Quark-Gluon Plasma Physics

4. Basics of Nucleon-Nucleon and Nucleus-Nucleus Collisions

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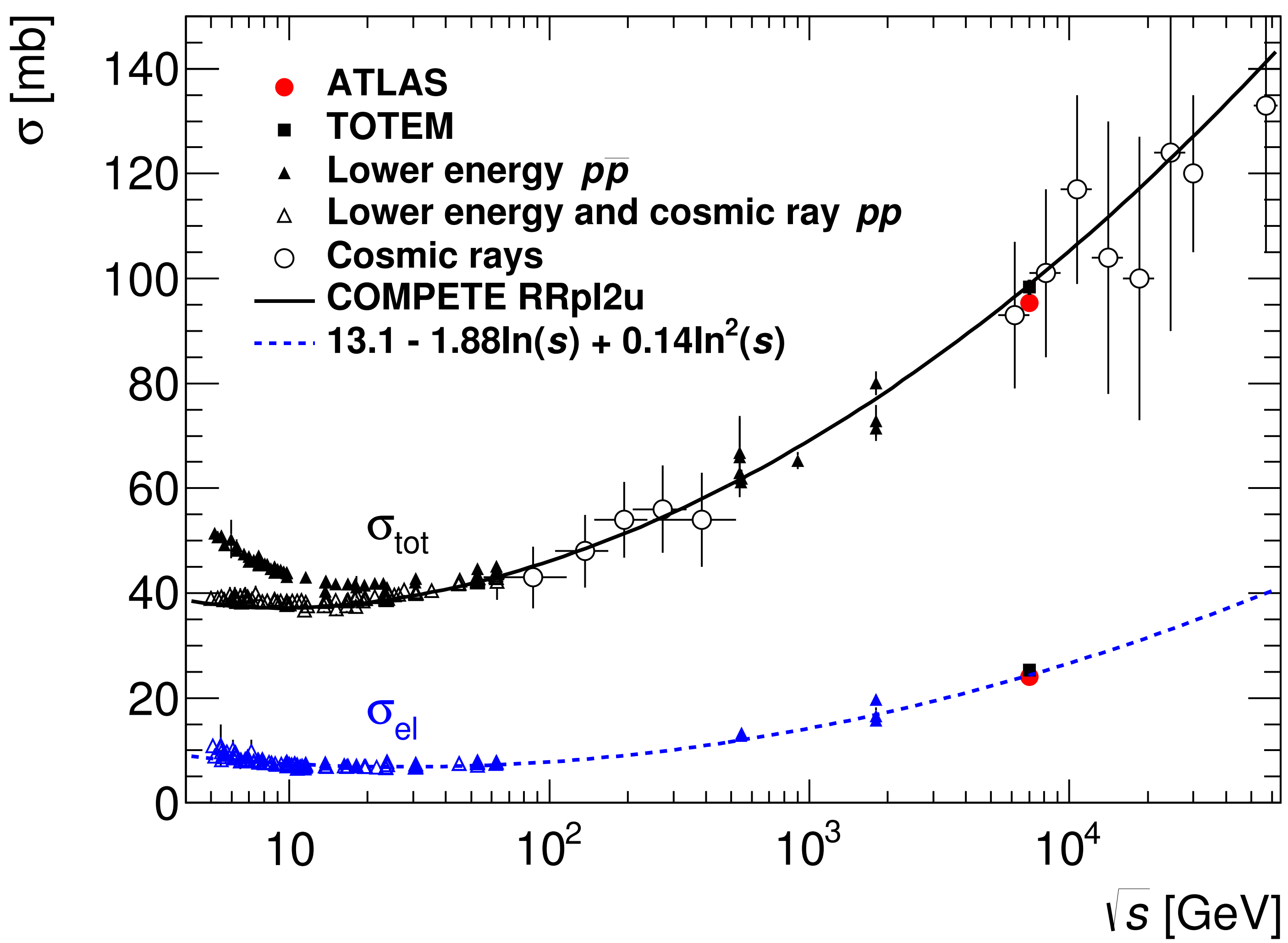
Part I: proton-proton collisions

Total p+p(pbar) Cross Section

ATLAS, arXiv:1408.5778



parameterization from Regge theory:

$$\sigma_{\text{tot}} = X s^{\epsilon} + Y s^{\epsilon'}$$

$$\epsilon = 0.08 - 0.1, \quad \epsilon' \approx -0.45$$

Above ~ $\sqrt{s}$ = 20 GeV all hadronic cross sections rise with increasing $\sqrt{s}$

Data show that

$$\sigma_{\text{tot}}(h + X) = \sigma_{\text{tot}}(\bar{h} + X)$$

(in line with Pomeranchuk's theorem)

Soft processes:
hard to calculate $\sigma_{\text{tot}}(\sqrt{s})$ in QCD

Modeling based on Regge theory: exchange of color-neutral object called *pomeron*

Diffractive collisions (I)

(Single) diffraction in p+p:
"Projectile" proton is excited to a hadronic state *X* with mass *M*

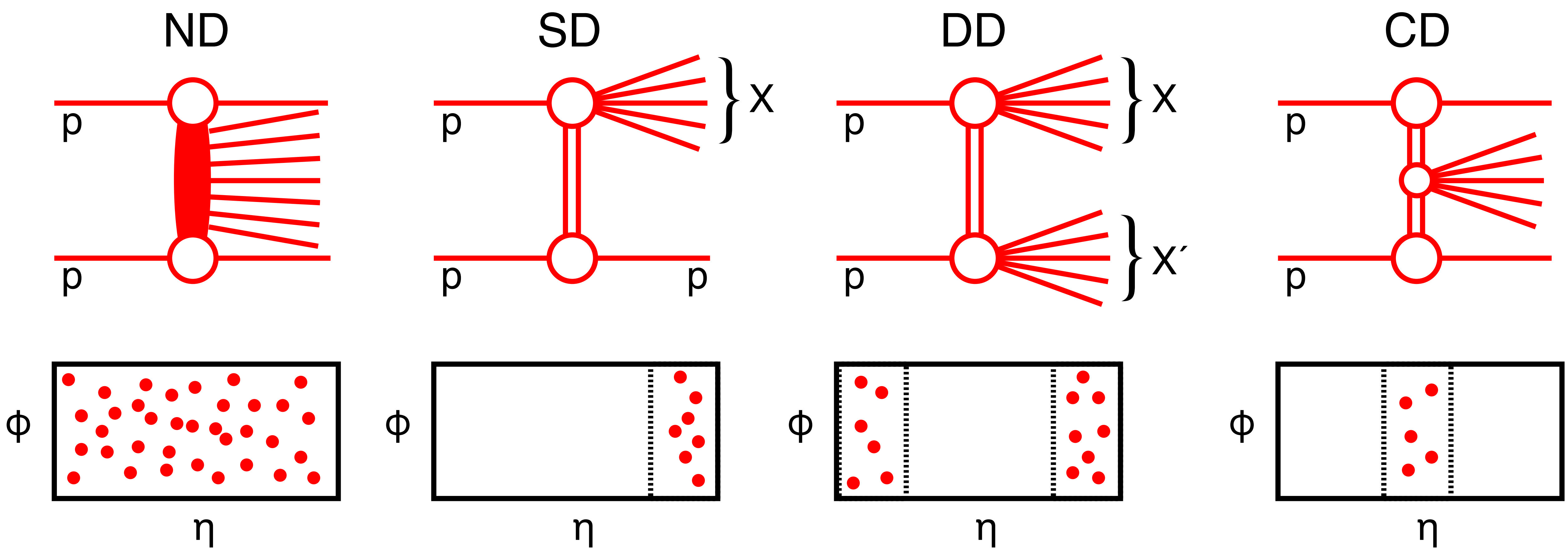
$$p_{\mathrm{proj}} + p_{\mathrm{targ}} \rightarrow X + p_{\mathrm{targ}}$$

The excited state X fragments, giving rise to the production of (a small number) of particles in the forward direction

Theoretical view:

- Diffractive events correspond to the exchange of a Pomeron
- The Pomeron carries the quantum numbers of the vacuum ($J^{PC} = 0^{++}$)
- Thus, there is no exchange of quantum numbers like color or charge
- In a QCD picture the Pomeron can be considered as a two- or multi-gluon state, see, e.g., O. Nachtmann (→ link)

Diffractive collisions (II)



non-diffractive collisions

single-diffractive dissociation

double-diffractive dissociation

central diffraction

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}, \quad \sigma_{\text{inel}} = \sigma_{\text{SD}} + \sigma_{\text{DD}} + \sigma_{\text{CD}} + \sigma_{\text{ND}}$$

Diffractive collisions (III)

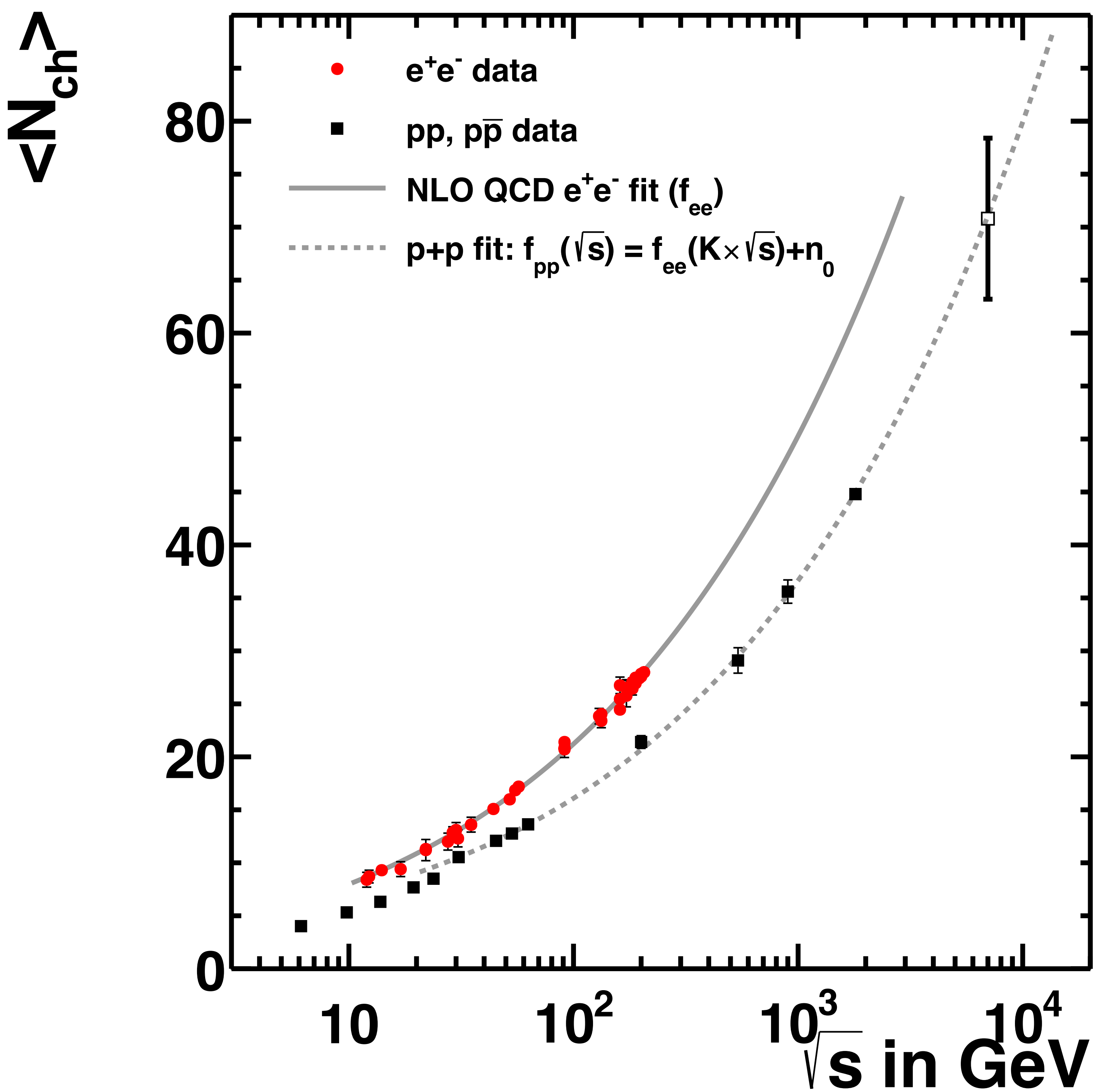
UA5, Z. Phys. C33, 175, 1986

$p + \bar{p}$	$\sqrt{s}$ = 200 GeV	$\sqrt{s}$ = 900 GeV
Total inelastic	(41.8 ± 0.6) mb	(50.3 ± 0.4 ± 1.0) mb
Single-diffractive	(4.8 ± 0.5 ± 0.8) mb	(7.8 ± 0.5 ± 1.8) mb
Double-diffractive	(3.5 ± 2.2) mb	(4.0 ± 2.5) mb
Non-diffractive	≈ 33.5 mb	≈ 38.5 mb

Fraction of diffractive dissociation events with respect to all inelastic collisions is about 20–30% (rather independent of $\sqrt{s}$)

See also ATLAS, arXiv:1201.2808

Charged-particle Multiplicity as a fct. of √s: Similarities between pp and e⁺e⁻



The increase of N_{ch} with $\sqrt{s}$ looks rather similar in p+p and e⁺e⁻

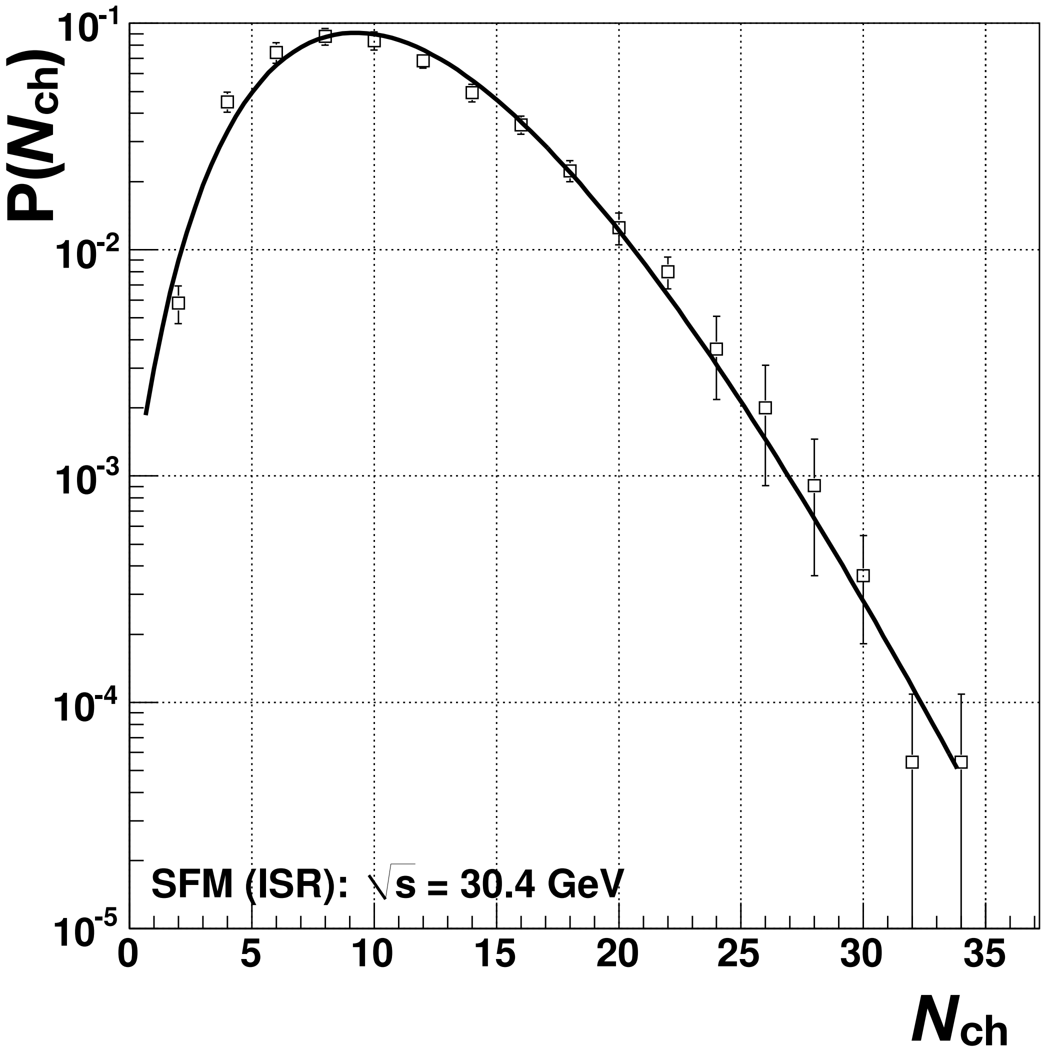
Roughly speaking, the energy available for particle production in p+p seems to be ~ 30–50%:

$$f(\sqrt{s}) := N_{ch}^{e+e-}(\sqrt{s})$$

$$\rightarrow N_{ch}^{p+p} = f(K\sqrt{s_{pp}}) + n_0$$

A fit yields: $K \approx 0.35, \quad n_0 \approx 2.2$

What is the distribution of the number of produced particles per collision?



