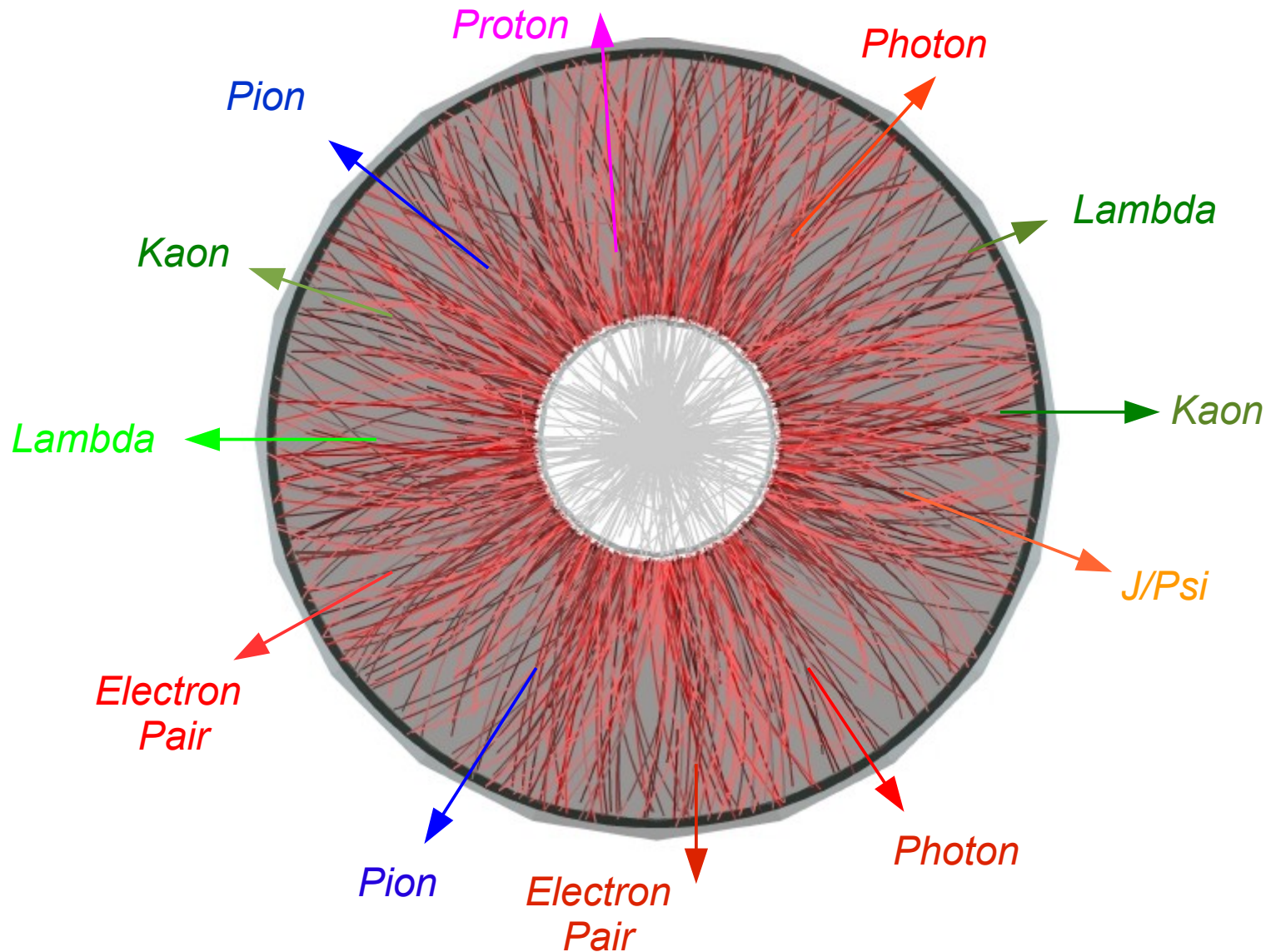


QGP Physics – from Fixed Target to LHC

5. Statistical Hadronization and Strangeness

**Prof. Dr. Klaus Reygers, Prof. Dr. Johanna Stachel
Physikalisches Institut, Universität Heidelberg
SS 2015**

5.1 Hadronization of the nuclear fireball



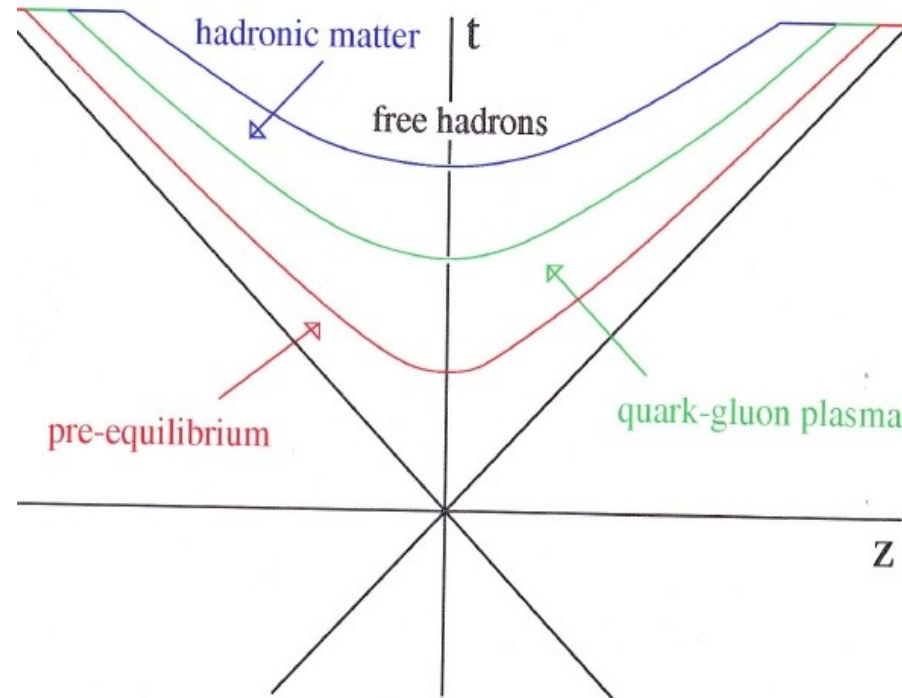
the fireball properties can be determined by measurement of the emitted particles
In this chapter as first species: hadrons with up,down, strange constituent quarks

The concept of hadrochemical freeze-out

nuclear fireball evolves
(as sketched in lecture 1)
it cools and expands,
when it hits T_c , it hadronizes,
maybe cools and expands further

and finally falls apart
when mean free path large as
compared to interparticle distance
“kinetic” or **“thermal freeze-out”**:
momentum distributions are frozen in
- no more elastic scattering

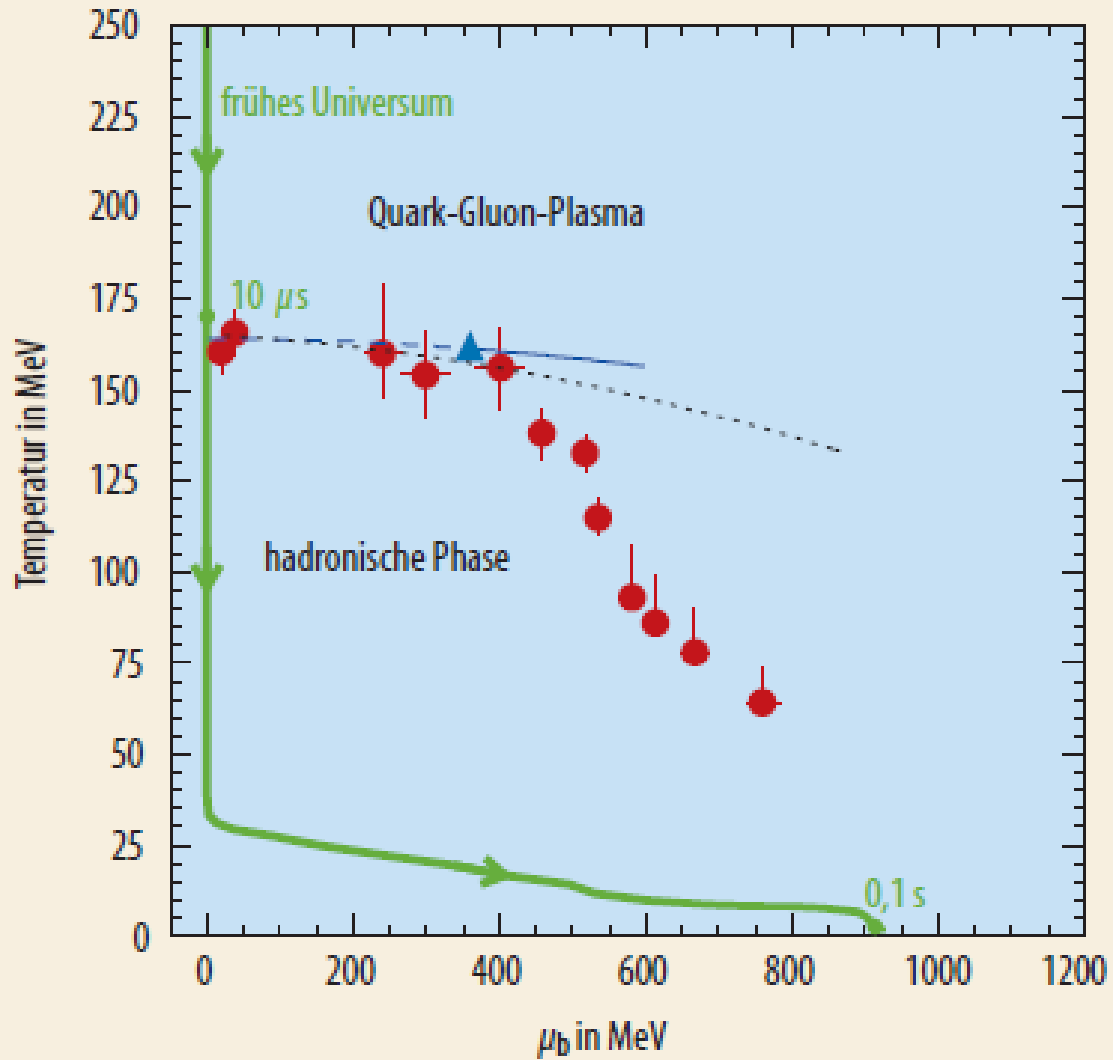
“chemical” or **“hadrochemical freeze-out”**:
abundancies of hadrons
are frozen in – no more inelastic
scattering



Note: chemical freeze-out can happen together with thermal freeze-out or before
the duration of these freeze-out processes is a priori not known

In the early universe freeze-out happened after order of 0.1 s

P.Braun-Munzinger, J. Wambach, Rev. Mod. Phys. 81 (2009)1031

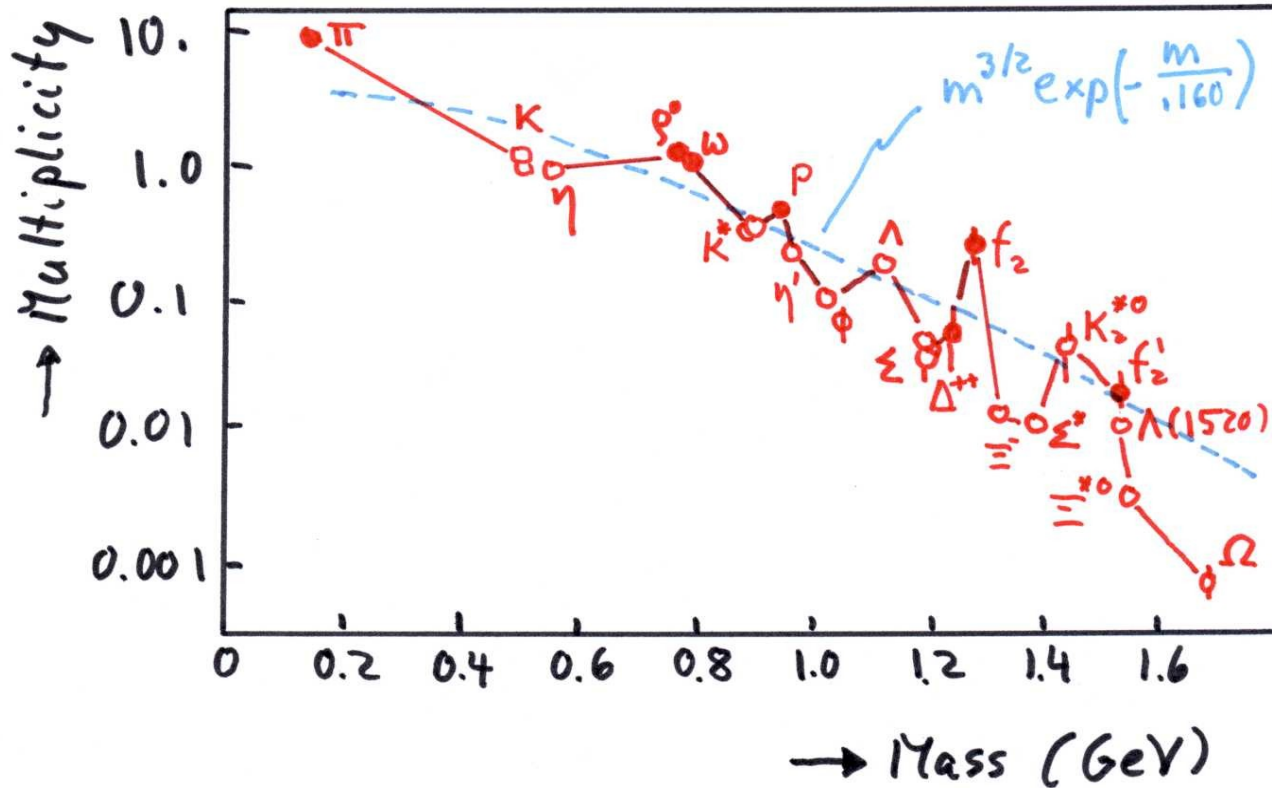


isentropic expansion and full chemical equilibrium between hadrons, leptons, photons

plus:
charge neutrality
net lepton number = net baryon number
constant entropy per baryon

5.1.1 Hadron production in elementary collisions

hadron production in e^+e^- collisions at $\sqrt{s} = 91.2$ GeV (LEP)

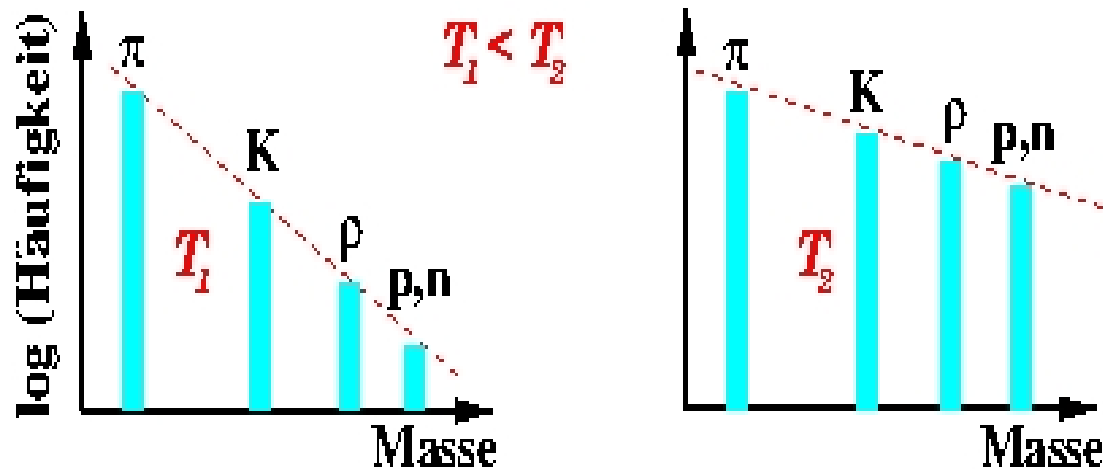


general trend: exponential decrease with mass

in addition: all hadrons with strange valence quarks produced less abundantly
"strangeness suppression"

