Quark-Gluon Plasma Physics

6. Space-time evolution of the QGP

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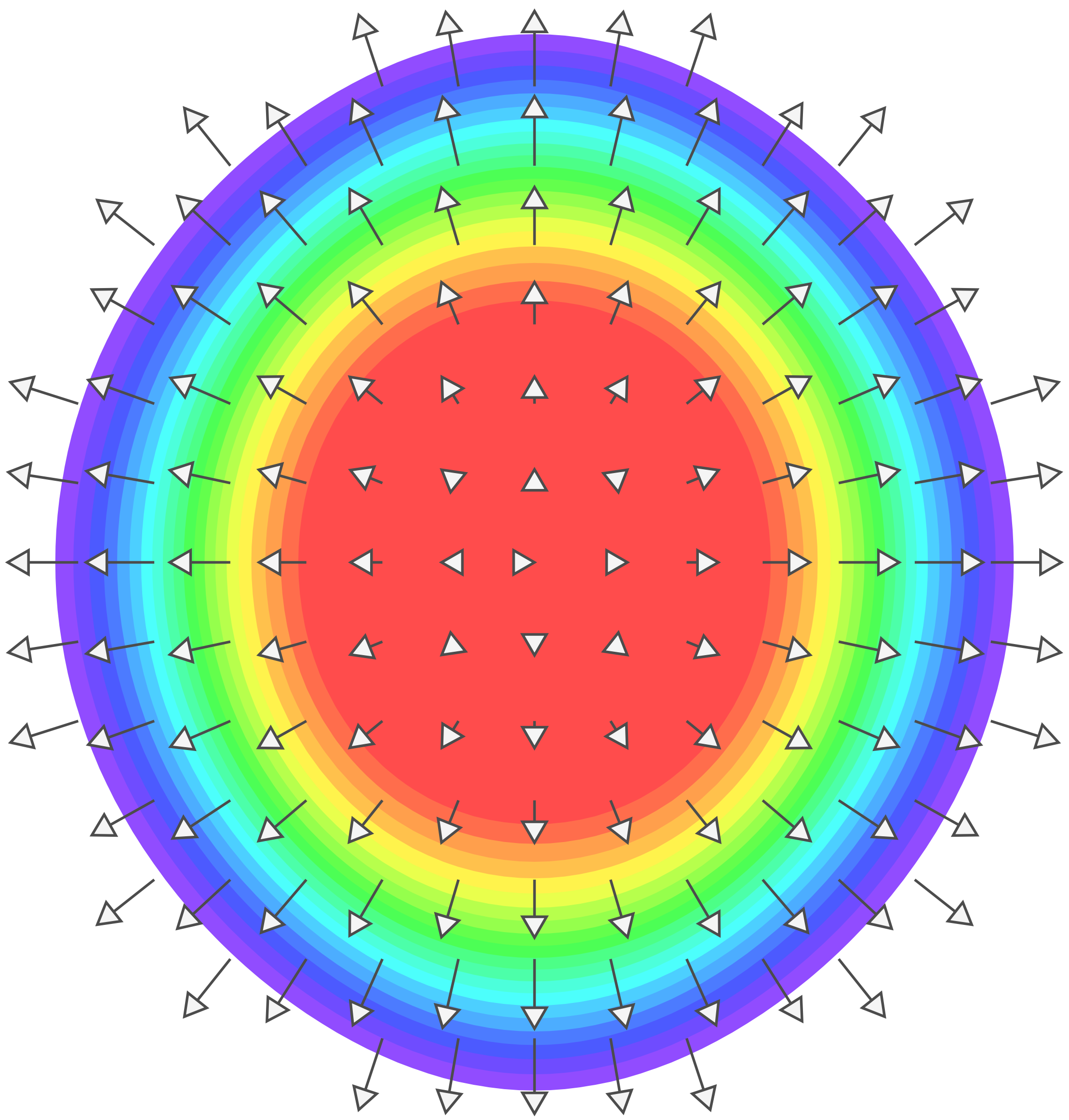
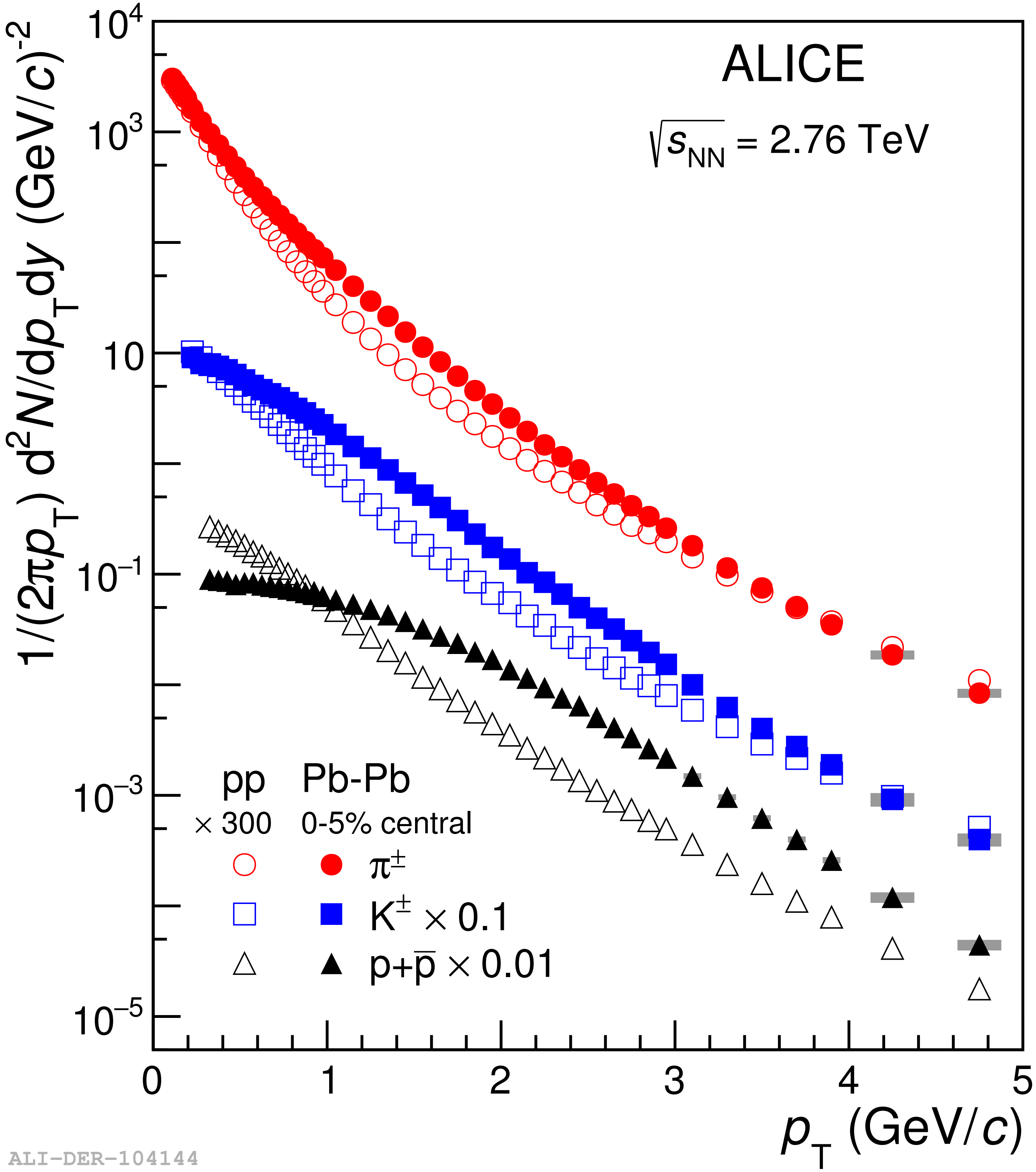
Basics of relativistic hydrodynamics

Evidence for collective behavior in heavy-ion collisions

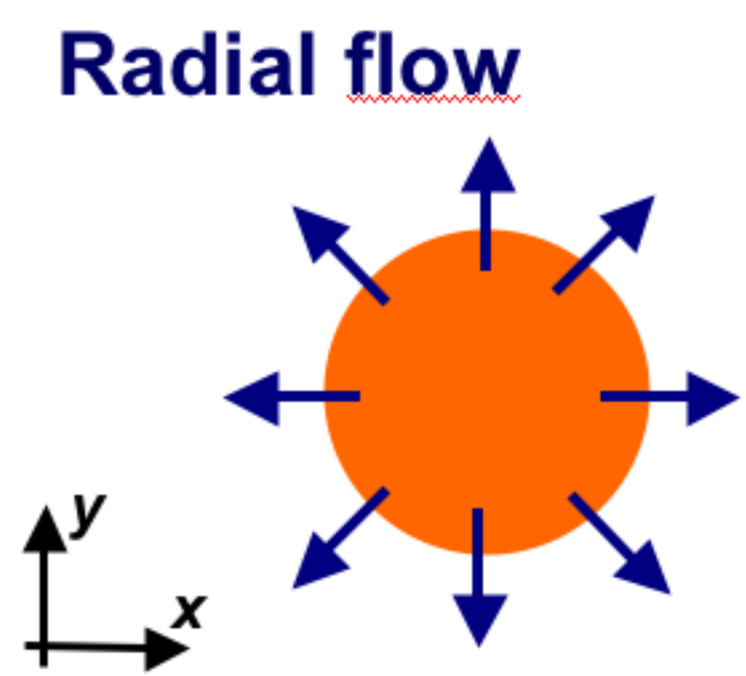


- Shape of low-p_T transverse momentum spectra for particles with different masses
- Azimuthal anisotropy of produced particles
- Source sizes from Hanbury Brown-Twiss correlations
- …

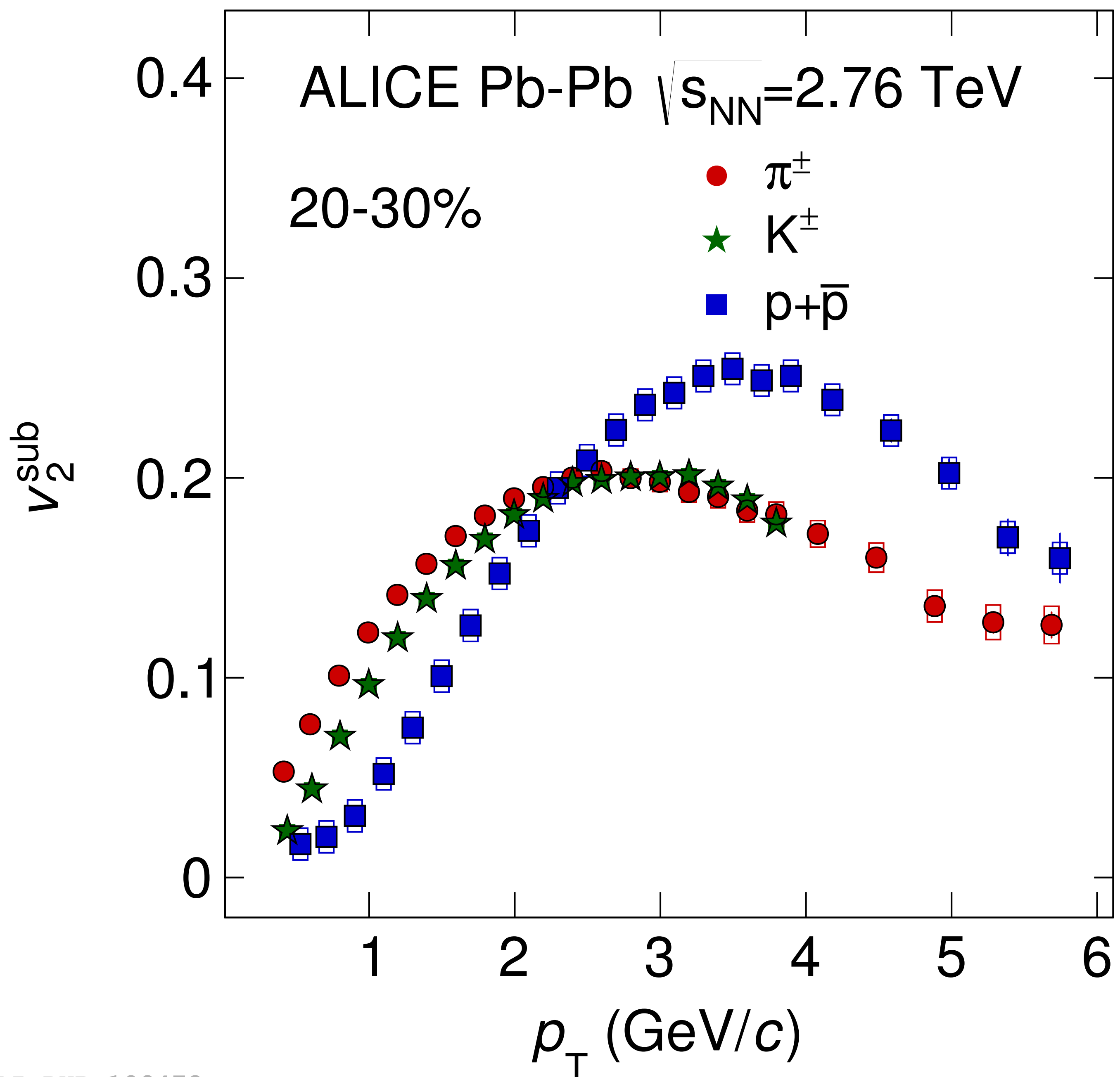
Evidence for radial flow



- Shape is different in pp and A-A
- Stronger effect for heavier particles

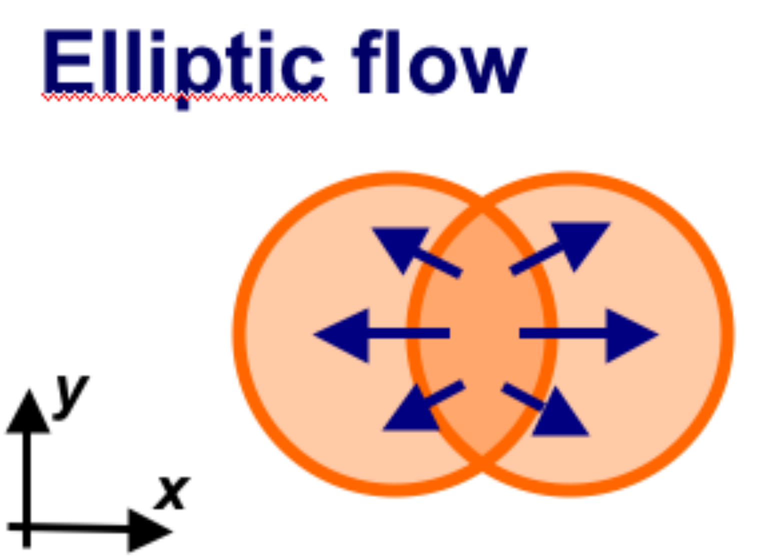


Evidence for elliptic flow



ALI-PUB-109472

Good explanation:
Azimuthal variation of the flow velocity



Basics of relativistic hydrodynamics

See e.g. Ollitrault, arXiv:0708.2433

Standard thermodynamics: P, T, μ constant over the entire volume

Hydrodynamics assumes *local* thermodynamic equilibrium: $P(x^\mu)$, $T(x^\mu)$, $\mu(x^\mu)$

Local thermodynamic equilibrium only possible if mean free path between two collisions much shorter than all characteristic scales of the system:

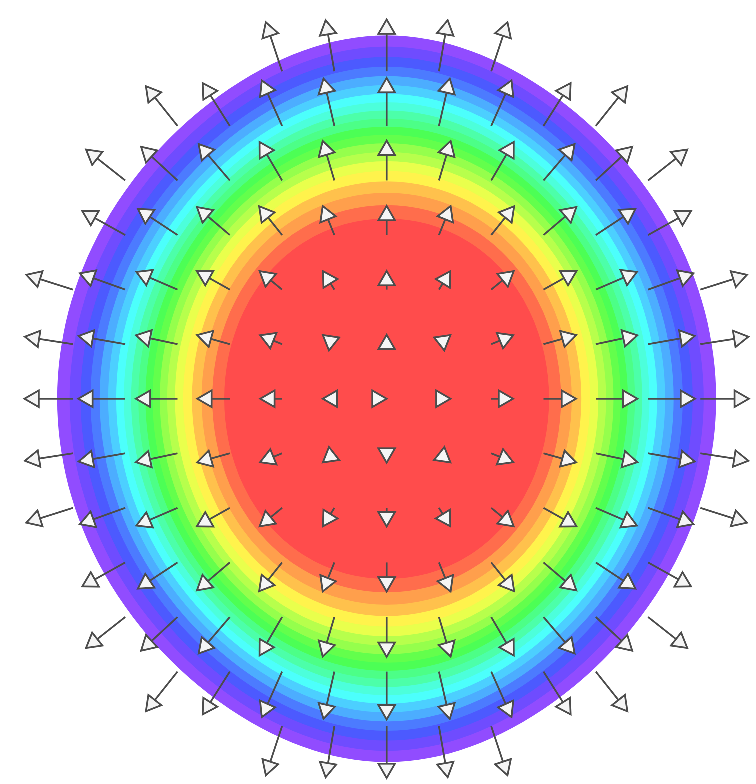
$$\lambda_{\mathrm{mfp}} \ll L$$

This is the limit of non-viscous hydrodynamics.