Independent sources: Poisson distribution

Observation:
Multiplicity distributions in pp, e^+e^-, and lepton-hadron collisions well described by a Negative Binomial Distribution (NBD)

Deviations from the NBD were discovered by UA5 at $\sqrt{s}$ = 900 GeV and later confirmed at the Tevatron at $\sqrt{s}$ = 1800 GeV (shoulder structure at $n \approx 2 \langle n \rangle$)

$$P^{\mathrm{NBD}}_{\mu,k}(n) = \frac{(n+k-1)\cdot(n+k-2)\cdot \ldots \cdot k}{\Gamma(n+1)} \left(\frac{\mu/k}{1+\mu/k}\right)^n \frac{1}{(1+\mu/k)^k}$$

$$\langle n \rangle = \mu, \;\; D := \sqrt{\langle n^2 \rangle - \langle n \rangle^2} = \sqrt{\mu\left(1+\frac{\mu}{k}\right)}$$

Limits of the NBD:
$k \to \infty$: Poisson distribution
integer k, $k<0$: Binomial distribution
($N = -k$, $p = -\langle n \rangle / k$)

π^0 transverse momentum distributions at different $\sqrt{s}$

Low p_T (< ~2 GeV/c):
"soft processes"

$$E\frac{d^3\sigma}{d^3p} = A(\sqrt{s}) \cdot e^{-\alpha p_T}, \ \alpha \approx 6/(\mathrm{GeV}/c)$$

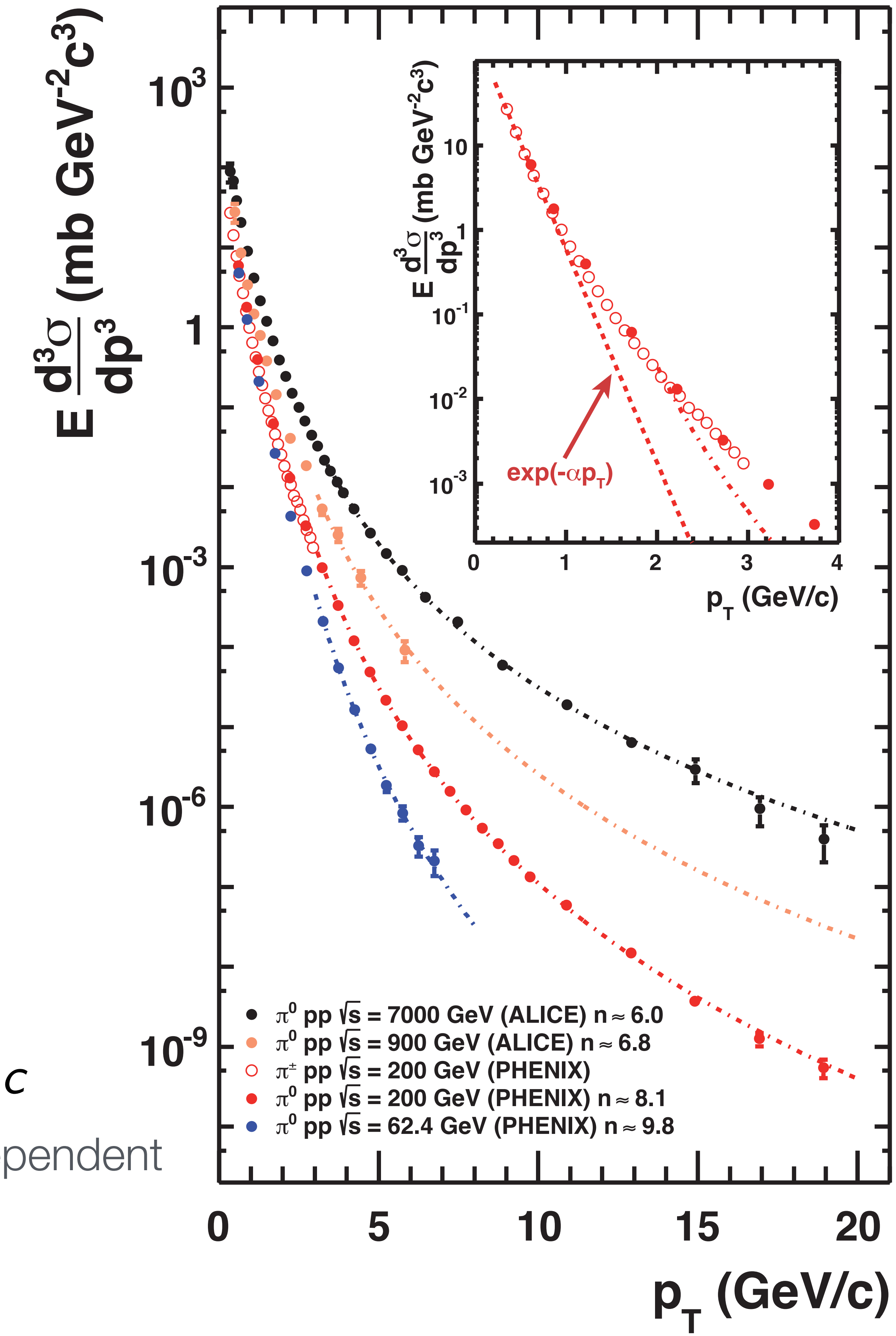
High p_T ("hard scattering"):

$$E\frac{d^3\sigma}{d^3p} = B(\sqrt{s}) \cdot \frac{1}{p_T^{n(\sqrt{s})}}$$

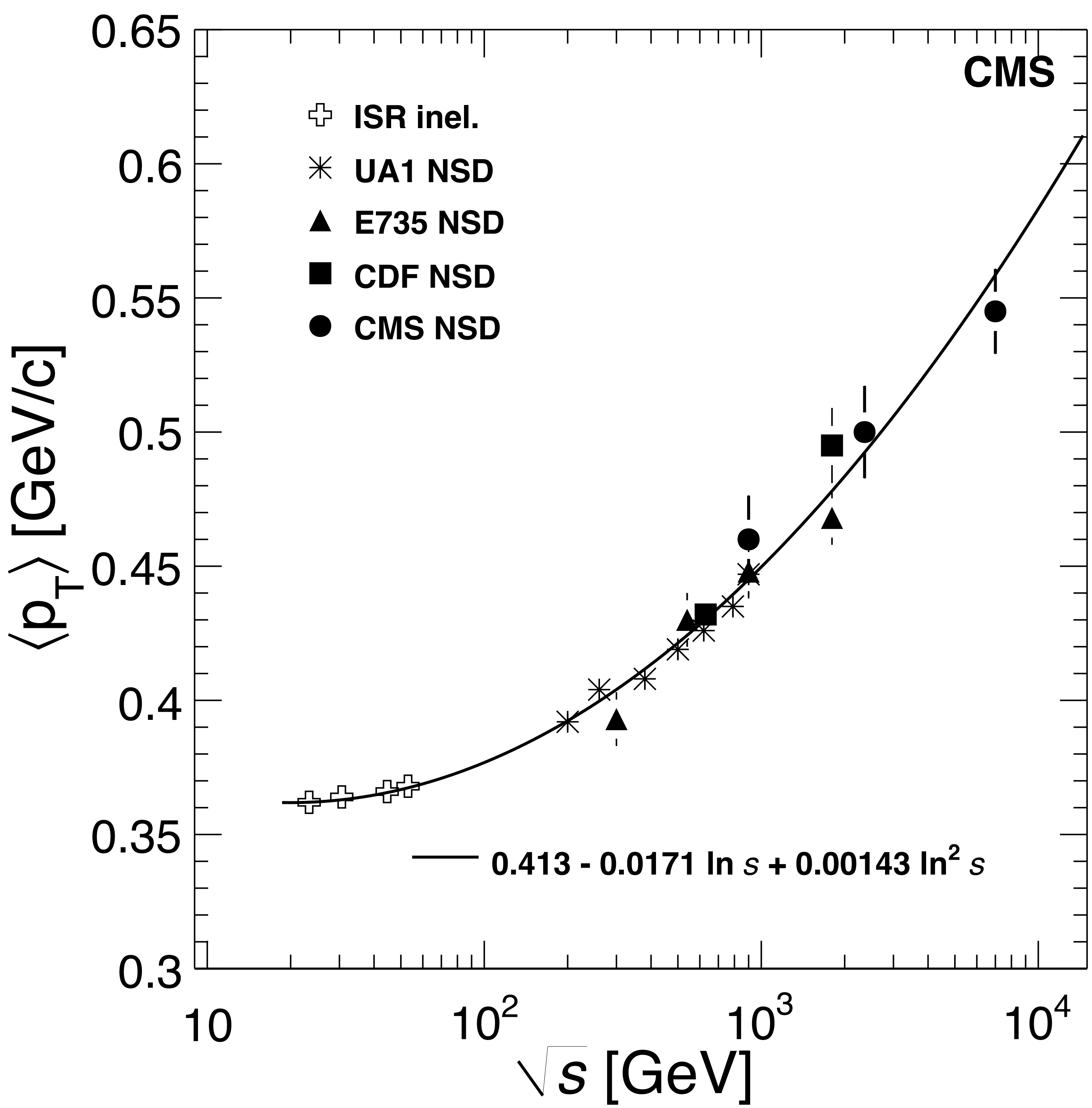
Average p_T:

$$\langle p_T \rangle = \frac{\int_0^\infty p_T \frac{dN_x}{dp_T} dp_T}{\int_0^\infty \frac{dN_x}{dp_T} dp_T} \approx 300 - 400 \mathrm{MeV}/c$$

pretty energy-independent for $\sqrt{s}$ < 100 GeV



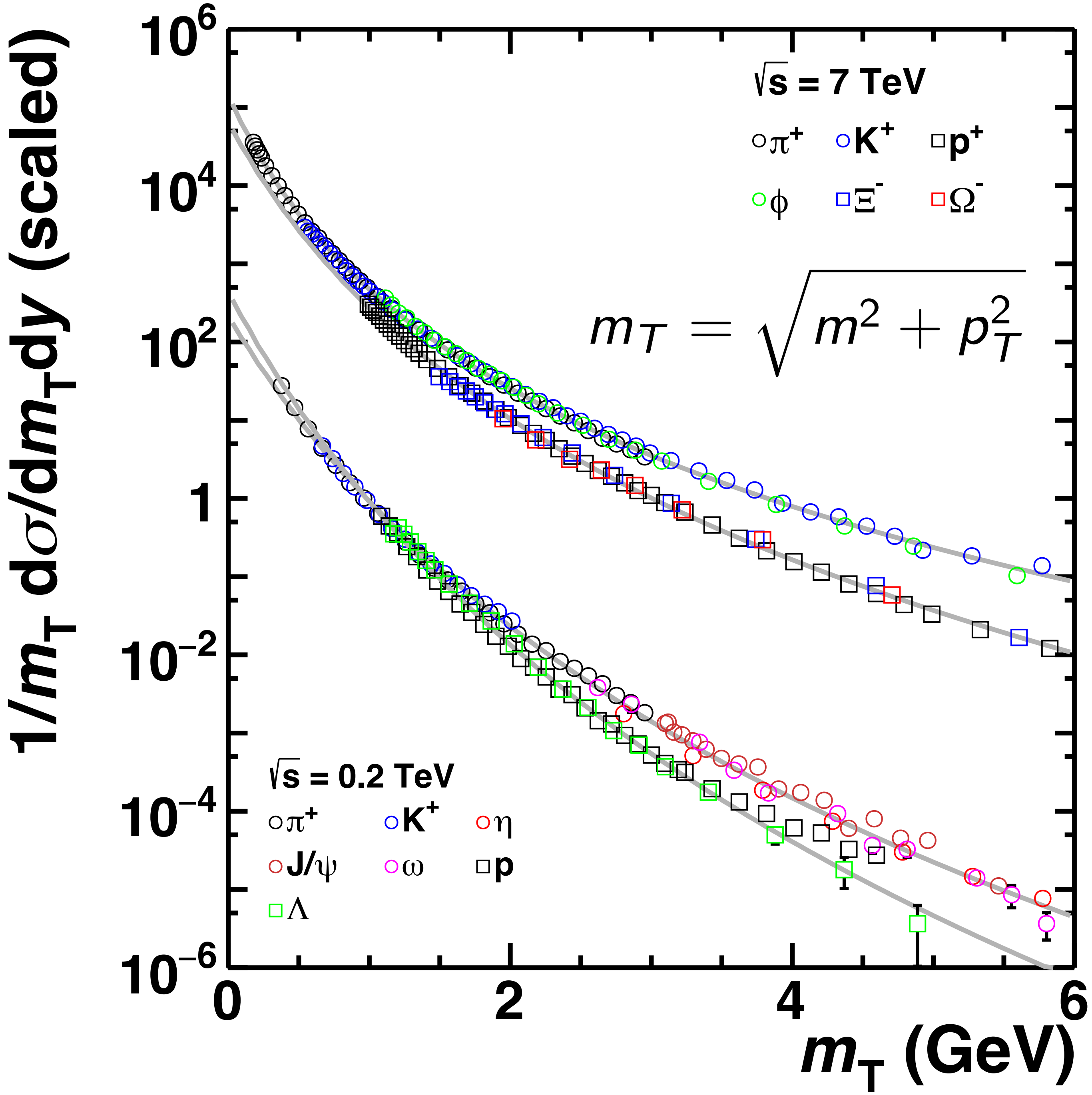
Mean p_T increases with $\sqrt{s}$



Increase of $\langle p_T \rangle$ with $\sqrt{s}$ (most likely) reflects increase in particle production from hard parton-parton scattering

CMS, PRL 105, 022002 (2010)
CDF, PRL 61, 1819 (1988)

m_T scaling in pp collisions



m_T scaling (early ref's):
Nucl. Phys. B70, 189–204 (1974)
Nucl.Phys. B120 (1977) 14-22

m_T scaling:
shape of m_T spectra the same for different hadron species

example: $\frac{dN/dm_T|_\eta}{dN/dm_T|_{\pi^0}} \approx 0.45$

possible interpretation:
thermodynamic models

$$E\frac{d^3n}{d^3p} \propto Ee^{-E/T}$$
$$\rightarrow \frac{1}{m_T}\frac{dn}{dm_T} \propto K_1\left(\frac{m_T}{T}\right)$$

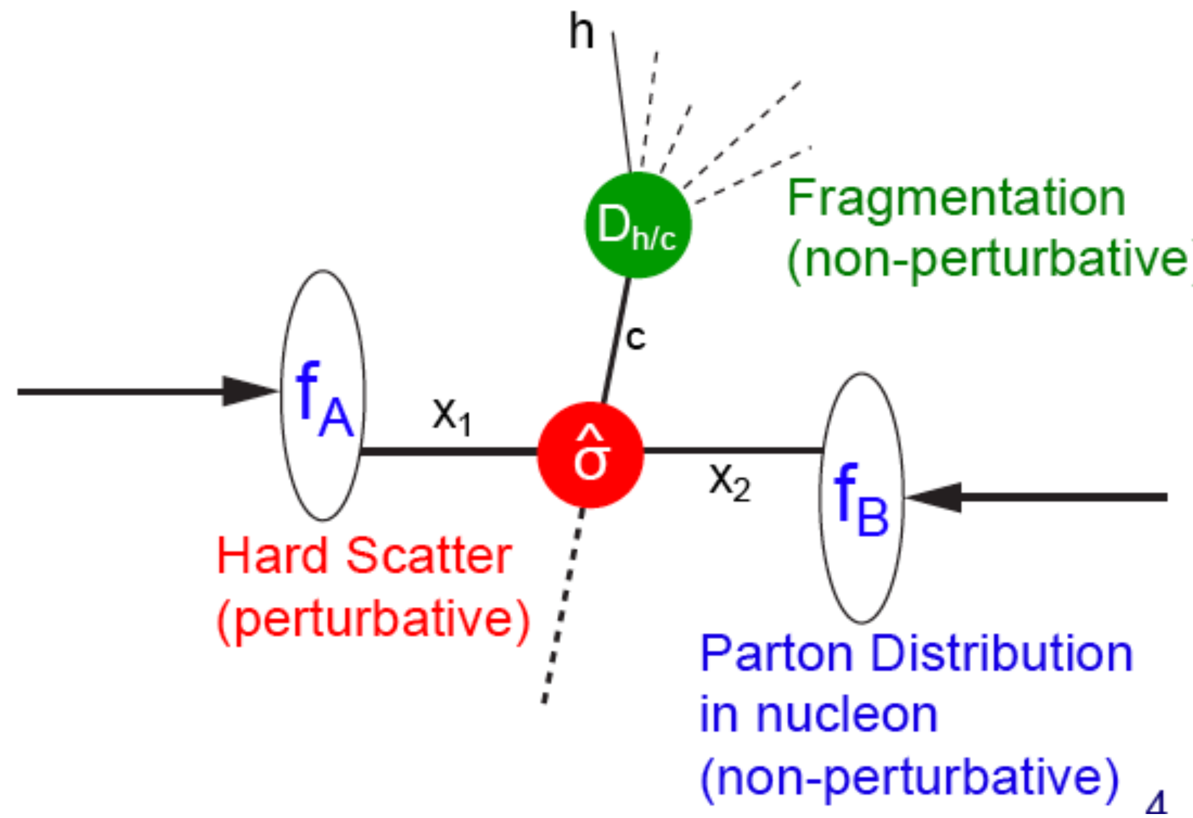
RHIC/LHC:
m_T scaling (approximately) satisfied, different universal function for mesons and baryons