Thermal energy leads to production of hadrons

equivalence of energy and mass



assume phase space is filled thermally (Boltzmann) at hadronization:

abundance of hadron species $\propto m^{3/2} \exp(-m/T)$

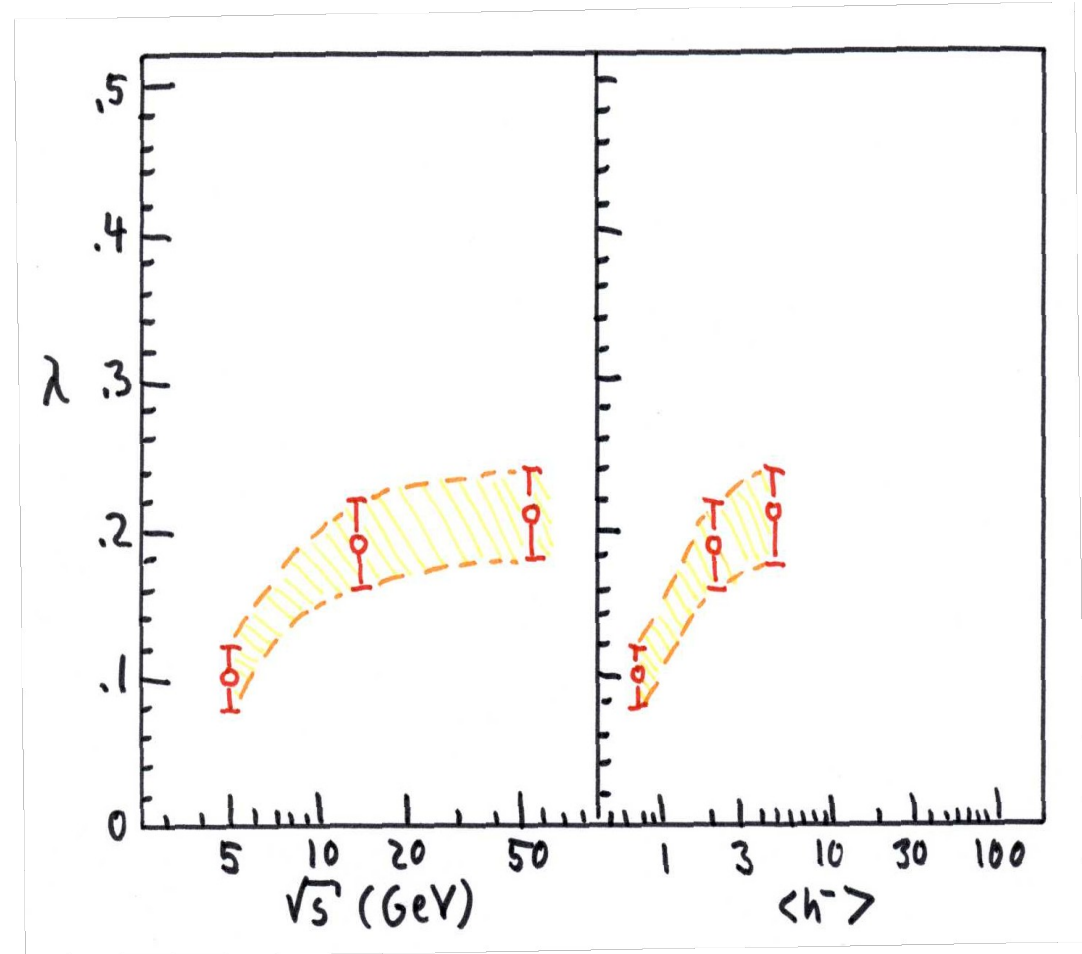
determined by temperature (and density) at time of production of hadrons = hadronization

Strangeness suppression in hadron-hadron and e+e- collisions

the general exponentially falling trend of abundancies with mass is superimposed by a characteristic suppression of all hadrons with valence strange quarks

quantified by the so-called Wroblewski factor: $\lambda = \frac{2s\bar{s}}{u\bar{u} + d\bar{d}}$

estimate from measured yields of hadrons the primary yields (before strong decays) and count the valence quarks



A.Wroblewski, Acta Phys. Pol. B16 (1985) 379

5.1.2 Hadron production in high energy heavy ion collisions

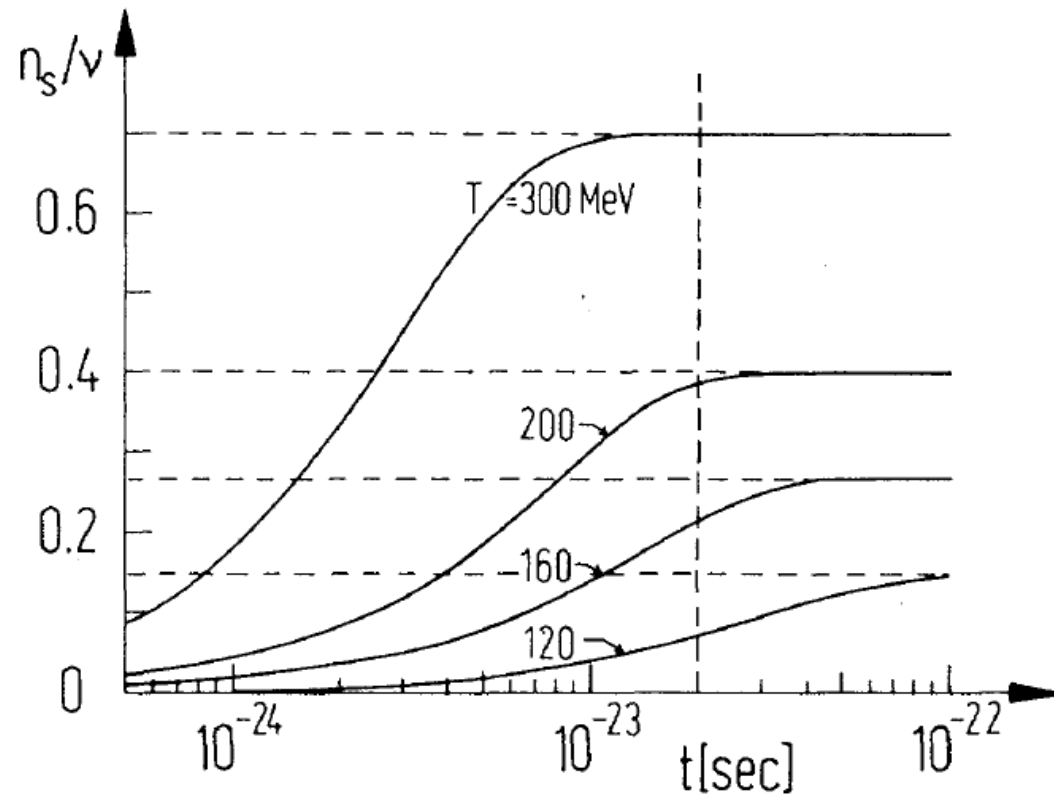
Expectation for strangeness production in heavy ion collisions where QGP is produced:

in QGP strangeness gets into equilibrium on a fast time scale

J. Rafelski, B. Müller,
Phys. Rev. Lett. 48 (1982) 1066

there should be more strangeness in heavy ion collisions than in elementary collisions if a QGP is formed

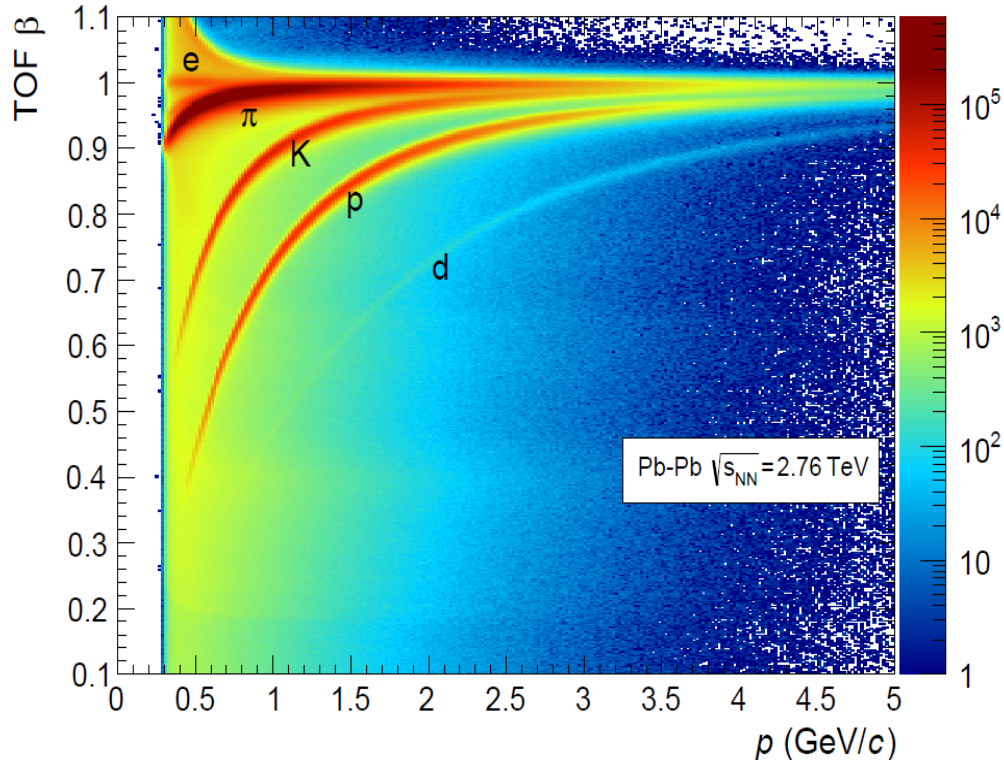
enhanced production of strange hadrons one of the earliest predicted signature of QGP



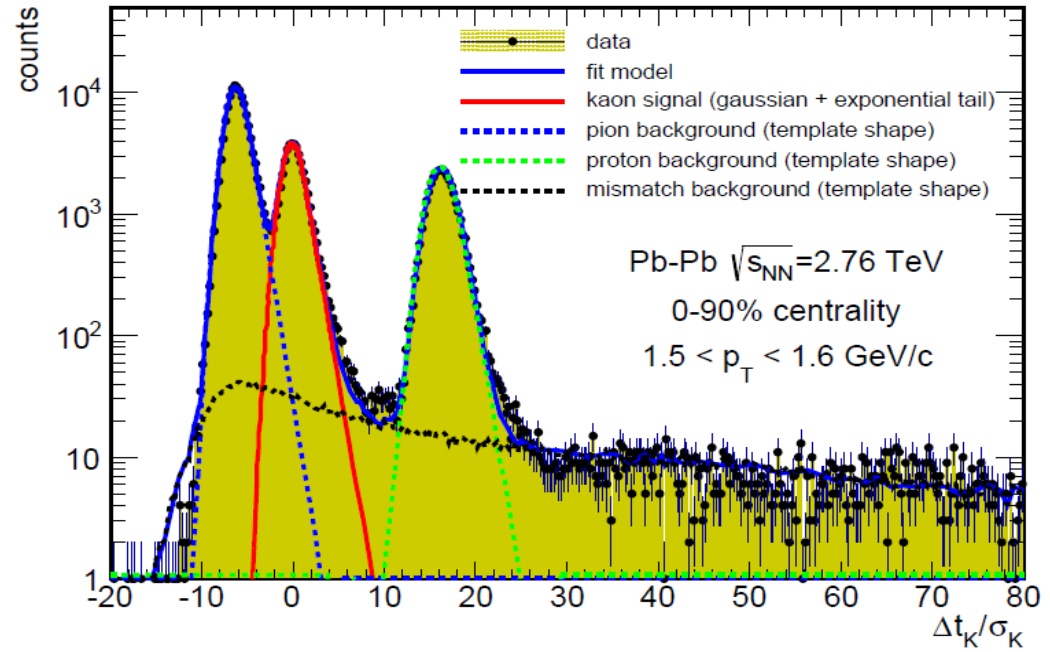
ratio of strange quark to baryon number abundance in a QGP for various temperatures

How to measure production yields of identified hadrons

identification via time-of-flight plus momentum measurement:



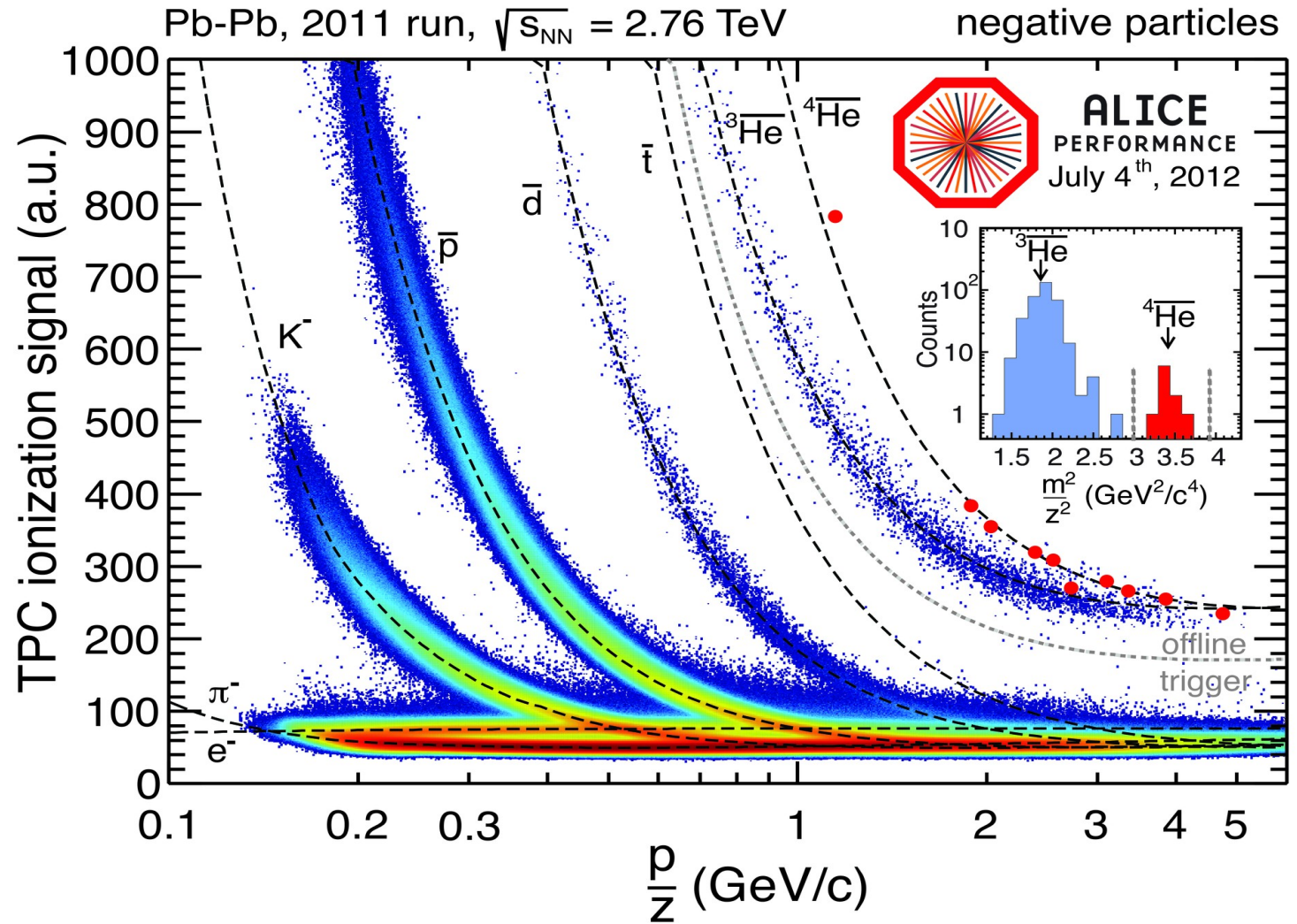
works well as long as $\beta/c < 1$
but then fades quickly



How to measure production yields of identified hadrons

Identification via specific energy loss:

example: ALICE TPC 150 space points per track



ALI-PERF-36713

How to measure production yields of identified hadrons

Identification via invariant mass of decay products (see lecture 2)

$$\begin{aligned} M^2 &= \left[\begin{pmatrix} E_1 \\ \vec{p}_1 \end{pmatrix} + \begin{pmatrix} E_2 \\ \vec{p}_2 \end{pmatrix} \right]^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 \\ &= m_1^2 + m_2^2 + 2E_1E_2 - 2\vec{p}_1 \cdot \vec{p}_2 \\ &= m_1^2 + m_2^2 + 2E_1E_2 - 2p_1p_2 \cos \vartheta \end{aligned}$$

How to measure production yields of identified hadrons

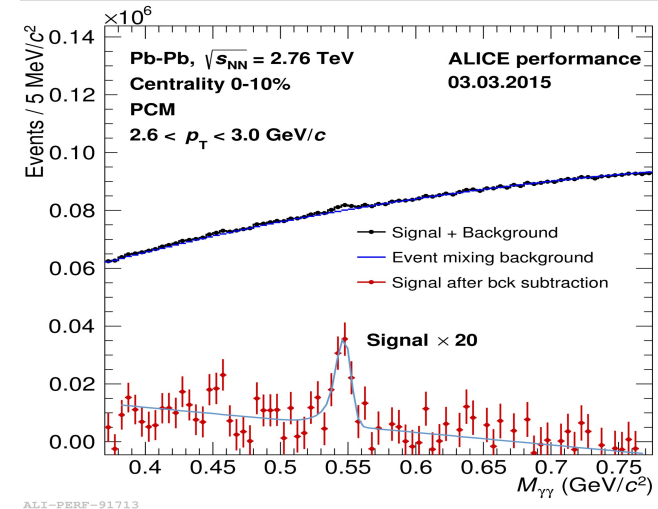
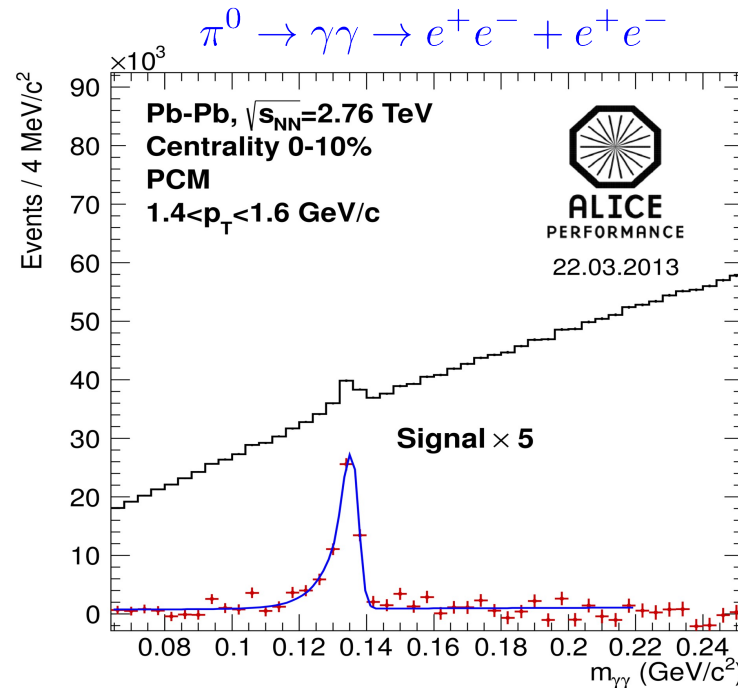
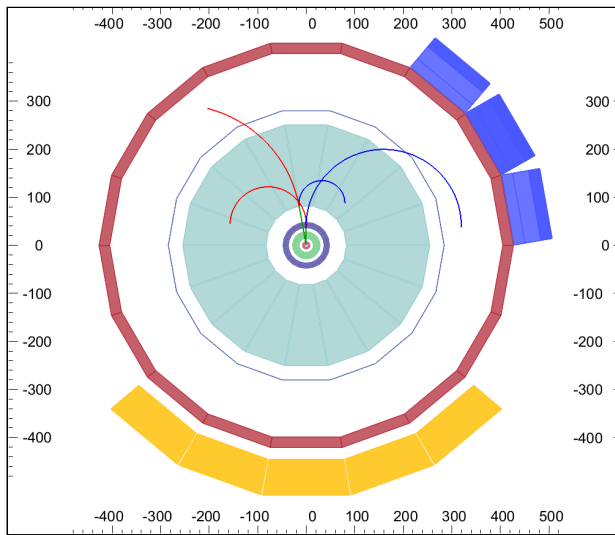
electromagnetic decays:

$$\pi^0 \rightarrow \gamma\gamma \quad m_{\pi^0} = 0.135\text{GeV}, \text{ BR} = 0.988, c\tau = 25.1 \text{ nm}$$

$$\eta \rightarrow \gamma\gamma \quad m_{\eta} = 0.548\text{GeV}, \text{ BR} = 0.393, c\tau = 0.2 \text{ nm}$$

happen practically in the interaction point/target

detect photons in calorimeter
or via e+e- from conversion in detector material



$\eta \rightarrow \gamma\gamma \rightarrow e^+e^- + e^+e^-$

How to measure production yields of identified hadrons

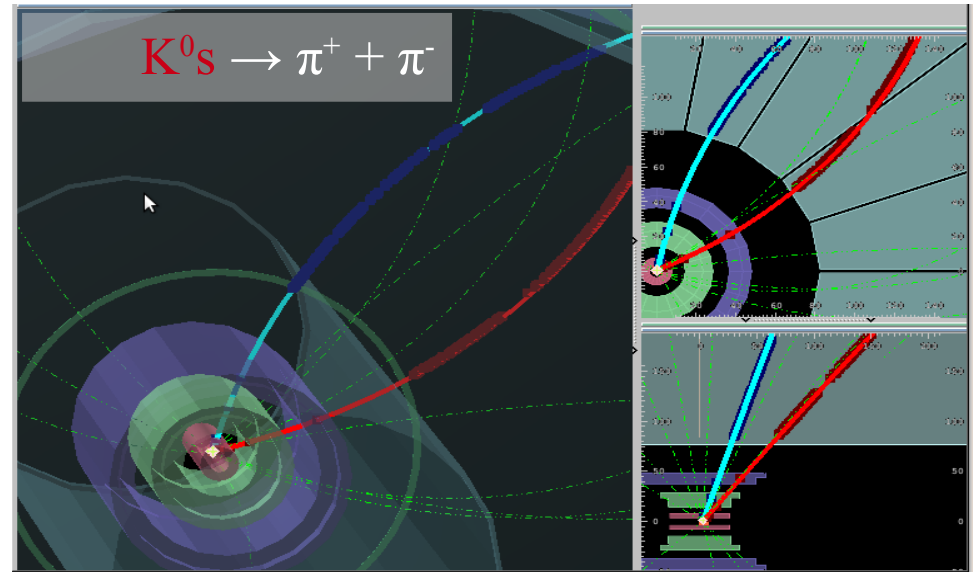
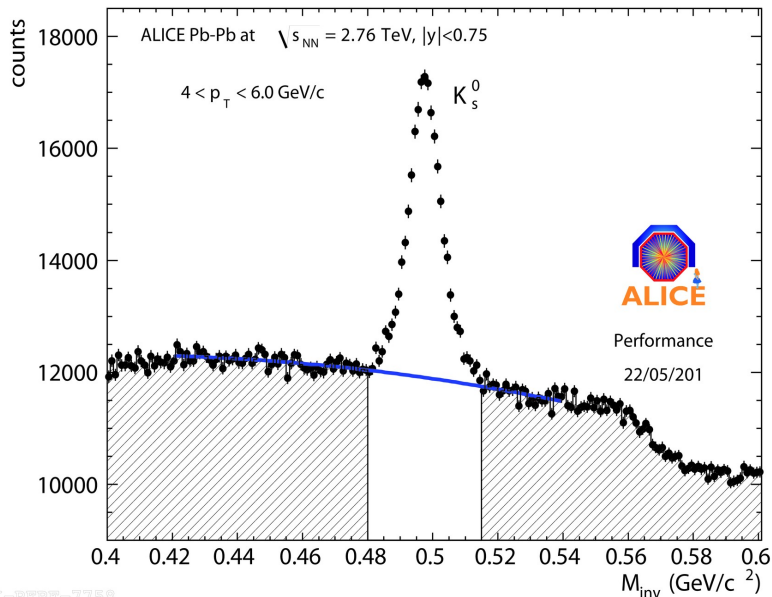
Identification via invariant mass of weak decay products

$$K_S^0 \rightarrow \pi^+ + \pi^- \quad (\text{B.R.}68\%) \quad c\tau = 2.68 \text{ cm}$$

$$\Lambda \rightarrow p + \pi^- \quad (\text{B.R.}64\%) \quad c\tau = 7.89 \text{ cm}$$

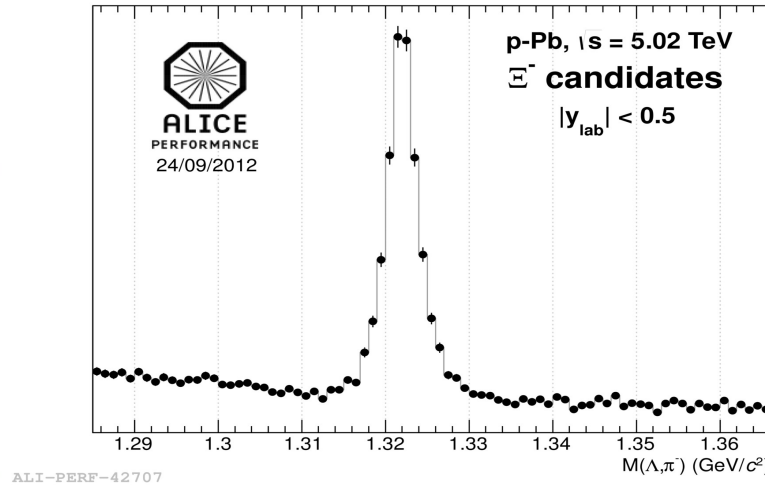
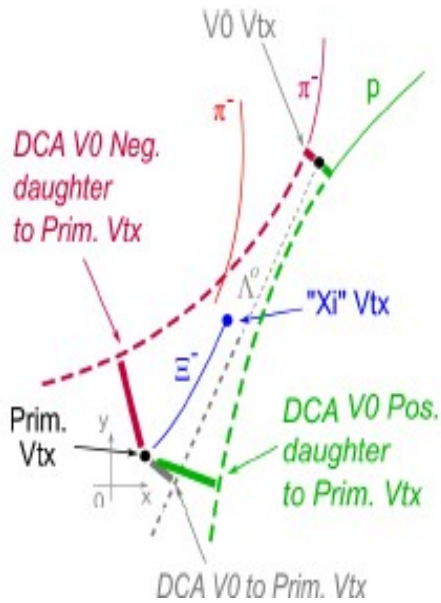
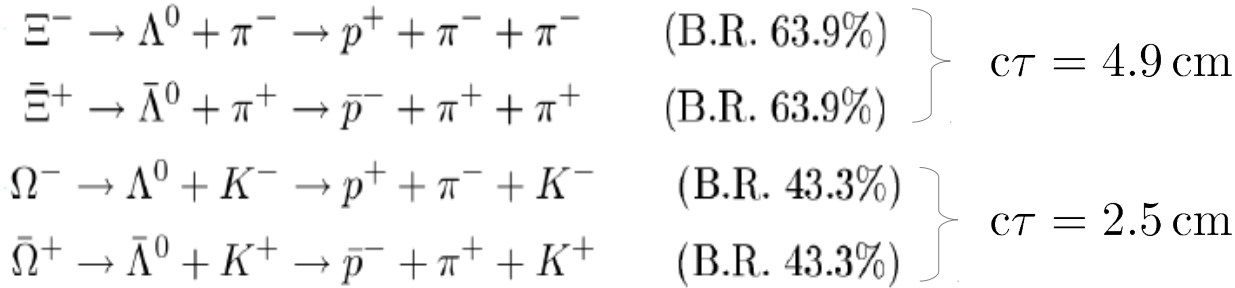
look for secondary decay vertex of a neutral object a few 10 cm away from interaction point

works up to very high momentum!

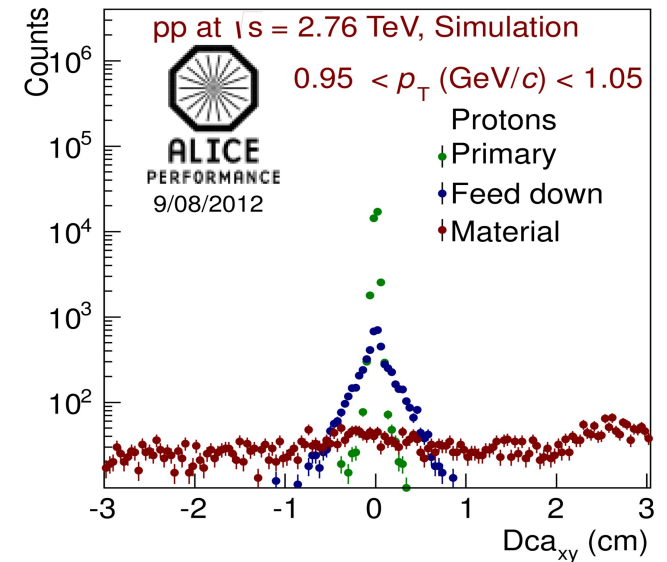
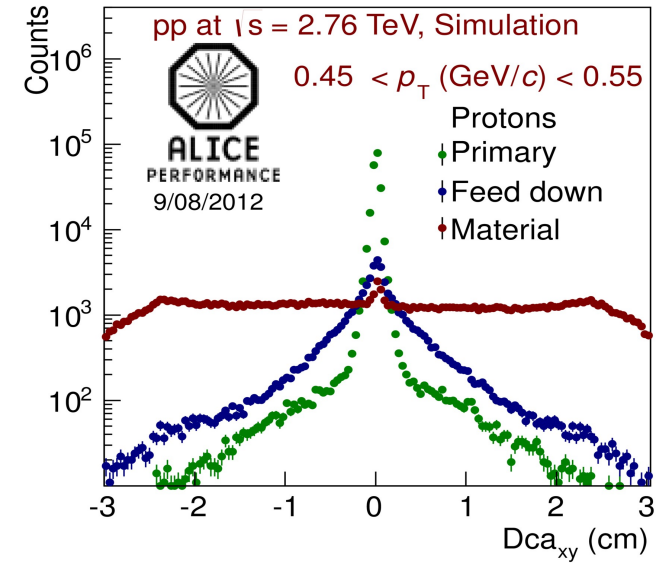


How to measure production yields of identified hadrons

Identification via invariant mass of weak decay products



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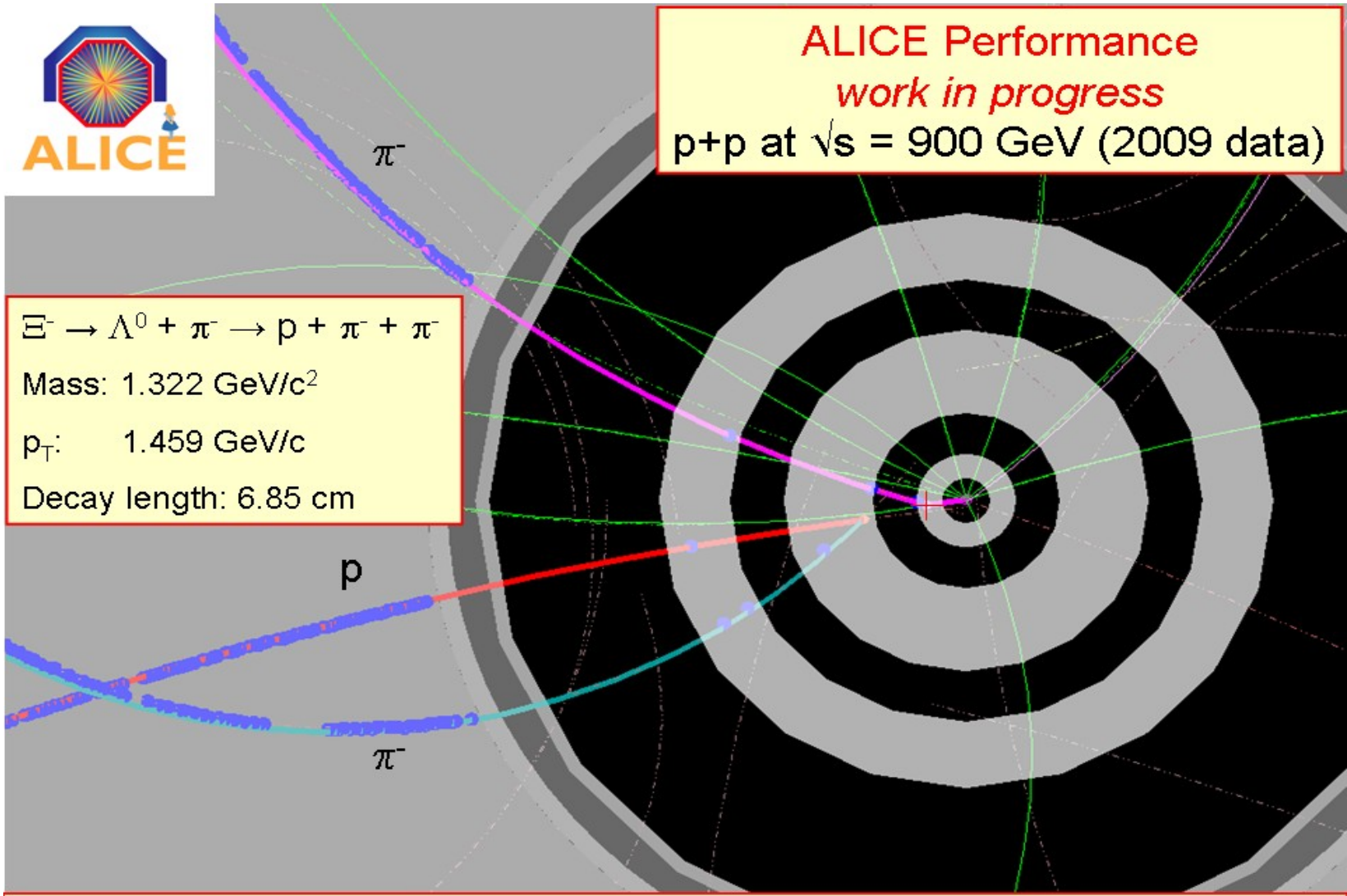
ALI-PERF-28409

How to measure production yields of identified hadrons



ALICE Performance
work in progress
p+p at $\sqrt{s} = 900$ GeV (2009 data)

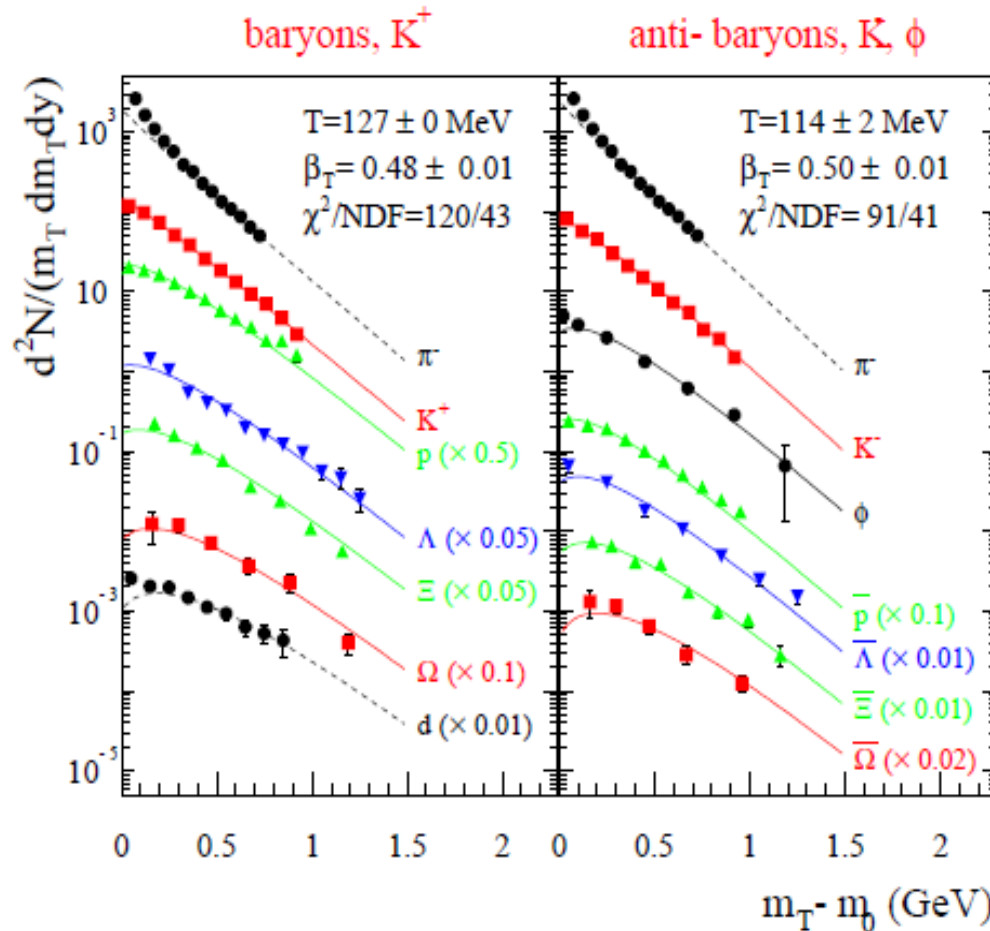
$\Xi^- \rightarrow \Lambda^0 + \pi^- \rightarrow p + \pi^- + \pi^-$
Mass: 1.322 GeV/c²
 p_T : 1.459 GeV/c
Decay length: 6.85 cm