4-velocity of a fluid element:

$$u = \gamma(1, \vec{\beta}), \quad u^\mu u_\mu = 1$$

$$\gamma = \frac{1}{\sqrt{1 - \vec{\beta}^2}}$$



Number conservation

Mass conservation in nonrelativistic hydrodynamics:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0 \qquad \text{[continuity equation]}$$

Lorentz contraction in the relativistic case: $\rho \quad \rightarrow \quad n\gamma = nu^0$

conserved quantity, e.g. baryon number

The continuity equation then reads: $$\frac{\partial (nu^0)}{\partial t} + \vec{\nabla}(n\vec{u}) = 0$$

nu^0 : baryon density

$n\vec{u}$: baryon flux

The conservation of *n* can be written more elegantly as

$$\partial_\mu (nu^\mu) = 0$$

For a general 4-vector a we have:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial t}, \vec{\nabla}), \quad \partial^\mu \equiv \frac{\partial}{\partial x_\mu} = (\frac{\partial}{\partial t}, -\vec{\nabla}), \qquad \partial_\mu a^\mu = (\frac{\partial}{\partial t}, \vec{\nabla}) \cdot (a^0, \vec{a}) = \frac{\partial a^0}{\partial t} + \vec{\nabla}\vec{a}$$

covariant derivative

contravariant derivative

Energy and momentum conservation

Analogous to the contravariant 4-vector $J^\mu = nu^\mu$ one can define conserved currents for the energy and the three moments components. These can be written as contravariant tensor:

$T^{\mu\nu}$ — energy-momentum tensor

ν : component of the 4-momentum

μ : component of the associated current

$$T^{\mu\nu} = \begin{pmatrix} \text{energy density} & \text{momentum density} \\ \text{energy flux density} & \text{momentum flux density} \end{pmatrix}$$

T^{00} : the energy density

T^{0j} : density of the j-th component of the momentum, j = 1, 2, 3

T^{i0} : energy flux along axis i

T^{ij} : flux along axis i of the j-th component of the momentum

Examples: $T^{00} = \frac{\partial E}{\partial x \partial y \partial z} \equiv \varepsilon, \quad T^{11} = \frac{\partial p_x}{\partial t \partial y \partial z}$ — force in x direction acting on an surface $\Delta y\, \Delta z$ perpendicular to the force → pressure

Equations of non-viscous hydrodynamics

Energy-momentum tensor in the fluid rest frame:

$$T_{\mathrm{R}}^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

pressure

rest frame: pressure is the same in all direction, constant energy density and momentum

For moving fluid cell (Lorentz transformation):
(without derivation)

$$T^{\mu\nu} = (\varepsilon + P) u^{\mu} u^{\nu} - P g^{\mu\nu}$$

$g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$

Energy, momentum and baryon number conservation then be written as

$$\partial_{\mu} T^{\mu\nu} = 0 \qquad \partial_{\mu}(n u^{\mu}) = 0$$

5 equations for 6 unknowns:
$(u_x, u_y, u_z, \varepsilon, P, n_{\mathrm{B}})$

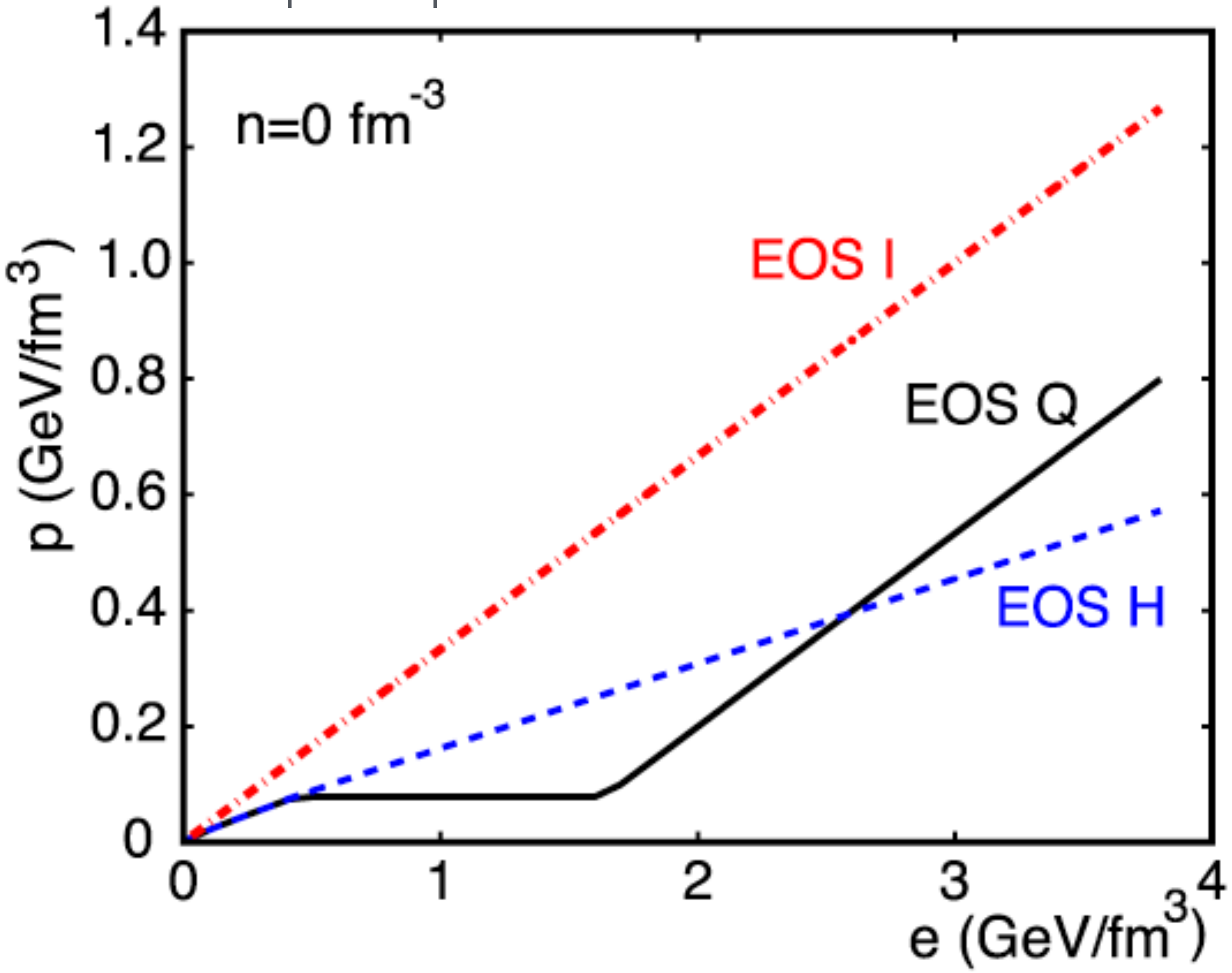
Ingredients of hydrodynamic models

- Equation of state (EoS) needed to close the system:

  $$P(\varepsilon, n_B)$$

- Via the EoS hydrodynamics allows one to relate observables with QCD thermodynamics
- Initial conditions (ε(*x*, *y*, *z*))
  - Glauber MC
  - Color glass condensate
- Transition to free-streaming particles
  - E.g. at given local temperature

Simple equations of state:



EOS I: ultra-relativistic gas $P = \varepsilon/3$

EOS H: resonance gas, $P \approx 0.15\,\varepsilon$

EOS Q: phase transition,
QGP ↔ resonance gas

Cooper-Frye freeze-out formula

Particle spectra from fluid motion:

Cooper, Frye, Phys. Rev. D10 (1974) 186

local thermal distribution

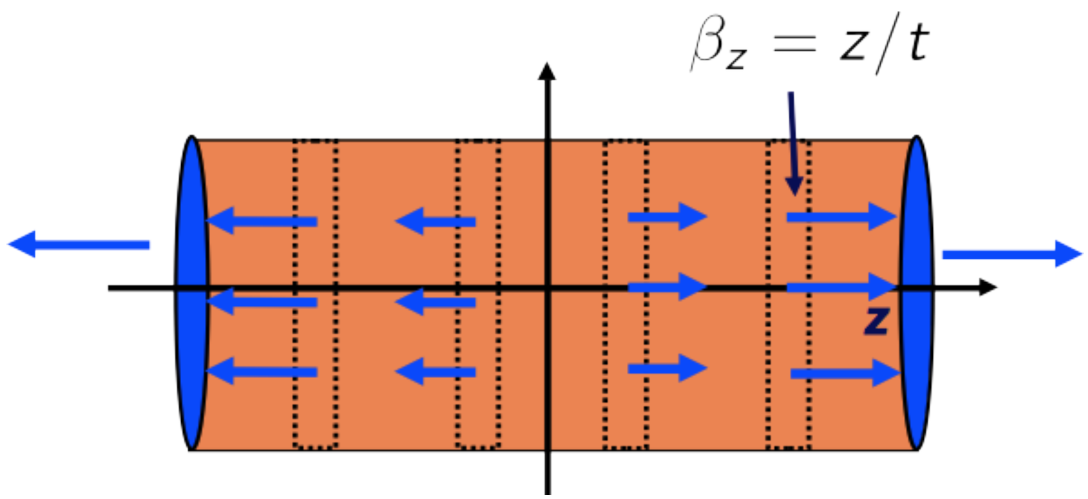
normal vector to the 3d freeze-out hyper surface Σ in space-time defined e.g. by $T = T_{\mathrm{fo}}$

$$E \frac{\mathrm{d}N}{\mathrm{d}^3 p} = \frac{1}{2\pi p_T} \frac{\mathrm{d}^3 N}{\mathrm{d}p_T \,\mathrm{d}y \,\mathrm{d}\varphi} = \int_{\Sigma_f} f(x,p) p^\mu \,\mathrm{d}\Sigma_\mu$$

$$= \frac{g}{(2\pi)^3} \int_{\Sigma_f} \frac{p^\mu \,\mathrm{d}\Sigma_\mu}{\exp\left(\frac{p_\mu \cdot u^\mu(x) - \mu(x)}{T(x)}\right) \pm 1}$$

In rest frame of the fluid cell: $u^\mu = (1,0,0,0) \rightsquigarrow p_\mu \cdot u^\mu = E$

Longitudinal expansion: Bjorken's scaling solution (I)



proper time:

$$\tau = t/\gamma = t\sqrt{1-\beta_z^2} = \sqrt{t^2 - z^2}$$

The Bjorken model is a 1d hydrodynamic model (expansion only in *z* direction). The initial conditions correspond to the one which one would get from free streaming particles starting at (*t*, *z*) = (0, 0).

Initial conditions in the Bjorken model:

preserved during the hydro evolution, i.e., $u^\mu(\tau) = \dfrac{x^\mu}{\tau}$

$$\varepsilon(\tau_0) = \varepsilon_0, \quad u^\mu = \frac{1}{\tau_0}(t, 0, 0, z) = \frac{x^\mu}{\tau_0}$$

initial energy density

In this case the equations of ideal hydrodynamics simplify to

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} + \frac{\varepsilon + p}{\tau} = 0$$

Longitudinal expansion: Bjorken's scaling solution (II)

For an ideal gas of quarks and gluons, i.e., for

$$\varepsilon = 3p, \quad \varepsilon \propto T^4$$

this gives

$$\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}, \quad T(\tau) = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}$$

The temperature drops to the critical temperature at the proper time

$$\tau_c = \tau_0 \left(\frac{T_0}{T_c}\right)^3$$

The QGP lifetime is therefore given by

$$\Delta\tau_{\mathrm{QGP}} = \tau_c - \tau_0 = \tau_0 \left[\left(\frac{T_0}{T_c}\right)^3 - 1\right]$$

Mixed phase in the Bjorken model

Entropy conservation in ideal hydrodynamics leads in the case of the Bjorken model (independent of the equation of state) to

$$s(\tau) = \frac{s_0 \tau_0}{\tau}$$

In case of an the ideal QGP:

$$s = \frac{\varepsilon + p}{T} = \frac{4}{3}\frac{\varepsilon}{T} = \frac{4}{3}\frac{\varepsilon_0}{T_0}\frac{\tau_0}{\tau}$$

If we consider a QGP/hadron gas phase transition we have a first oder phase transition and a mixed phase with temperature T_c. The entropy in the mixed phase is given by

ξ(τ): fraction of fireball in hadron gas phase

$$s(\tau) = s_{HG}(T_c)\xi(\tau) + s_{QGP}(T_c)(1 - \xi(\tau)) = \frac{s_{QGP}(T_c)\tau_c}{\tau}$$

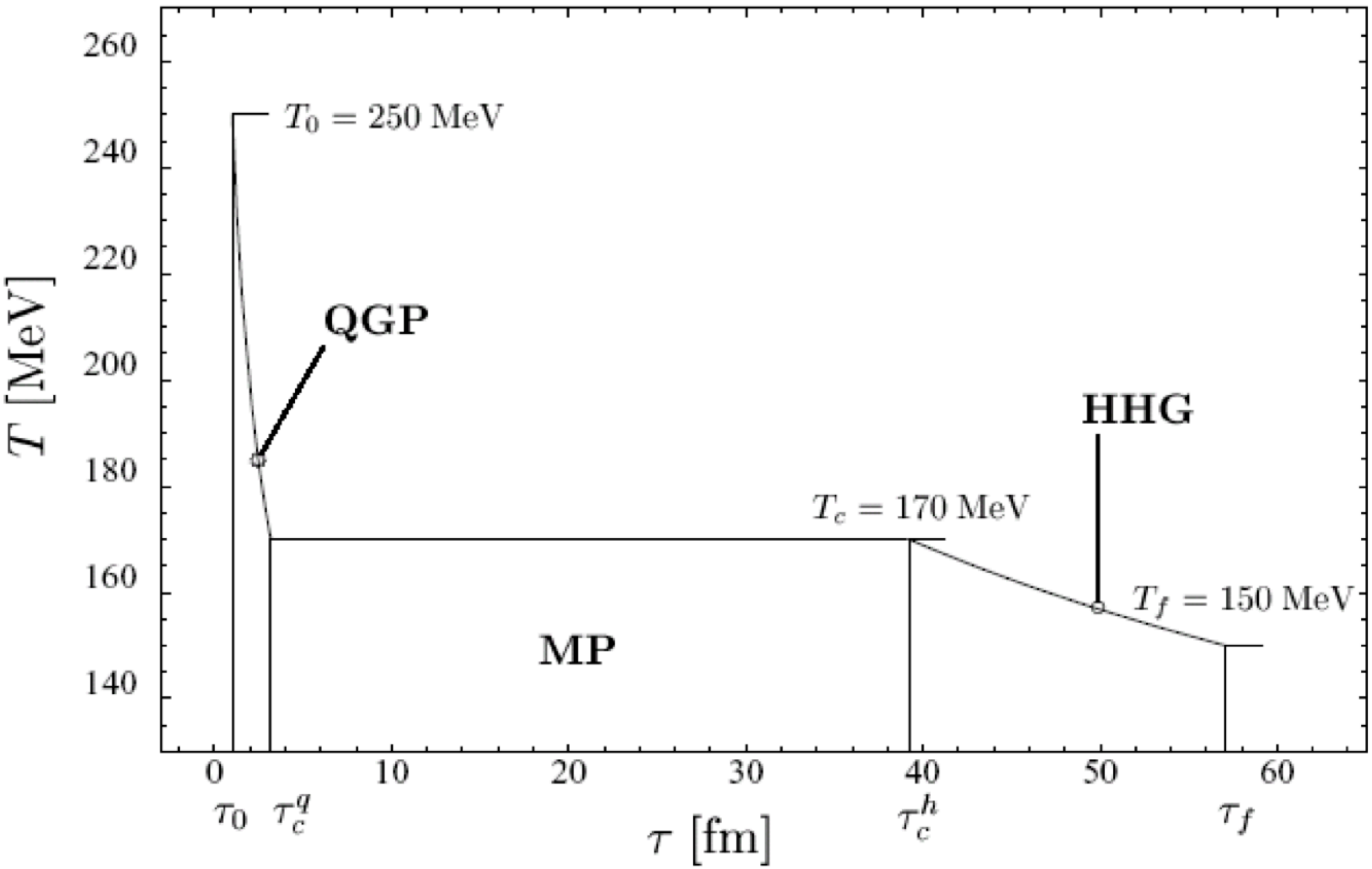
This equation determines the time dependence of $\xi(\tau)$ and the time τ_h at which the mixed phase vanishes:

end of mixed phase

$$\xi(\tau) = \frac{1 - \tau_c/\tau}{1 - g_{HG}/g_{QGP}} \quad \rightsquigarrow \quad \tau_h = \tau_c \frac{g_{QGP}}{g_{HG}}$$

the hadron gas close to T_c can be described with $g_{HG} \approx 12$

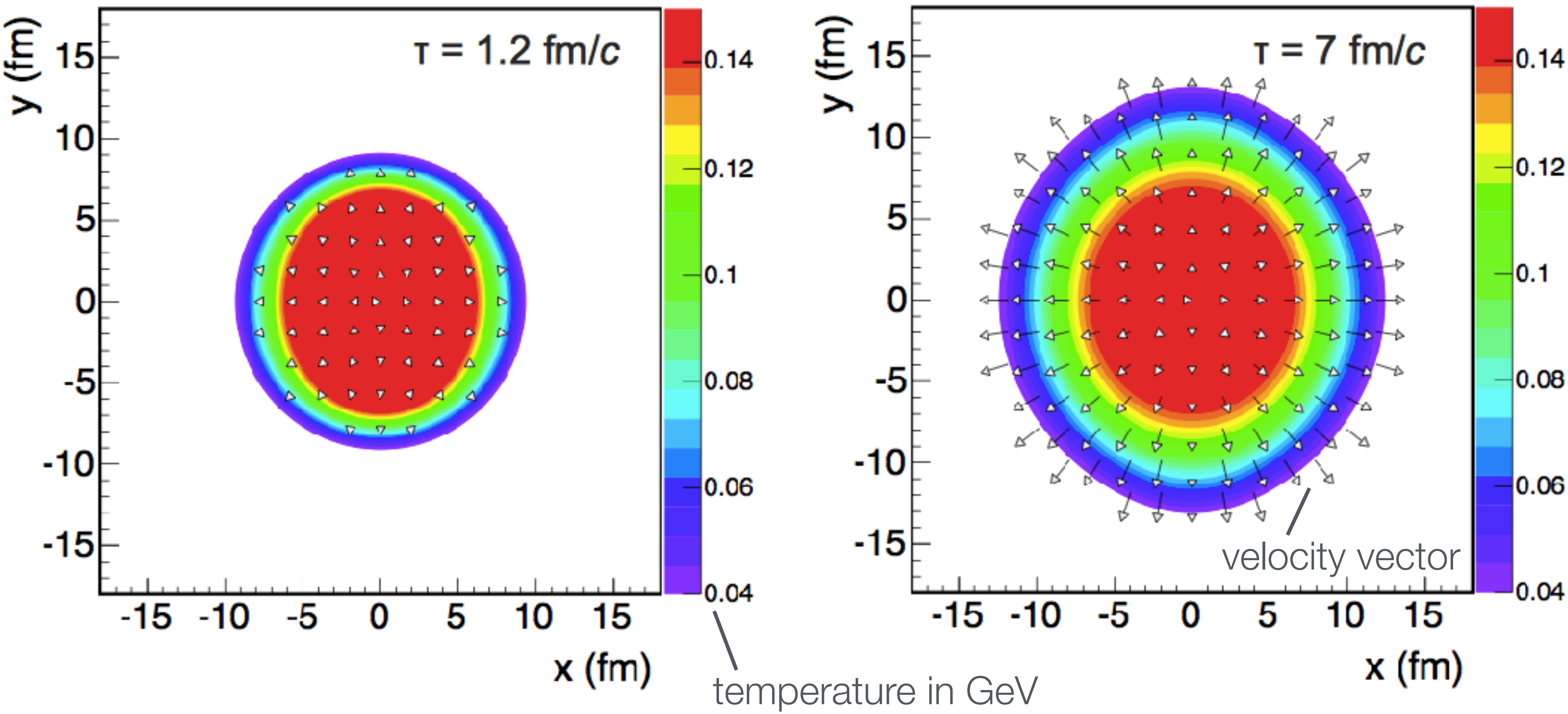
Temperature evolution in the Bjorken model