Do deviations from m_T scaling in pp at low p_T indicate onset of radial flow? (1312.4230)

Theoretical modeling: General considerations

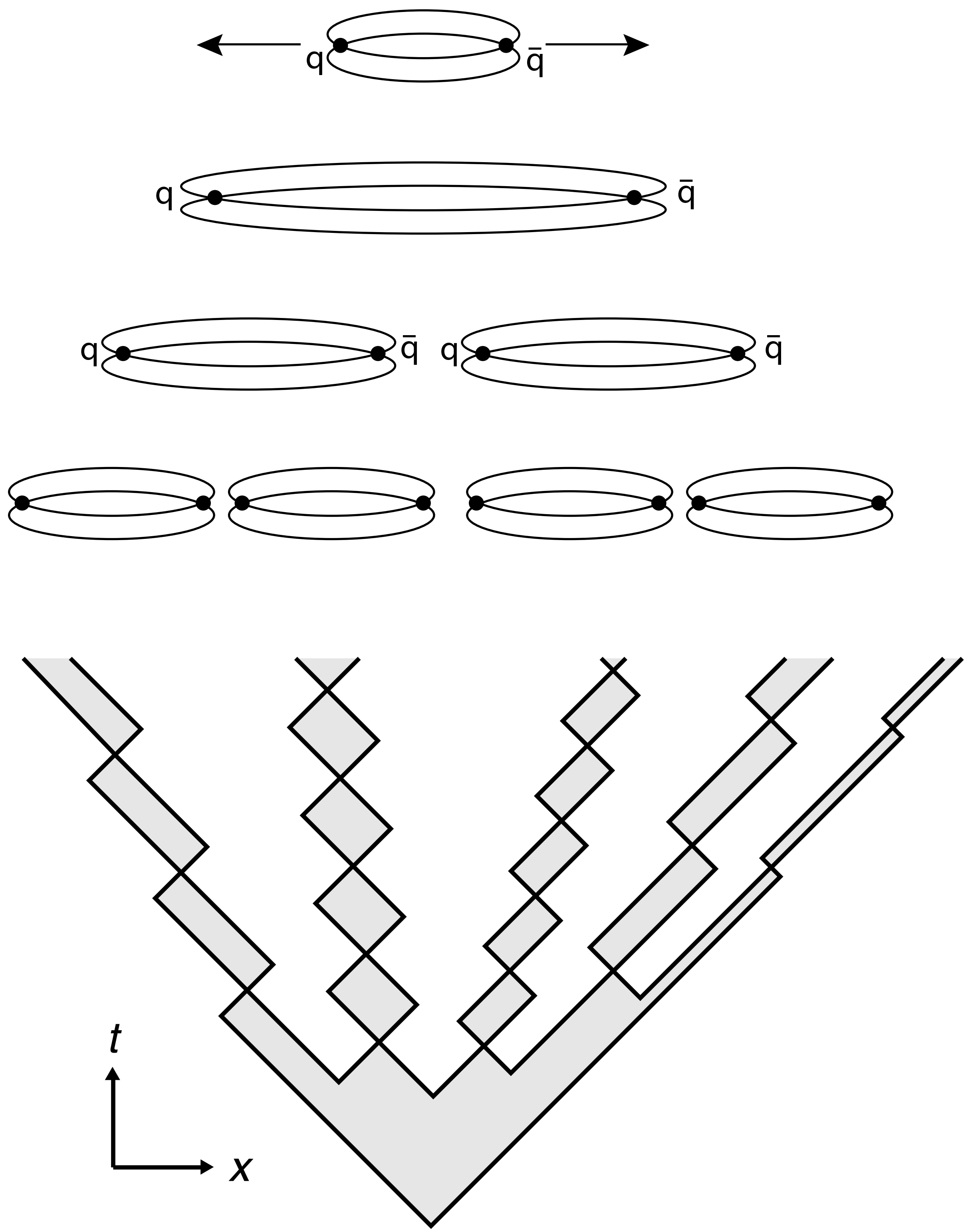
- Description of particle production amenable to perturbative methods only at sufficiently large p_T (so that α_s becomes sufficiently small)
  - parton distributions (PDF)
  - parton-parton cross section from perturbative QCD (pQCD)
  - fragmentation functions (FF)

$$E\frac{d^3\sigma}{d^3p} = \int \mathrm{PDF} \otimes \mathrm{pQCD} \otimes \mathrm{FF}$$



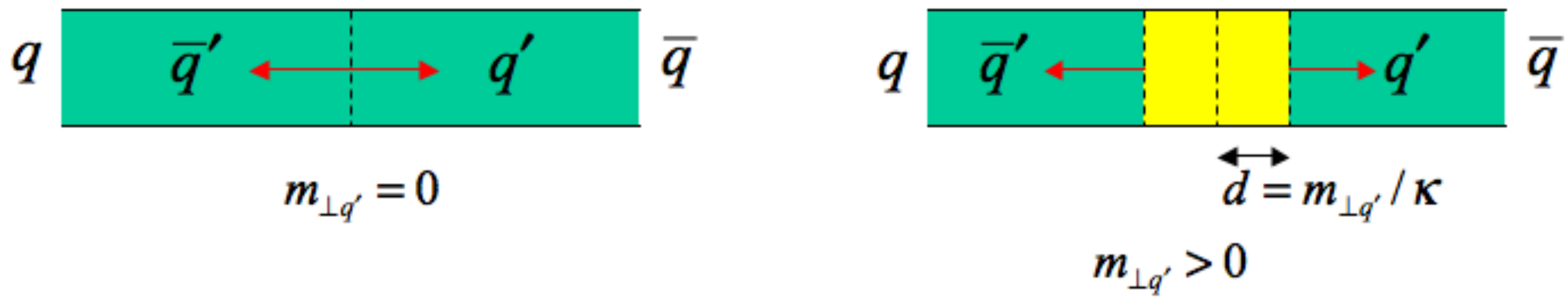
- Low-p_T:
  Need to work with (QCD inspired) models, and confront them with data
  - e.g. Lund string model

Modeling particle production as string breaking (I)



- Color flux tube between two quarks breaks due to quark-antiquark pair production in the intense color field
- Lund model:
  The basic assumption of the symmetric Lund model is that the vertices at which the quark and the antiquark are produced lie approximately on a curve on constant proper time
- Result: flat rapidity distribution of the produced particles

Modeling particle production as string breaking (II)



In terms of the transverse mass of the produced quark ($m_{T,q'} = m_{T,q'bar}$) the probability that the break-up occurs is:

$$P \propto \exp\left(-\frac{\pi m^2_{\perp q'}}{k}\right) = \exp\left(-\frac{\pi p^2_{\perp q'}}{k}\right) \exp\left(-\frac{\pi m^2_{q'}}{k}\right)$$

This leads to a transverse momentum distribution for the quarks of the form:

$$\frac{1}{p_T}\frac{\mathrm{d}N_\mathrm{quark}}{\mathrm{d}p_T} = \mathrm{const.} \cdot \exp\left(-\pi p_T^2/k\right) \quad \rightsquigarrow \quad \sqrt{\langle p_T^2 \rangle_\mathrm{quark}} = \sqrt{k/\pi}$$

For pions (two quarks) one obtains: $\sqrt{\langle p_T^2 \rangle_\mathrm{pion}} = \sqrt{2k/\pi}$

With a string tension of 1 GeV/fm this yields $\langle p_T \rangle_\mathrm{pion} \approx 0.37$ GeV/c, in approximate agreement with data

Modeling particle production as string breaking (III)

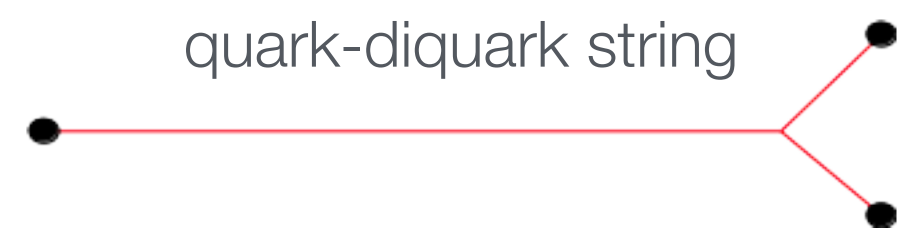
Convolution of the string breaking mechanism with fluctuations of the string tension described by a Gaussian give rise to exponential p_T spectra

Phys. Lett. B466, 301–304 (1999)

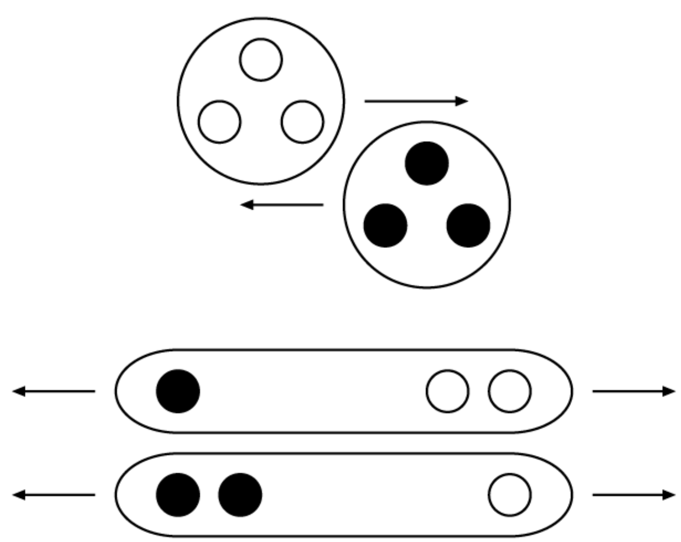
The tunneling process implies heavy-quark suppression:

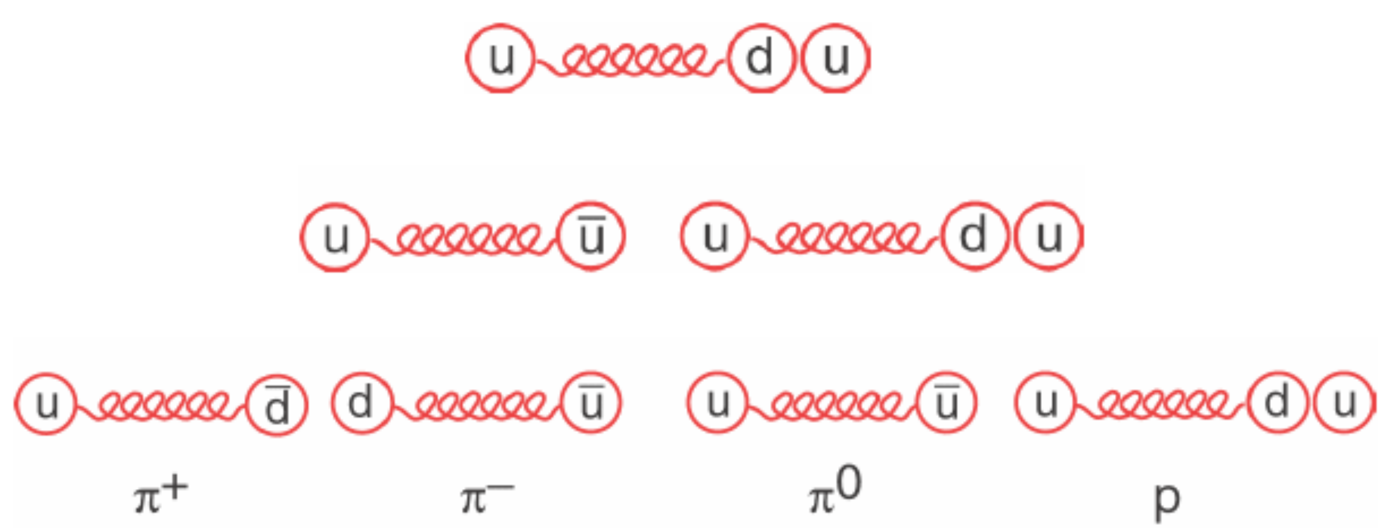
$$u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$$

The production of baryons can be modeled by replacing the q-qbar pair by an quark-diquark pair



Collisions of hadrons described as excitation of quark-diquarks strings:

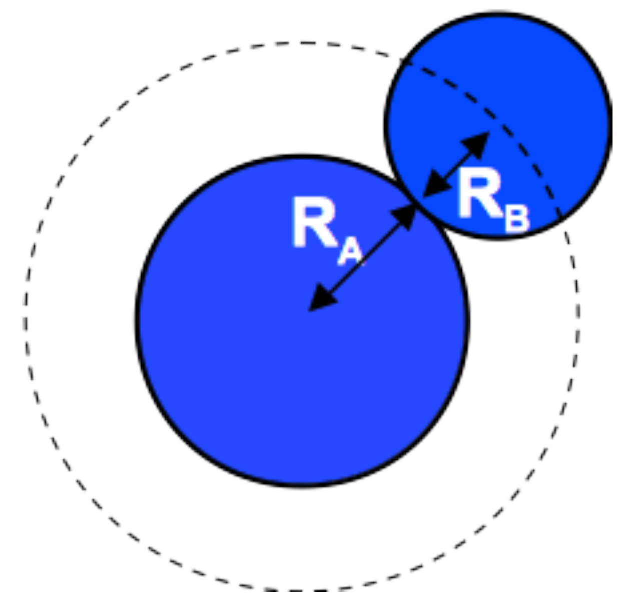




Part II: nucleus-nucleus collisions

Ultra-Relativistic Nucleus-Nucleus Collisions: Importance of Nuclear Geometry

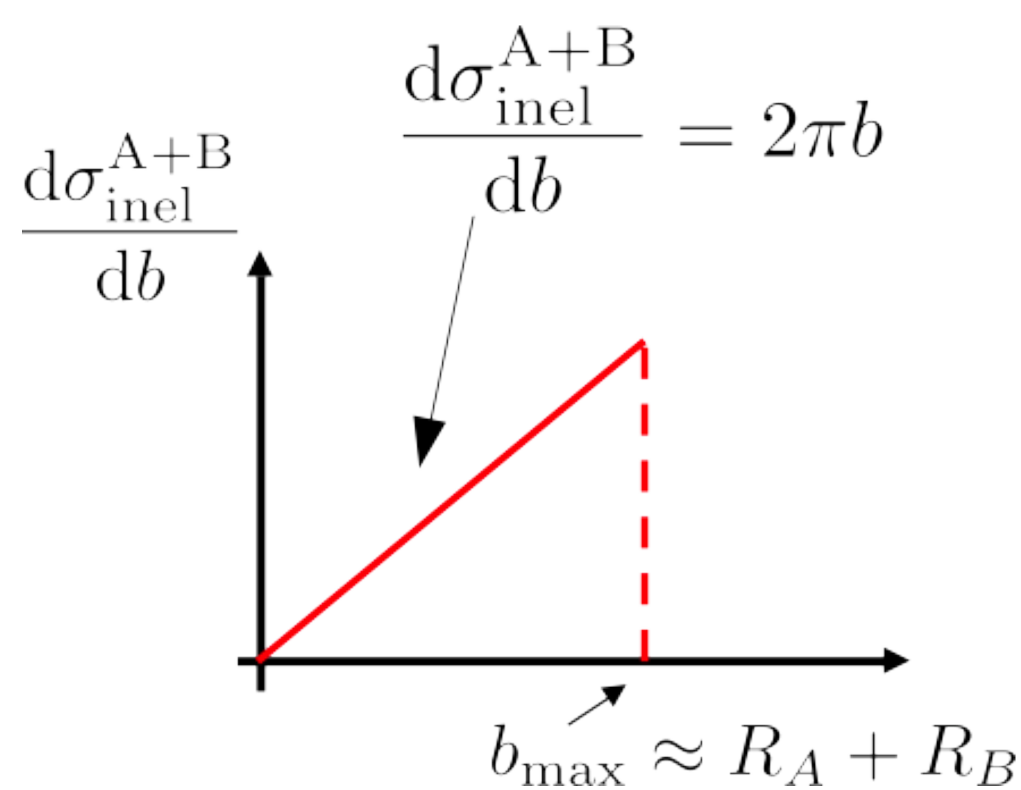
- Ultra-relativistic energies
  - De Broglie wave length much smaller than size of the nucleon
  - Wave character of the nucleon can be neglected for the estimation of the total cross section