Run 104892, raw data chunk 09000104892020.130, event in chunk 1840

hadron production in central PbPb collisions at the CERN SPS

NA49

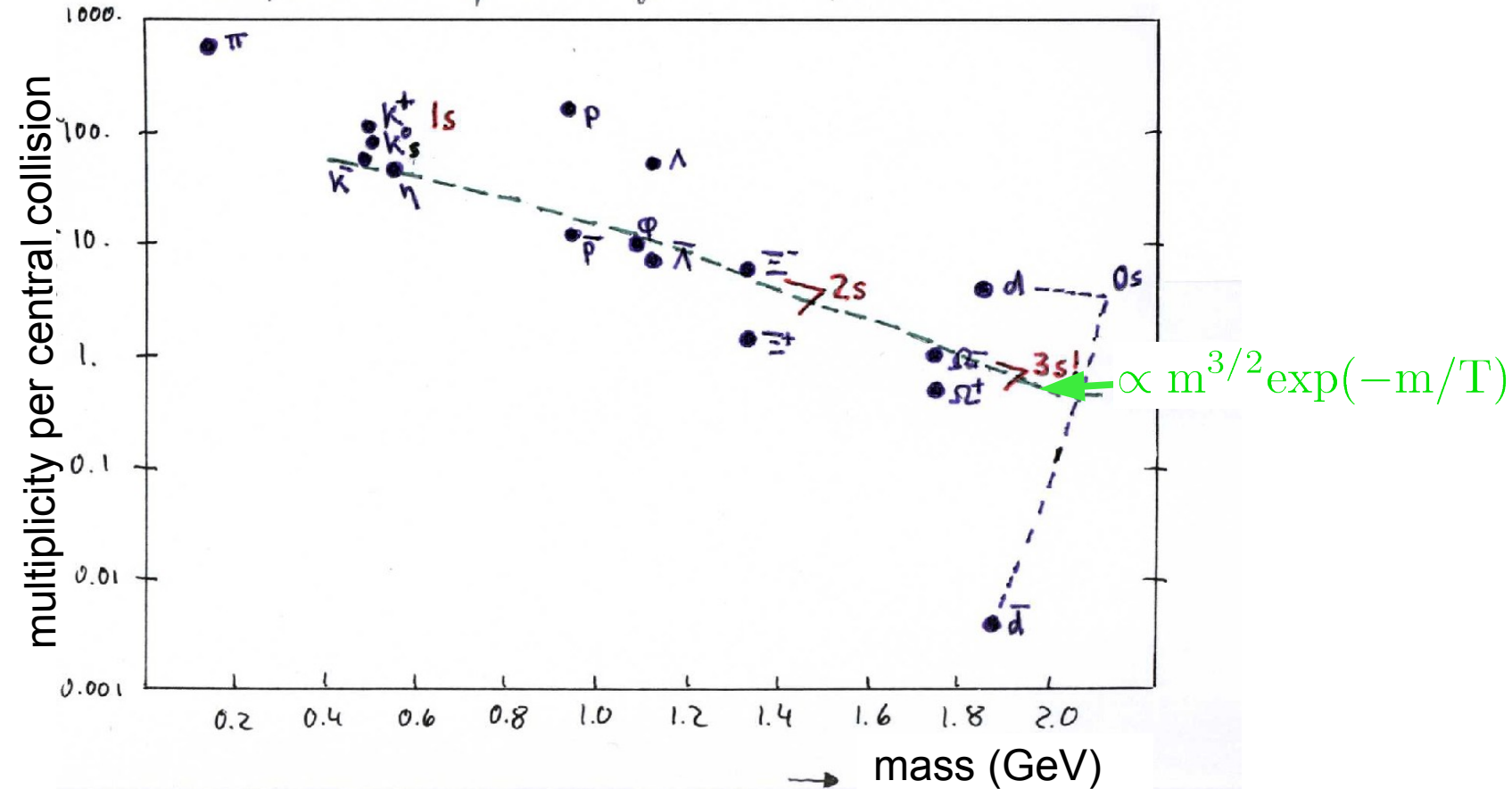


measure p_t spectra and integrate/extrapolate yield over all values of p_t from 0 to infinity

between 5 different experiments a comprehensive data set for 158 A GeV PbPb collisions

First look at particle multiplicities for CERN SPS PbPb central collisions

compilation of data from 6 experiments

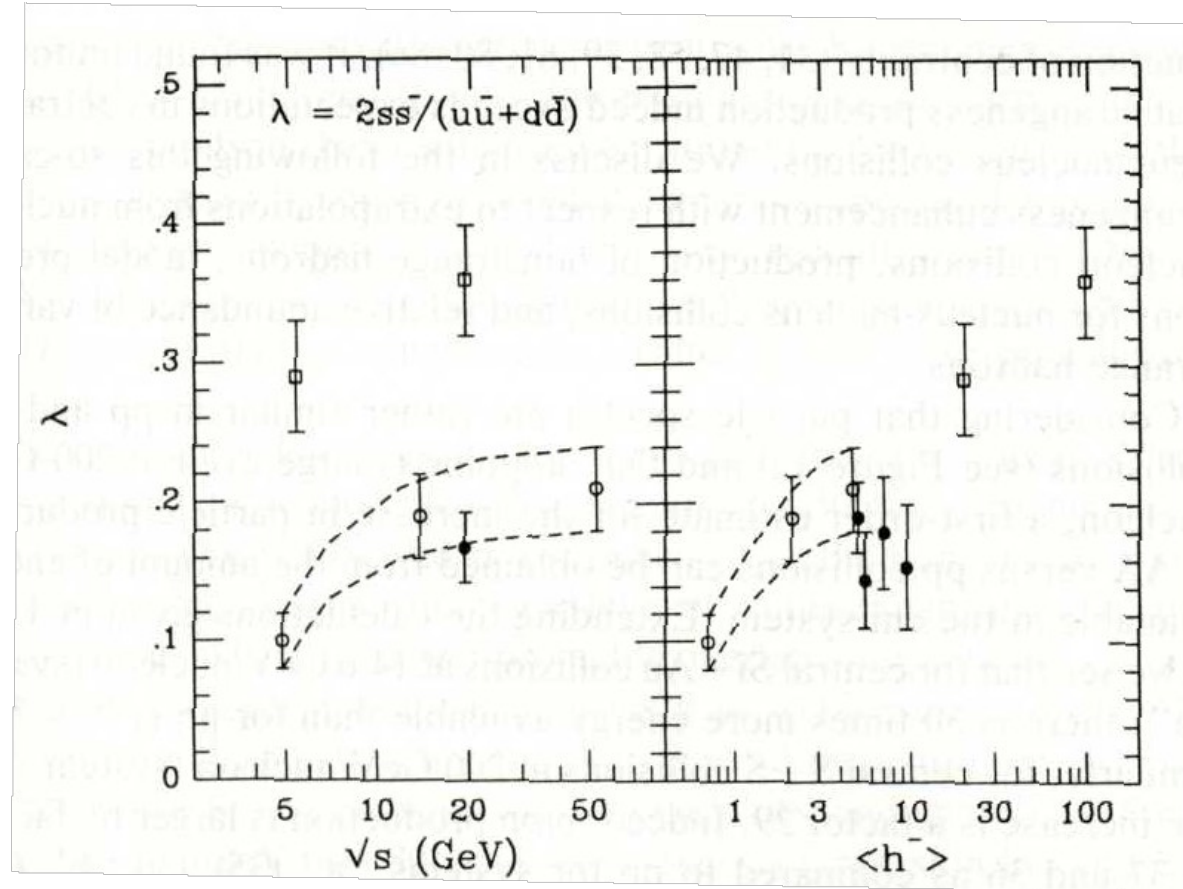


measured multiplicities can be understood under assumption that all particles are produced simultaneously at temperature of 170 MeV

(will come to characteristic splitting of baryons and antibaryons later)

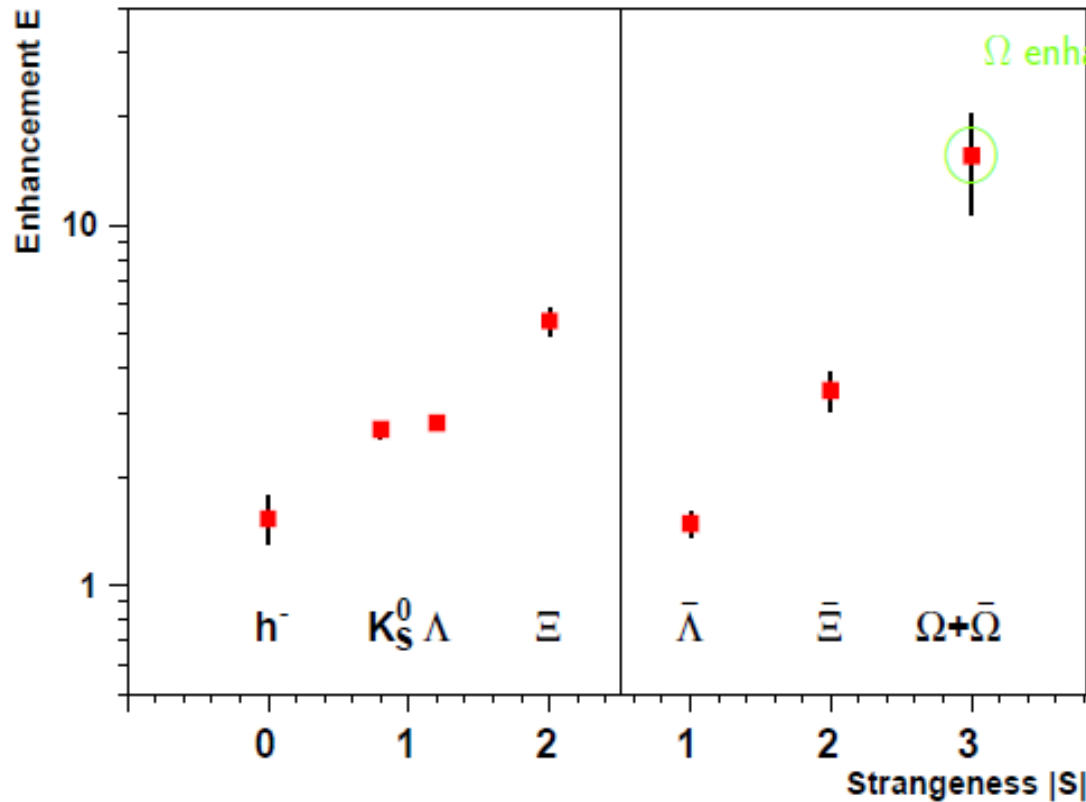
First indication from SiSi and SS collisions:

in nuclear collisions about factor of 2 more strangeness produced than in pp



Strangeness enhancement in PbPb collisions relative to pp

WA97, 158 A GeV/c Pb + Pb Collisions, Phys. Lett. B449 (1999) 401



$$\text{Enhancement } E = \frac{\text{yield(PbPb)}/N_{\text{part}}(\text{PbPb})}{\text{yield(pBe)}/N_{\text{part}}(\text{pBe})}$$

why Be target?

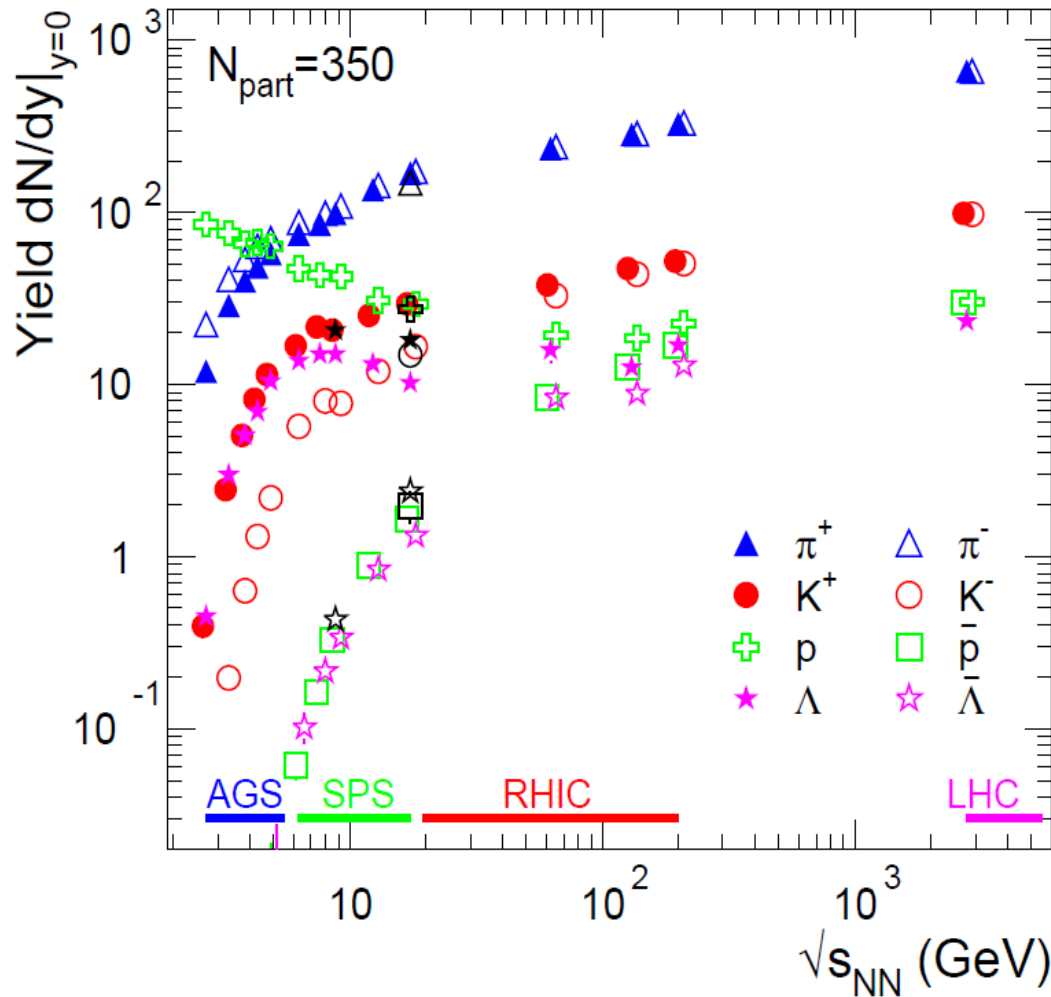
- i) simpler than liquid hydrogen
- ii) on average one pp or pn collision – better, if isospin effects important

general feature: in high energy nuclear collisions hadrons with strange quarks produced more abundantly than in pp collisions

the enhancement grows with the number of strange valence quarks

Particle production in central AA collisions

from A. Andronic



a summary of 25 years of experimental research

systematic trends with beam energy:
 mesons rise and level off
 baryons drop
 antibaryons rise steeply

can we understand all of these?