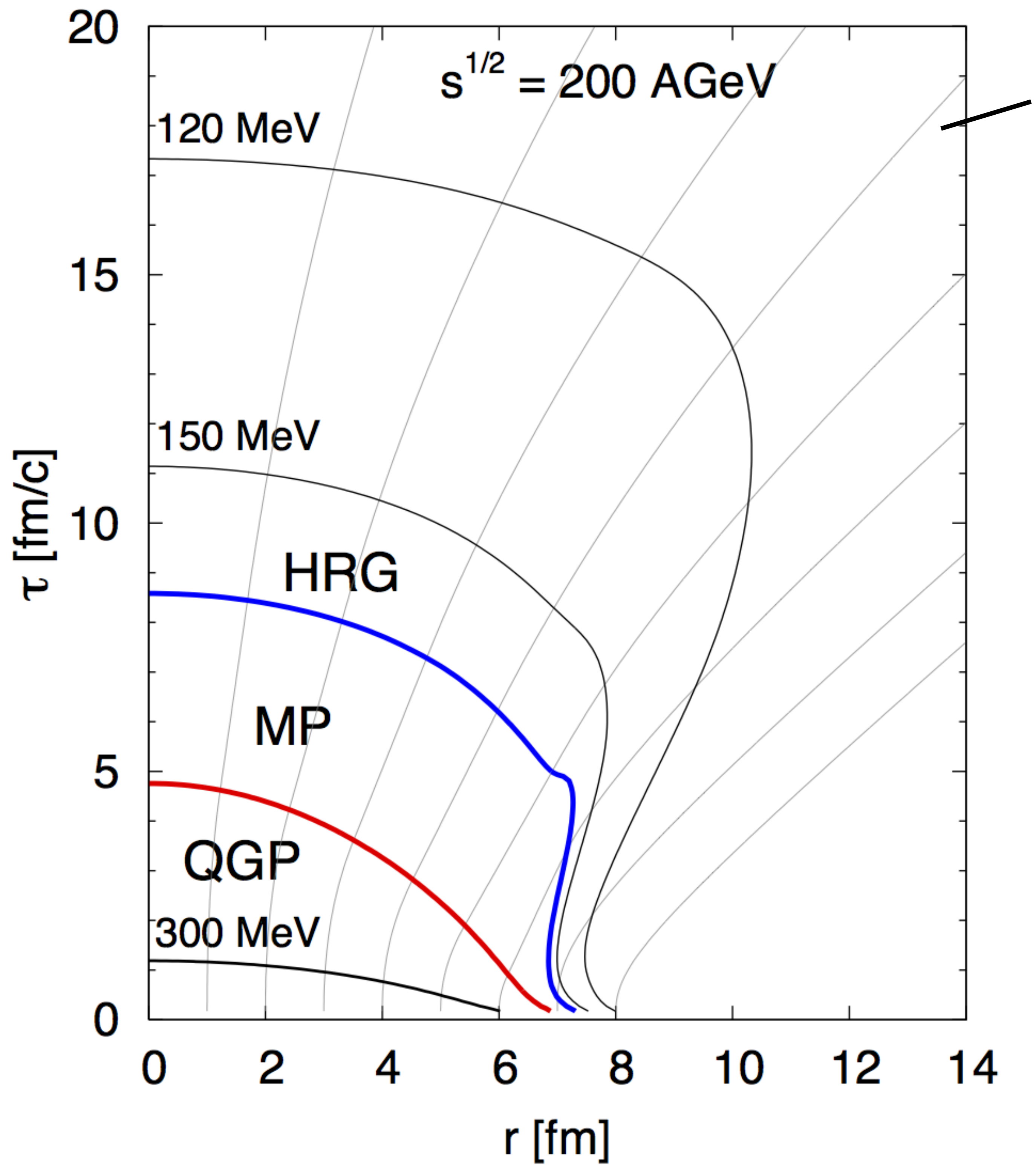
Transverse expansion

Transverse expansion of the fireball in a hydro model (temperature profile)

2+1 d hydro: Bjorken flow in longitudinal direction



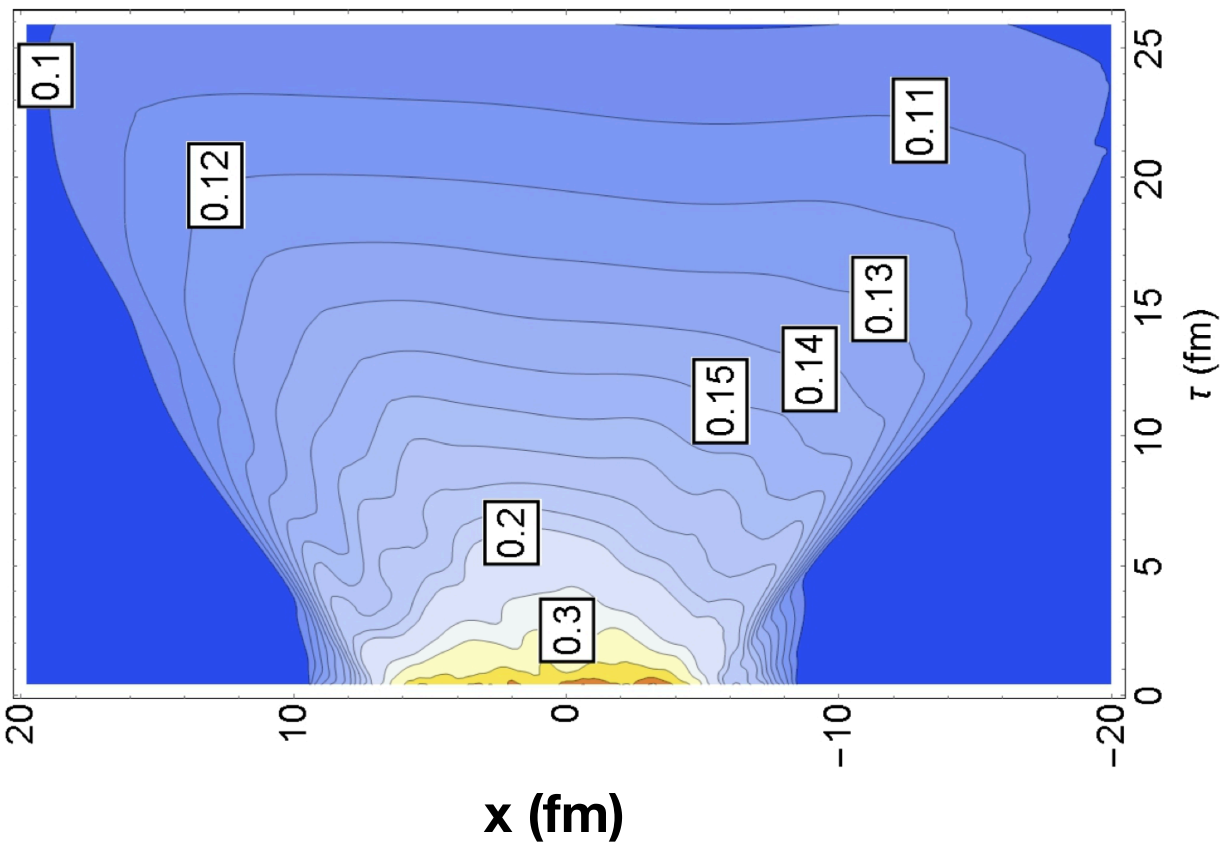
Temperature Contours and Flow lines



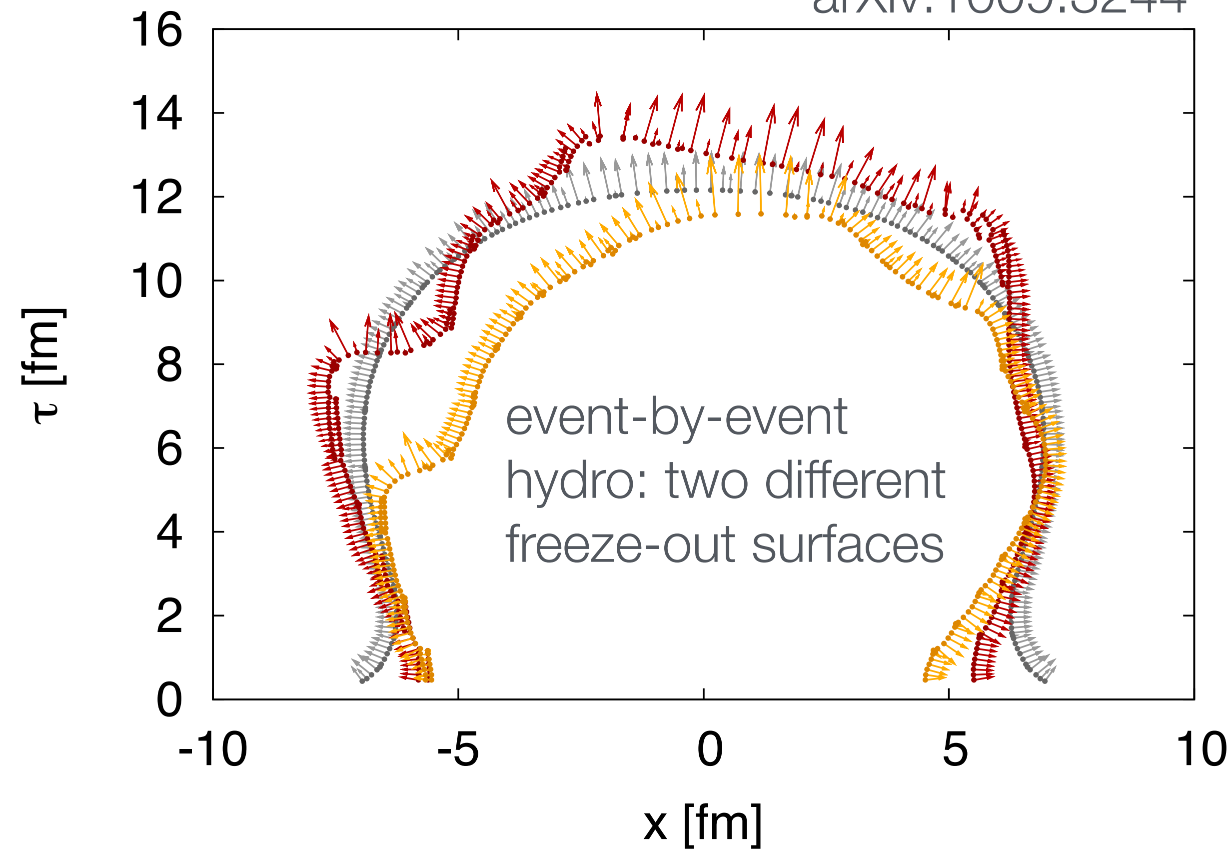
flow lines indicate motion of fluid cells

Hydrodynamic modeling of heavy-ion collisions: State of the art

- Equation of state from lattice QCD
- (2+1)D or (3+1)D viscous hydrodynamics
- Fluctuating initial conditions (event-by-event hydro)
- Hydrodynamic evolution followed by hadronic cascade