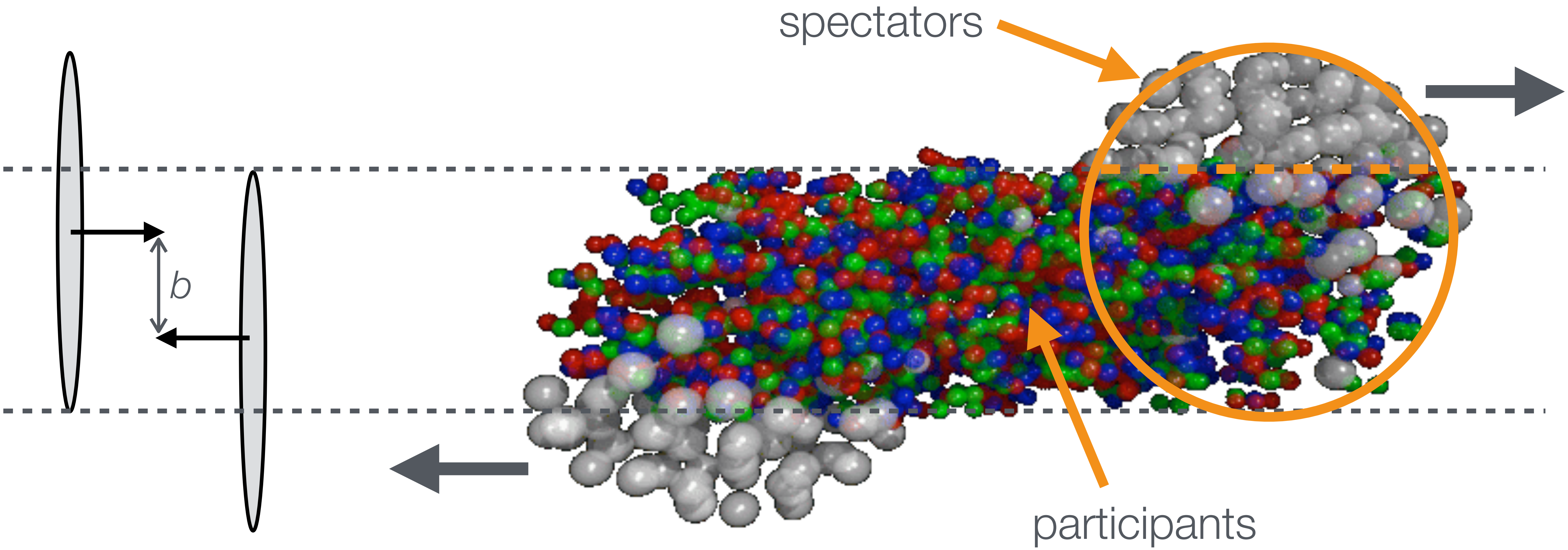
- Nucleus-Nucleus collision can be considered as a collision of two black disks

$$R_A \approx r_0 \cdot A^{1/3}, \quad r_0 = 1.2\,\mathrm{fm}$$

$$\sigma_{\mathrm{inel}}^{\mathrm{A+B}} \approx \sigma_{\mathrm{geo}} \approx \pi r_0^2 (A^{1/3} + B^{1/3})^2$$

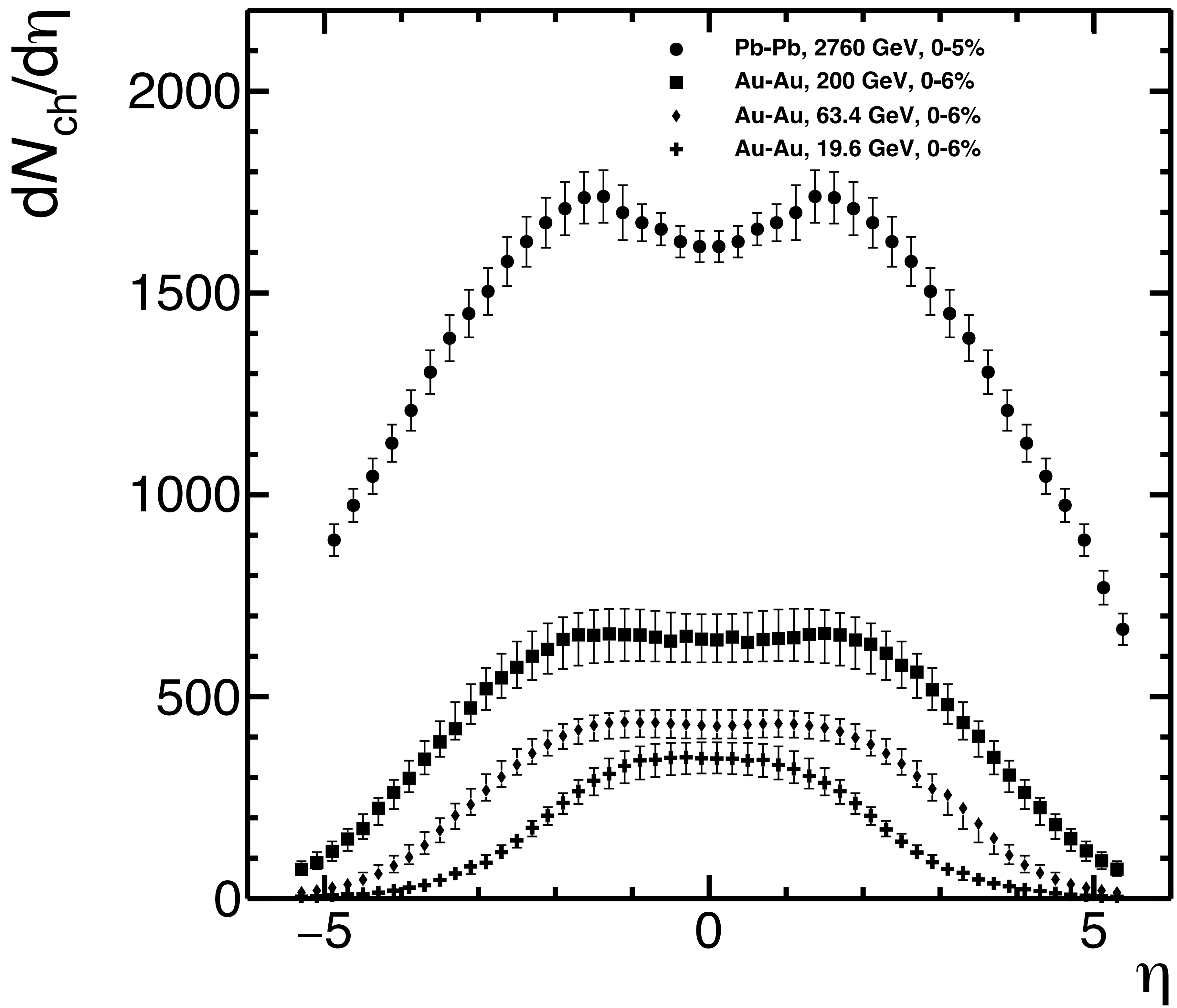


Participants and spectators

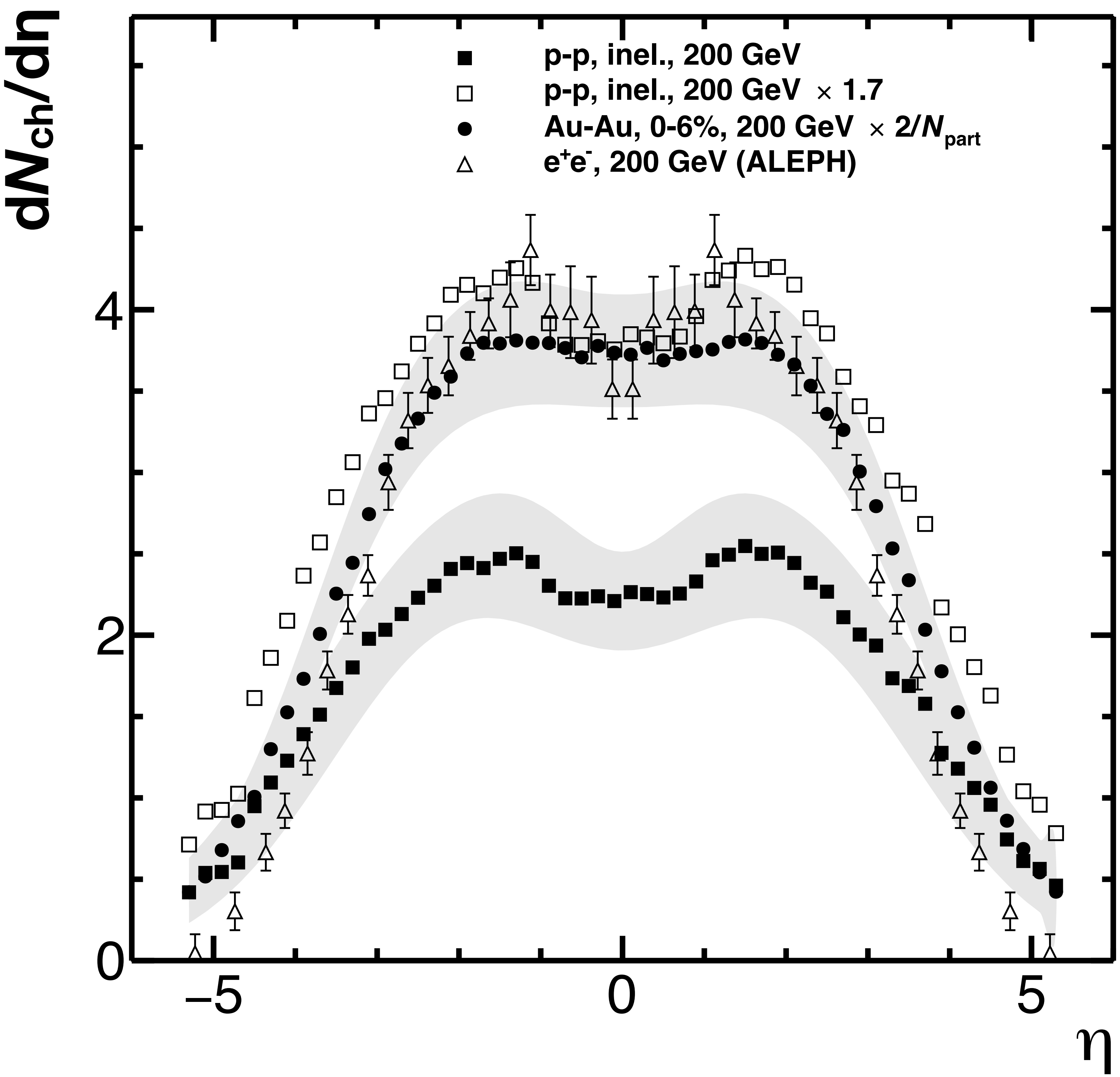


- N_{coll}: number of inelastic nucleon-nucleon collisions
- N_{part}: number of nucleons which underwent at least one inelastic nucleon-nucleon collisions

Charged particle pseudorapidity distributions for different $\sqrt{s_{NN}}$

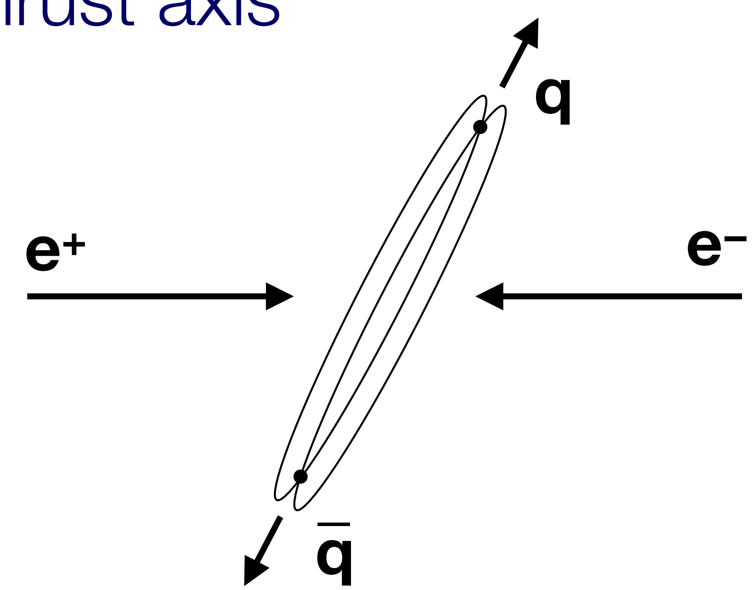


Charged-particle Pseudorapidity Distributions: Comparison e+e-, pp, and AA



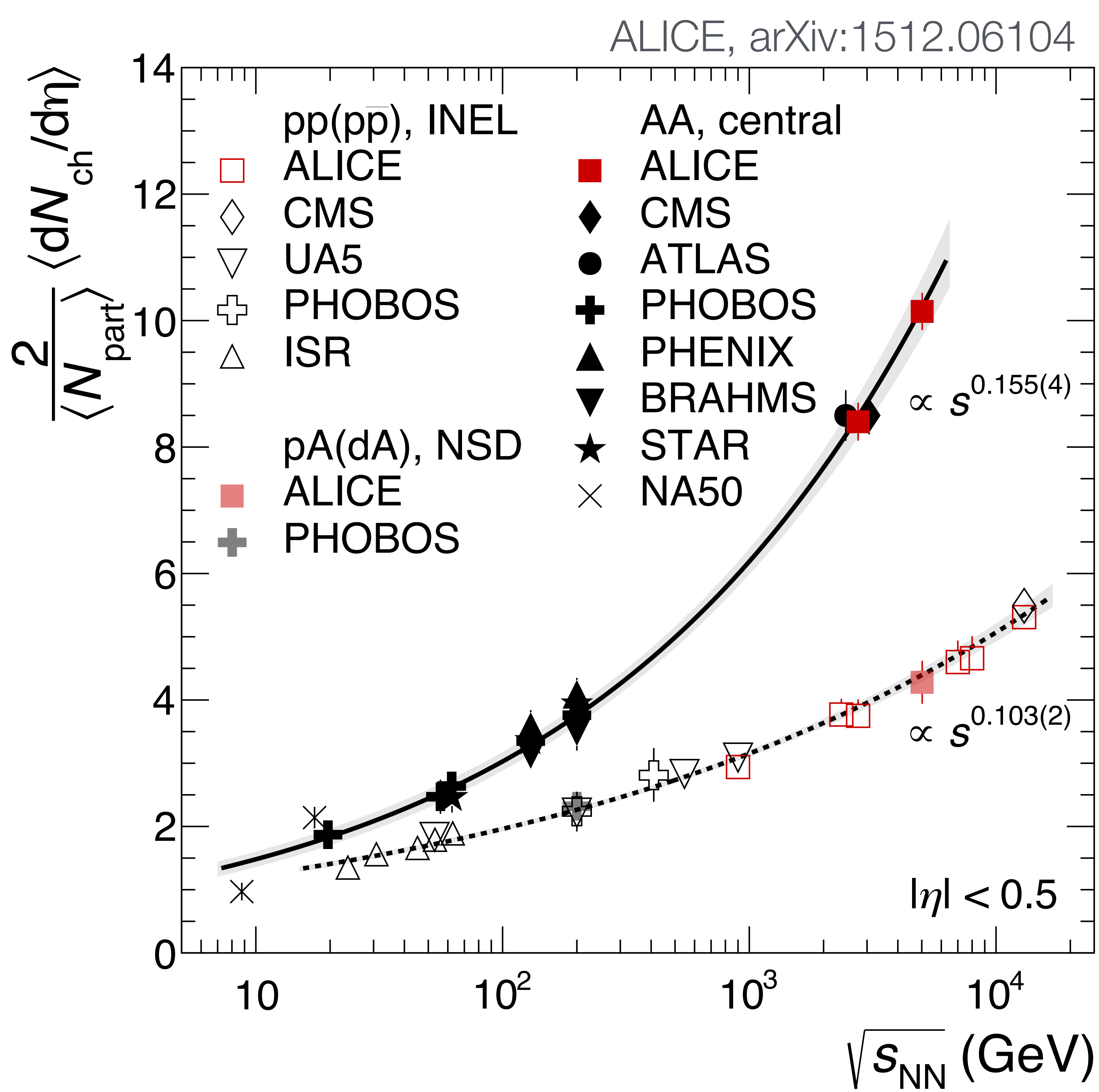
Multiplicity per participant higher in AA than in pp