5.2 Statistical model description of hadron yields

5.2.1 Choice of statistical ensemble

Grand Canonical Ensemble (GC): in large system, with large number of produced particles, conservation of additive quantum numbers (B, S, I₃) can be implemented on average by use of chemical potential μ

→ asymptotic realization of exact canonical approach
much simpler to compute

Canonical Ensemble (C): in small system, with small particle multiplicity, conservation laws must be implemented locally on event-by-event basis (Hagedorn 1971, Shuryak 1972, Rafelski/Danos 1980, Hagedorn/Redlich 1985)
severe phase space reduction for particle production “**canonical suppression**”

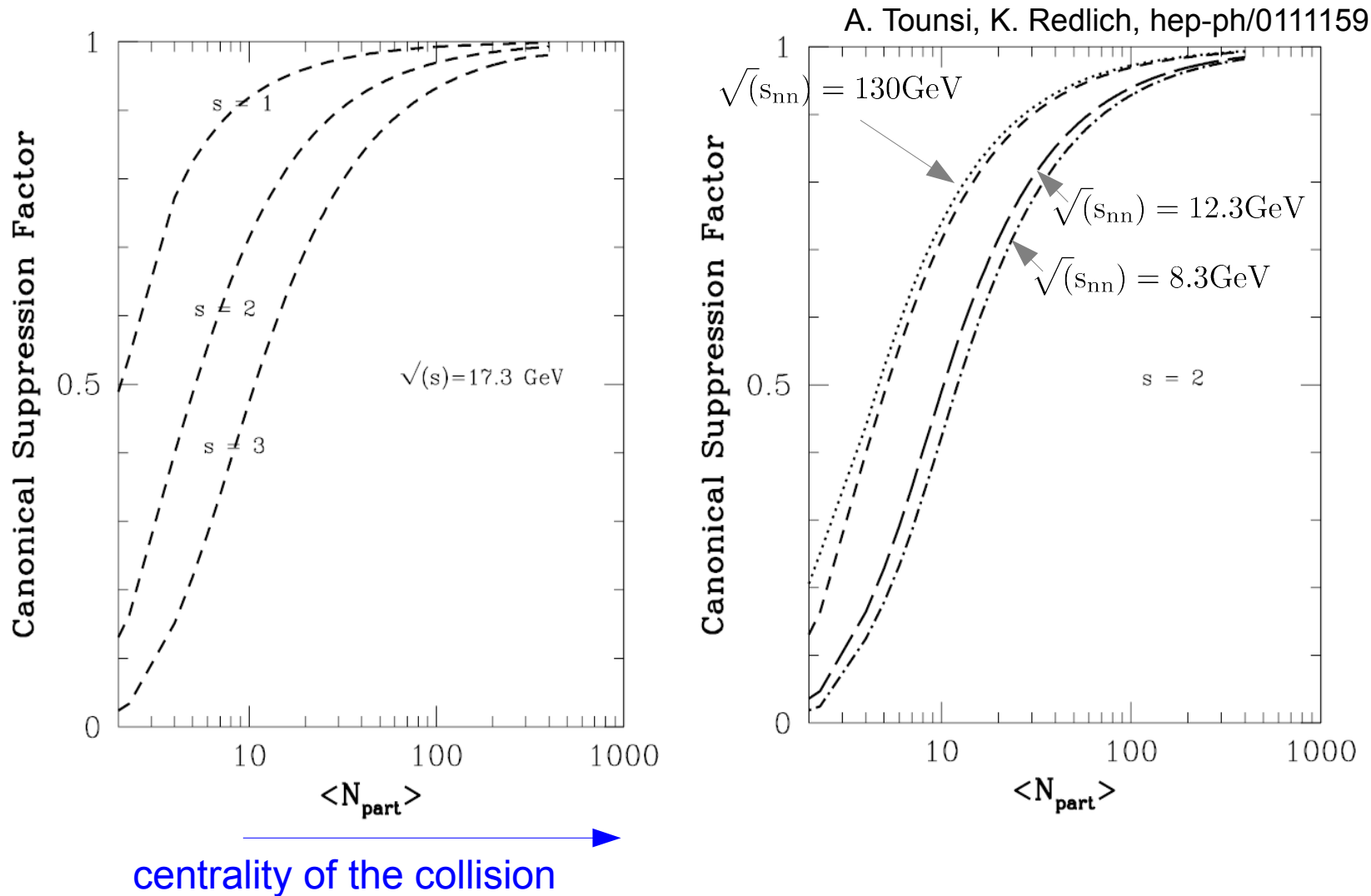
Results of C and GC can be related in a simple way: (Tounsi/Redlich 2001)

$$\langle N_K \rangle^C = \langle N_K \rangle^{GC} \underbrace{\frac{I_1(2\langle N_K \rangle^{GC})}{I_0(2\langle N_K \rangle^{GC})}}_{F_{CS}}$$

here 'K' stands generically for all hadrons
with S = -1
and analogously for S = -2 and S = -3

$$F_{CS,2} = \frac{I_2}{I_0} \quad F_{CS,3} = \frac{I_3}{I_0}$$

Difference between computations in the canonical and grand canonical ensemble



already for moderately central PbPb collisions (100 of possible 416 nucleons in overlap region) deviations small – 10% level for SPS energy

Canonical corrections relevant in:

- low energy HI collisions (Cleymans/Redlich/Oeschler 1998/1999)
- very peripheral HI collisions (Hamieh, Redlich, Tounsi 2000)
- high energy hh or e+e- collisions (Becattini/Heinz 1996/1997)
- considering heavy quarks in HI collisions (see later)

5.2.2 Grand canonical ensemble and application to data from high energy heavy ion collisions

partition function: $\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$

particle densities: $n_i = N/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp((E_i - \mu_i)/T) \pm 1}$

for every conserved quantum number there is a chemical potential:

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

but can use conservation laws to constrain V, μ_S, μ_{I_3}

baryon number: $V \sum_i n_i B_i = Z + N \quad \rightarrow V$

strangeness: $V \sum_i n_i S_i = 0 \quad \rightarrow \mu_S$

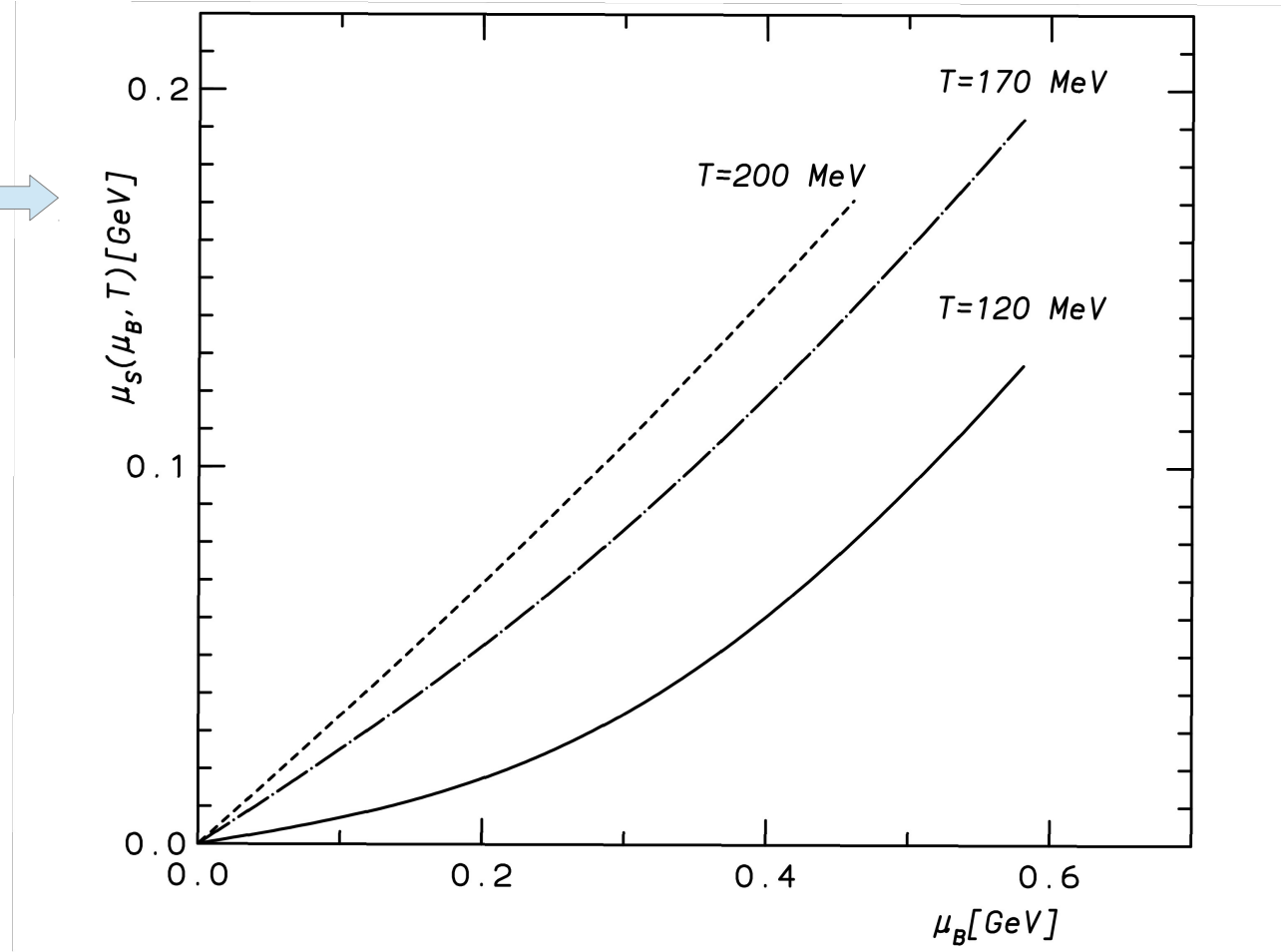
charge: $V \sum_i n_i I_i^3 = \frac{Z - N}{2} \quad \rightarrow \mu_{I_3}$

only 2 free parameters left \rightarrow

fit at each energy provides values for T and μ_b

Dependence of μ_s on T and μ_b

for every pair of (T, μ_b)
there is only one value of
 μ_s that ensures total
strangeness $S = 0$
not a free parameter at all



Some technical details:

van der Waals type interaction via excluded volume correction
a volume is assigned to each hadron and densities have to be corrected
good guess: $r=0.3$ fm to account for hard core nucleon-nucleon repulsion
thermodynamically consistent treatment following Rischke, Gorenstein, Stöcker, Greiner, 1991

$$p^{excl.}(T, \mu) = p^{id.gas}(T, \hat{\mu}); \quad \text{with } \hat{\mu} = \mu - v_{eigen} p^{excl.}(T, \mu)$$

finite volume correction a la Balian and Bloch

$$f = 1 - \frac{3\pi}{4pR} + \frac{1}{(pR)^2}$$

width of all resonances included by integrating over Breit-Wigner distributions
Weinhold, Friman, Nörenberg, 1996

$$\ln Z_R = N \frac{V d_R}{2\pi^2} T \exp[\mu/T] \int_{s_{min}}^{s_{max}} ds s K_2(\sqrt{s}/T) \frac{1}{\pi} \frac{m_R \Gamma_R}{(s - m_R^2)^2 + m_R^2 \Gamma_R^2}$$

for a review see: Braun-Munzinger, Redlich, Stachel, in QGP3,
R. Hwa ed. (Singapore 2004) 491-599; nucl-th/0304013

Comparison to experimental data

compute primary thermal occupation probability for each particle species

spectrum of hadrons involves for state-of-the-art calculation 426 hadronic species (PDG2008)

beyond 3 GeV mass knowledge still very incomplete (effects see later)
maybe even below unknown baryonic states?

implement all strong decays according to PDG

(example: for $T=160$ MeV, 80% of all pions come from strong decays)

do experimental data include weak decays? If yes, do same in calculation

compute for a grid of (T, μ_b) χ^2 between statistical ensemble calculation and data

note: ratios of particle yields may have smaller systematic errors

data sets from different experiments may not correspond to exactly the same collisions centrality, correct for this

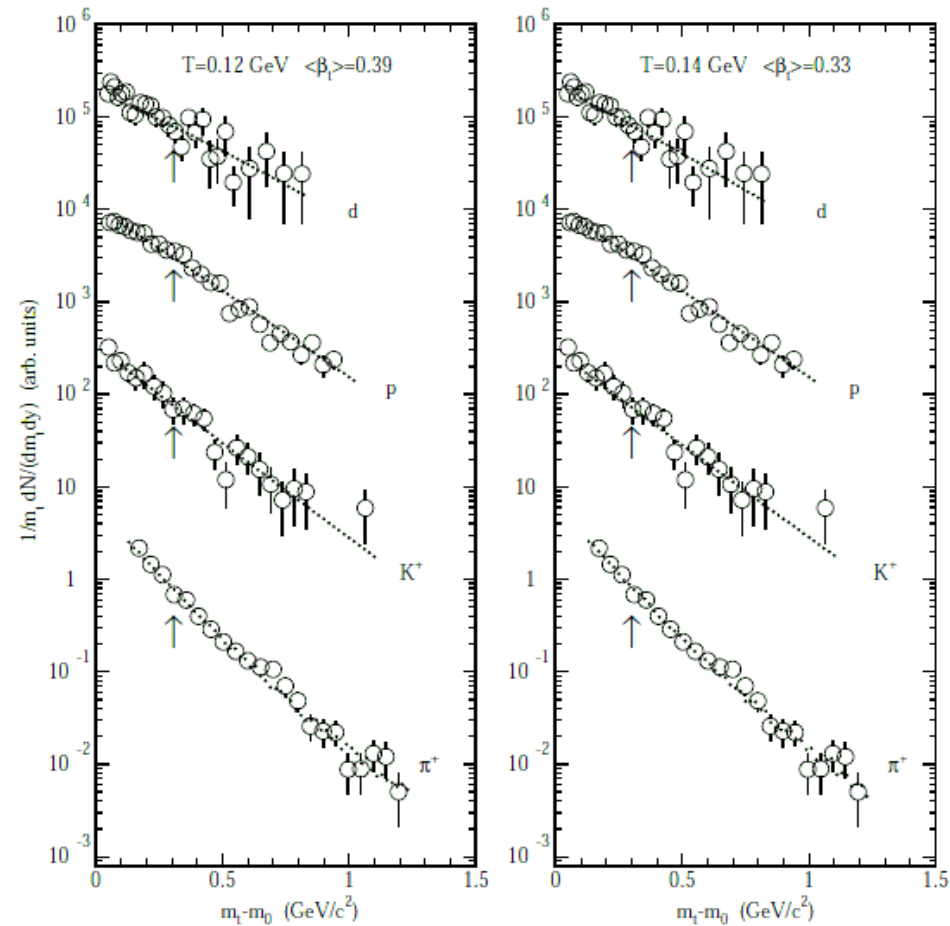
minimize χ^2 to obtain for each beam energy and collision system best set (T, μ_b)

First thermal model results in 1994 for AGS Si+Au data

Integrate hadron spectra over p_t and rapidity

pion spectra exhibit increase at low p_t due to decay of Delta resonance

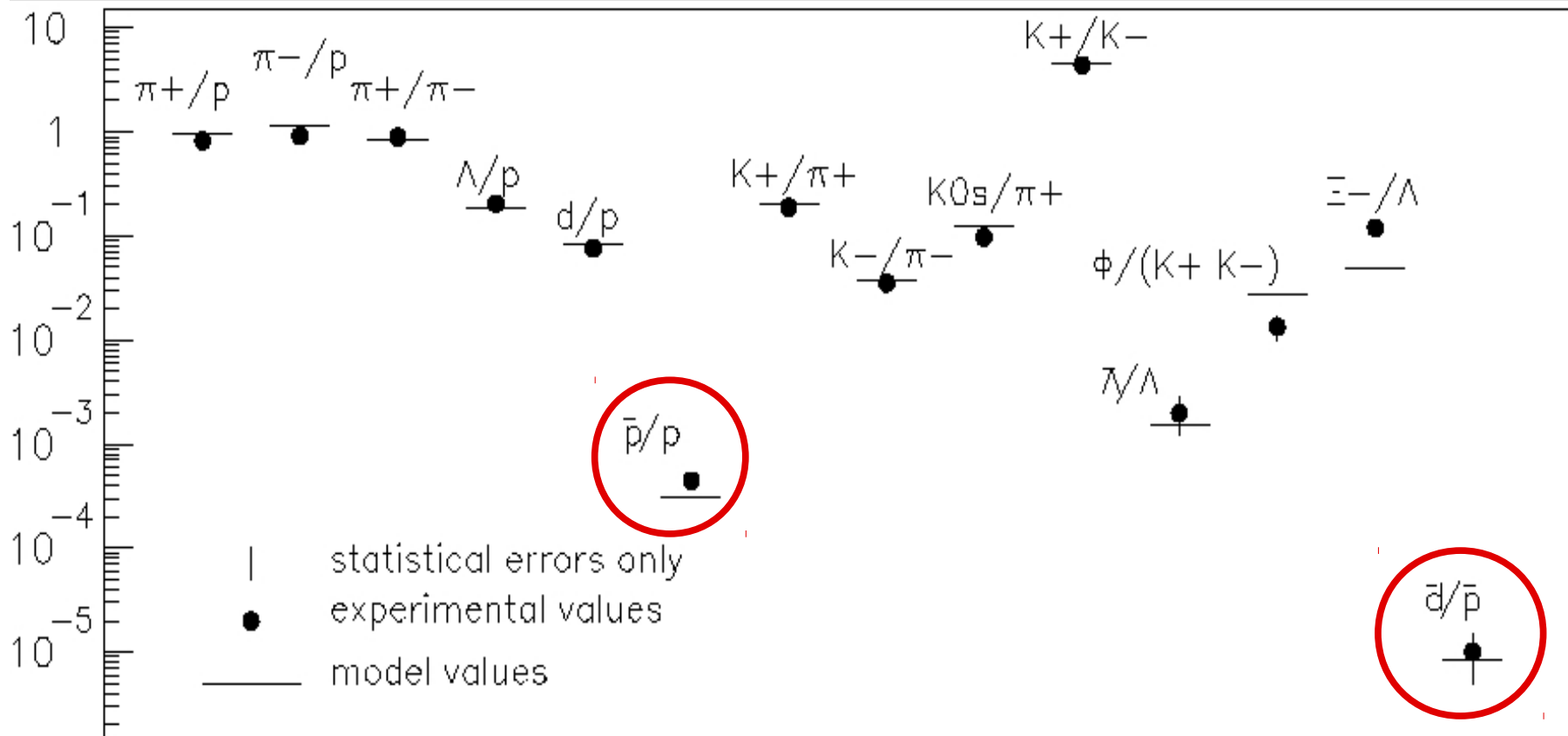
understanding of spectral shapes in chapter 6 (interplay of temperature, collective expansion and decays)



Fit to AGS Data – reproduces yields for strange and nonstrange hadrons down to \bar{d}

14.6 A GeV/c central Si + Au collisions and GC statistical model
 $T = 120$ MeV, $\mu_b = 540$ MeV

P. Braun-Munzinger, J. Stachel, J.P. Wessels, N. Xu, PLB 344 (1995) 43; nucl-th/9410026

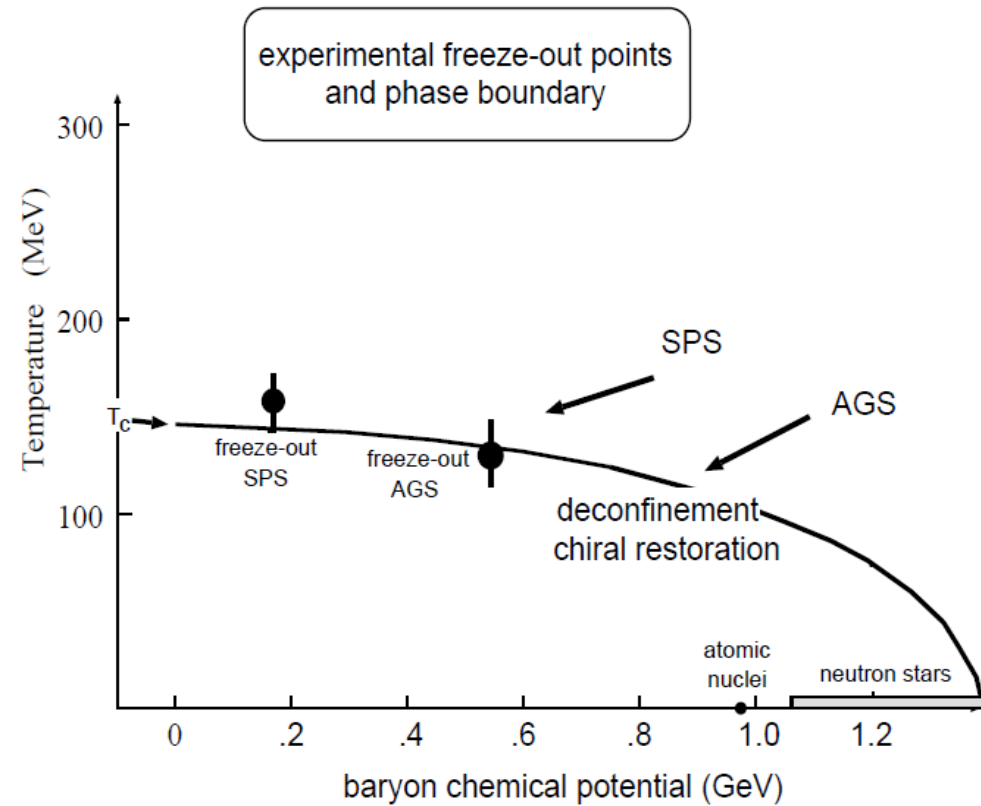


dynamic range: 9 orders of magnitude!