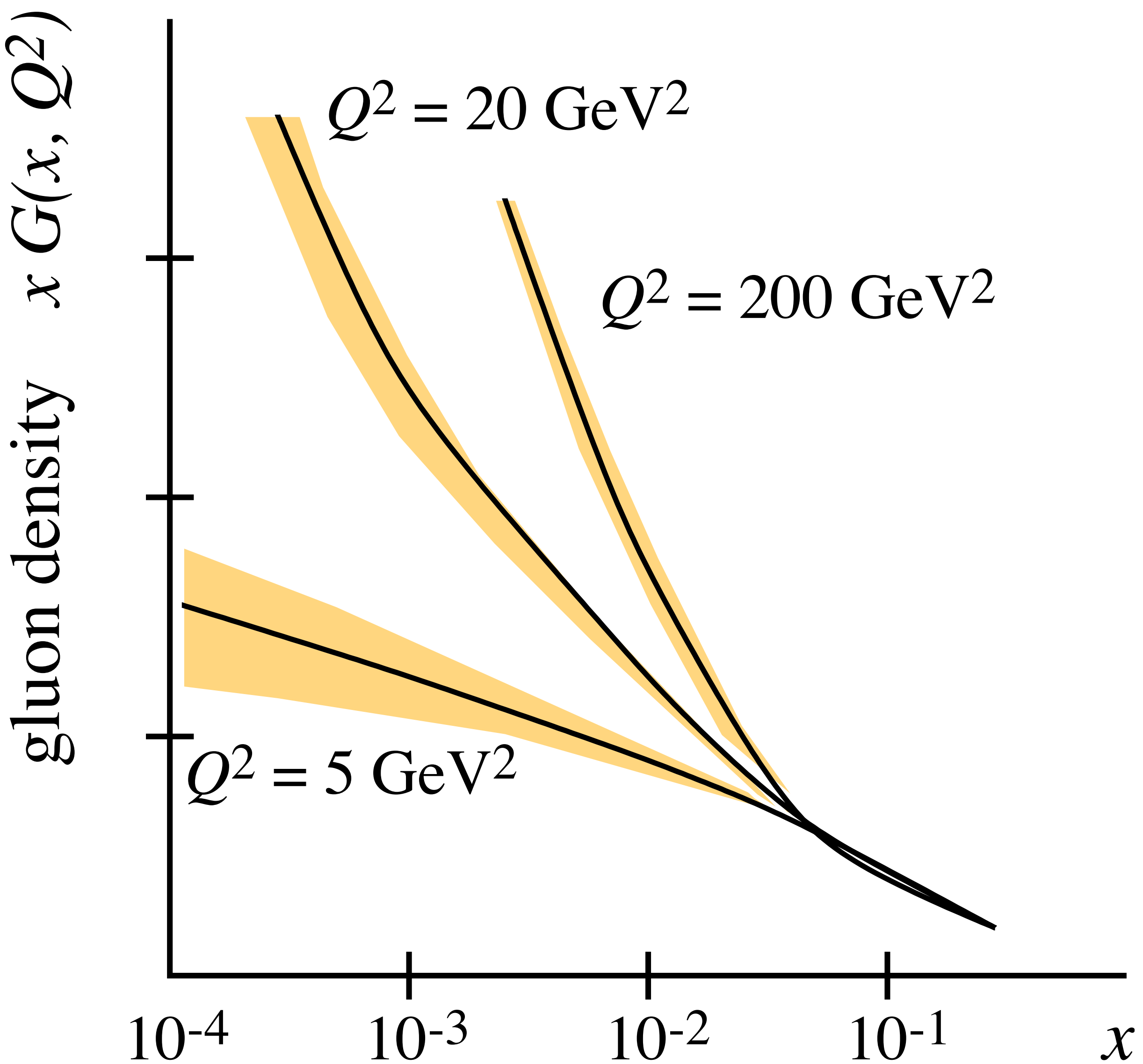


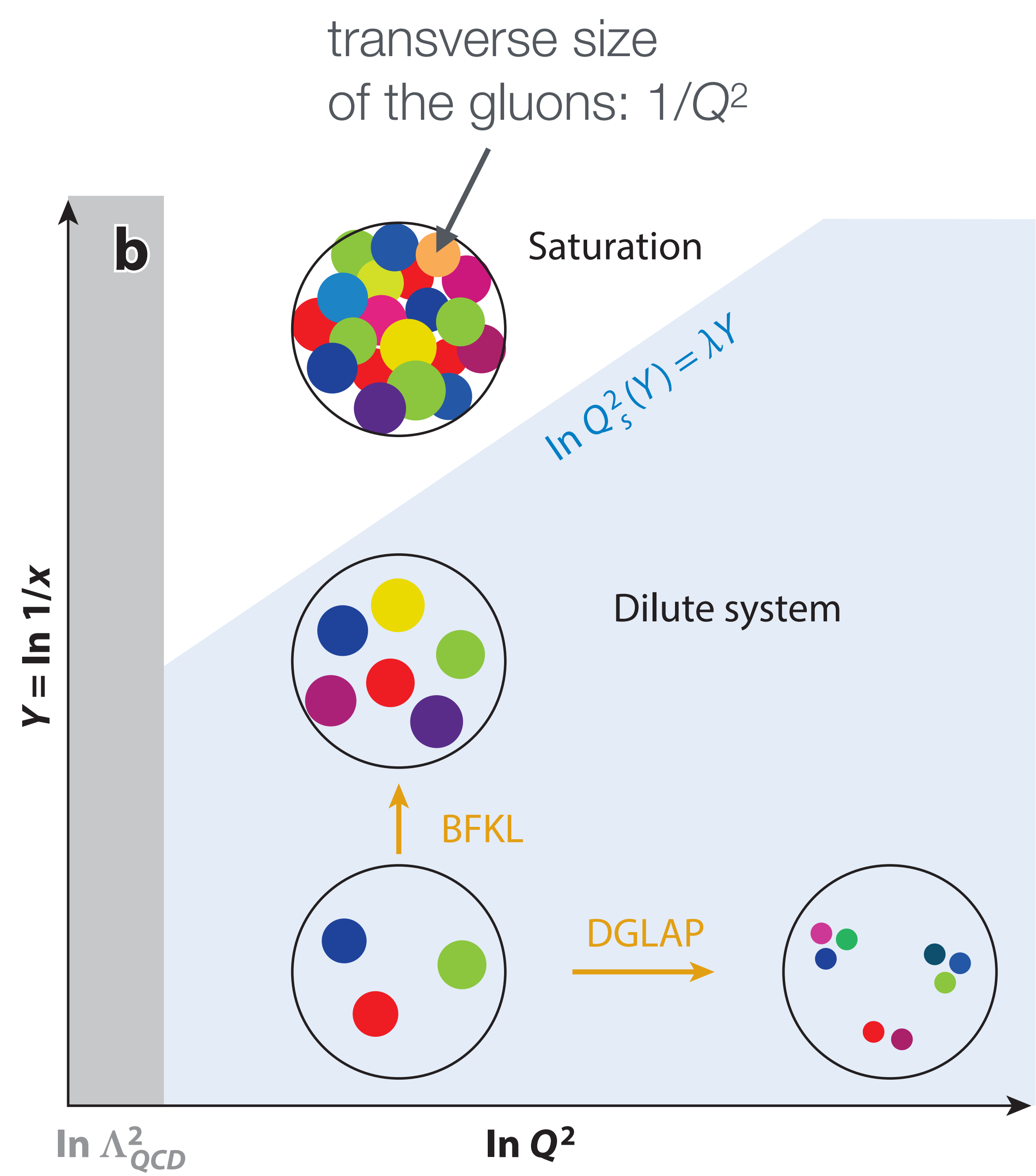
arXiv:1009.3244



Initial conditions from gluon saturation models (I)

Annu. Rev. Nucl. Part. Sci. 2010.60:463



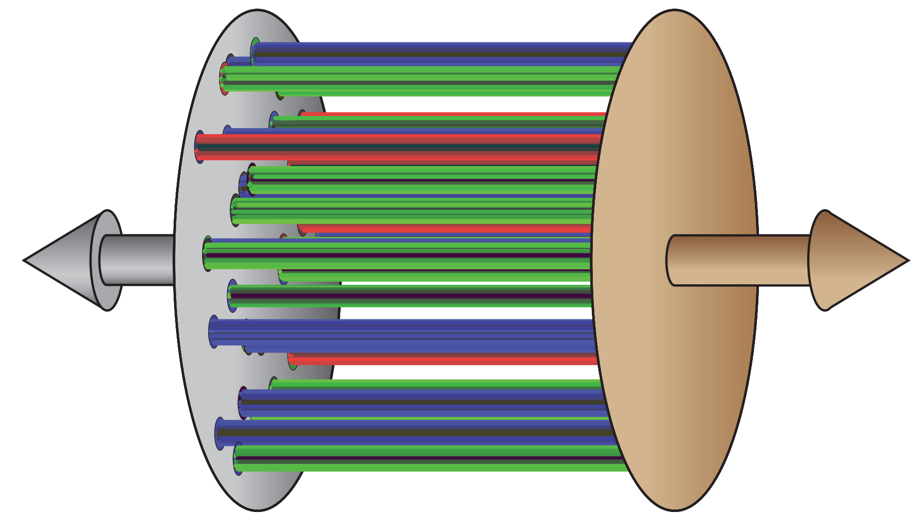


Growth of gluons saturates at an occupation number $1/\alpha_s$. This defines a (semihard) scale $Q_s(x)$, i.e., a typical gluon transverse momentum.

$$\frac{1}{2(N_c^2 - 1)} \frac{xG(x, Q_s^2)}{\pi R^2 Q_s^2} = \frac{1}{\alpha_s(Q_s^2)}$$

Initial conditions from gluon saturation models (II)

- Color glass condensate:
  Effective field theory, which describes universal properties of saturated gluons in hadron wave functions
- CGC dynamics defines field configurations at early times
  - Strong longitudinal chromoelectric and chromomagnetic fields screened on transverse distance scales $1/Q_s$.



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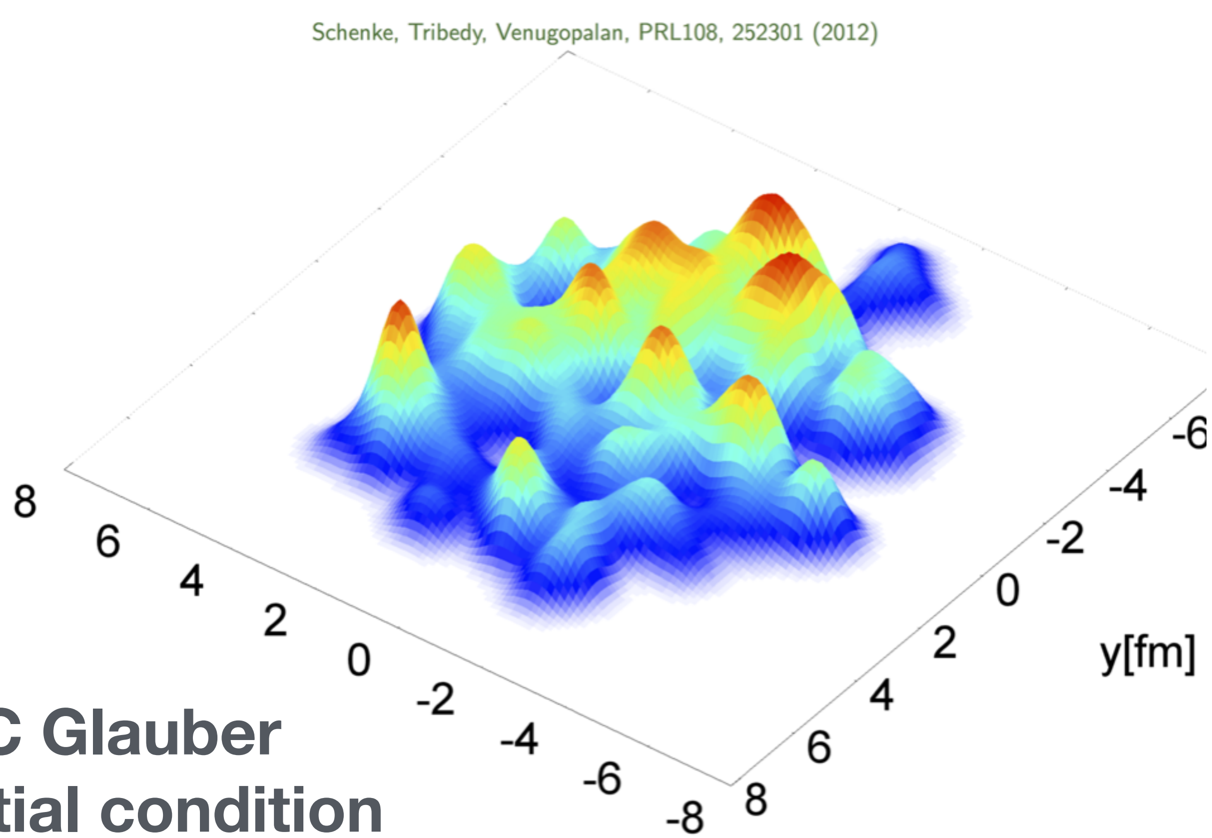
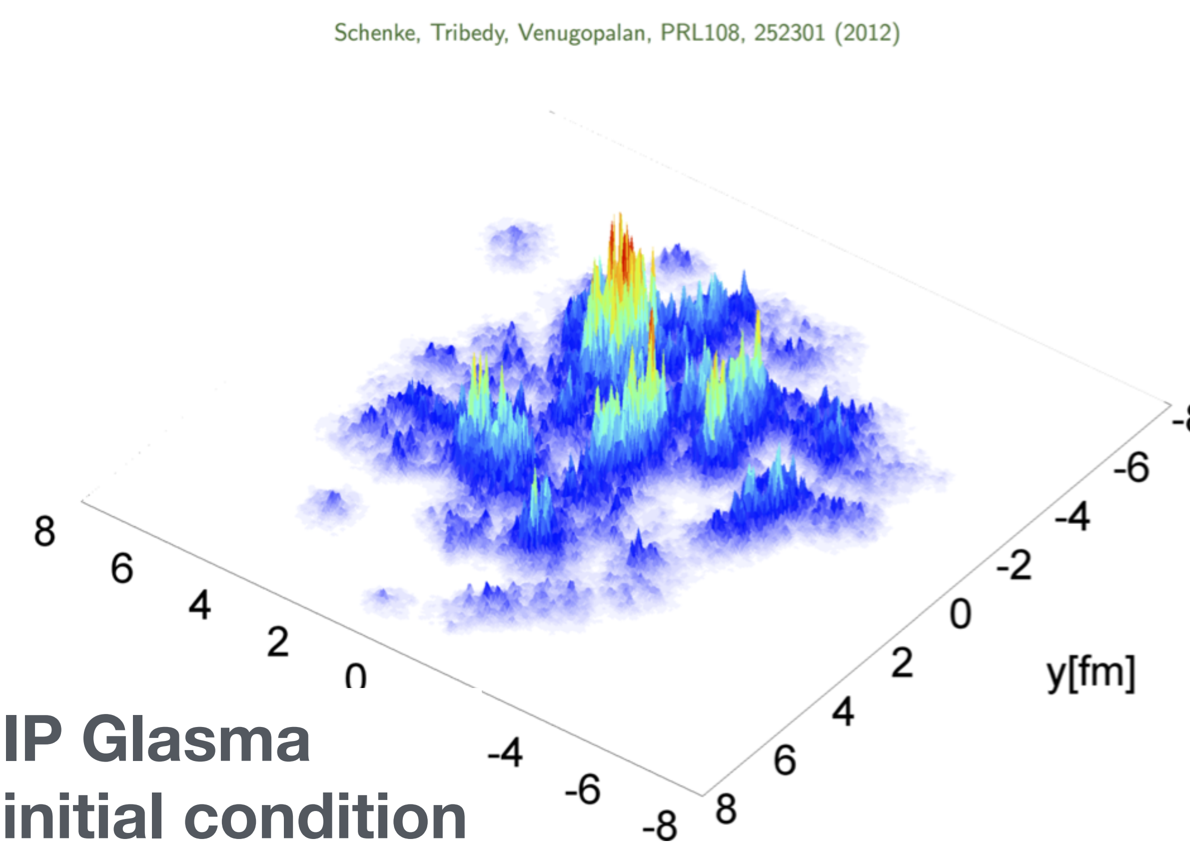


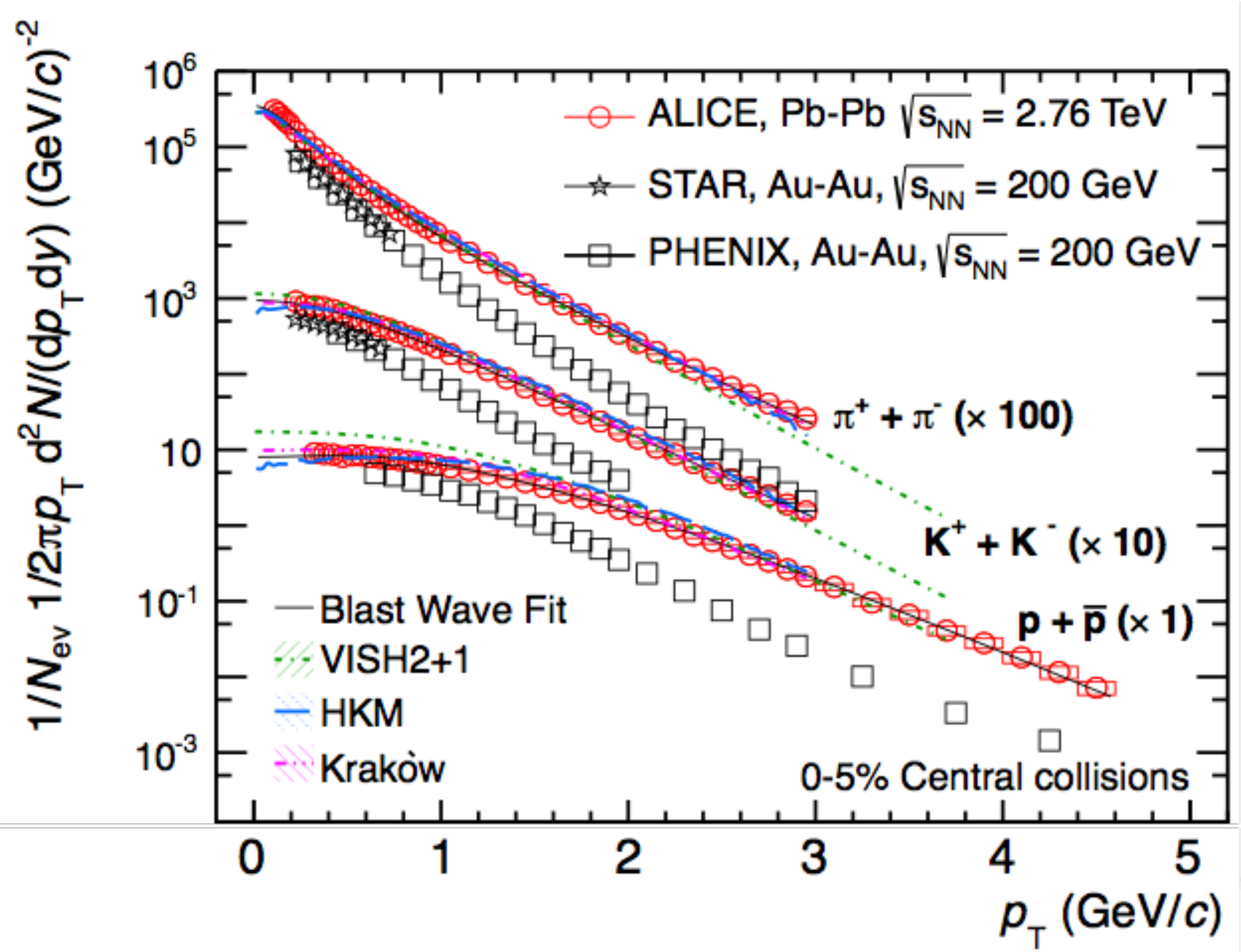
MC Glauber initial condition



IP Glasma initial condition

Spectra and Radial flow

Comparison of π, K, p spectra with hydro models



The blast-wave model: A Simple model to describe the effect of radial flow on particle spectra

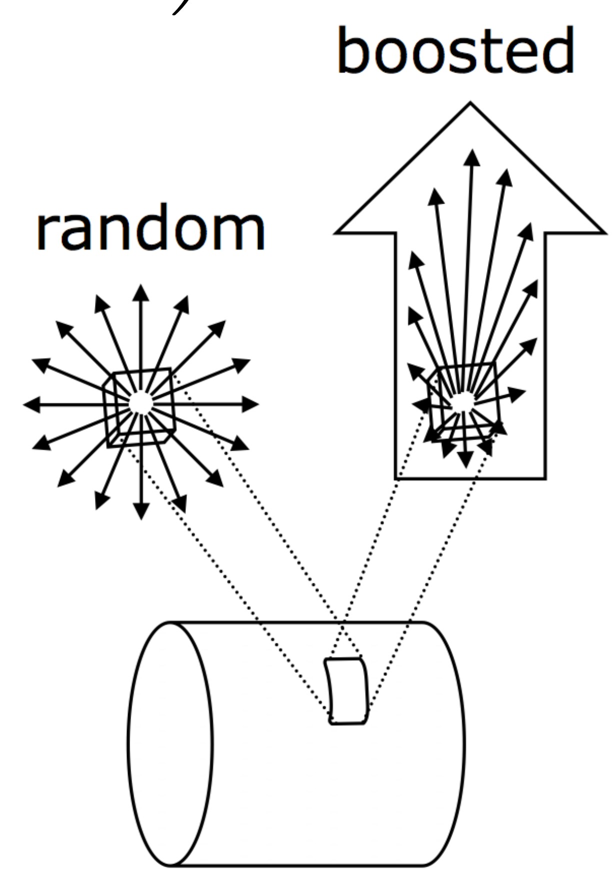
Transverse velocity profile: $$\beta_T(r) = \beta_s \left(\frac{r}{R}\right)^n$$