e^+e^-:
pseudorapidity along the thrust axis



AA and e^+e^- η distributions strikingly similar

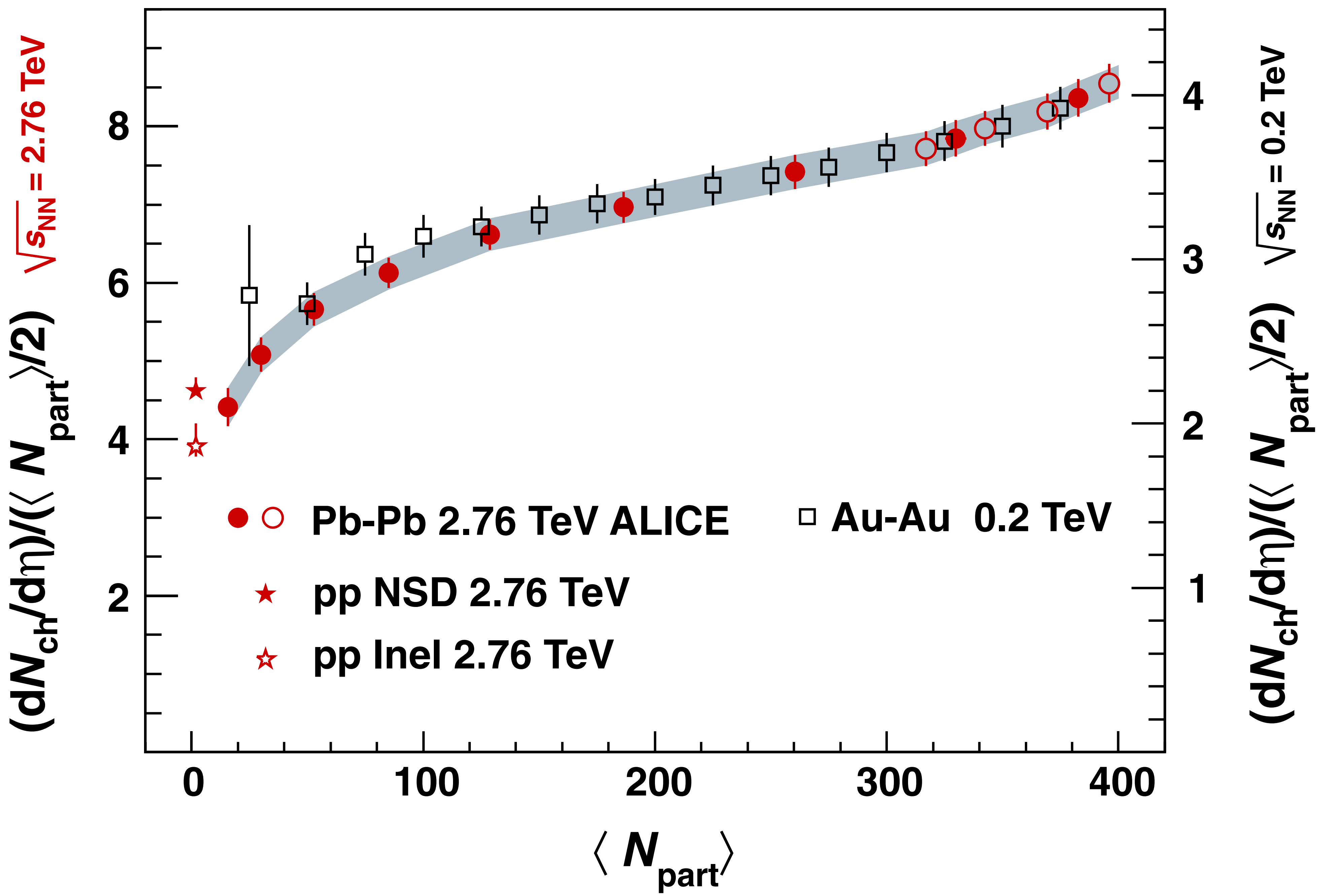
$dN_{ch}/d\eta$ vs $\sqrt{s_{NN}}$ in pp and central A-A collisions



- $dN_{ch}/d\eta$ scales with s^{α}
- Increase in central A+A stronger than in p+p

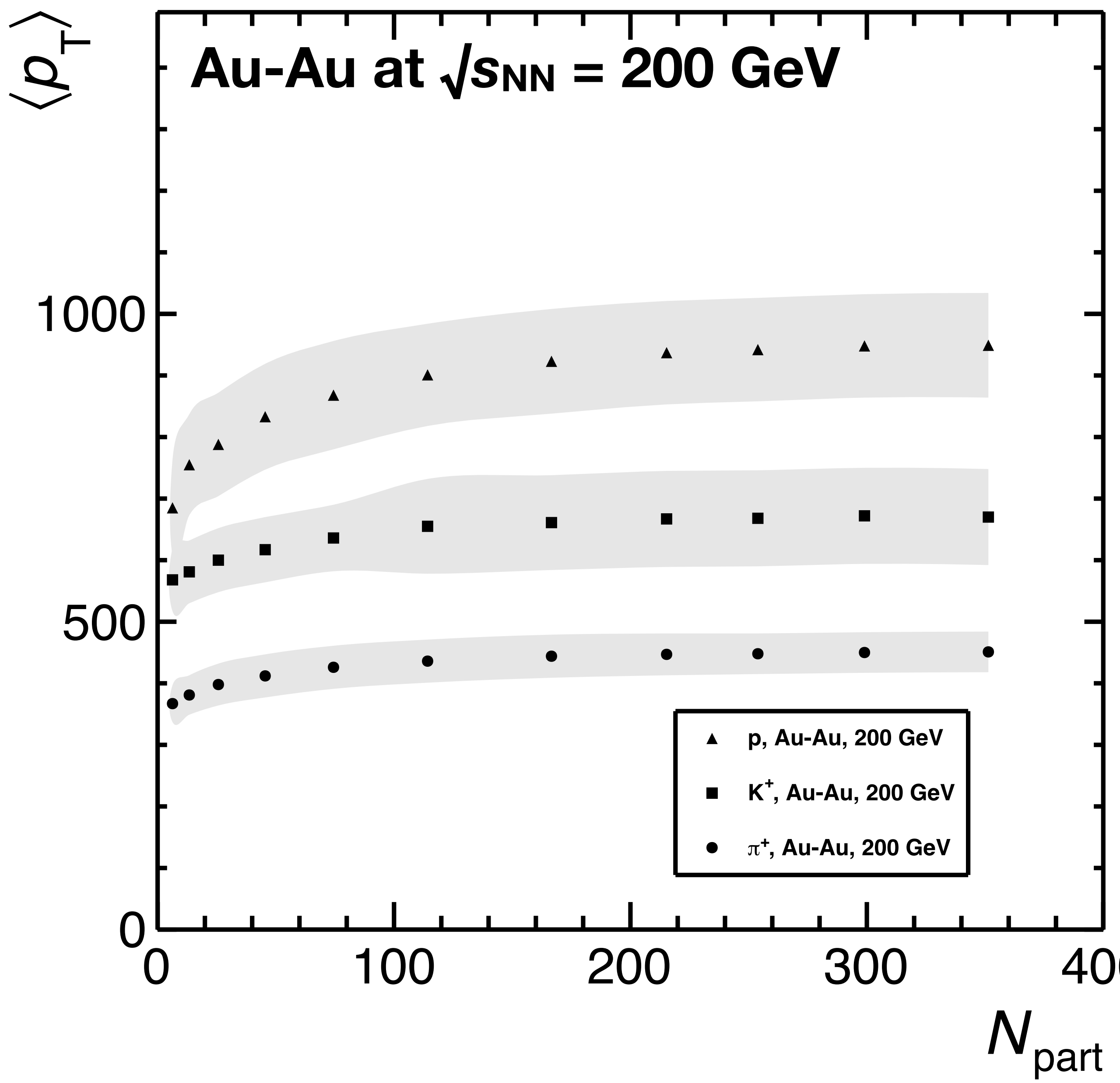
Centrality dependence of dN_{ch}/dη

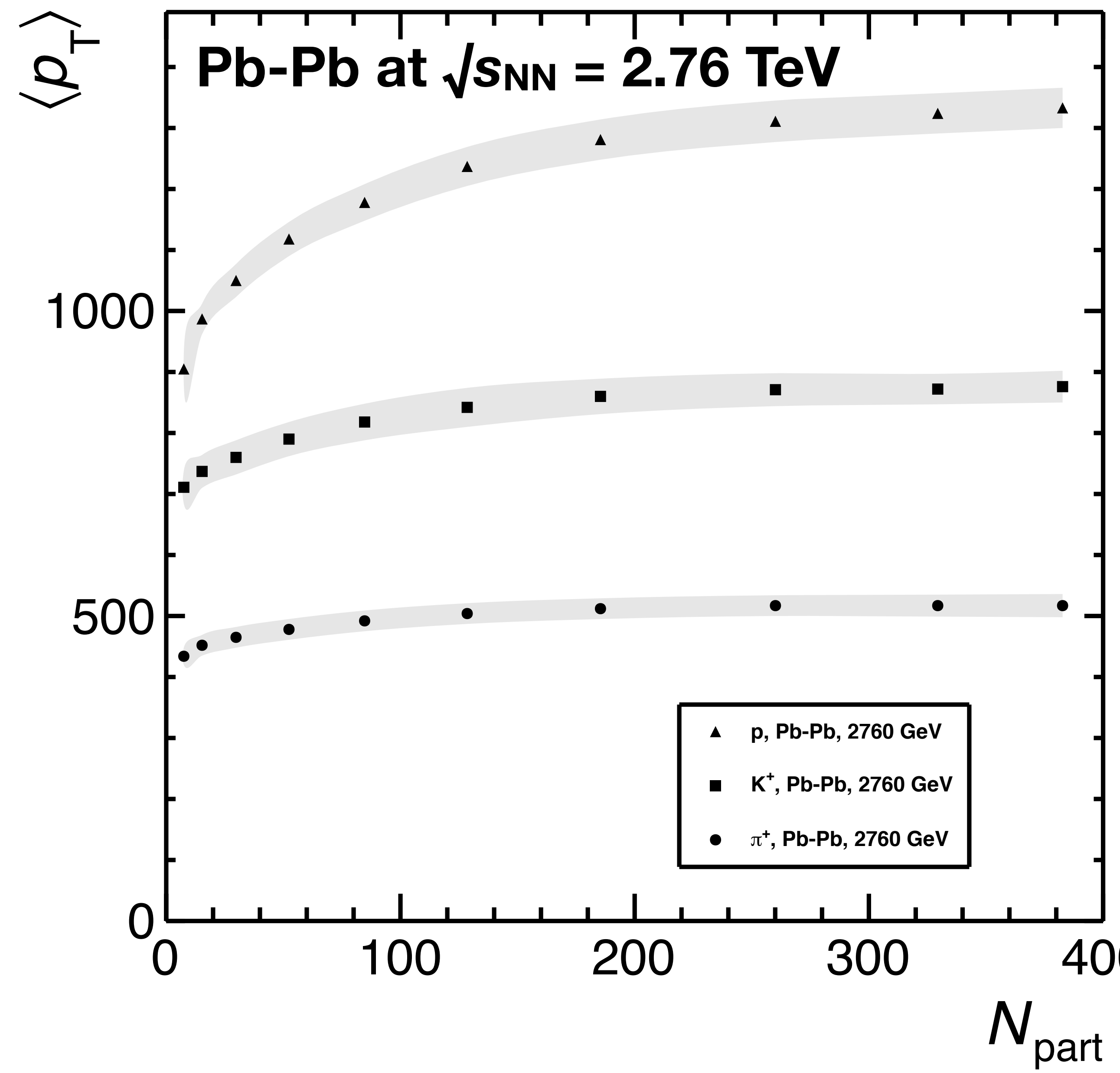
ALICE, arXiv:1012.1657



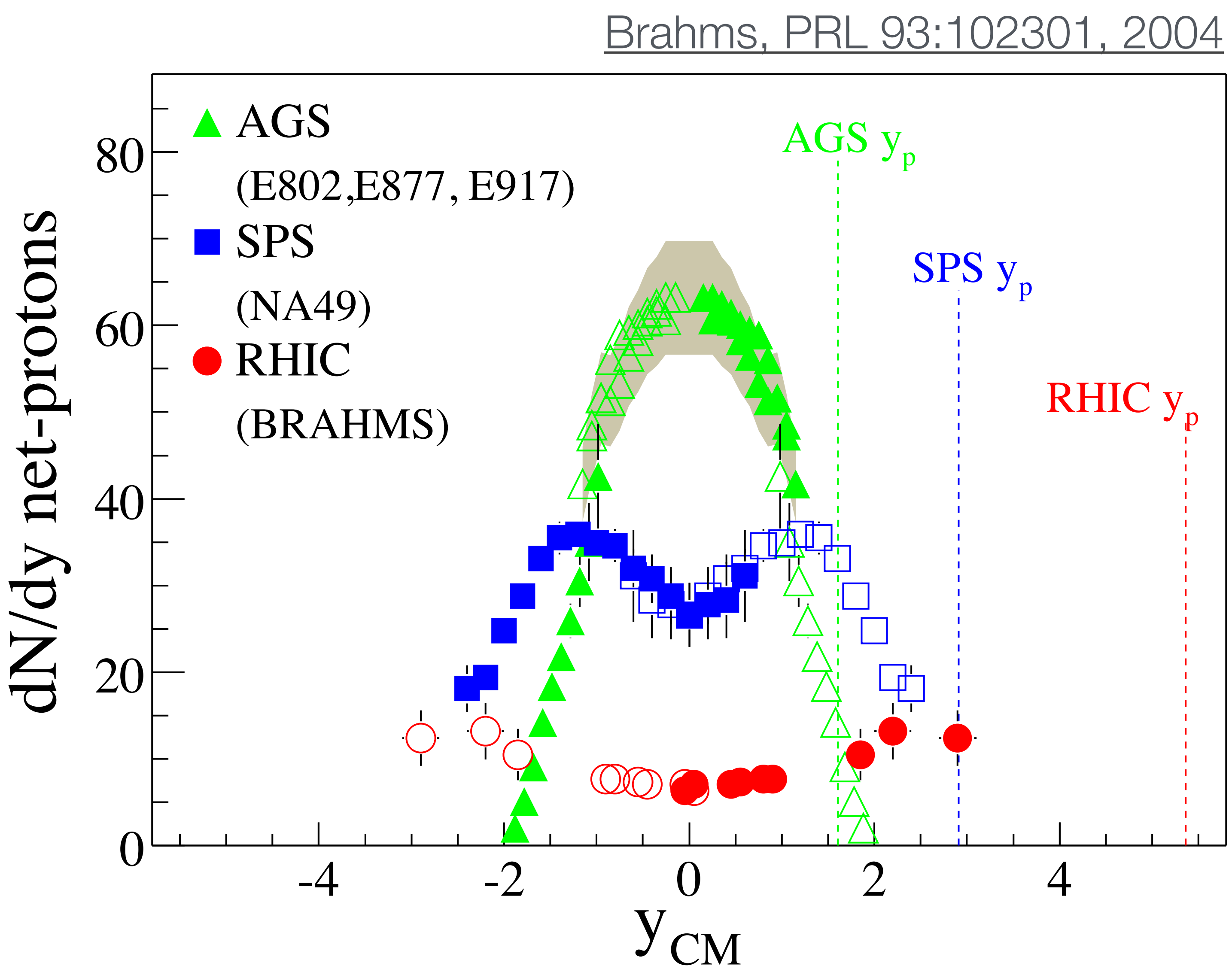
- dN_{ch}/dη / N_{part} increases with centrality
- Relative increase similar at RHIC and the LHC: Importance of geometry!