First attempt to establish connection to phase boundary

equation of state from a bag model,
transition 1st order by construction
(see above), anchored at $\mu_b = 0$ to
the lattice QCD results at that time

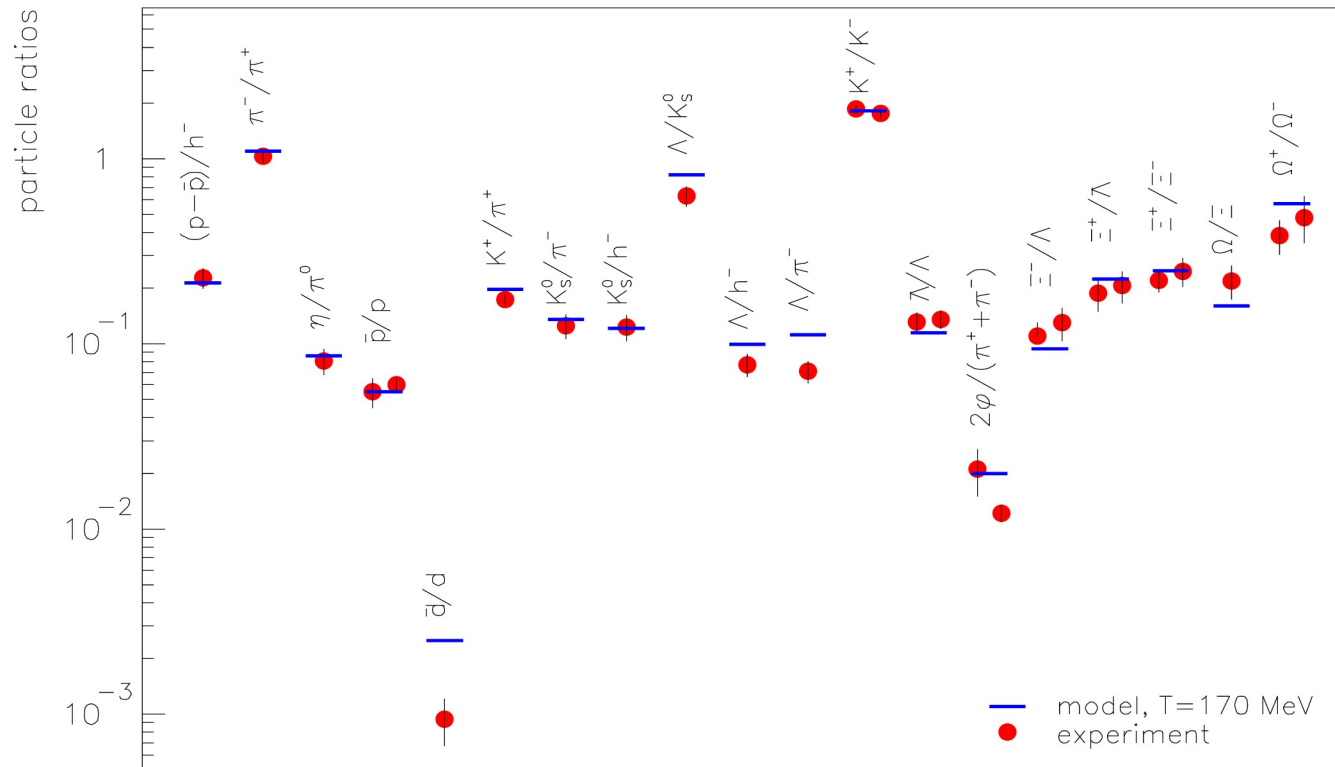
only 2 data points at AGS and SPS
for light beams of Si and S
looks at this for many more collisions
systems in the following



Braun-Munzinger, Stachel, Nucl. Phys. A606 (1996) 320

CERN SPS data: 158 A GeV/c Pb Pb collisions

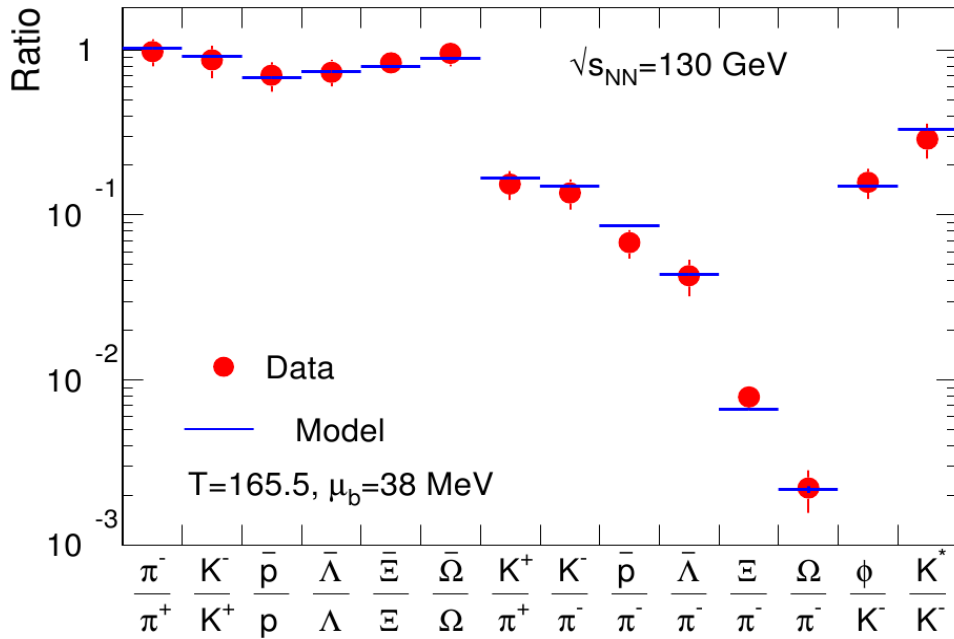
good fit with: $T = 0.170 \pm 0.005 \text{ GeV}$ $\mu_b = 0.255 \pm 0.010 \text{ GeV}$



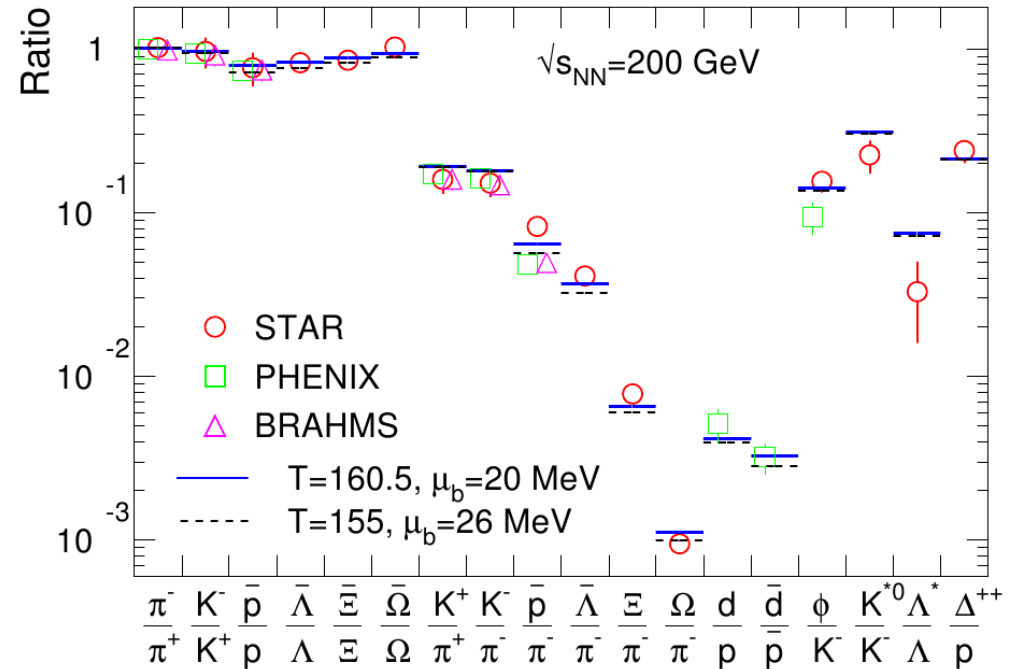
P. Braun-Munzinger, I. Heppe, J. Stachel, PLB 465 (1999) 15
and reanalysis 2004 with more data

Hadron yields at RHIC compared to statistical model (GC)

130 GeV data in excellent agreement with thermal model **predictions**



prel. 200 GeV data fully in line with thermal model **predictions**
some experimental discrepancies

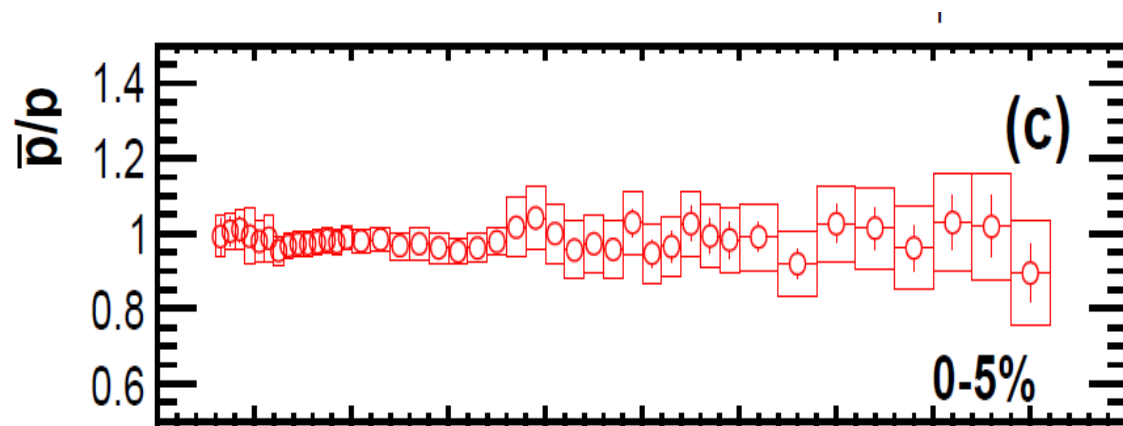


chemical freeze-out at: $T = 165 \pm 5$ MeV

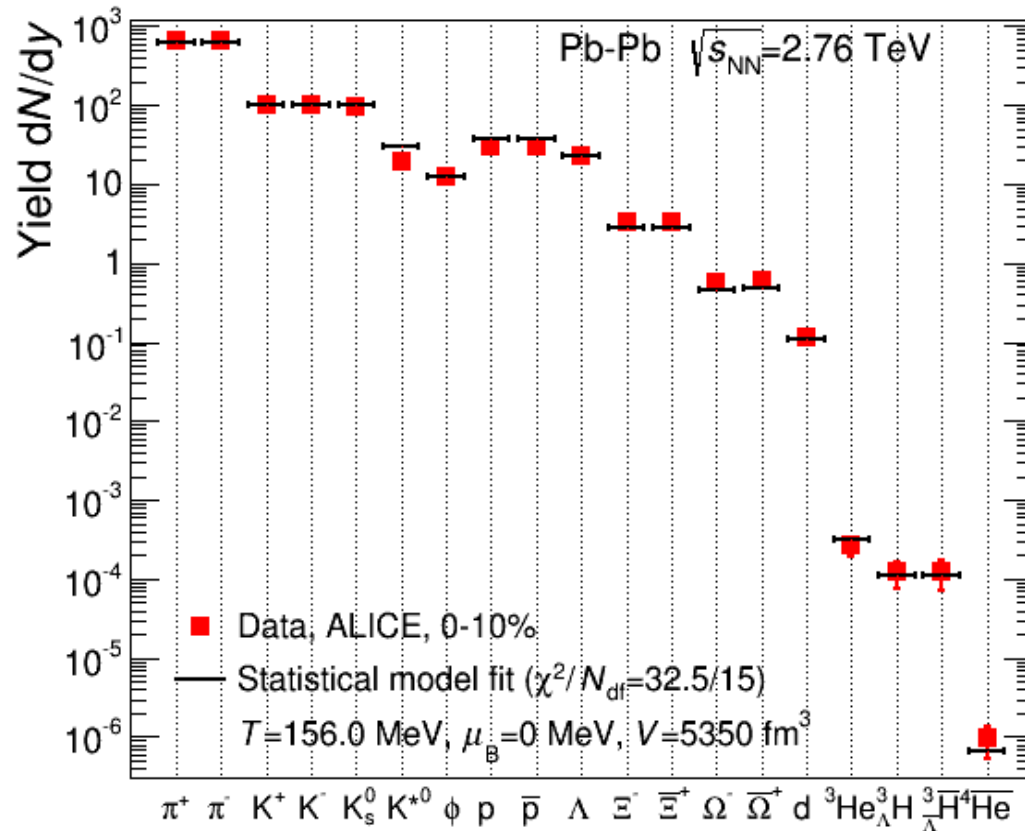
P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel, Phys. Lett. B518 (2001) 41
 A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167

Hadron yields at the LHC – PbPb at 2.76 TeV/nucleon pair

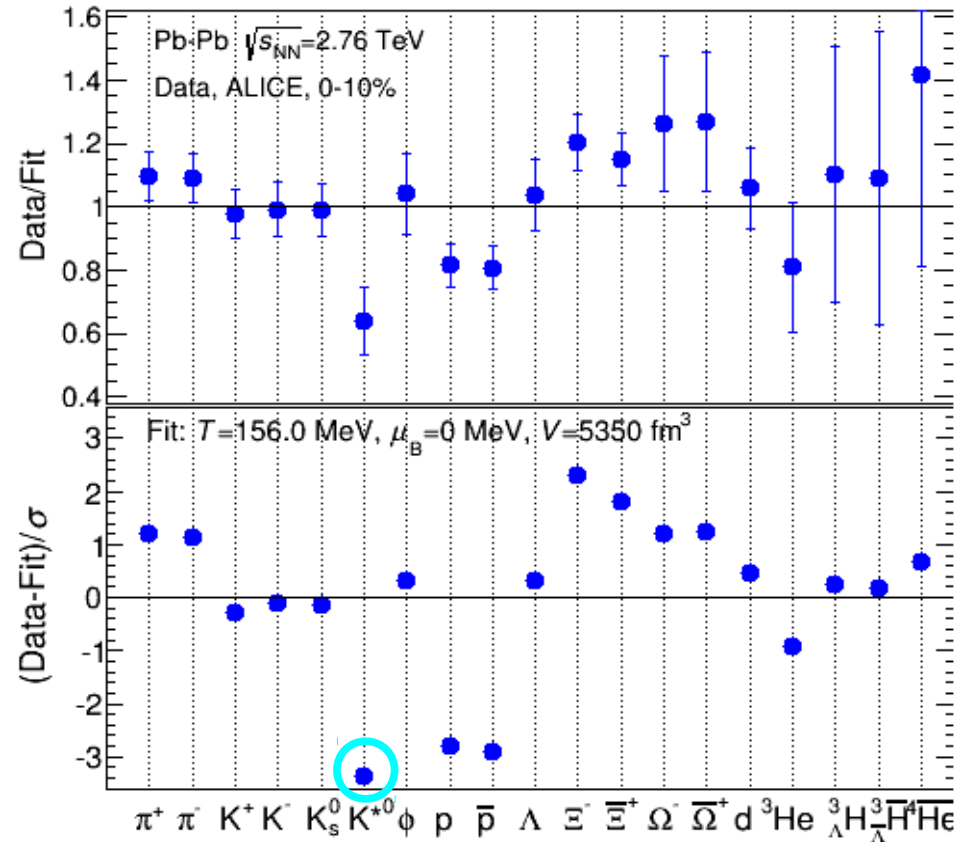
- matter and anti-matter produced in equal proportions at LHC
- consistent with net-baryon free central region, ($\mu_b < 1$ MeV)