Superposition of thermal sources with different radial velocities:

$$\frac{1}{m_T}\frac{dn}{dm_T} \propto \int_0^R r\,dr\,m_T I_0\left(\frac{p_T \sinh\rho}{T}\right) K_1\left(\frac{m_T \cosh\rho}{T}\right)$$

$$\rho := \mathrm{arctanh}(\beta_T) \quad \text{"transverse rapidity"}$$

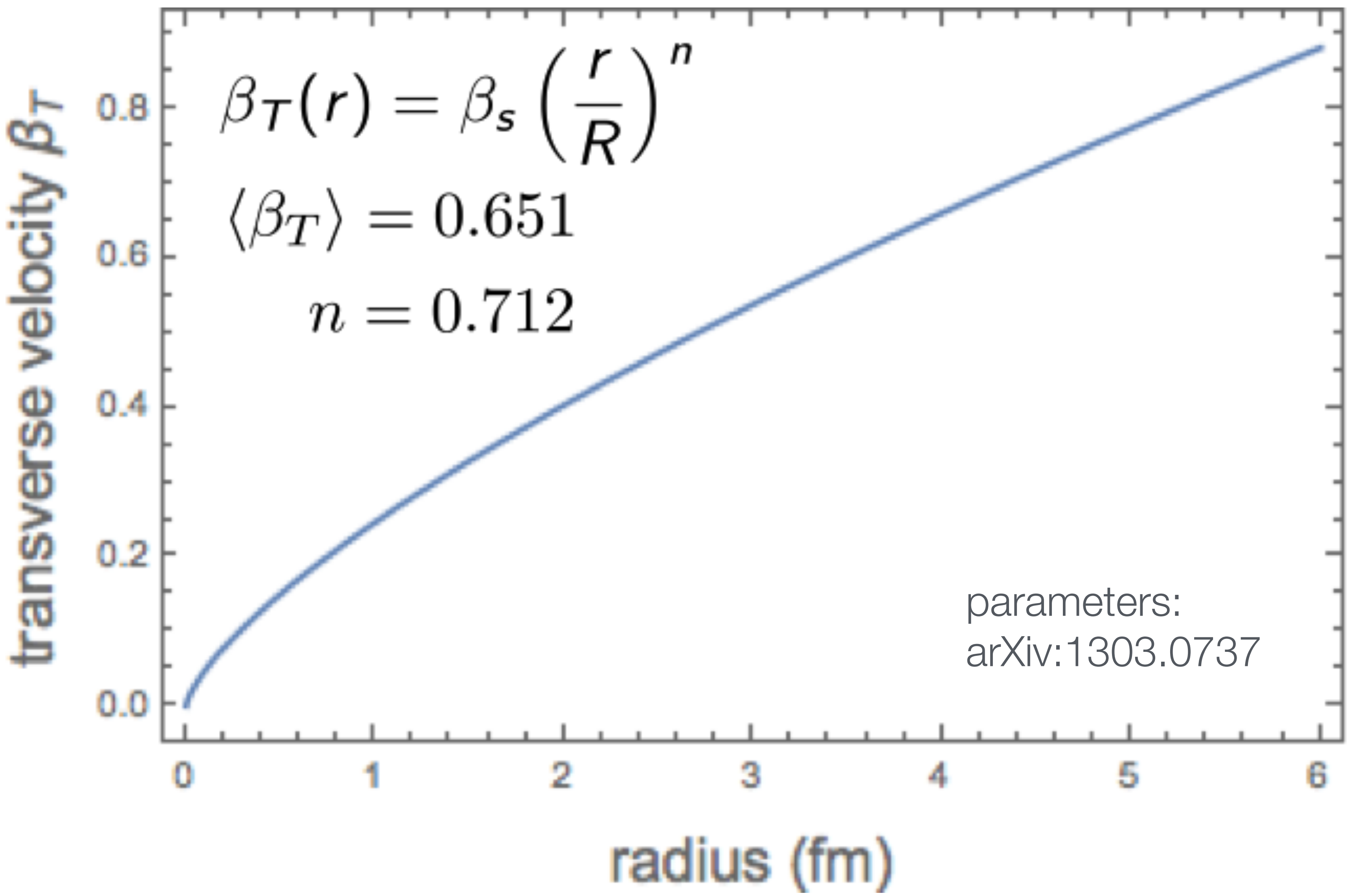
$$I_0,\ K_1 : \ \text{modified Bessel functions}$$



Schnedermann, Sollfrank, Heinz, Phys.Rev.C48:2462-2475,1993

Freeze-out at a 3d hyper-surface, typically instantaneous, e.g.: $$t_f(r,z) = \sqrt{\tau_f^2 + z^2}$$

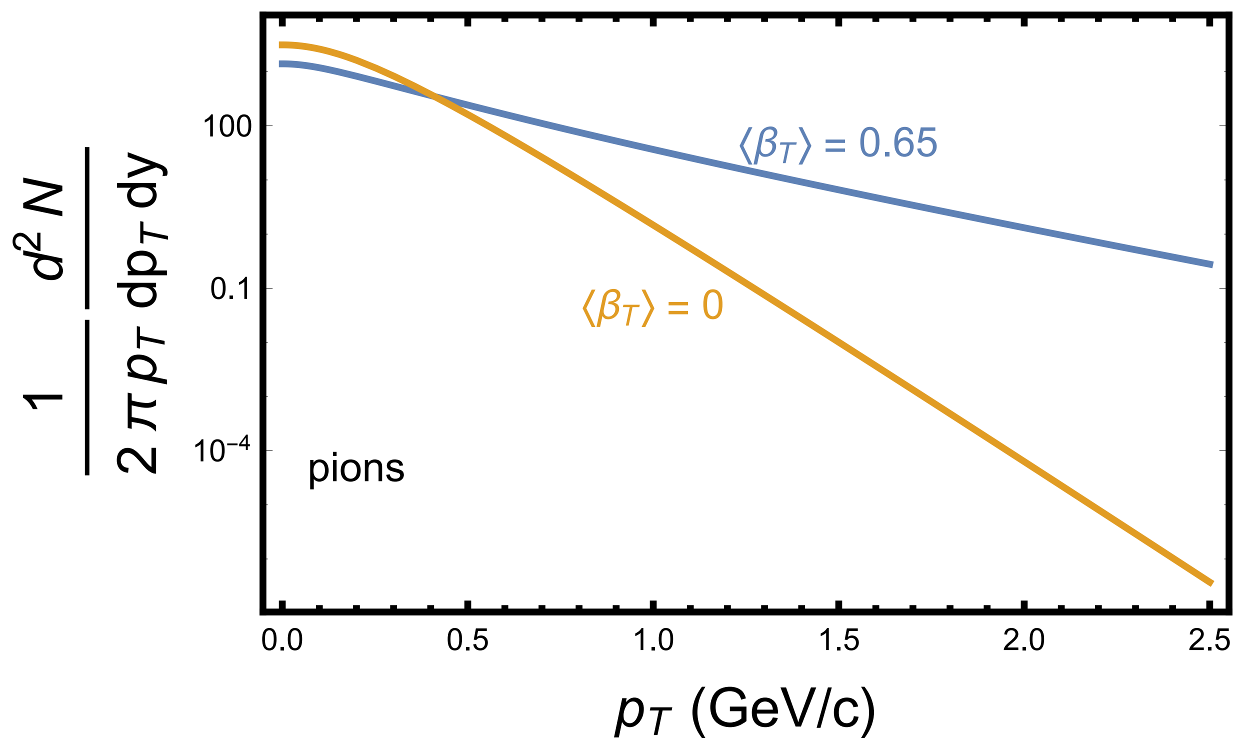
Example: Radial Flow Velocity Profile from Blast-wave Fit to 2.76 TeV Pb-Pb Spectra (0-5%)



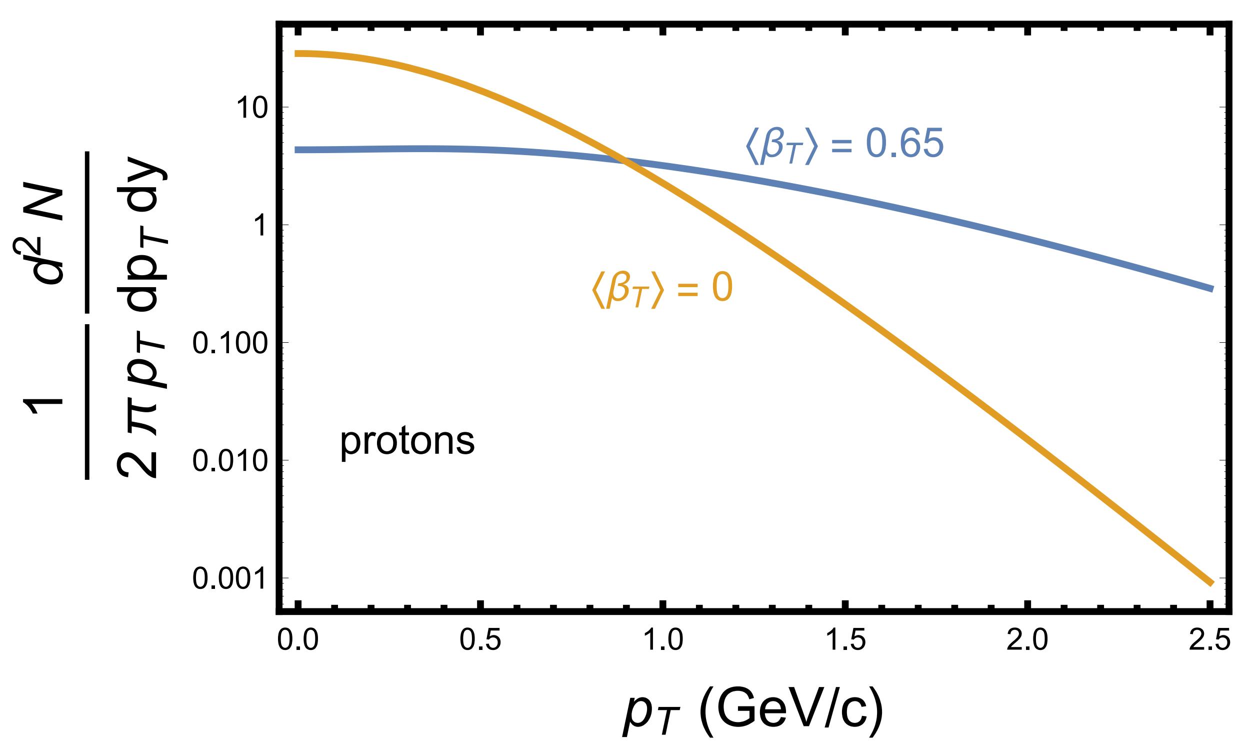
$$\langle\beta_T\rangle = \frac{\int_0^R \int_0^{2\pi} r dr d\varphi \beta_T(r)}{\int_0^R \int_0^{2\pi} r dr d\varphi} = \frac{2}{n+2}\beta_s$$

$$\langle\beta_T\rangle = 0.651, n = 0.712$$
$$\rightarrow \quad \beta_s = 0.8$$

Example: Pion and Proton p_T Spectra from blast-wave model



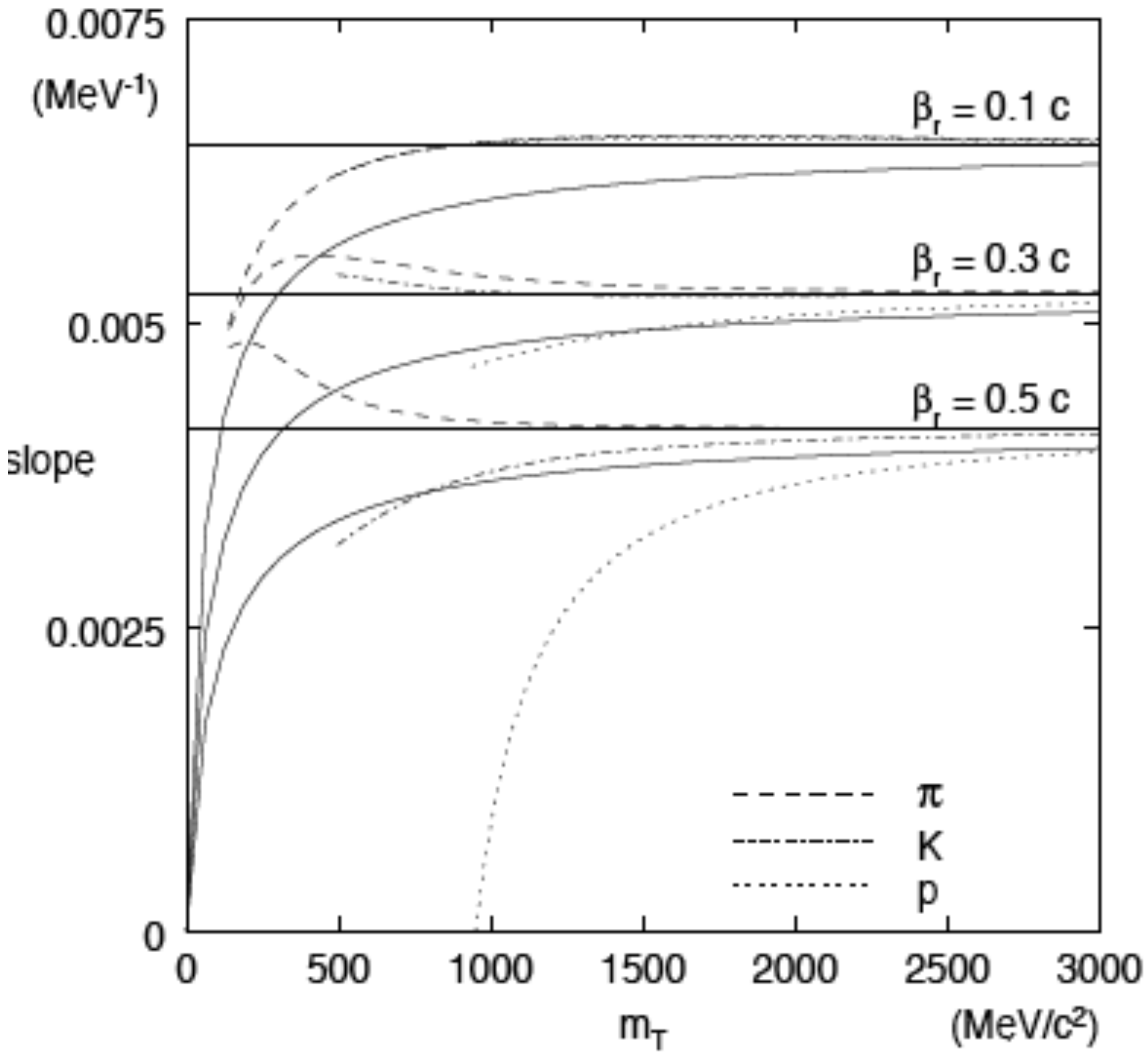
Parameters for 0-5% most central Pb-Pb collisions at 2.76 TeV, arXiv:1303.0737



Larger p_T kick for particles with higher mass:

$$p = \beta_{\text{source}} \gamma_{\text{source}} m + \text{"thermal"}$$

Local slope of m_T spectra with radial flow



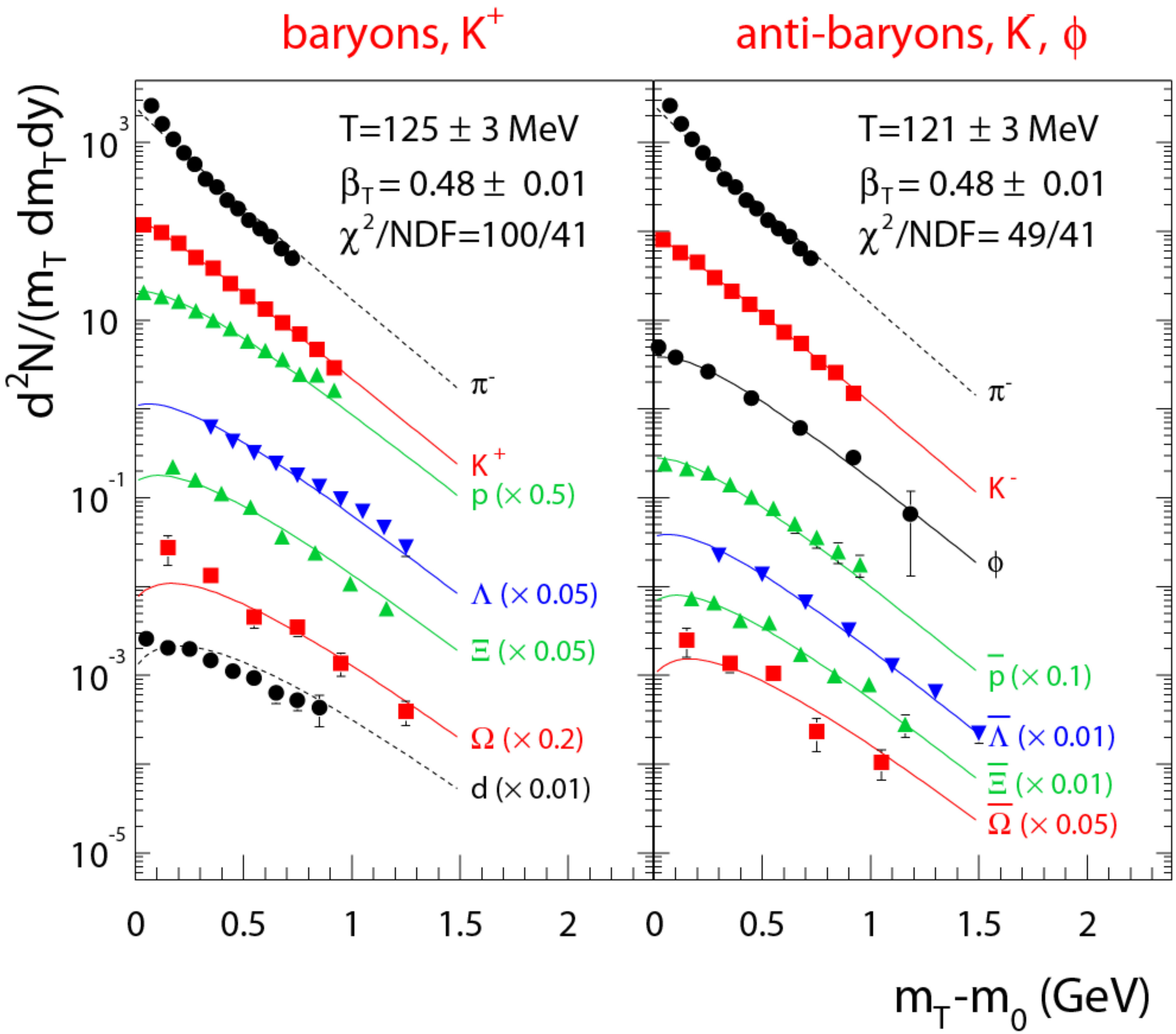
m_T slopes with transverse flow for pions for fixed transverse expansion velocity β_r

$$\lim_{m_t \to \infty} \frac{d}{dm_T} \ln \left(\frac{1}{m_T} \frac{dn}{dm_T} \right) = -\frac{1}{T} \sqrt{\frac{1-\beta_r}{1+\beta_r}}$$