Average p_T of pions, kaons, and protons in Au-Au@200 GeV and Pb-Pb@2.76 TeV





Nuclear stopping power (Au-Au at $\sqrt{s_{NN}} = 200$ GeV)



Average rapidity loss:

Initial rapidity:

$$y_p = 5.36$$

Net baryons after the collision:

$$\langle y \rangle = \frac{2}{N_{part}} \int_0^{y_p} y \frac{dN_{B-\bar{B}}}{dy} dy$$

Average rapidity loss:

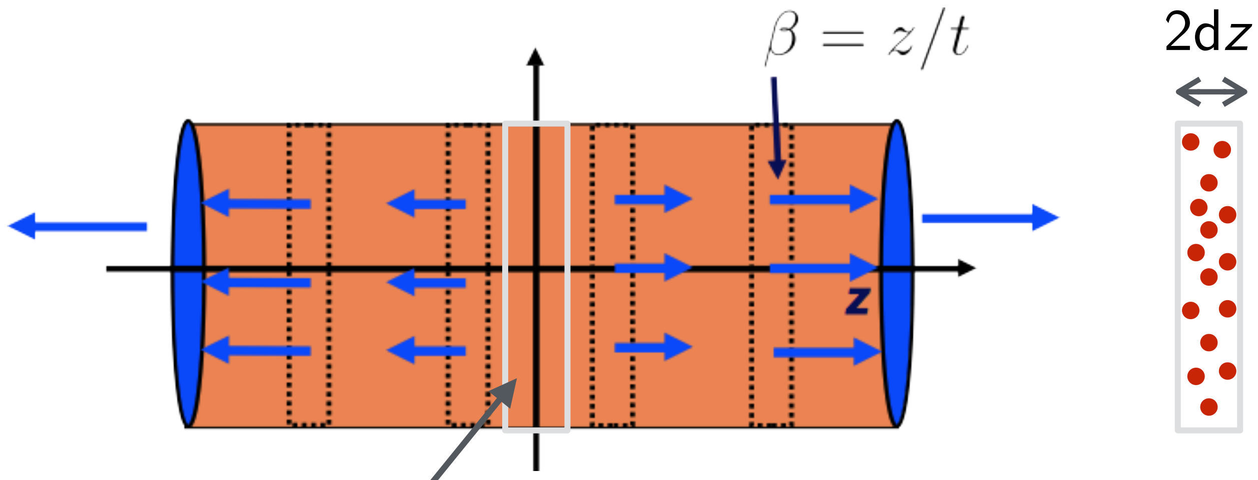
$$\langle \delta y \rangle = y_p - \langle y \rangle \approx 2$$

Average energy per (net) baryon:

$$E_p = 100\,\text{GeV}, \qquad \langle E \rangle = \frac{1}{N_{part}} \int_{-y_p}^{y_p} \underbrace{\langle m_T \rangle \cosh y}_{E} \frac{dN_{B-\bar{B}}}{dy} dy \approx 27 \pm 6\,\text{GeV}$$

Average energy loss of a nucleon in central Au+Au@200GeV is 73 ± 6 GeV

Bjorken's formula for the initial energy density



Consider total energy in slice at $z = 0$ at time τ_0

Assumptions:

- Particles (quarks and gluons) materialize at proper time τ_0
- Position z and longitudinal velocity (i.e. rapidity) are correlated
  - As if particles streamed freely from the origin

$z = \tau \sinh y$

$$\varepsilon = \frac{E}{V} = \frac{1}{A}\frac{dE}{dz}\bigg|_{z=0} = \frac{1}{A}\frac{dE}{dy}\bigg|_{y=0}\frac{dy}{dz}\bigg|_{z=0} = \frac{1}{A}\frac{dE}{dy}\bigg|_{y=0}\frac{1}{\tau} = \frac{\langle m_T \rangle}{A\cdot\tau}\frac{dN}{dy}\bigg|_{y=0}$$

A = transverse area

$$\varepsilon = \frac{1}{A\cdot\tau_0}\frac{dE_T}{dy}\bigg|_{y=0}, \quad \tau_0 \approx 1\,\mathrm{fm}/c$$

However, this formula neglects longitudinal work:

- dE/dy drops as a fct. of time
- Bjorken formula underestimates ε

J.D. Bjorken, Phys.Rev. D27 (1983) 140-151, 2952 citations on inspirehep.net on May 3, 2019

Energy density in central Pb-Pb collisions at the LHC

$$\varepsilon = \frac{1}{A \cdot \tau_0} \frac{dE_T}{dy}\bigg|_{y=0}$$

$$= \frac{1}{A \cdot \tau_0} J(y,\eta) \frac{dE_T}{d\eta}\bigg|_{\eta=0}$$

with $J(y,\eta) \approx 1.09$

Transverse area:

$$A = \pi R_{\mathrm{Pb}}^2 \quad \text{with } R_{\mathrm{Pb}} \approx 7\,\mathrm{fm}$$

Central Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV:

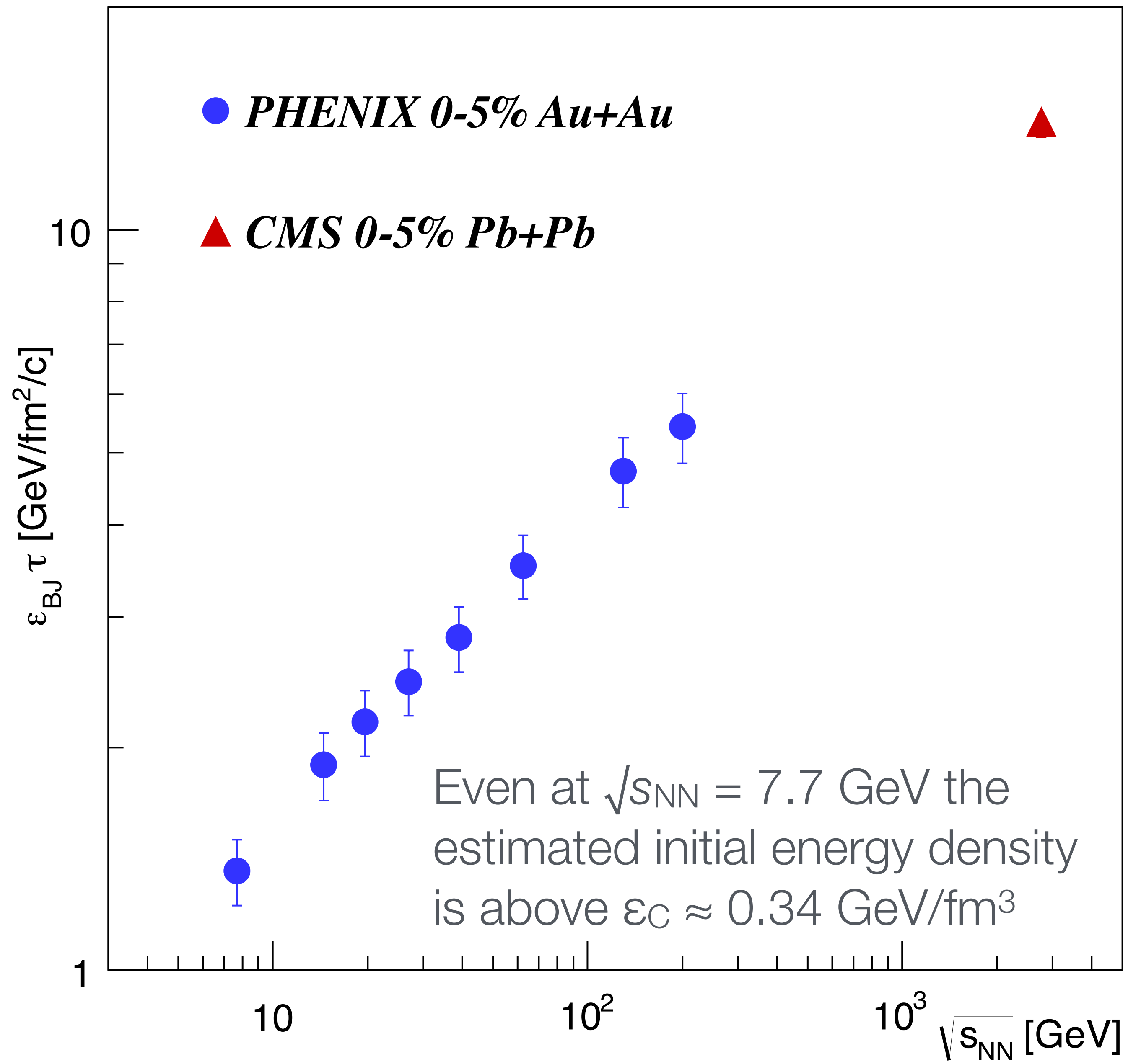
$$dE_T/d\eta = 2000\,\mathrm{GeV}$$

Energy density:

$$\varepsilon_{\mathrm{LHC}} = 14\,\mathrm{GeV/fm^3}$$

$$\approx 2.6 \times \varepsilon_{\mathrm{RHIC}} \text{ for } \tau_0 = 1\,\mathrm{fm}/c$$

PHENIX, arXiv:1509.06727

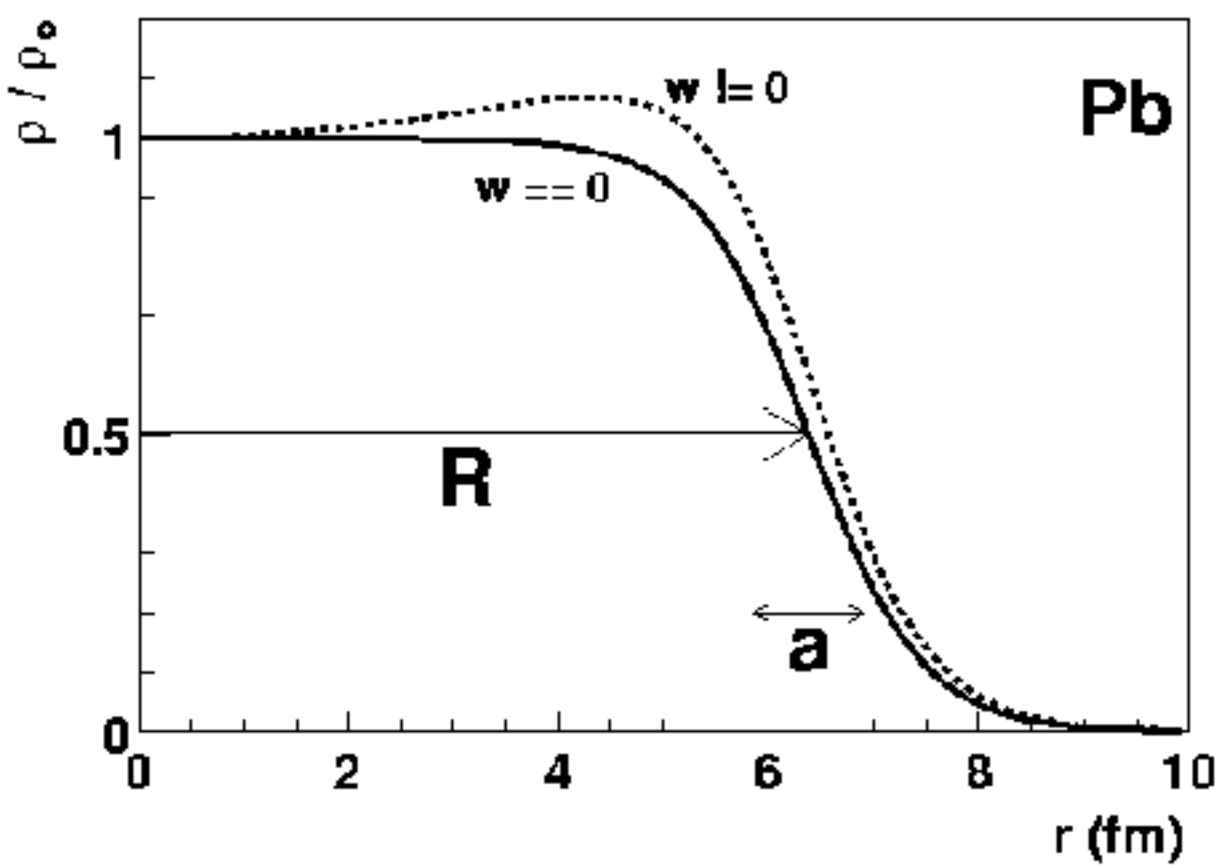


Even at $\sqrt{s_{NN}} = 7.7$ GeV the estimated initial energy density is above $\varepsilon_C \approx 0.34$ GeV/fm³

Glauber modeling:
An interface between theory and experiment

Starting point: nucleon density

$$\rho(r) = \frac{\rho_0 \left(1 + wr^2/R^2\right)}{1 + \exp((r - R)/a)}$$



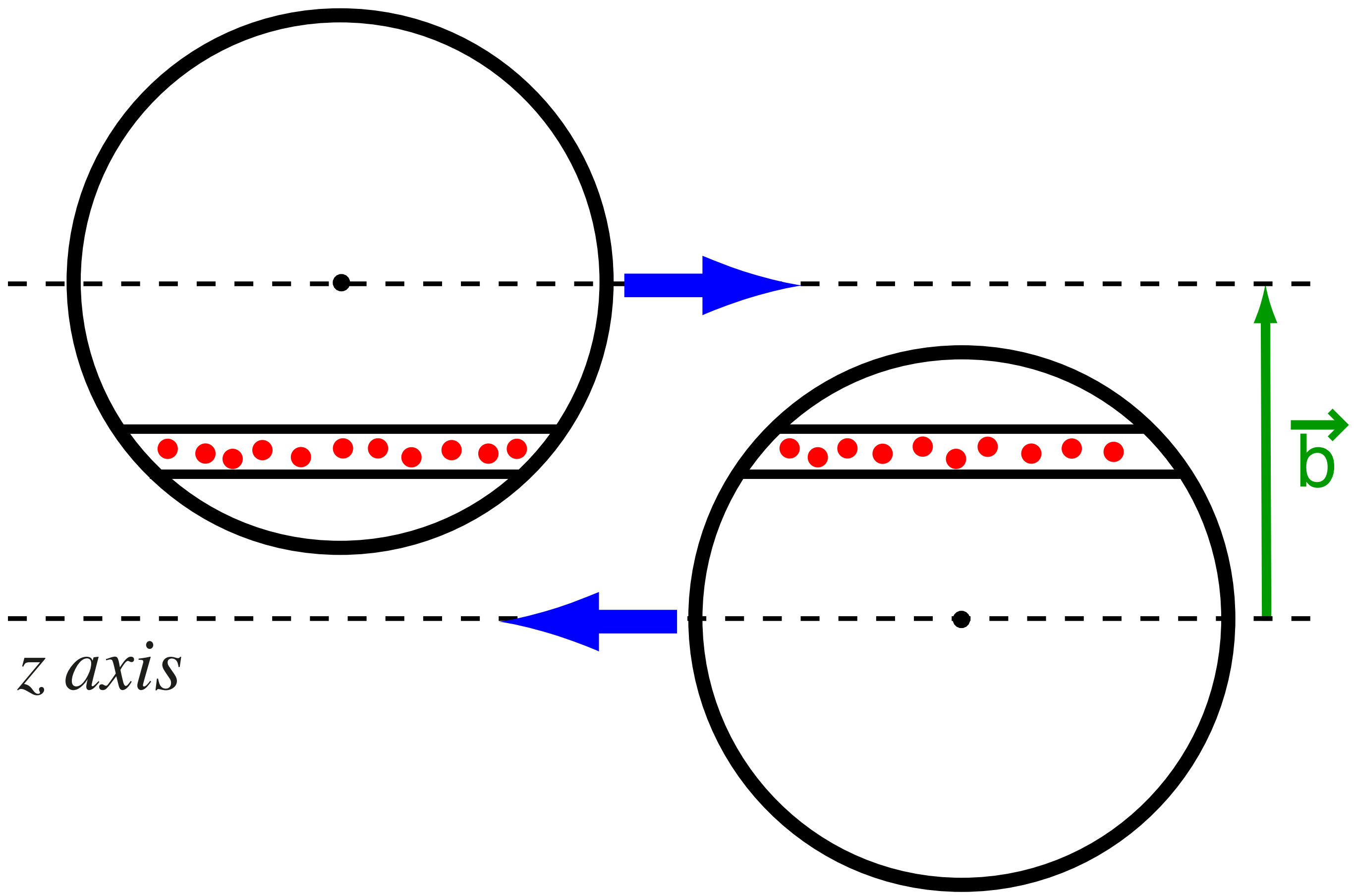
Nucleus	A	R (fm)	a (fm)	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	**197**	**6.38**	**0.535**	**0**
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

Woods-Saxon parameters typically from e^--nucleus scattering (sensitive to charge distribution only)

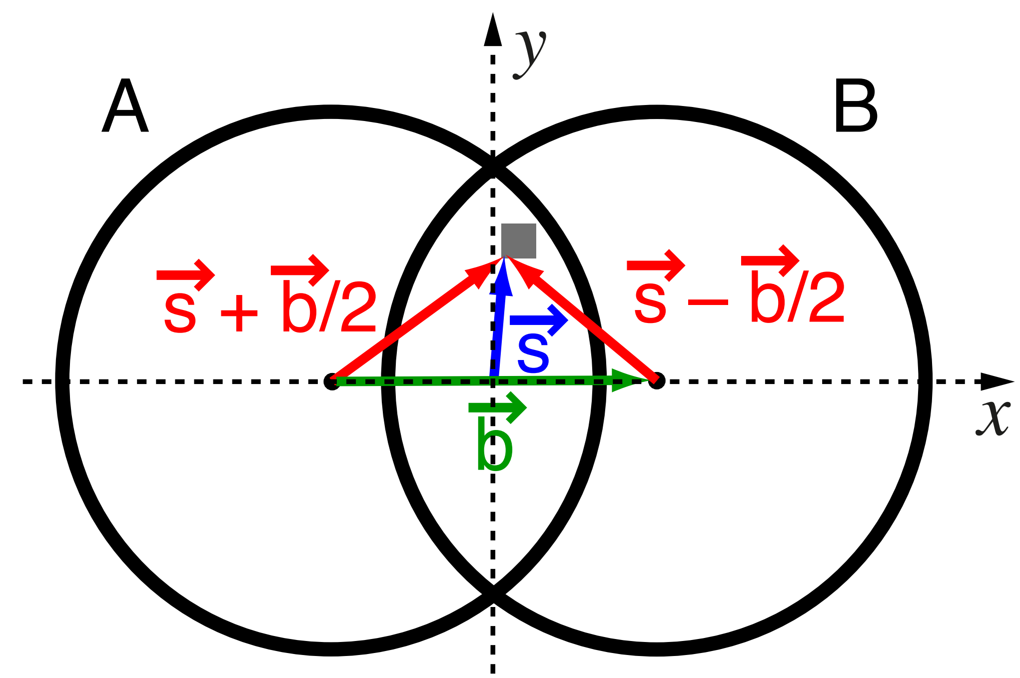
Difference between neutron and proton distribution small and typically neglected