Hadron yields at the LHC compared to statistical model



data very well reproduced
 except 2.7 sigma deviation for protons – to be understood
 without protons in the fit:
 T unchanged, reduced $\chi^2 = 1.0$



ignore deviation – strongly decaying resonance

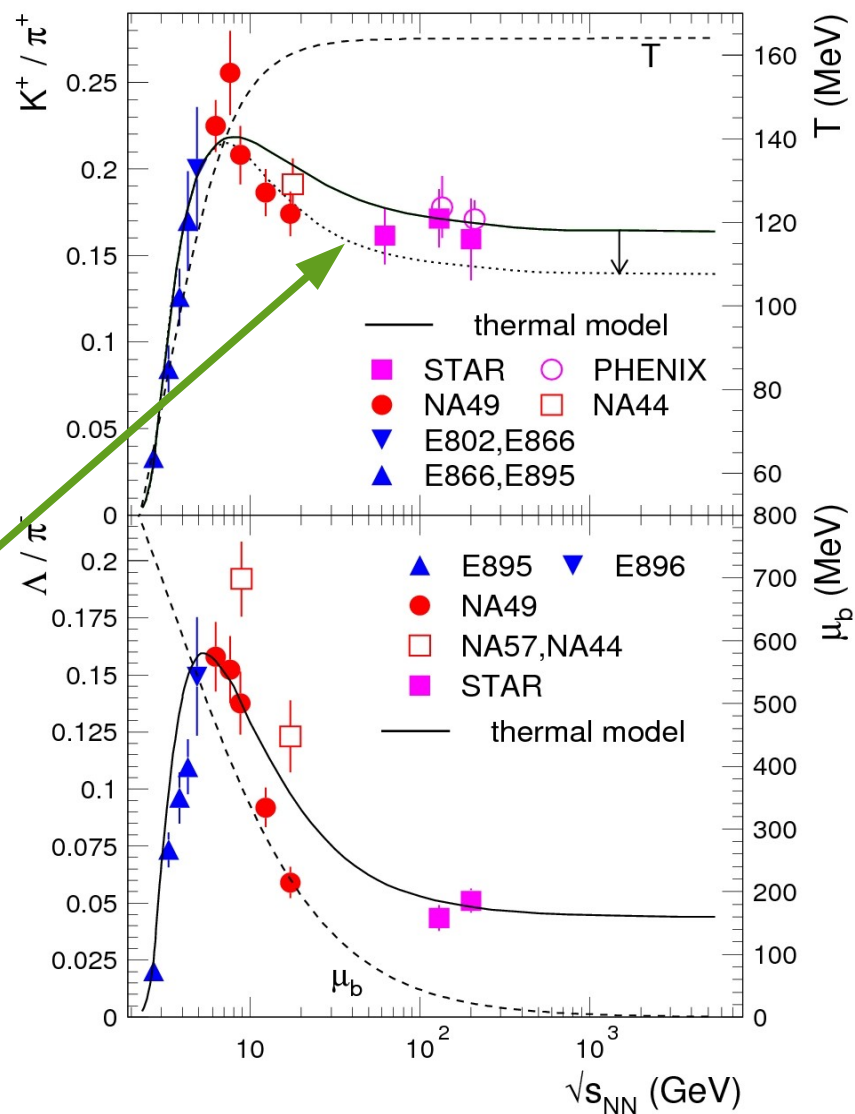
Effect of still incomplete knowledge of hadron spectrum

could proton discrepancy be effect of incomplete hadron spectrum?

based on Hagedorn spectrum:
 estimate effect by extending mass spectrum beyond 3 GeV based on $T_{\text{Hagedorn}} = 200 \text{ MeV}$ and assumption how states decay

strongest contribution to kaon from K^*
 producing one K
 all high mass resonances produce multiple pions

-> further reduction of K^+/π^+
 similar expectation for p/π

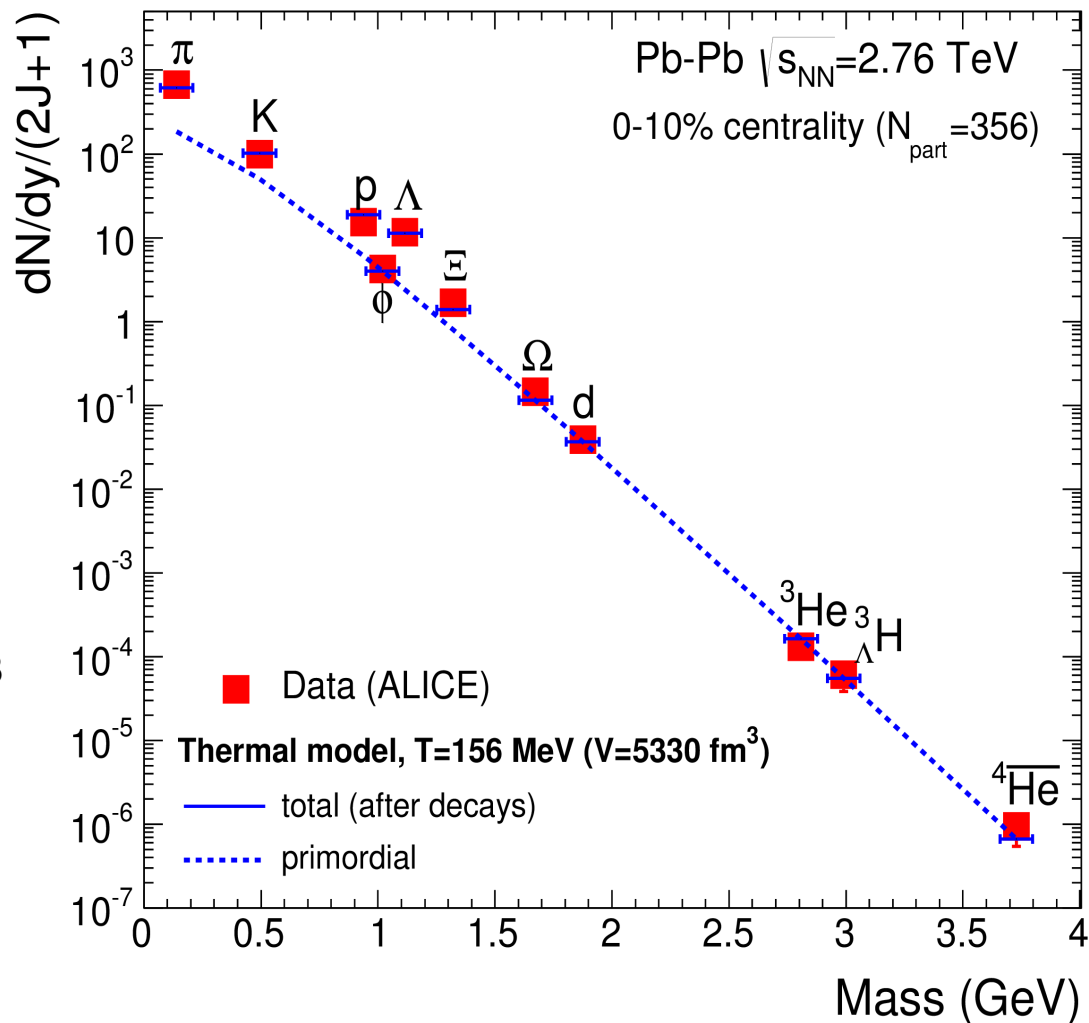


Loosely bound states at hadronization of the hot fireball

agreement over 9 orders of magnitude with QCD statistical operator prediction
 (- strong decays need to be added)

works equally well for nuclei and loosely bound (anti)hyper-nuclei
 prediction P. Braun-Munzinger, J.S., J. Phys. G28 (2002) 1971, J. Phys. G21 (1995) L17

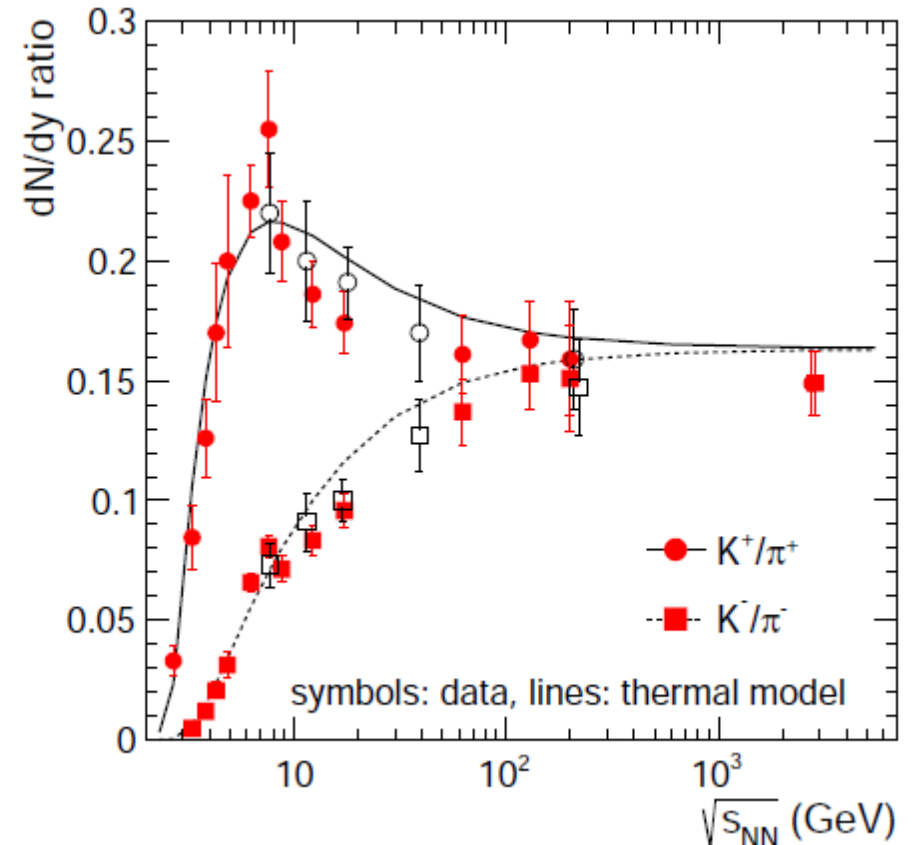
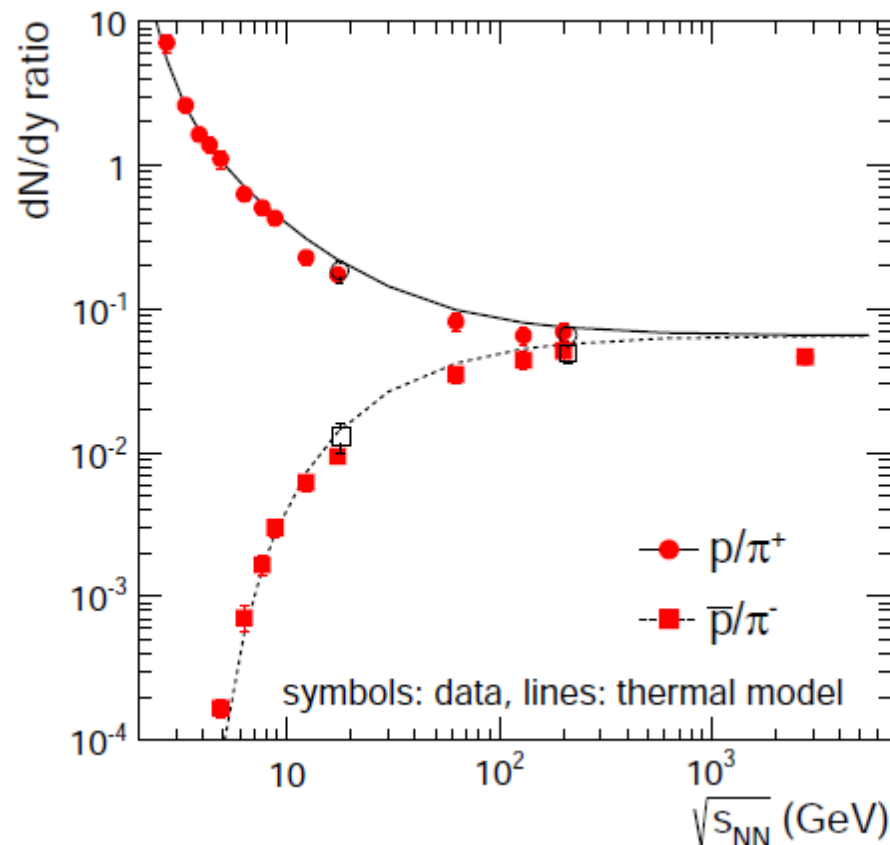
strong indication of isentropic expansion in hadronic phase



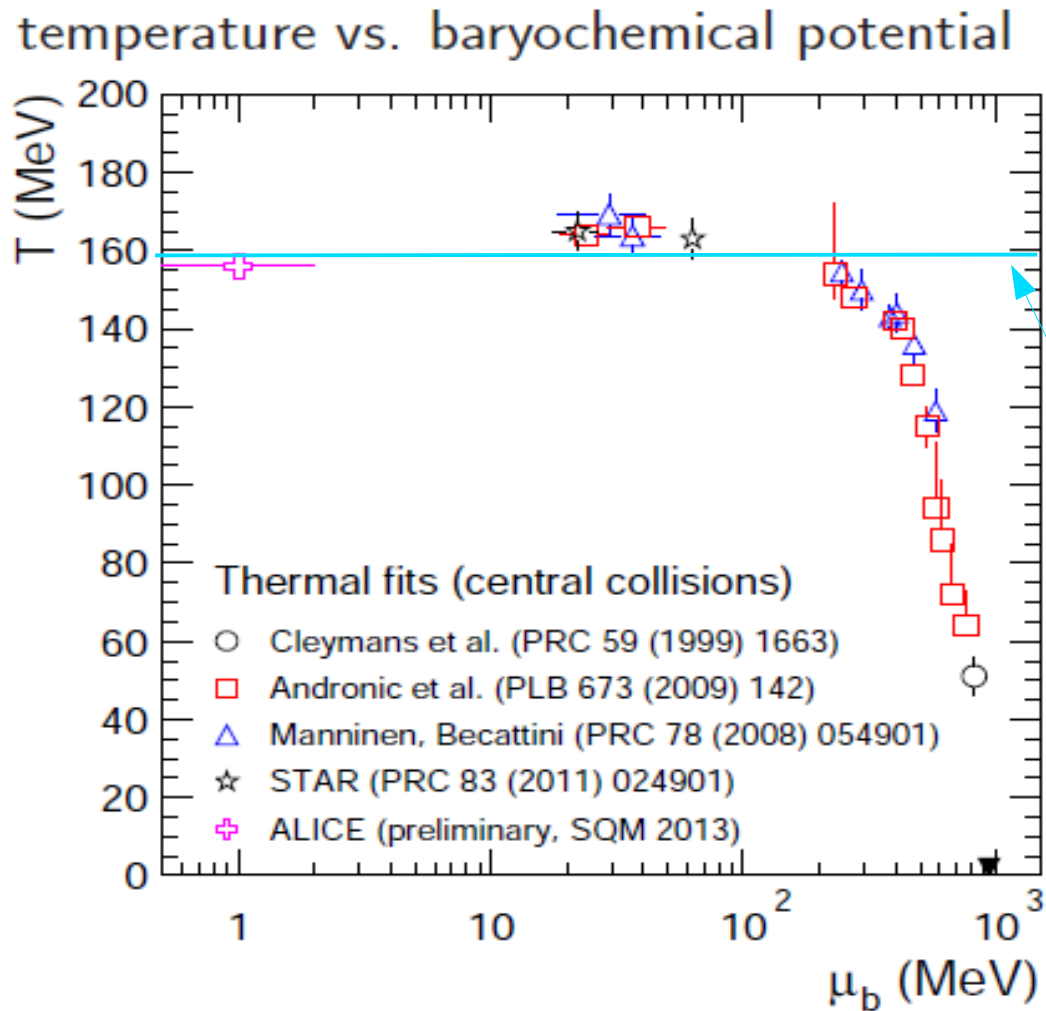
Beam energy dependence of hadron yields from AGS to LHC

fits work equally well at lower beam energies

following the obtained T and μ_b evolution, features of proton/pion and kaon/pion ratios reproduced in detail



Energy dependence of temperature and baryochemical pot.