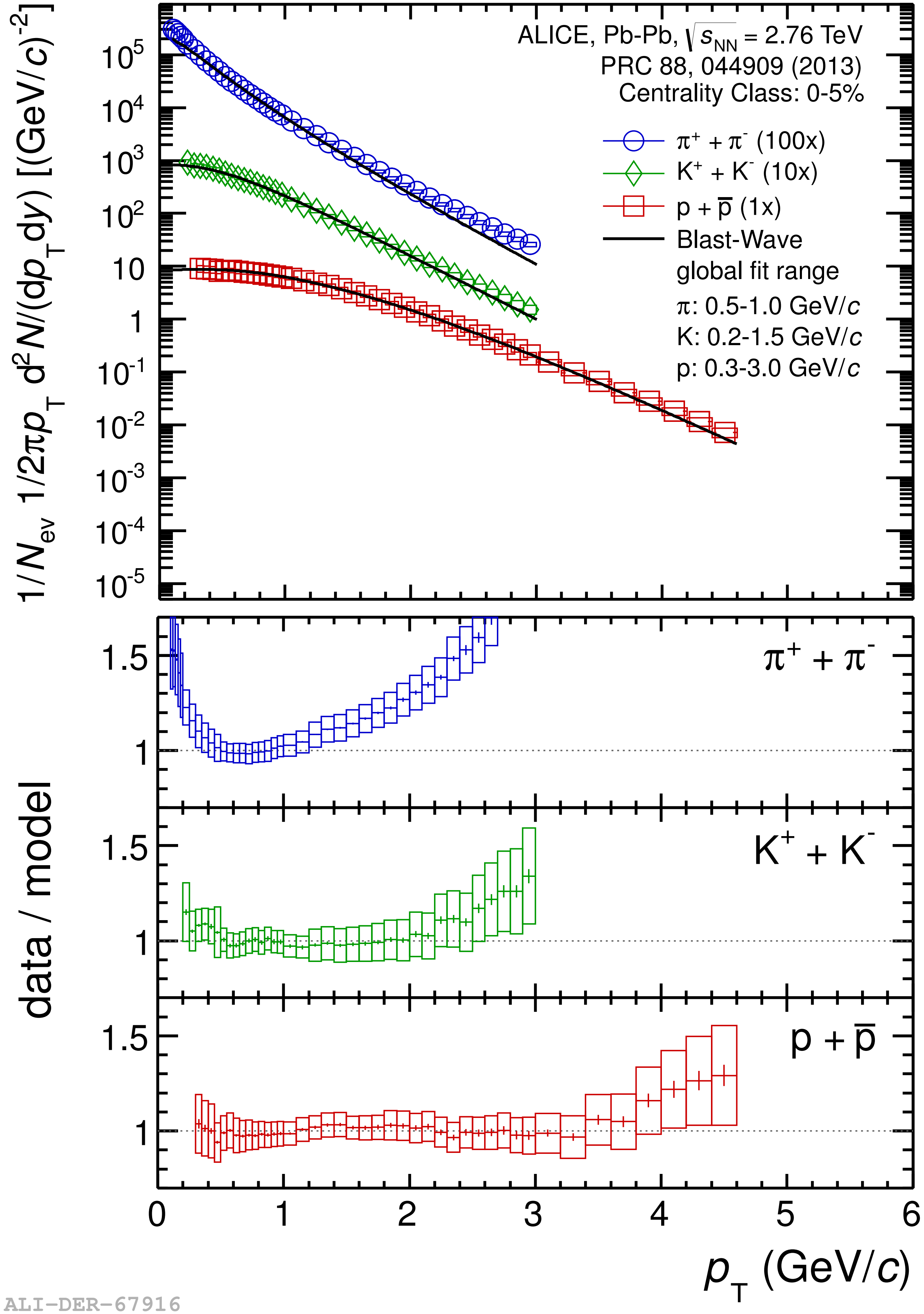
The apparent temperature, i.e., the inverse slope at high m_T, is larger than the original temperature by a blue shift factor:

$$T_{\text{eff}} = T \sqrt{\frac{1+\beta_r}{1-\beta_r}}$$

Blast-wave fit for CERN SPS data (NA49)



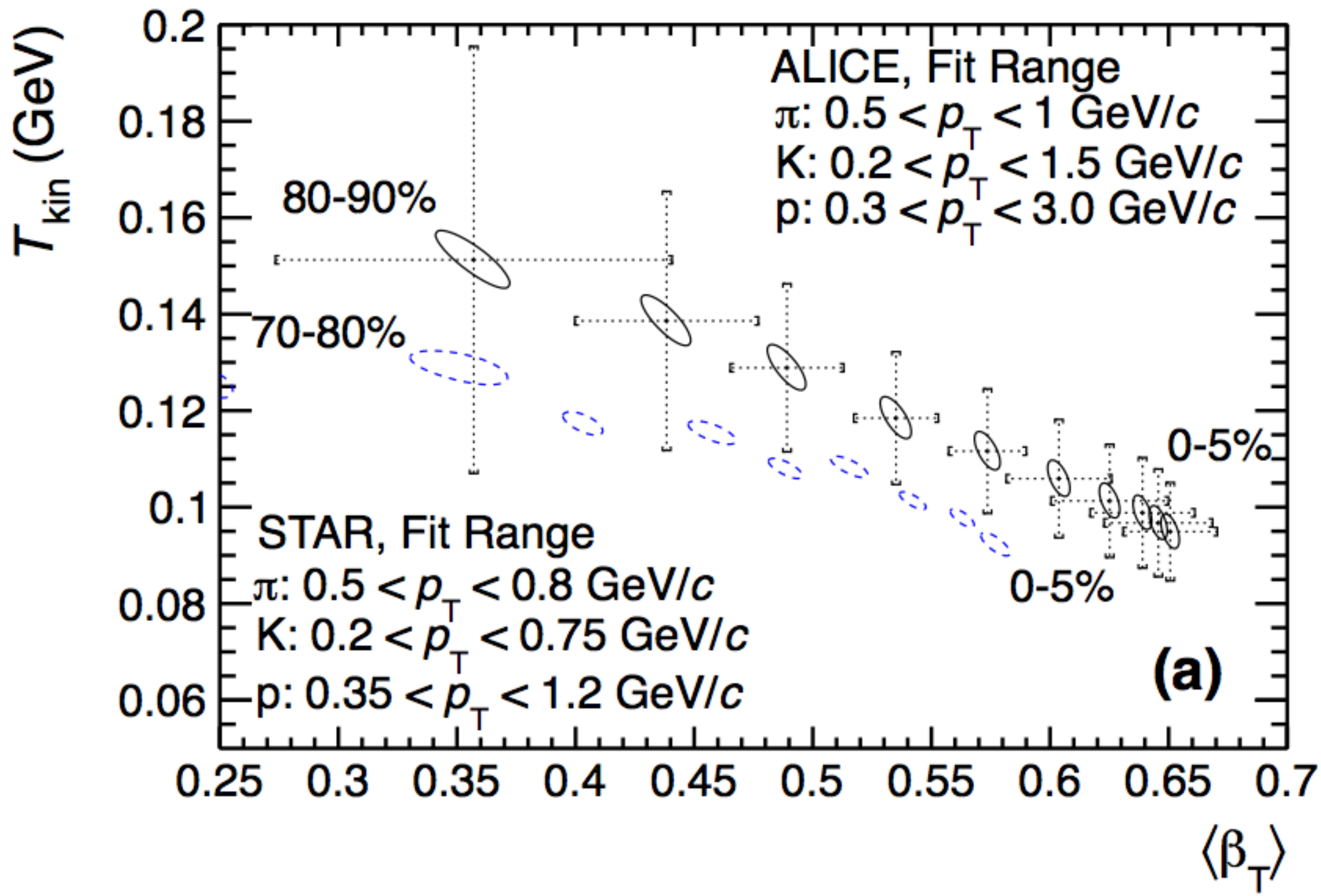
Blast-wave fit LHC



Works well for K and p

For pions, the contribution from resonance decays at low p_T and hard scattering at high p_T probably explains the discrepancy

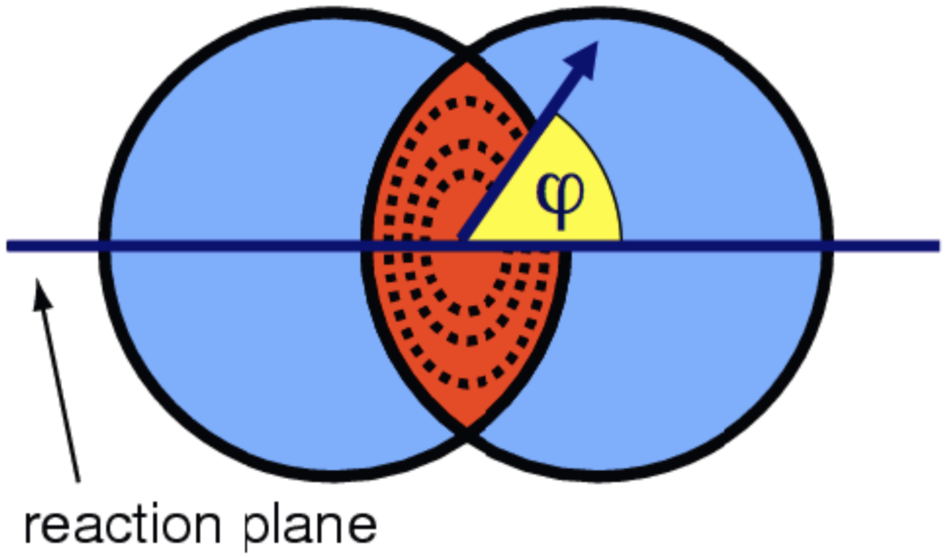
T und ⟨β⟩ for different centralities at RHIC and the LHC

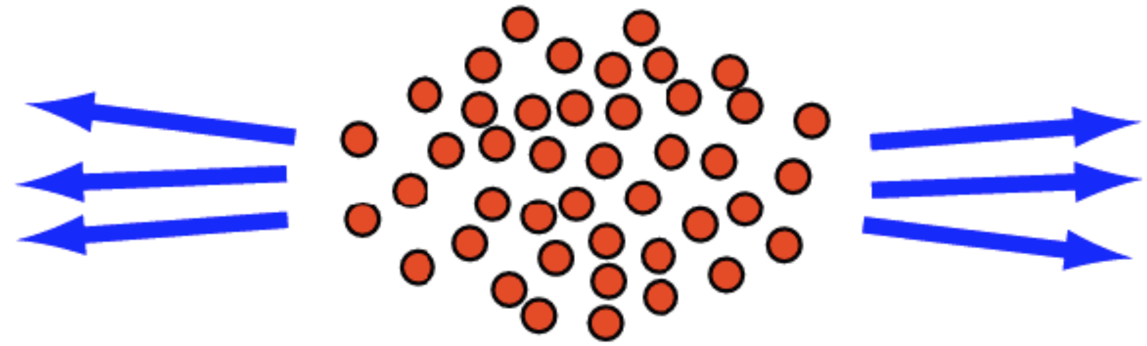


10% larger flow velocities in central collisions at the LHC than at RHIC

Elliptic flow and higher flow harmonics

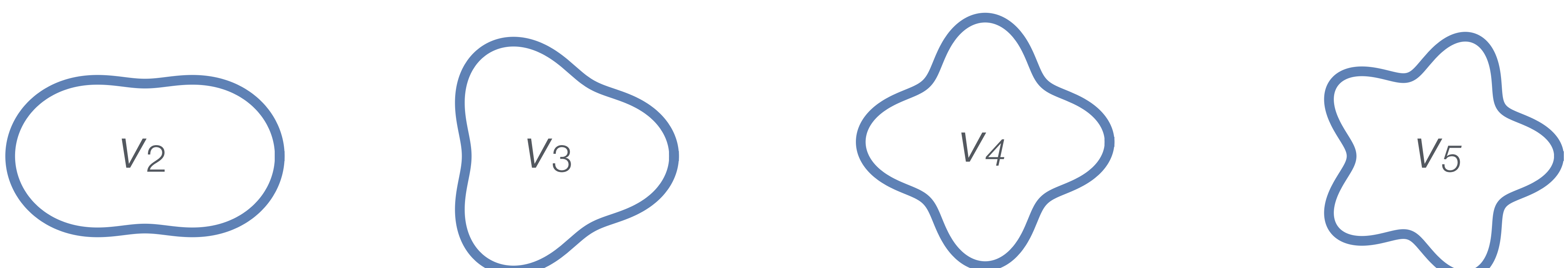
Azimuthal distribution of produced particles





$$\frac{dN}{d\varphi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]$$

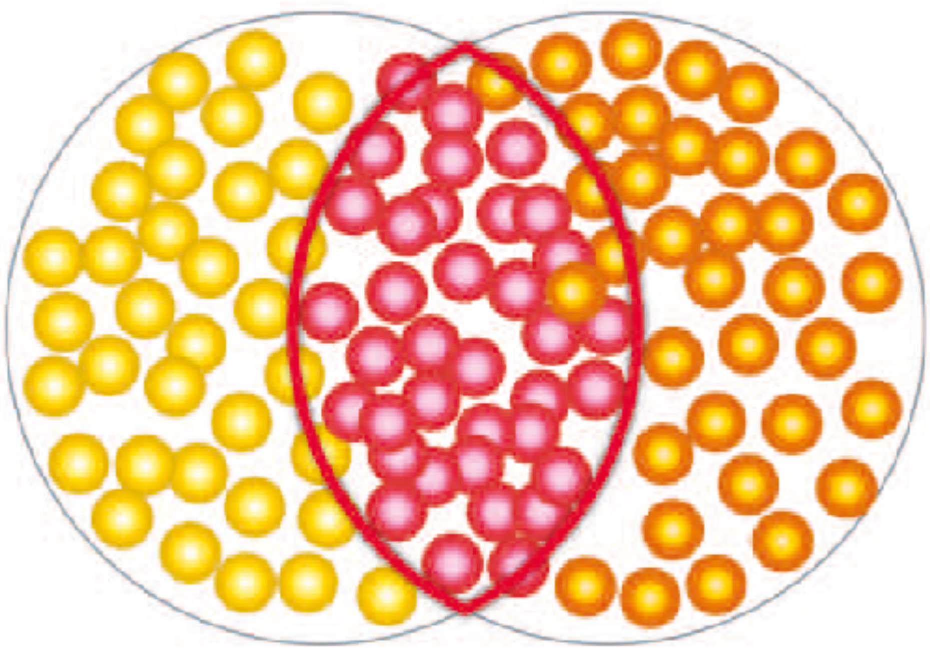
Fourier coefficients: $v_n(p_T, y) = \langle \cos[n(\varphi - \Psi_n)] \rangle$

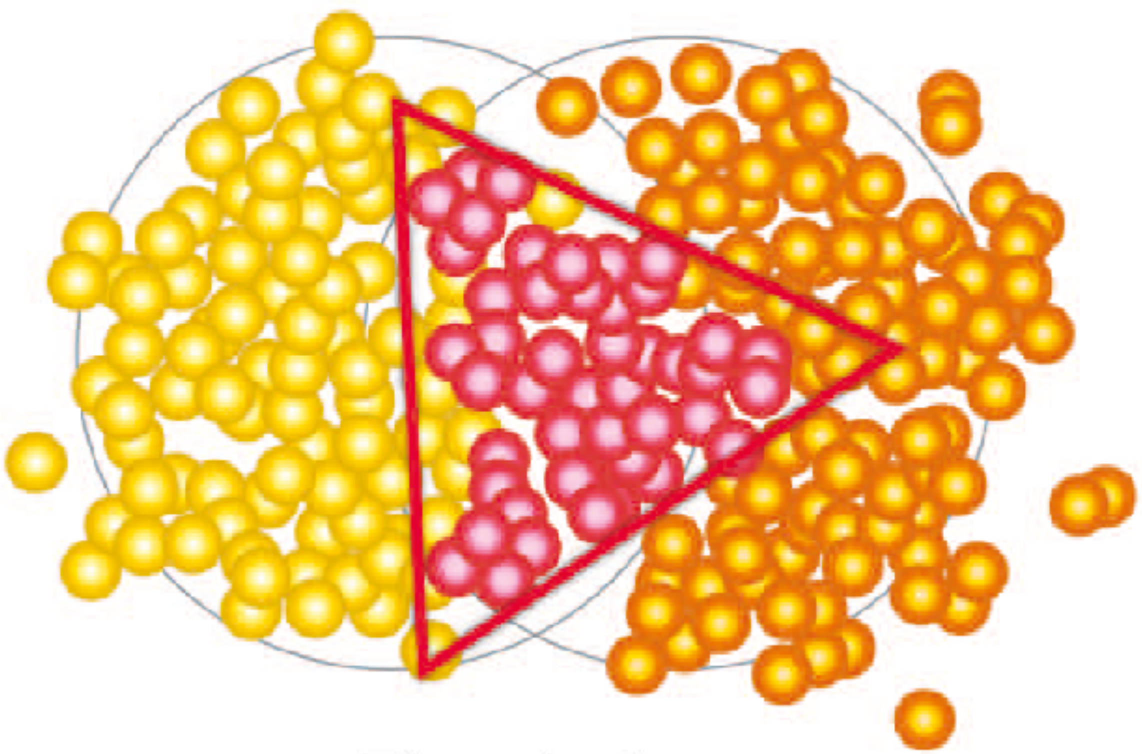


elliptic flow　　triangular flow　　$f(\varphi) = 1 + 2v_n \cos(n\varphi)$

Origin of odd flow components (v_3, v_5, …)