Nuclear Thickness Function

side view:



transverse plane:



Projection of nucleon density on the transverse plane ("nuclear thickness fct."):

$$T_\mathrm{A}(\vec{s}') = \int \mathrm{d}z \; \rho_\mathrm{A}(z, \vec{s}')$$

(analogous for nucleus B)

Number of nucleon-nucleon encounters per transverse area element:

$$\mathrm{d}\,T_\mathrm{AB} = T_\mathrm{A}(\vec{s} + \vec{b}/2) \cdot T_\mathrm{B}(\vec{s} - \vec{b}/2) \; \mathrm{d}^2 s$$

Nuclear Overlap function and the number of nucleon-nucleon collisions

Nuclear overlap function:

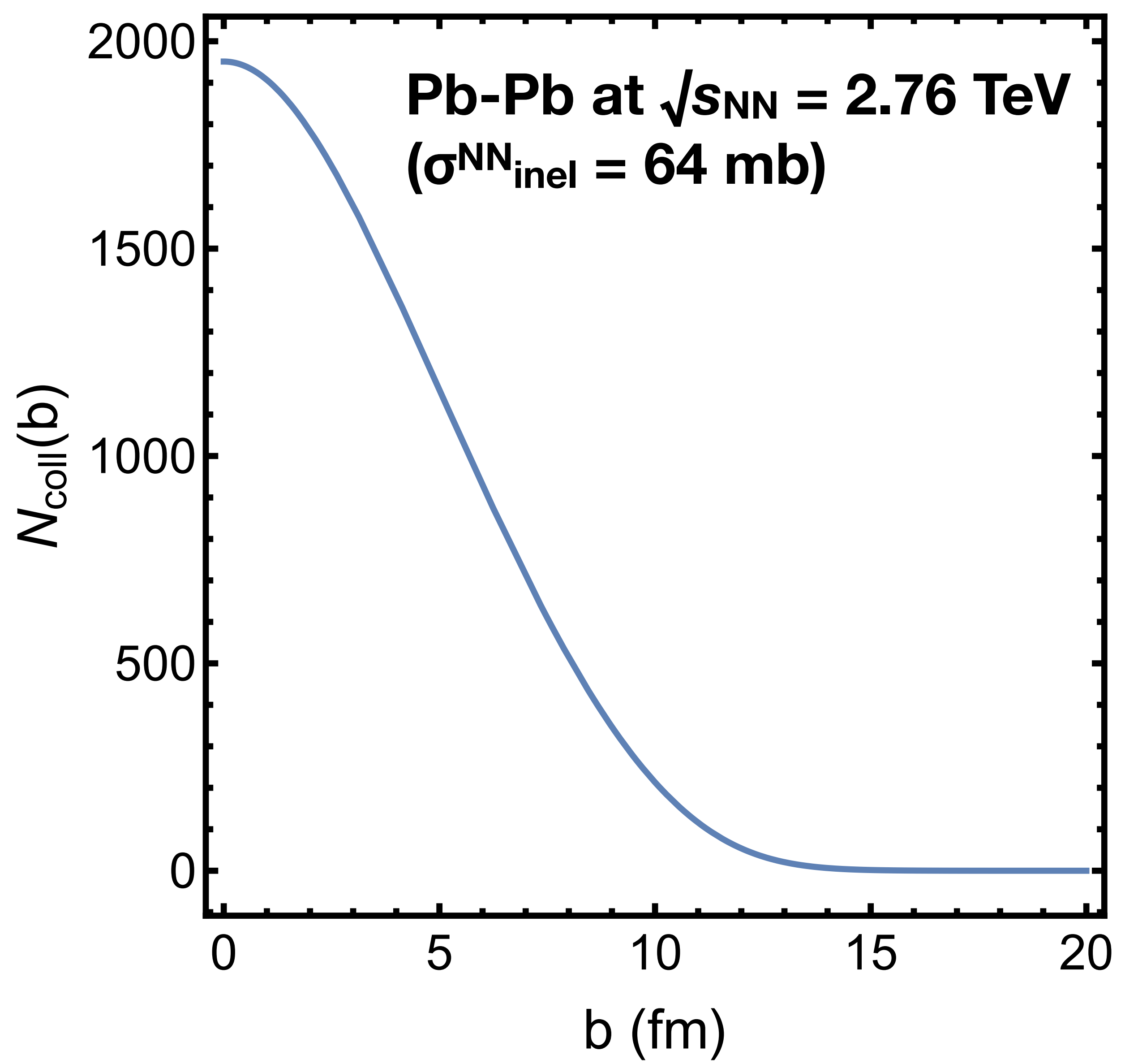
$$T_{\mathrm{AB}}(\vec{b}) = \int T_{\mathrm{A}}(\vec{s} + \vec{b}/2) \cdot T_{\mathrm{B}}(\vec{s} - \vec{b}/2)\, \mathrm{d}^2 s$$

Nuclear overlap function resembles integrated luminosity of a collider:

$$N_{\mathrm{coll}}(b) = T_{\mathrm{AB}}(b) \cdot \sigma_{\mathrm{inel}}^{\mathrm{NN}}$$

Or, more generally for a process with cross section σ_{int}:

$$N_{\mathrm{int}}(b) = T_{\mathrm{AB}}(b) \cdot \sigma_{\mathrm{int}}$$



Probability for an Inelastic A+B collision

Def's (different normalization of the thickness functions):

$$\hat{T}_\mathrm{A}(\vec{s}') = T_\mathrm{A}(\vec{s}')/A \qquad \hat{T}_\mathrm{B}(\vec{s}') = T_\mathrm{B}(\vec{s}')/B \qquad \hat{T}_\mathrm{AB}(\vec{b}) = T_\mathrm{AB}(\vec{b})/(AB)$$

We can then write:

$$N_\mathrm{coll}(b) = AB\,\hat{T}_\mathrm{AB}(b) \cdot \sigma_\mathrm{inel}^\mathrm{NN} \qquad p_\mathrm{NN} = \hat{T}_\mathrm{AB}(\vec{b}) \cdot \sigma_\mathrm{inel}^\mathrm{NN}$$

probability for a certain nucleon from nucleus A to collide with a certain nucleon from nucleus B

Probability for k nucleon-nucleon coll.:

$$P(k, \vec{b}) = \binom{AB}{k} p_\mathrm{NN}^k (1 - p_\mathrm{NN})^{AB-k}$$

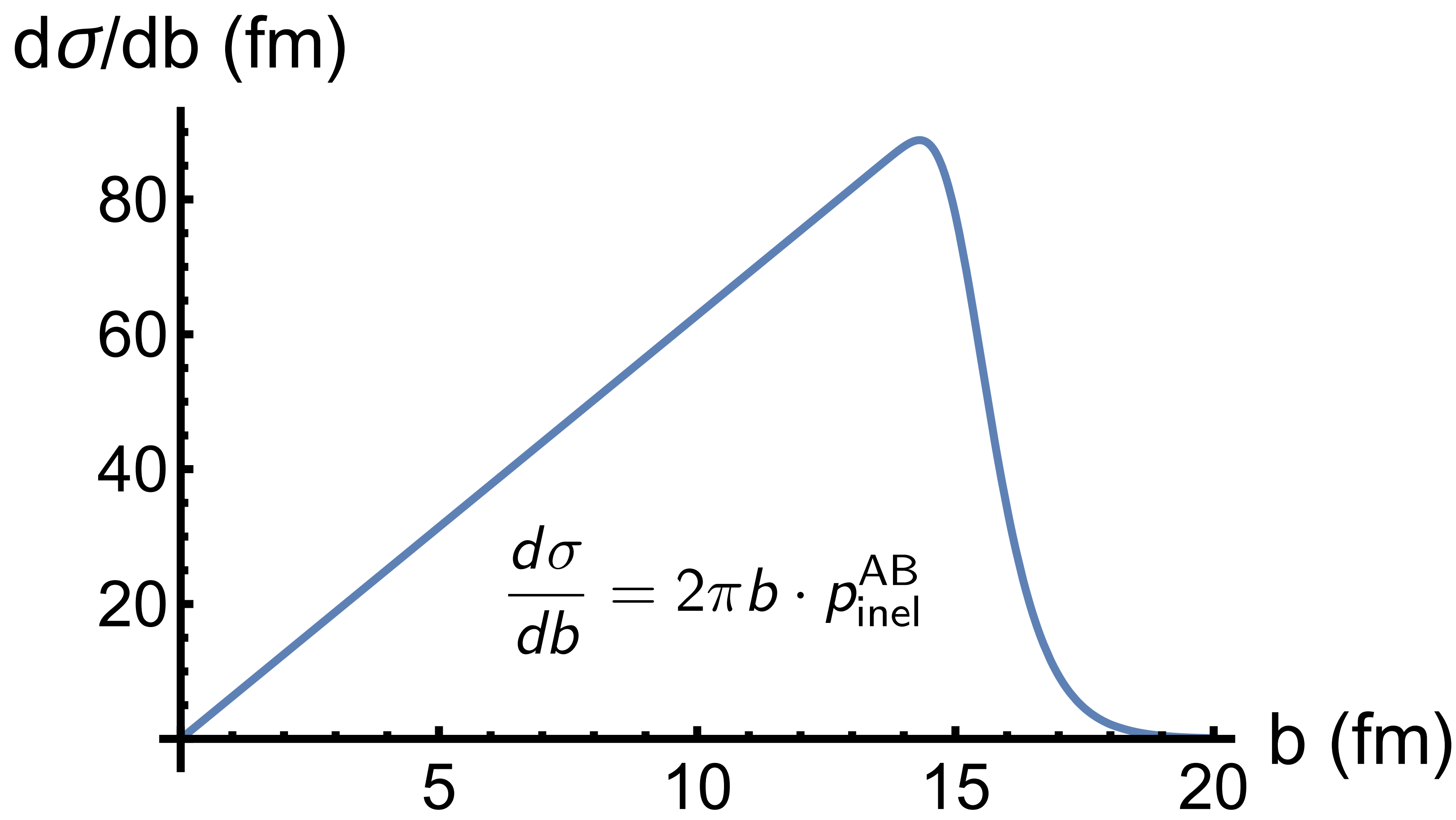
$$(1-x)^n = \exp(n \ln(1-x)) \overset{x \to 0}{\approx} \exp(-nx)$$

Probability for $k = 0$ is $(1 - p_\mathrm{NN})^{AB}$. Thus:

$$p_\mathrm{inel}^\mathrm{AB}(\vec{b}) = 1 - (1 - \hat{T}_\mathrm{AB}(\vec{b}) \cdot \sigma_\mathrm{inel}^\mathrm{NN})^{AB} \approx 1 - \exp(-AB\,\hat{T}_\mathrm{AB}(\vec{b}) \cdot \sigma_\mathrm{inel}^\mathrm{NN})$$

Poisson limit of the binomial distribution

dσ/db for Pb-Pb



Total cross section: $\sigma_{\mathrm{inel}}^{\mathrm{AB}} = \int_0^\infty \frac{d\sigma}{db}\, db \approx 784\,\mathrm{fm}^2 = 7.84\,\mathrm{b}$

Number of Participants

Probability that a test nucleon of nucleus A interacts with a certain nucleon of nucleus B:

$$p_{\mathrm{NN,A}}(\vec{s}) = \hat{T}_{\mathrm{B}}(\vec{s} - \vec{b}/2)\sigma_{\mathrm{inel}}^{\mathrm{NN}}$$

Probability that the test nucleon does not interact with any of the *B* nucleons of nucleus B:

$$(1 - p_{\mathrm{NN,A}}(\vec{s}))^{B}$$

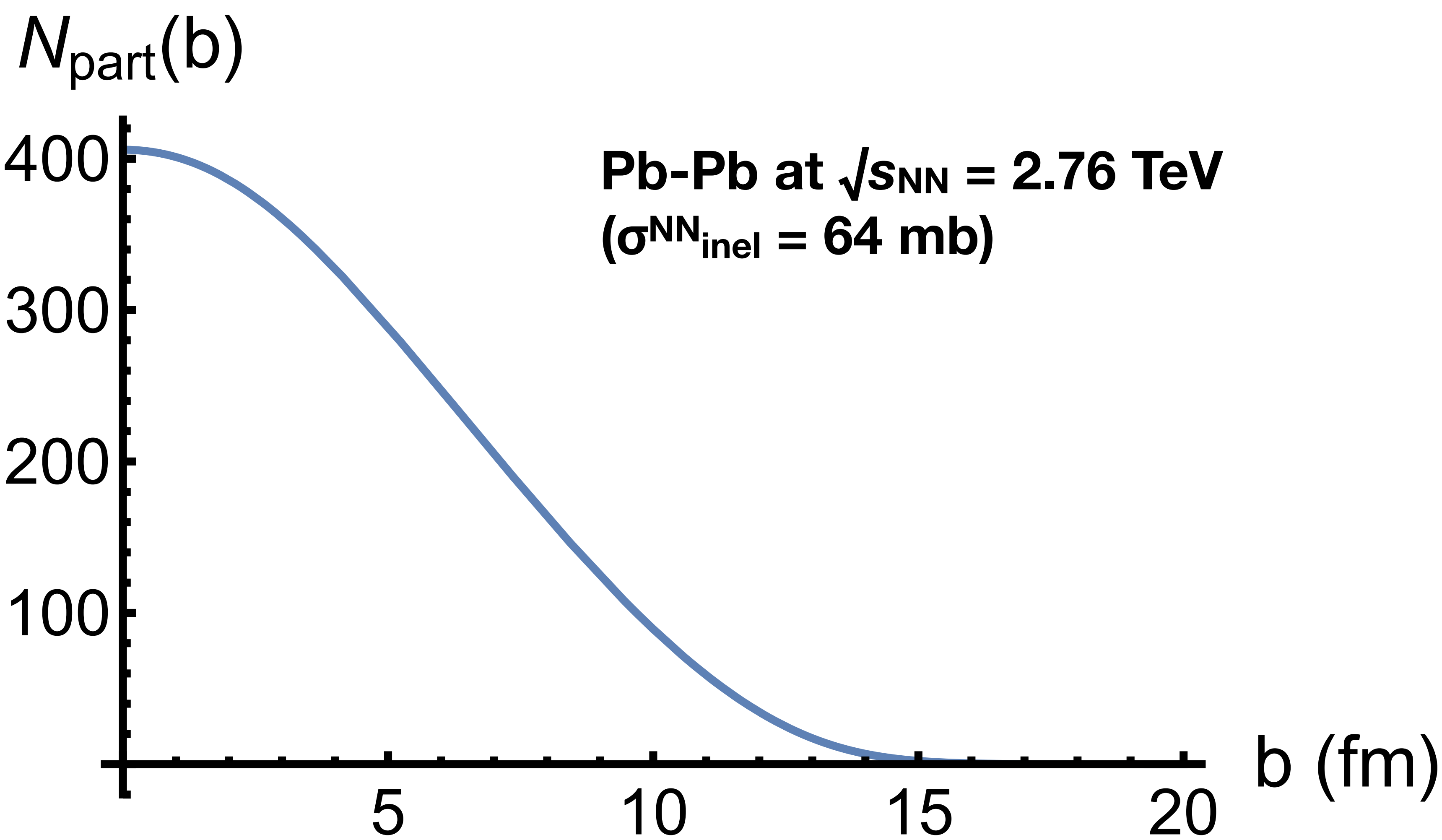
Probability that the test nucleon makes at least on interaction:

$$1 - (1 - p_{\mathrm{NN,A}}(\vec{s}))^{B} \approx 1 - \exp(-B p_{\mathrm{NN,A}}(\vec{s}))$$

