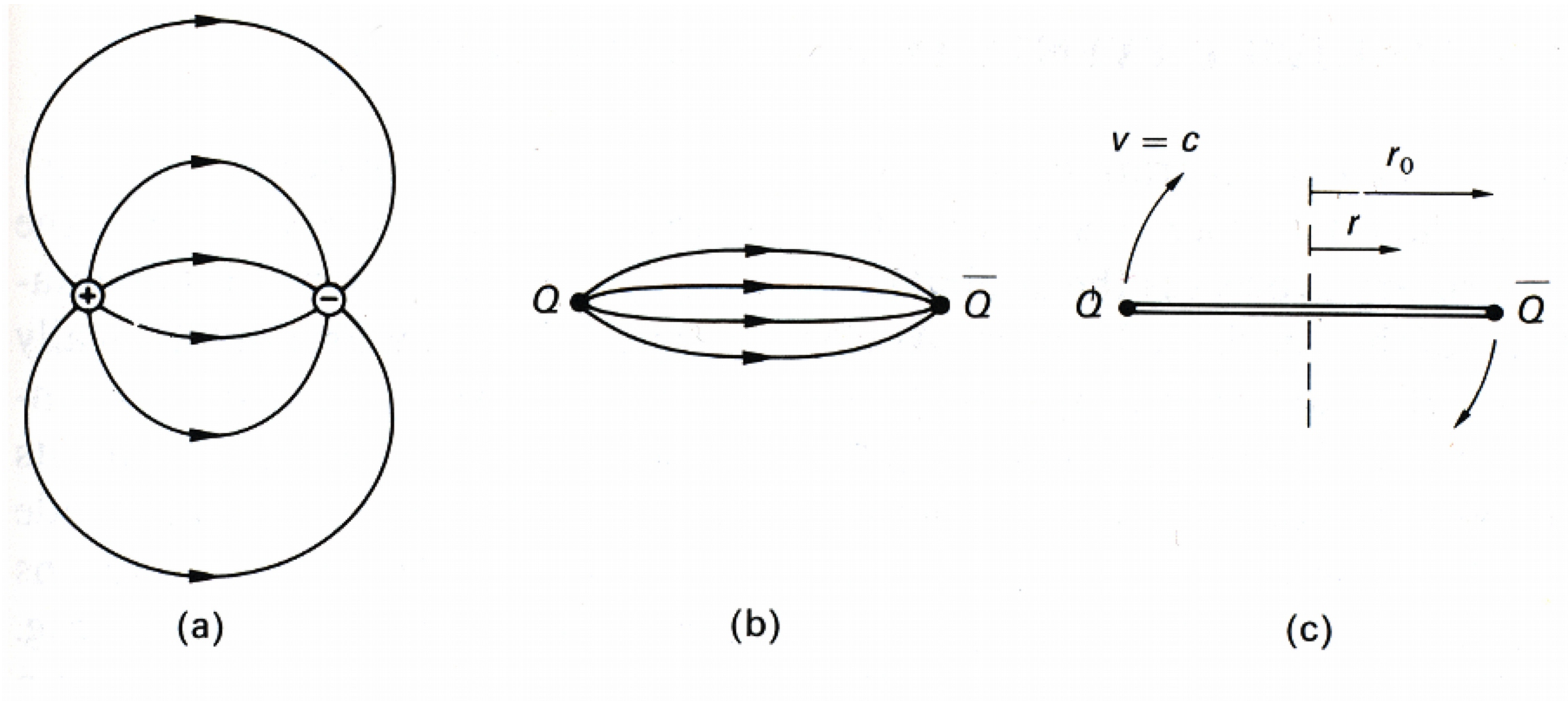
- QCD perturbation theory cannot be used to describe particle production at low p_T (α_s is too large)
- Phenomenology of low-p_T particle production reasonably well described by the Lund string model (in e^+e^- as well as in p+p)
- Centrality in A+A collisions often characterized by N_{part} and N_{coll} (from Glauber calculations)
- p+A: particle yields scale approximately with N_{part}. In A+A this is only a very rough approximation.
- Bjorken's estimate for the initial energy density of the fireball

$$\varepsilon = \frac{1}{A \cdot \tau_0} \frac{\mathrm{d}E_\mathrm{T}}{\mathrm{d}y}\bigg|_{y=0}$$

- Already in central A+A collisions at CERN SPS energies this estimate yields energy densities above the critical energy density of $\varepsilon_c \approx 0.7$ GeV/fm^3 expected for the QGP transition.
- ε_{RHIC}(Au+Au, 200 GeV, central) = 5.4 GeV/fm^3, $\varepsilon_{LHC} \approx 2.6 \cdot \varepsilon_{RHIC}$

