Problem 16: Leading particle effect ("trigger bias")

Consider a parton transverse momentum distribution

$$\frac{1}{\hat{p}_T}\frac{\mathrm{d}n_p}{\mathrm{d}\hat{p}_T} \propto \frac{1}{\hat{p}_T^n} =: f(\hat{p}_T)$$

and a parton-to-pion fragmentation function

$$D(z) = Be^{-bz} \quad \text{with} \quad b = 8.$$

The invariant yield of pions differential in the parton transverse momentum $\hat{p}_T$ and the transverse momentum fraction $z = p_T/\hat{p}_T$ (where p_T is the pion transverse momentum) is then given by

$$\frac{1}{\hat{p}_T}\frac{\mathrm{d}^2 n_\pi}{\mathrm{d}\hat{p}_T\,\mathrm{d}z} = f(\hat{p}_T)\cdot D(z).$$

a) What is the average transverse momentum fraction z of pions in the parton fragmentation?

b) Show that the pion invariant yield as a function of the pion p_T and z is given by

$$\frac{1}{p_T}\frac{\mathrm{d}^2 n_\pi}{\mathrm{d}p_T\,\mathrm{d}z} = f(p_T/z)\cdot D(z)\cdot z^{-2}.$$

c) Show that the average z for pions measured at a given p_T is roughly given by $\langle z\rangle \approx (n-1)/b$. At LHC energies $n \approx 5$ so that $\langle z\rangle_{\mathrm{LHC}} \approx 0.5$.
Hint:

$$\int_0^1 z^k e^{-bz} = \frac{1}{b^{k+1}}\left(\Gamma(k+1) - \Gamma(k+1,b)\right) \qquad (k \geq 0,\, b \geq 0)$$

where $\Gamma(a,x)$ is the incomplete Gamma function.

Problem 17: Thermal photon equilibration time in the QGP

Suppose we create a quark-gluon plasma with a temperature T which initially contains no photons, only quarks, antiquarks, and gluons. The equilibrium photon phase space density is

$$f_{\mathrm{eq}} = \frac{\mathrm{d}n_\gamma}{\mathrm{d}^3p\mathrm{d}^3x} = \frac{2}{(2\pi)^3}\frac{1}{e^{E/T}-1}.$$

The photon absorption rate is proportional to the photon phase space density. We can therefore write

$$\dot{f}(t) = p - kf(t)$$

where p is the photon production rate and $f(t)$ the photon phase space density at time t. This is solved by

$$f(t) = f_{\mathrm{eq}}(1 - e^{-t/\tau})$$

with $k = 1/\tau$. What is the photon equilibration time τ at $T = 200\,\mathrm{MeV}$ for photons with $E = 1\,\mathrm{GeV}$ for a photon production rate given by

$$E\frac{\mathrm{d}n_{\mathrm{th}}}{\mathrm{d}^4x\,\mathrm{d}^3p} = C\frac{5}{9}\frac{\alpha\alpha_{\mathrm{s}}}{2\pi^2}T^2 e^{-E/T} \quad \text{with} \quad C = K\log\left(1+\frac{2.912}{4\pi\alpha_{\mathrm{s}}}\right)?$$

Use $K = 2$ and $\alpha_s = 0.3$. How does this time compare to the typical lifetime of a QGP created in heavy-ion collisions?

Problem 18: Quark flavor ratio in string fragmentation

In pp collisions at the LHC it is observed that the ratio of strange to non-strange particles increases with the produced charged-particle multiplicity (ALICE, Nature Physics, 2017). We discuss in this problem whether this may be qualitatively understood within the QCD string model. The transverse momentum spectrum of quarks produced in the fragmentation of a QCD string is given by

$$\frac{1}{p_T}\frac{\mathrm{d}n_q}{\mathrm{d}p_T} \propto e^{-\pi(m_q^2+p_T^2)/\langle k\rangle}$$

where $\langle k\rangle \approx 1\,\mathrm{GeV/fm}$ is the string tension.

a) What is the ratio $r = (s\bar{s})/(u\bar{u} + d\bar{d})$ of the yield of strange to non-strange quark pairs assuming constituent quark masses ($m_u = m_d = 0.325\,\mathrm{GeV}/c^2$, $m_s = 0.45\,\mathrm{GeV}/c^2$) for the quarks?

b) Suppose the string tension k can fluctuate. How big is the increase $r(k)/r(\langle k\rangle)$ and the increase in the number of non-strange quark pairs for $k = 2\cdot\langle k\rangle$?