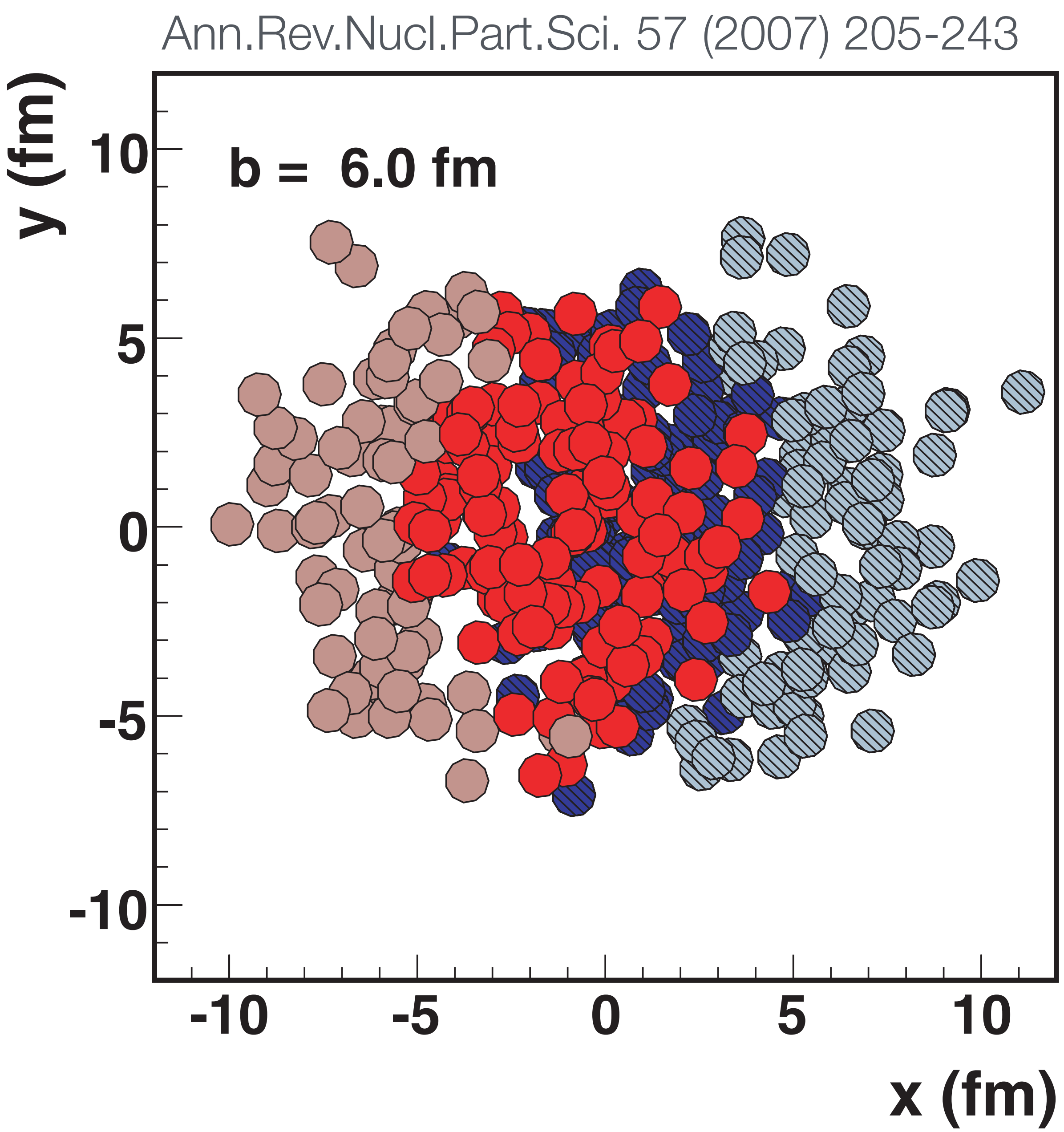
Number of participants:

$$\begin{aligned} N_{\mathrm{part}}(\vec{b}) &= N_{\mathrm{part}}^{\mathrm{A}}(\vec{b}) + N_{\mathrm{part}}^{\mathrm{B}}(\vec{b}) \\ &= \int T_{\mathrm{A}}(\vec{s} + \vec{b}/2) \cdot \left[1 - \exp(-T_{\mathrm{B}}(\vec{s} - \vec{b}/2)\sigma_{\mathrm{inel}}^{\mathrm{NN}})\right] \mathrm{d}^2 s \\ &+ \int T_{\mathrm{B}}(\vec{s} - \vec{b}/2) \cdot \left[1 - \exp(-T_{\mathrm{A}}(\vec{s} + \vec{b}/2)\sigma_{\mathrm{inel}}^{\mathrm{NN}})\right] \mathrm{d}^2 s \end{aligned}$$

N_{part} vs Impact Parameter b



Glauber Monte Carlo Approach



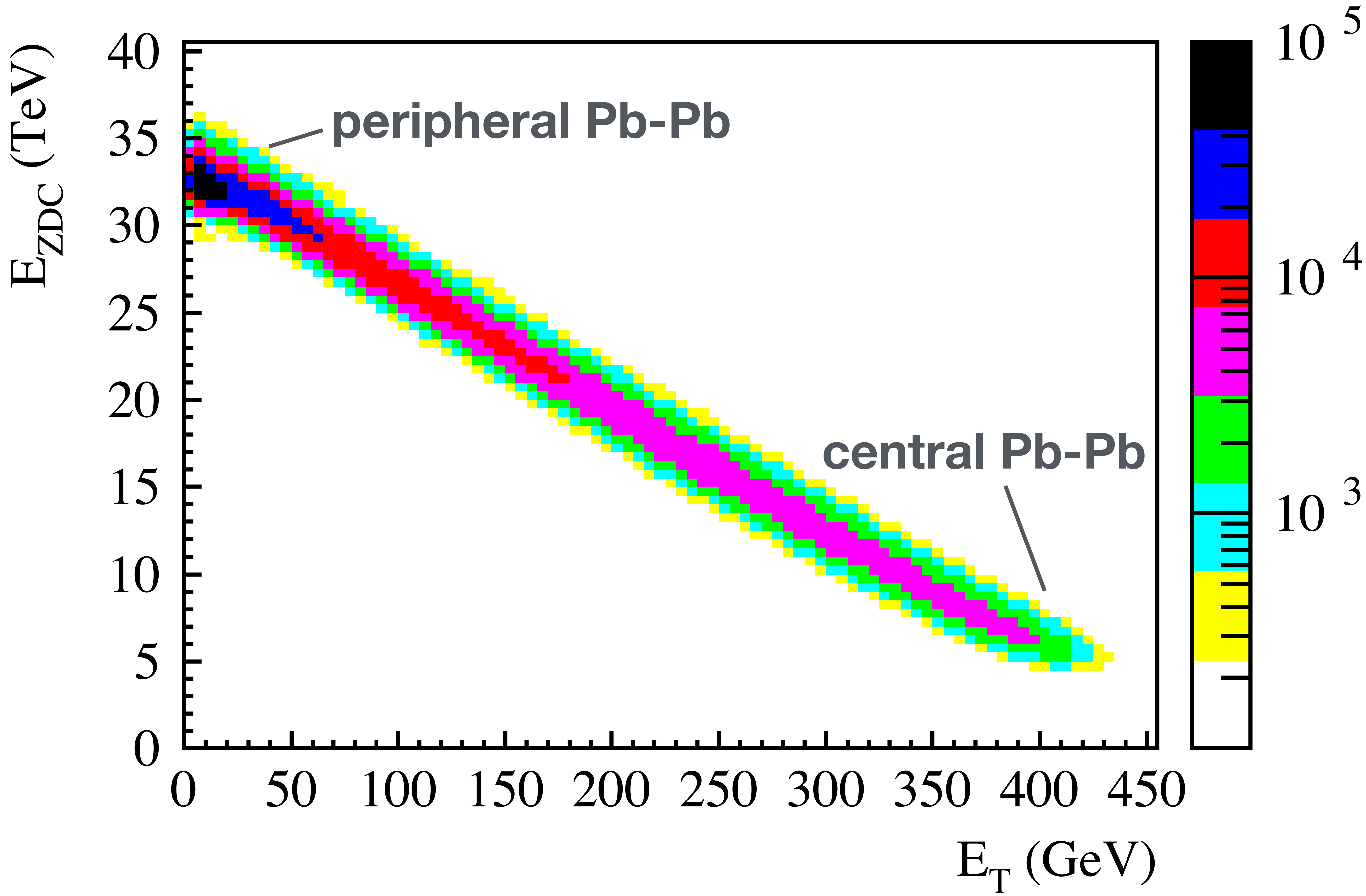
- Randomly select impact parameter b
- Distribute nucleons of two nuclei according to nuclear density distribution
- Consider all pairs with one nucleon from nucleus A and the other from B
- Count pair as inel. n-n collision if distance d in x-y plane satisfies:

$$d < \sqrt{\sigma_{\mathrm{inel}}^{\mathrm{NN}} / \pi}$$

- Repeat many times: $\langle N_{\mathrm{part}} \rangle (b)$ $\langle N_{\mathrm{coll}} \rangle (b)$

Centrality selection: Forward and transverse energy

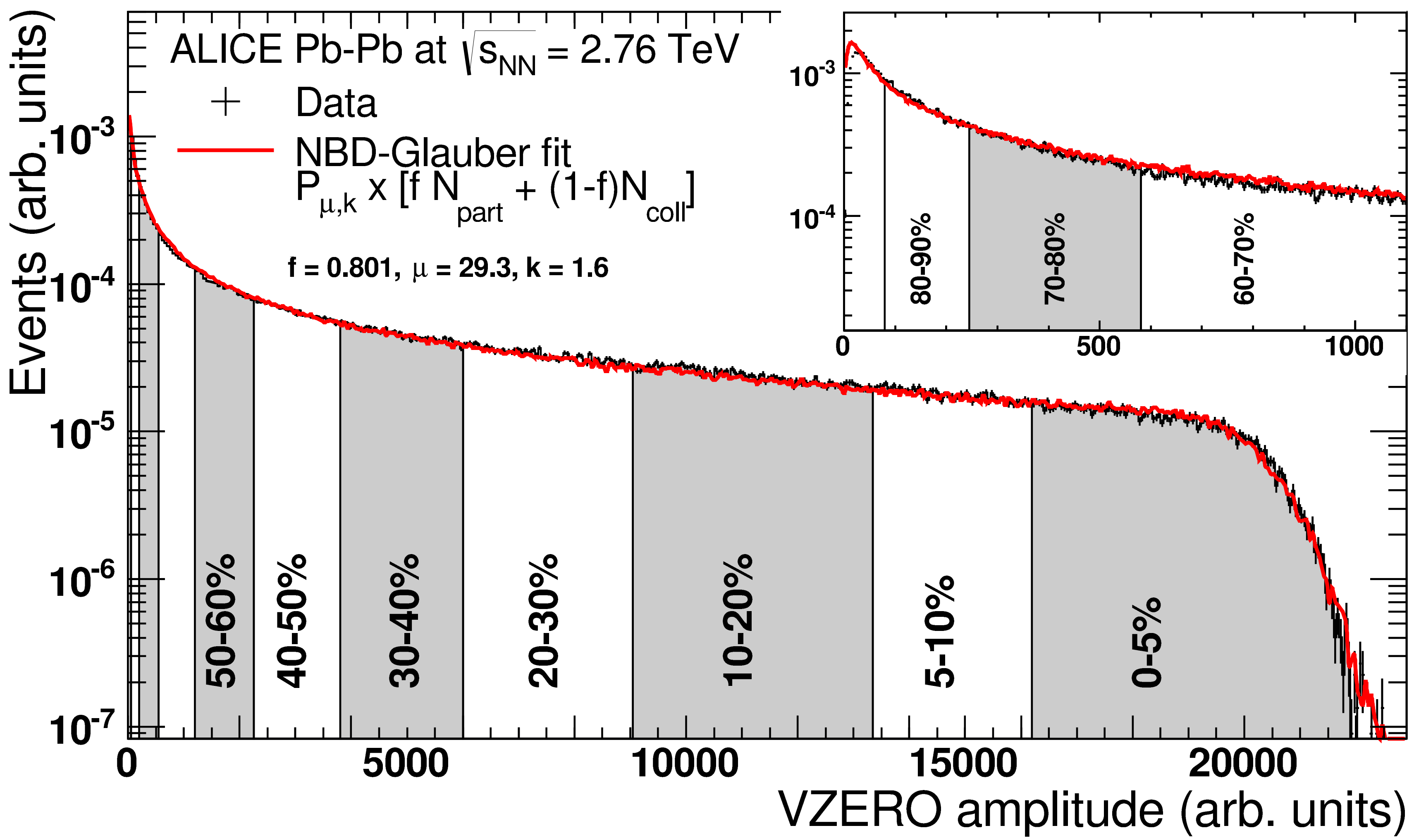
Example: Pb-Pb, fixed-target experiment (WA98, CERN SPS)



Both E_T and E_{ZDC} can be used to define centrality classes

Centrality Selection: Charged-Particle Multiplicity

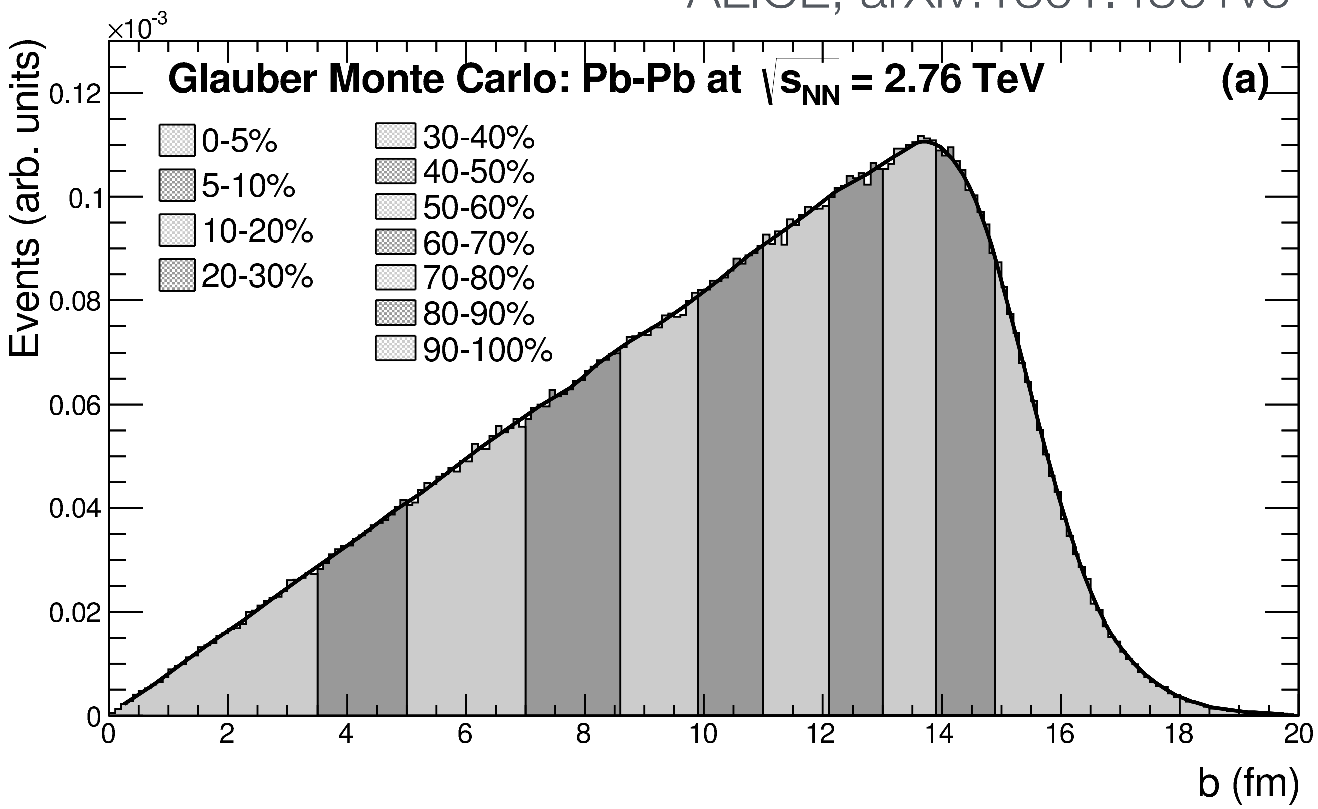
ALICE, arXiv:1301.4361v3



- Measure charged particle multiplicity
  - ALICE: VZERO detectors ($2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$)
  - Assumption: $\langle N_{ch} \rangle(b)$ increases monotonically with decreasing b
- Define centrality class by selecting a percentile of the measured multiplicity distribution (e.g. 0-5%)
  - Need Glauber fit to define “100%” (background at low multiplicities)

How ⟨N_{part}⟩, ⟨N_{coll}⟩, and ⟨b⟩ are Assigned to an Experimental Centrality Class?

ALICE, arXiv:1301.4361v3



- Glauber Monte Carlo
  - Find impact parameter interval [b_1, b_2] which corresponds to the same percentile
  - Average $N_{part}(b)$, $N_{coll}(b)$, etc over this interval
- Example: Pb-Pb at $\sqrt{s_{NN}}$ = 2.76 TeV
  - σ_{NN}(inel) = (64 ± 5) mb

Centrality	b_{min} (fm)	b_{max} (fm)	⟨N_{part}⟩	RMS	(*sys.*)	⟨N_{coll}⟩	RMS	(*sys.*)	⟨T_{AA}⟩ 1/mbarn	RMS 1/mbarn	(*sys.*) 1/mbarn
0–5%	0.00	3.50	382.7	17	3.0	1685	140	190	26.32	2.2	0.85
5–10%	3.50	4.94	329.4	18	4.3	1316	110	140	20.56	1.7	0.67
10–20%	4.94	6.98	260.1	27	3.8	921.2	140	96	14.39	2.2	0.45
20–40%	6.98	9.88	157.2	35	3.1	438.4	150	42	6.850	2.3	0.23
40–60%	9.88	12.09	68.56	22	2.0	127.7	59	11	1.996	0.92	0.097
60–80%	12.09	13.97	22.52	12	0.77	26.71	18	2.0	0.4174	0.29	0.026
80–100%	13.97	20.00	5.604	4.2	0.14	4.441	4.4	0.21	0.06939	0.068	0.0055