Extra slides

Classical String Model: Rotating Strings



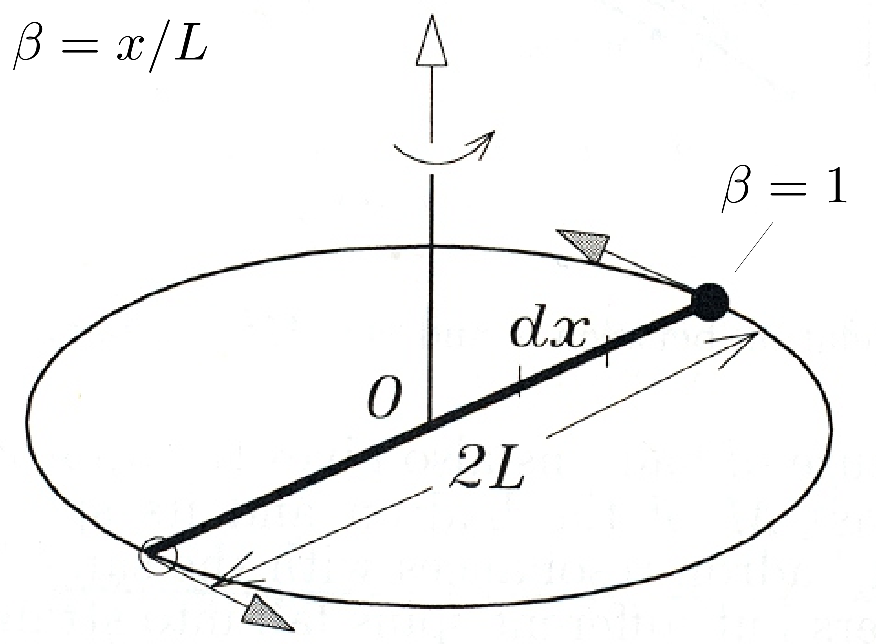
- Quarks considered as massless
- Rotation of the string produces the spin of the hadron

Part I: Nucleon-Nucleon Collisions

Classical String Model: Relation between Mass and Angular Momentum (I)

Mass density of the non-rotating string:

$$\mathrm{d}M = k\,\mathrm{d}x, \; k = \text{string tension}$$



Total energy (= mass) of the string:

$$M = 2\int_0^L \gamma\, k\, \mathrm{d}x$$

$$= 2\int_0^L \frac{k\,\mathrm{d}x}{\sqrt{1-(x/L)^2}} = \pi\, k\, L$$

Angular momentum: $\mathrm{d}J = x\,\mathrm{d}p$

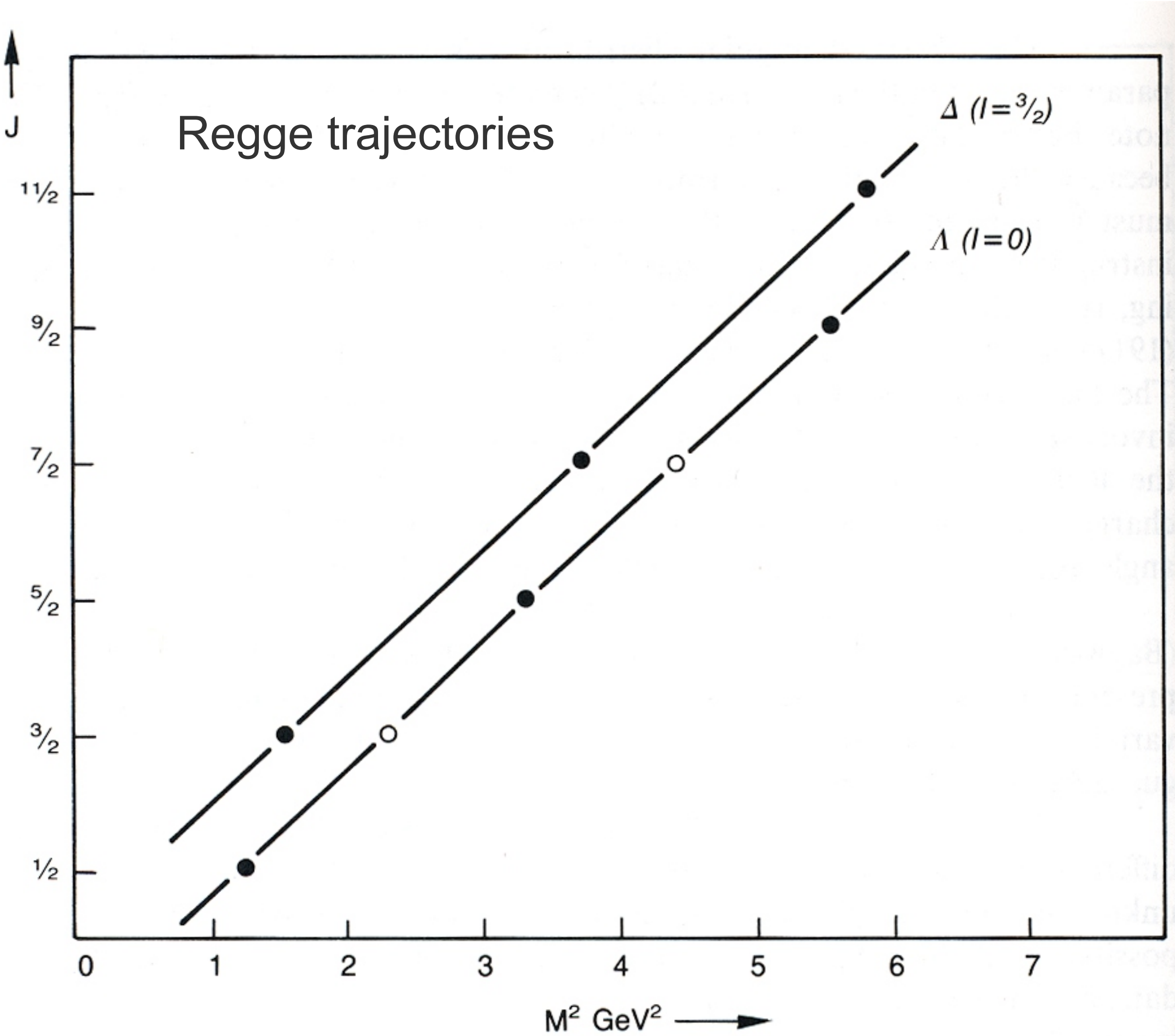
$$J = 2\int_0^L x\,\beta\,\gamma\, k\,\mathrm{d}x$$