- v_2 is related to the geometry of the overlap zone
- Higher moments result from fluctuations of the initial energy distribution

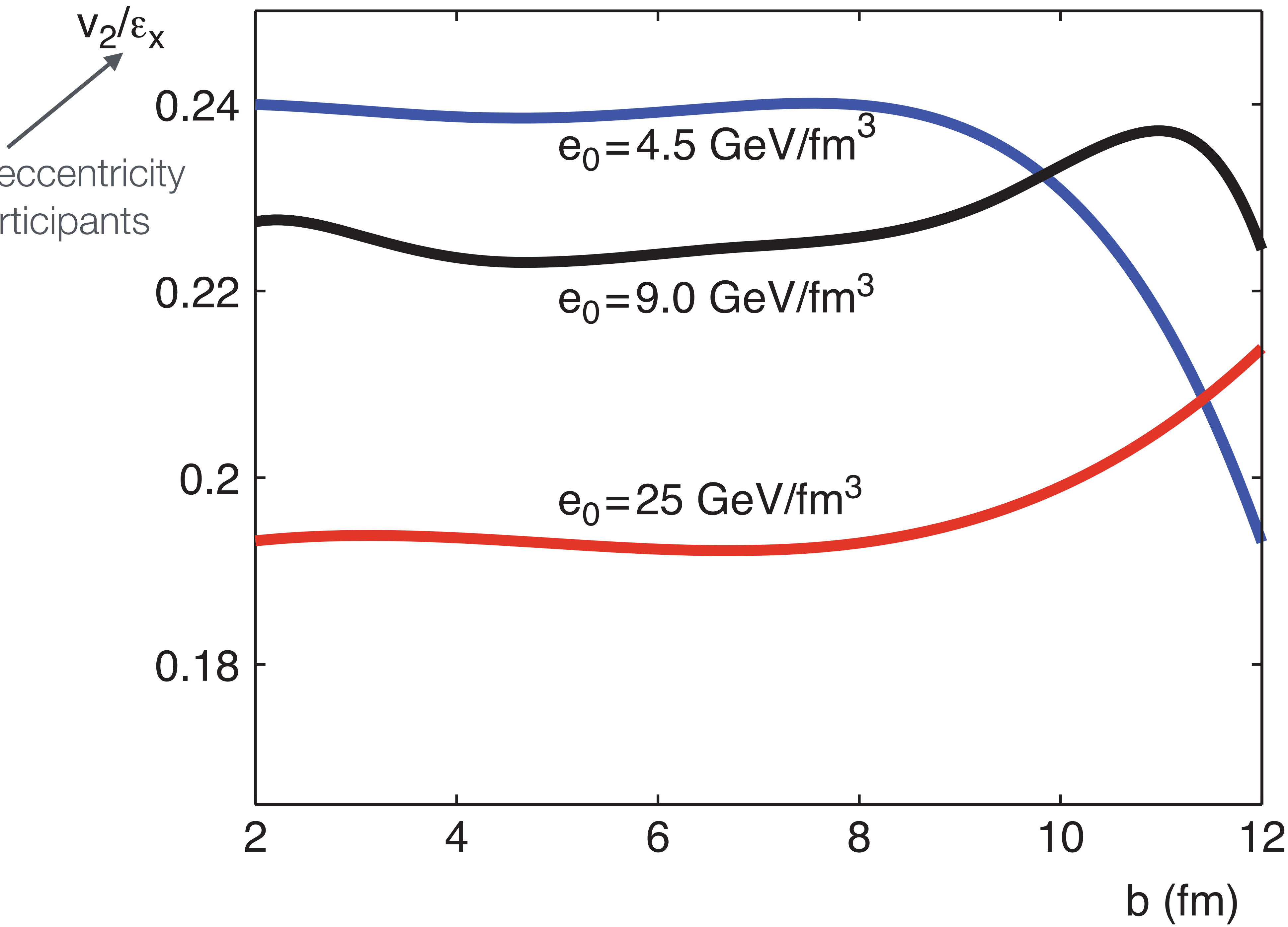




Müller, Jacak, http://dx.doi.org/10.1126/science.1215901

Hydrodynamic models: v_2/ε approx. constant

εx: initial eccentricity of the participants



Ideal hydrodynamics gives $v_2 \approx$ 0.2 – 0-25 ε

How the v_n are measured (1): Event plane method (more or less obsolete by now)

Event flow vector Q_n e.g., measured at forward rapidities:

$$Q_n = \sum_k e^{in\varphi_k} = |Q_n| e^{in\Psi_{n,rec}} = Q_{n,x} + iQ_{n,y}$$

Event plane angle reconstructed in a given event:

$$\Psi_{n,rec} = \frac{1}{n} \mathrm{atan2}(Q_{n,y}, Q_{n,x})$$

Reconstructed event plane angle fluctuates around “true” reaction plane angle. The reconstructed v_n is therefore corrected for the event plane resolution:

$$v_n = \frac{v_n^{\mathrm{rec}}}{R_n}, \qquad v_n^{\mathrm{rec}} = \langle \cos[n(\varphi - \Psi_n^{\mathrm{rec}})] \rangle, \qquad R_n = \text{"resolution correction"}$$

What the event plane methods measures depends on the resolution which depends on the number of particles used in the event plane determination:

$$\langle v^\alpha \rangle^{1/\alpha} \quad \text{where} \quad 1 \leq \alpha \leq 2$$

Therefore other methods are used today where possible.

How the v_n are measured (2): Cumulants

Two-particle correlations:

$$\langle\langle e^{i2(\varphi_1-\varphi_2)}\rangle\rangle = \langle\langle e^{i2(\varphi_1-\Psi_{RP}-(\varphi_2-\Psi_{RP}))}\rangle\rangle,$$
$$= \langle\langle e^{i2(\varphi_1-\Psi_{RP})}\rangle\langle e^{-i2(\varphi_2-\Psi_{RP})}\rangle\rangle = \langle v_2^2\rangle$$

if correlations are only due to collective flow

Cumulants:

two-particle correlations

average over all particles within an event, followed by averaging over all events

if correlations are only due to collective flow

$$c_n\{2\} \equiv \left\langle\left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle\right\rangle = \left\langle v_n^2\right\rangle$$
$$c_n\{4\} \equiv \left\langle\left\langle e^{in(\varphi_1+\varphi_2-\varphi_3-\varphi_4)}\right\rangle\right\rangle - 2\left\langle\left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle\right\rangle^2 = \left\langle -v_n^4\right\rangle$$

$c_n\{4\}$ is a measure of genuine 4-particle correlations, i.e., it is insensitive to two-particle non-flow correlations. It can, however, still be influenced by higher-order non-flow contributions.

$$v_n\{2\}^2 := c_n\{2\}, \qquad v_n\{4\}^4 := -c_n\{4\}$$

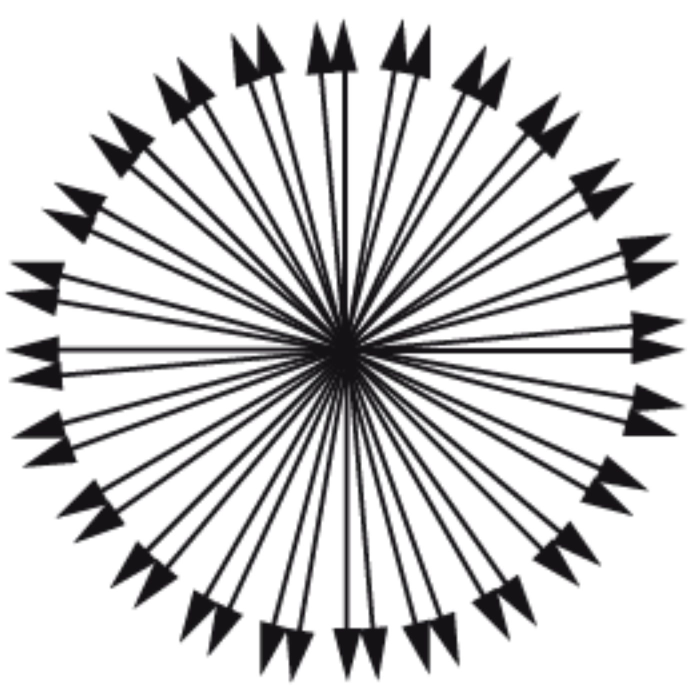
Non-flow effects

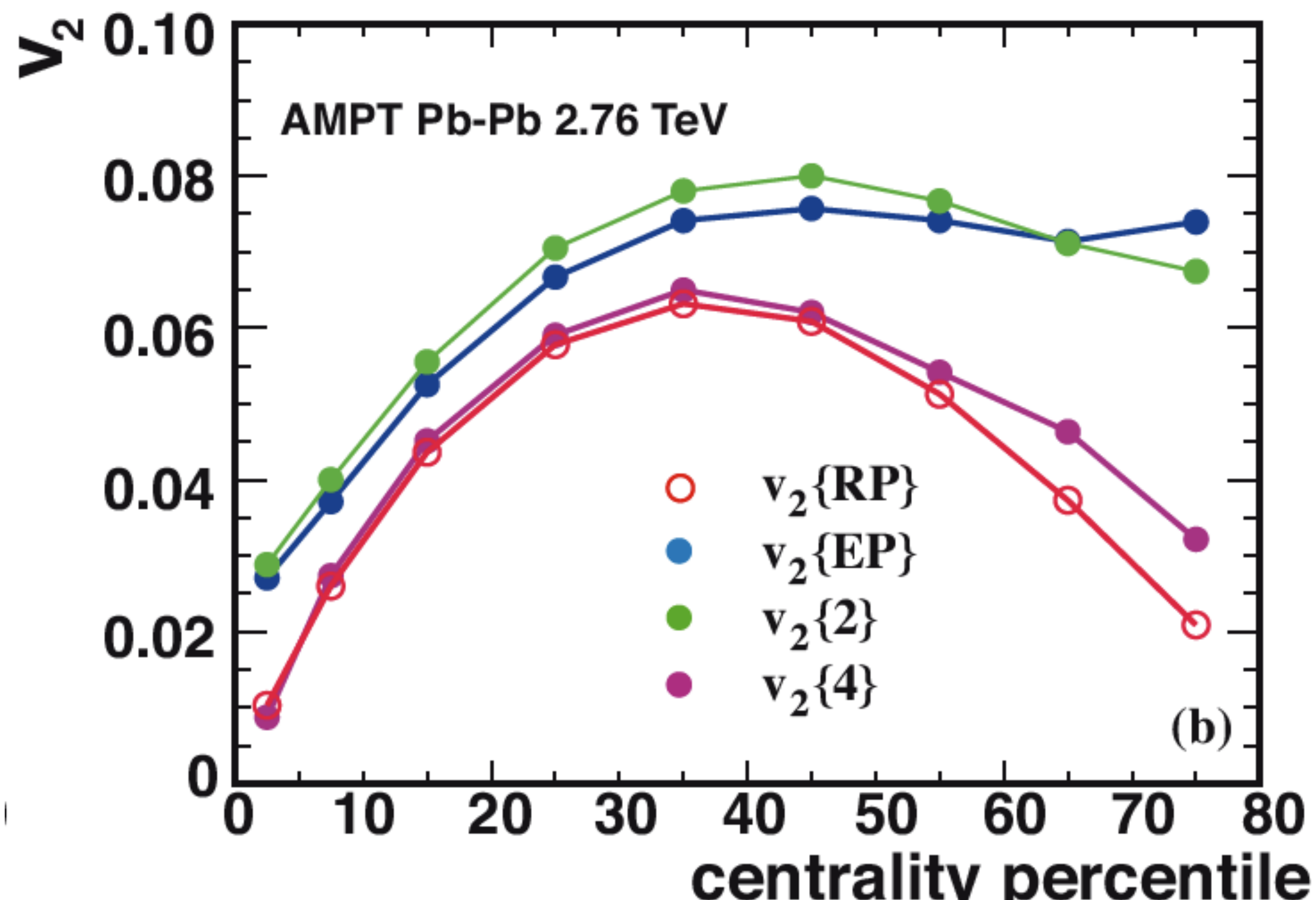
Not only flow leads to azimuthal correlations.
Examples: resonance decays, jets, …

$$v_n\{2\}^2 = \langle v_n^2 \rangle + \delta_n$$

Different methods have different sensitivities to nonflow effects. The 4-particle cumulant method is significantly less sensitive to nonflow effects than the 2-particle cumulant method

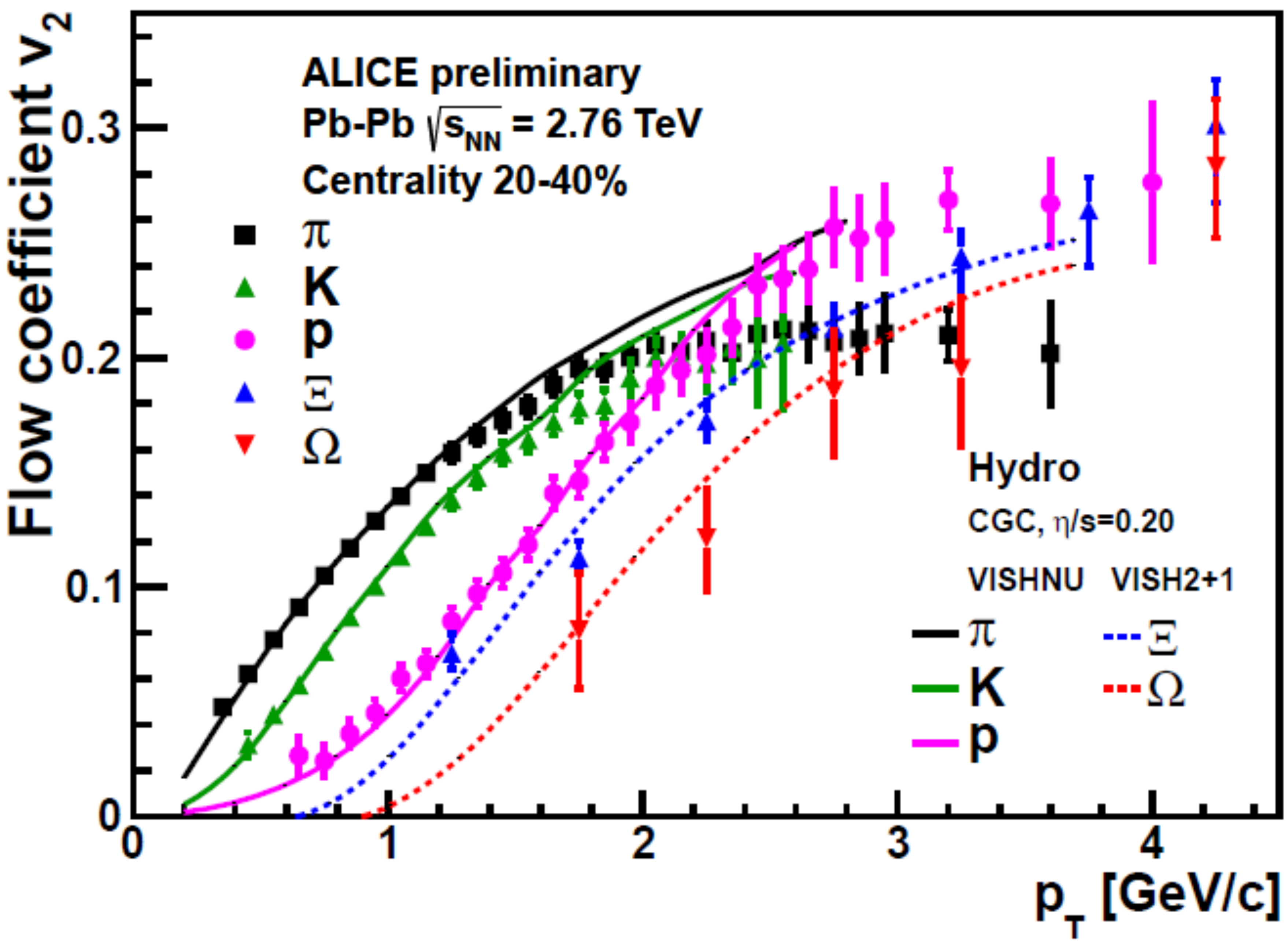
Example:
$v_2 = 0$, $v_2\{2\} > 0$





Elliptic flow of identified hadrons: Reproduced by viscous hydro with $\eta/s = 0.2$

final results: arXiv:1405.4632



Dependence of v_2 on particle mass ("mass ordering") is considered as strong indication for hydrodynamic space-time evolution