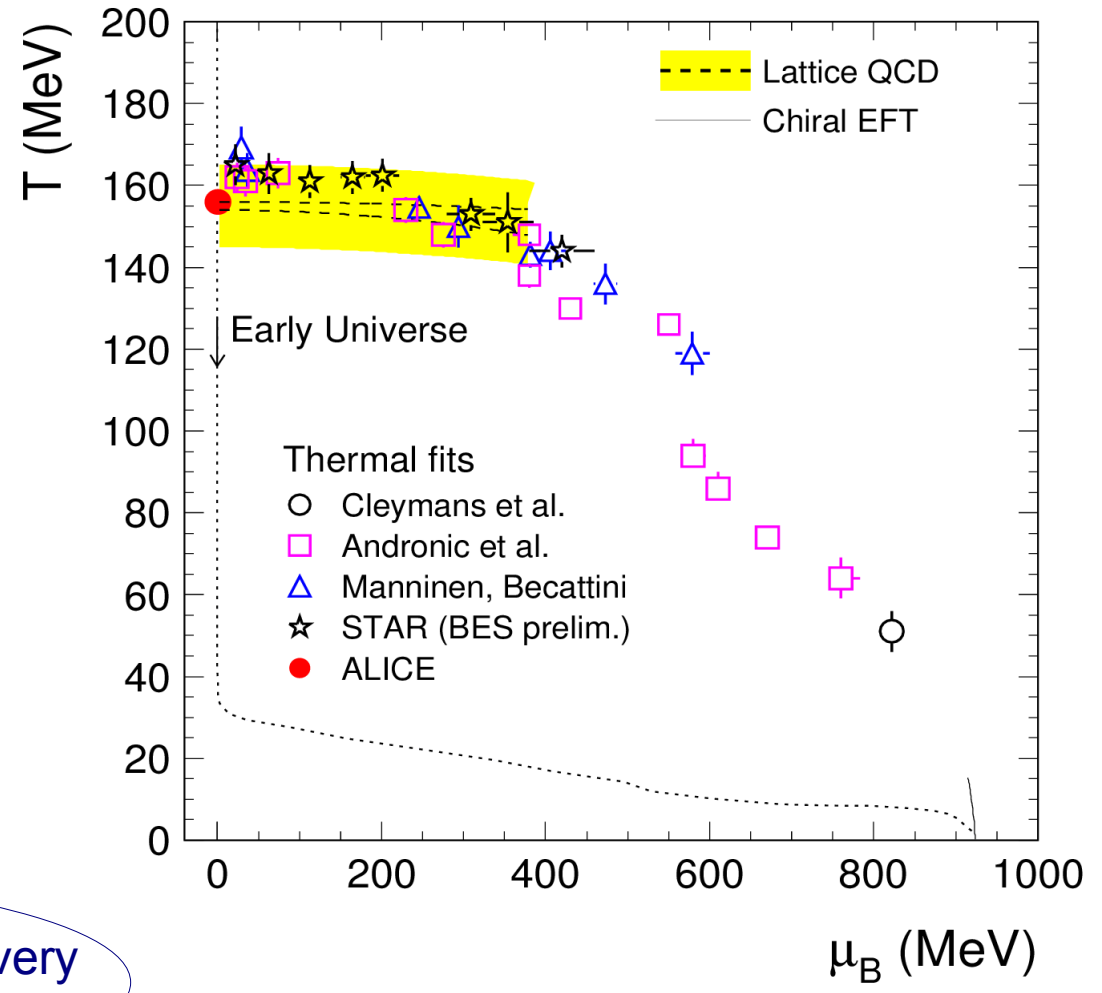
hadron yields for Pb-Pb central collisions from LHC down to RHIC, SPS, AGS and even SIS energies well described by a statistical ensemble
- there is a limiting temperature for a hadronic system

$$T_{\text{lim}} = 159 \pm 3 \text{ MeV}$$

reached for $\sqrt{s_{\text{NN}}} \geq 10 \text{ GeV}$

The QCD phase diagram – experiment and lattice QCD

experimental hadronic freeze-out points coincide with pseudocritical temperature T_c from state-of-the-art lattice QCD for high collision energies



why do hadron yields freeze out at or very close to hadronization temperature?

5.3. How is chemical equilibration achieved?

2-particle collisions not enough – takes about one order of magnitude too long

even when system is initialized in equilibrium at $T = 170$ MeV, it falls out of equilibrium quickly

simple example:

use a data driven estimate of rate of cooling

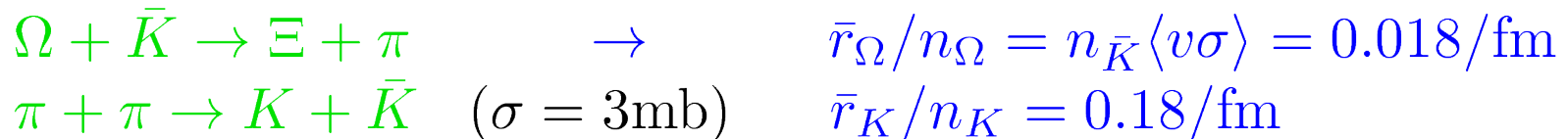
near chemical freeze-out (can be explained later) $|\dot{T}/T| = \tau_T^{-1} = (13 \pm 1)\%/fm$

typical densities at T_{ch} : $\rho_\pi = 0.174/fm^3$ (incl.res.), $\rho_K = 0.030/fm^3$, $\rho_\Omega = 0.0003/fm^3$
to maintain equilibrium during 5 MeV temperature drop need a relative rate of change

of densities of $|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}| = \tau_\Omega^{-1} - \tau_K^{-1} = 1.10 - 0.55/fm = 0.55/fm$

so Ω density needs to change by 100 % in 1 fm/c

typical reactions with large cross section (10 mb) and rel. velocities of 0.6 give



much too slow to maintain equilibrium even over drop of T of 5 MeV!

much harder to get into equilibrium!

A possible scenario for rapid equilibration

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004) 61

near phase boundary multiparticle reactions become important
dynamics associated with collective excitations
(key word: critical opalescence at phase transition)
propagation and scattering of these collective excitations expressed in form of multi-hadron scattering

will see: this drives the system into equilibrium very rapidly

Evaluation of multi-strange baryon yield as most challenging test case

consider situation at $T_{\text{ch}} = 176$ MeV first

rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{\text{in}}, n_{\text{out}}) = \bar{n}(T)^{n_{\text{in}}} |\mathcal{M}|^2 \phi$$

with

$$\phi = \prod_{k=1}^{n_{\text{out}}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

the phase space factor ϕ depends on \sqrt{s}

needs to be weighted by the probability $f(s)$ that multi-particle scattering occurs at a given value of \sqrt{s}

evaluate numerically in Monte-Carlo using thermal momentum distribution

typical reaction $\Omega + \bar{N} \rightarrow 2\pi + 3K$

assume cross section equal to the measured one for $p + \bar{p} \rightarrow 5\pi$

at proper energy above threshold, i.e. $\sqrt{s} = 3.25$ GeV \rightarrow 6.4 mb

compute matrix element and use for rate of $2\pi + 3K \rightarrow \bar{N} + \Omega$

$$r_\Omega = n_\pi^5 (n_K/n_\pi)^3 |\mathcal{M}|^2 \phi$$

Evaluation of multi-strange baryon yield

reaction $2\pi + 3K \rightarrow \bar{N} + \Omega$ leads to

$$r_{\Omega} = 0.00014 \text{fm}^{-4} \quad \text{or} \quad r_{\Omega}/n_{\Omega} = 1/\tau_{\Omega} = 0.46/\text{fm}$$

can achieve final density starting from only pions and kaons at $t=0$ in $2.2 \text{ fm}/c$

similarly one obtains

$$\text{for } 3\pi + 2K \rightarrow \Xi + \bar{N} \quad \text{or} \quad \tau_{\Xi} = 0.71 \text{ fm}$$

and

$$\text{for } 4\pi + K \rightarrow \Lambda + \bar{N} \quad \text{or} \quad \tau_{\Lambda} = 0.66 \text{ fm}$$

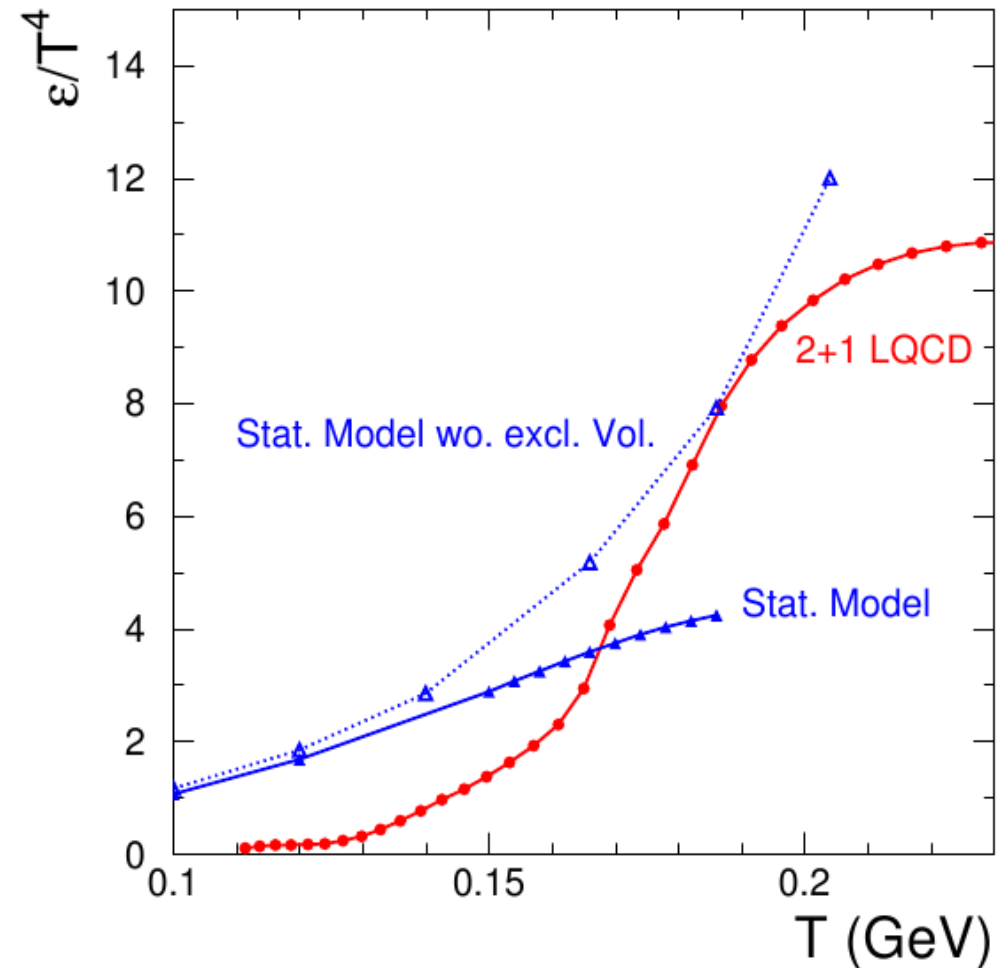
Why do all particle yields show one common freeze-out T?

density of particles varies rapidly (factor 2 within 8 MeV) with T near the phase transition due to increase in degrees of freedom.

also: system spends time at $T_c \rightarrow$ volume has to triple (entropy cons.)

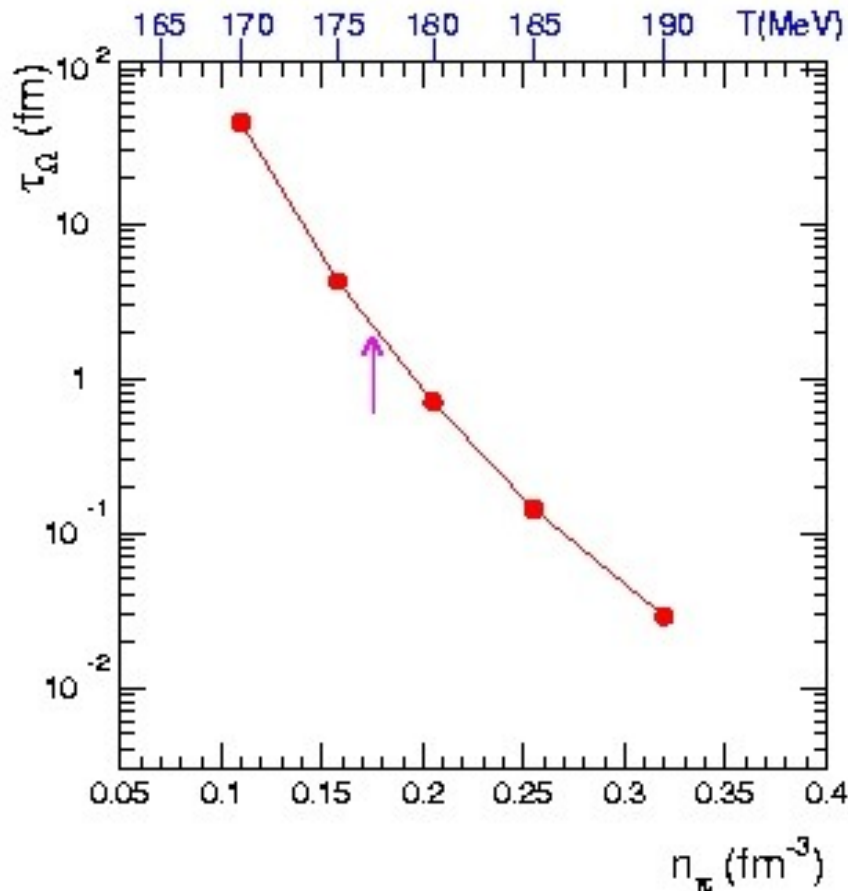
multi-particle collisions are strongly enhanced at high density and lead to chem. equilibrium very near to T_c independently of cross section

all particles can freeze out within narrow temperature interval



Lattice QCD by F. Karsch et al.

Density dependence of characteristic time for multi-strange baryon production



- near phase transition particle density varies rapidly with T (see previous slide)
 - for SPS energies and above reaction such as $2\pi + KKK \rightarrow \Omega \bar{N}$ bring multi-strange baryons close to equilibrium rapidly
 - in region around T_c equilibration time $\tau_\Omega \propto T^{-60}$!
 - increase n_π by 1/3: $\tau = 0.2$ fm/c
(corresponds to increase in T by 8 MeV)
 - decrease n_π by 1/3: $\tau = 27$ fm/c
- all particles freeze out within a very narrow temperature window due to the extreme temperature sensitivity of multi-particle reactions

5.4 Direct comparison of LHC data and lattice QCD

fluctuations of conserved charges (baryon number, strangeness, charge) sensitive to criticality related to spontaneous breaking of chiral symmetry

$$\frac{\chi_N}{T^2} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_N^2} \quad \frac{\chi_{NM}}{T^2} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_N \partial \hat{\mu}_M}$$

with $\hat{P} = P/T^4$, $\hat{\mu}_N = \mu_N/T$ and $N, M = (B, S, Q)$

- exhibit characteristic properties governed by universal part of free energy in IQCD chiral transition in O(4) critical region

can we see signs of this criticality in experimental data?

- direct measurement of higher moments of fluctuations of conserved charges very challenging, for LHC under way (very statistics hungry!)
- 2nd order cumulants of conserved charges can be obtained from measured inclusive distributions in case probability distribution of number of particles and antiparticles are Poissonian and uncorrelated

P. Braun-Munzinger, A. Kalweit, K. Redlich, J.S. arXiv:1412.8614

Fluctuations of net charges

susceptibilities of a conserved charge related to variance of net charge distribution

$$\hat{\chi}_N = \frac{1}{VT^3} (\langle N^2 \rangle - \langle N \rangle^2)$$

if e.g. baryon and antibaryon distributions are Poisson, the net baryon distribution is a Skellam distribution

variance of Skellam distribution given by total mean number of baryons +

antibaryons $\frac{\chi_N}{T^2} = \frac{1}{VT^3} (\langle N_q \rangle + \langle N_{-q} \rangle)$

can be directly computed from measured ALICE data using rapidity densities of measured baryon and antibaryon yields:

$$\frac{\chi_B}{T^2} \simeq \frac{1}{VT^3} [\langle p \rangle + \langle N \rangle + \langle \Lambda + \Sigma^0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + \langle \Xi^- \rangle + \langle \Xi^0 \rangle + \langle \Omega^- \rangle + \text{antiparticles}],$$

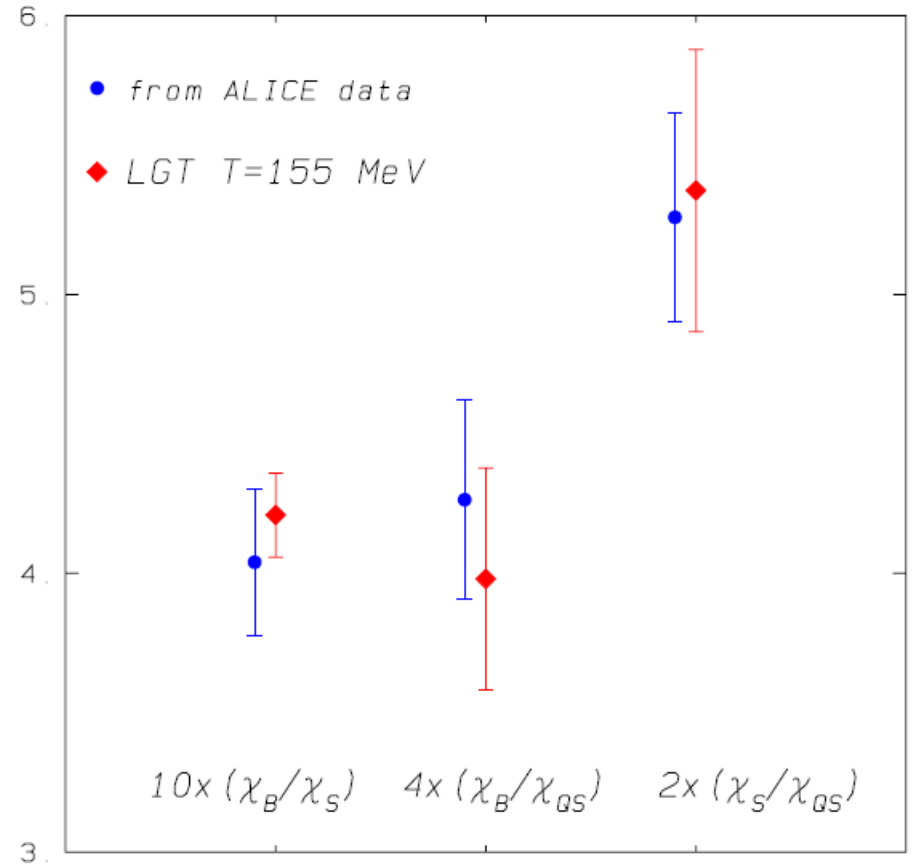
and equivalent for strangeness using strange hadron yields or electric charge/strangeness correlations using charged strange hadron yields

Comparison chiral susceptibilities data and IQCD

ALICE data: $\frac{\chi_B}{T^2} \simeq \frac{1}{VT^3} (203.7 \pm 11.4)$
 $\frac{\chi_S}{T^2} \simeq \frac{1}{VT^3} (504.2 \pm 16.8)$
 $\frac{\chi_{QS}}{T^2} \simeq \frac{1}{VT^3} (191.1 \pm 12).$

- in ratios of susceptibilities volume and temperature drop out
- compare to lattice QCD at pseudo-critical temperature for $\mu_b = 0$ and extrapolated to continuum limit

A.Basavov et al., PRL 113 (2014) 072001 and PRD 86 (2012) 034509



very good agreement data and IQCD
strongly suggests: fluctuations are of thermal origin and indeed established at the phase boundary