$$= 2\int_0^L \frac{x^2/L\;k\,\mathrm{d}x}{\sqrt{1-(x/L)^2}} = k\,L^2\pi/2$$

Resulting relation between mass and angular momentum:

$$J = \frac{1}{2\,\pi\,k} M^2$$

Classical String Model: Relation between Mass and Angular Momentum (II)

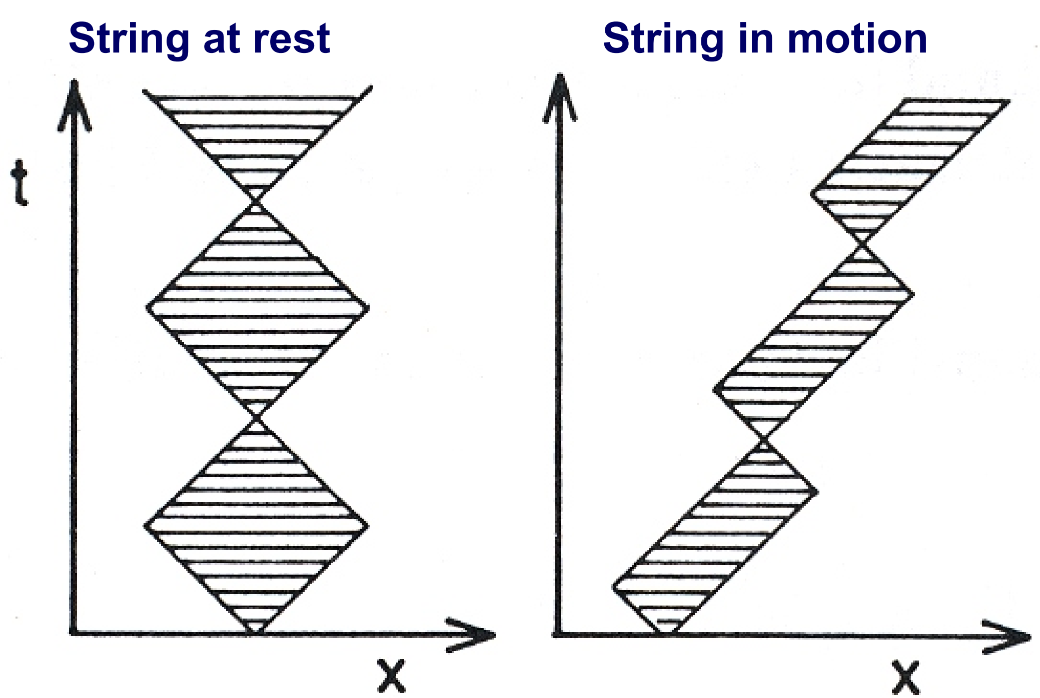


Data show the expected relation between angular momentum and mass

Value for string tension:

$$k \approx 1\,\mathrm{GeV/fm}$$

Classical String Model: Strings in One Dimension



"Yo-Yo" string

- Massless quarks, connected by a string
- Linear potential
- Equation of motion:

$$\mathrm{d}p/\mathrm{d}t = \pm k$$

- Solution:

$$p = p_0 - k \cdot t$$

$$(\sqrt{s} = 2p_0)$$

- Area *A* of the string in *x*-*t* plane is Lorentz invariant:

$$A = s/k^2$$