Viscosity

Pitch drop experiment, started in Queensland, Australia in 1927

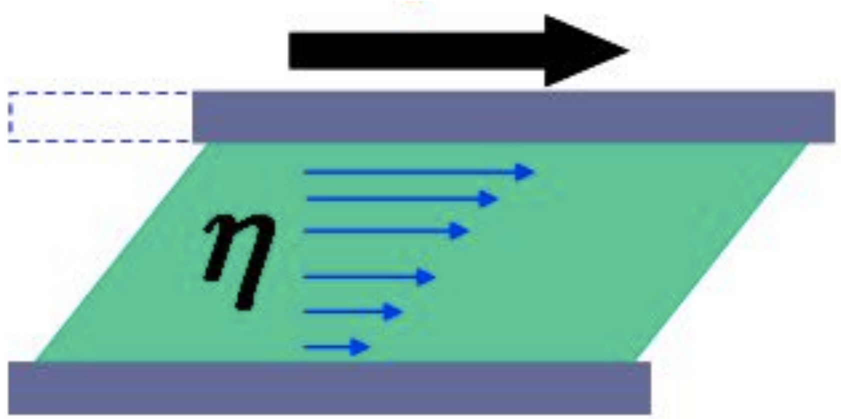
Date	Event	Duration		
		Years	Months	
1927	Hot pitch poured			
October 1930	Stem cut			
December 1938	1st drop fell	8.1	98	
February 1947	2nd drop fell	8.2	99	
April 1954	3rd drop fell	7.2	86	
May 1962	4th drop fell	8.1	97	
August 1970	5th drop fell	8.3	99	
April 1979	6th drop fell	8.7	104	
July 1988	7th drop fell	9.2	111	
November 2000	8th drop fell[A]	12.3	148	
April 2014	9th drop[B]	13.4	156	

https://en.wikipedia.org/wiki/Pitch_drop_experiment

Meaningful comparison of different fluids: η/s

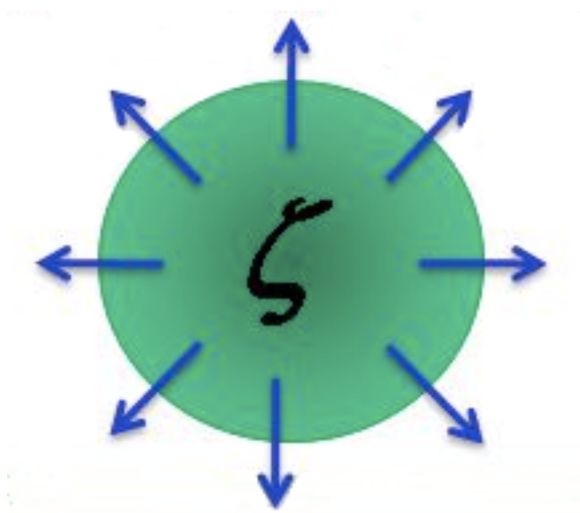
Shear and bulk viscosity

Shear viscosity



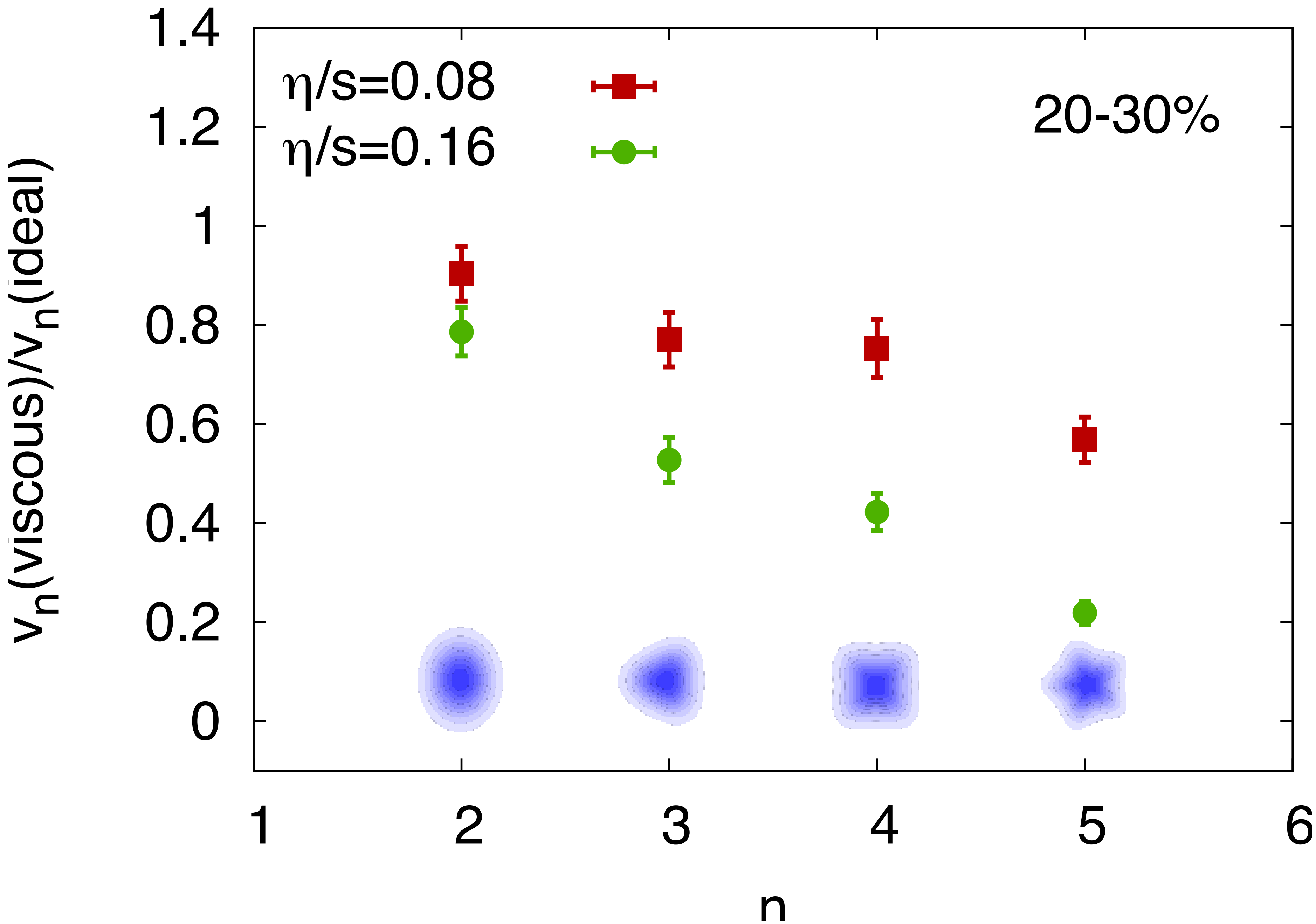
Acts against buildup of flow anisotropies (v_2, v_3, v_4, v_5, ,..)

Bulk viscosity



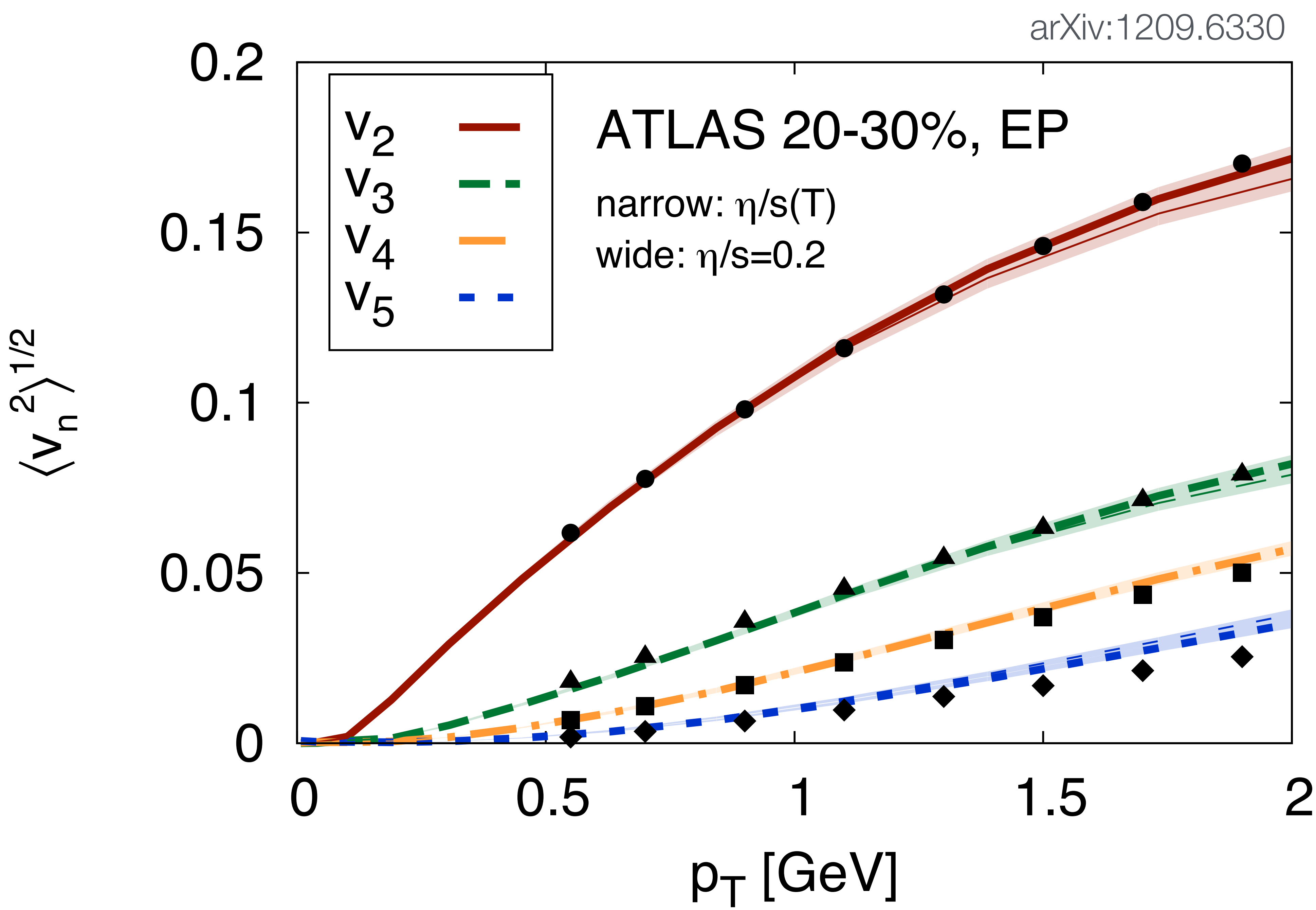
Acts against buildup of radial flow

Higher flow harmonics are particularly sensitive to η/s



Major uncertainty in extracting η/s from data: uncertainty of initial conditions

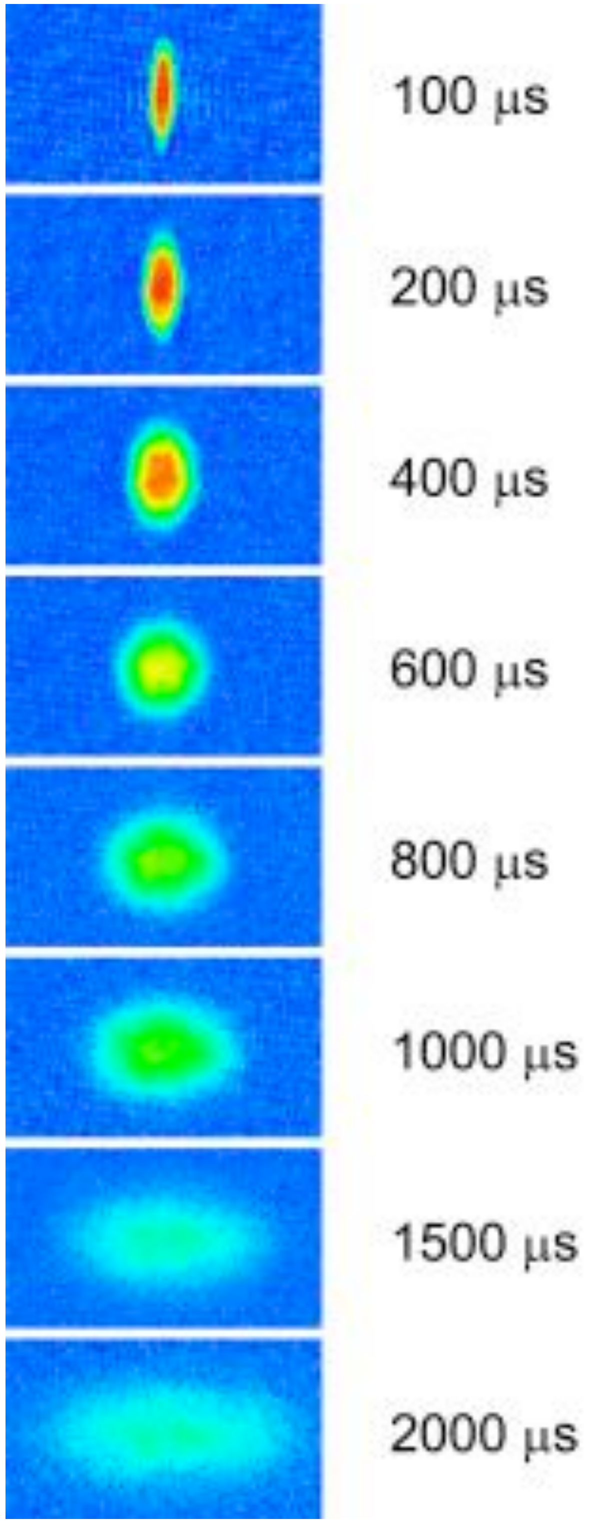
η/s from comparison to data



Current status (Pb-Pb at $\sqrt{s_{NN}}$ = 2.76 TeV): arXiv:1301.2826

$$(\eta/s)_{\text{QGP}} \approx 0.2 = 2.5 \times \frac{1}{4\pi} \qquad (20\% \text{ stat. err., } 50\% \text{ syst. err.})$$

Universal aspects of the underlying physics



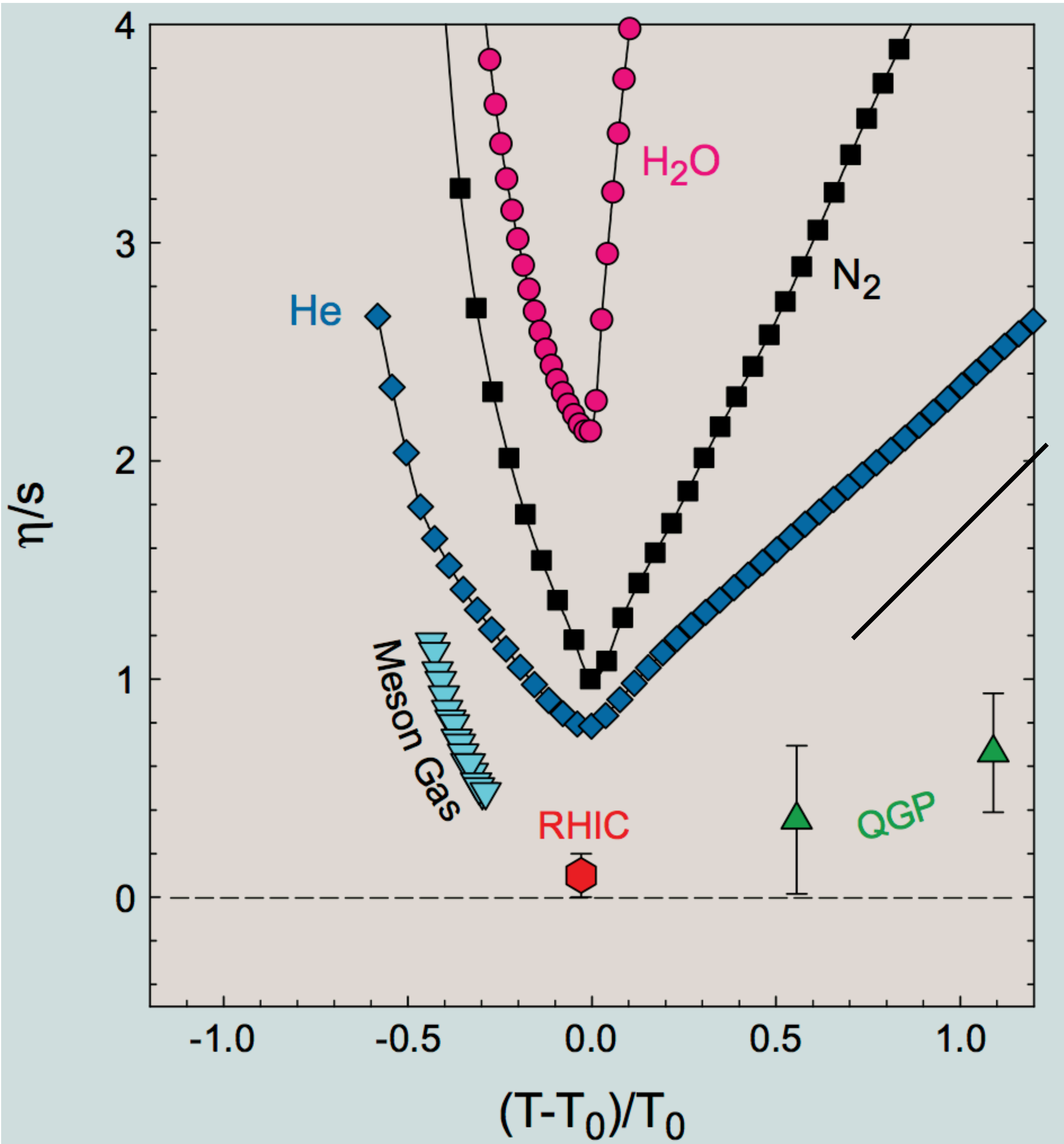
- Strongly-interacting degenerate gas of fermionic ^{6}Li atoms at 0.1 µK
- Cigar-shaped cloud initially trapped by a laser field
- Anisotropic expansion upon abruptly turning off the trap: Elliptic flow!
- η/s can be extracted: [PhD thesis Chenglin Cao]

$$(\eta/s)_{^6\mathrm{Li\,gas}} \approx 0.4 = 5 \times \frac{1}{4\pi}$$

The ultimate goal is to unveil the universal physical laws governing seemingly different physical systems (with temperature scales differing by 19 order of magnitude)

John Thomas, https://www.physics.ncsu.edu/jet/research/stronginter/index.html

Temperature-dependence of η/s for different gases



η/s appears to be minimal at a phase transition

QGP is a candidate for being the most perfect fluid

Conjectured lower bound from string theory

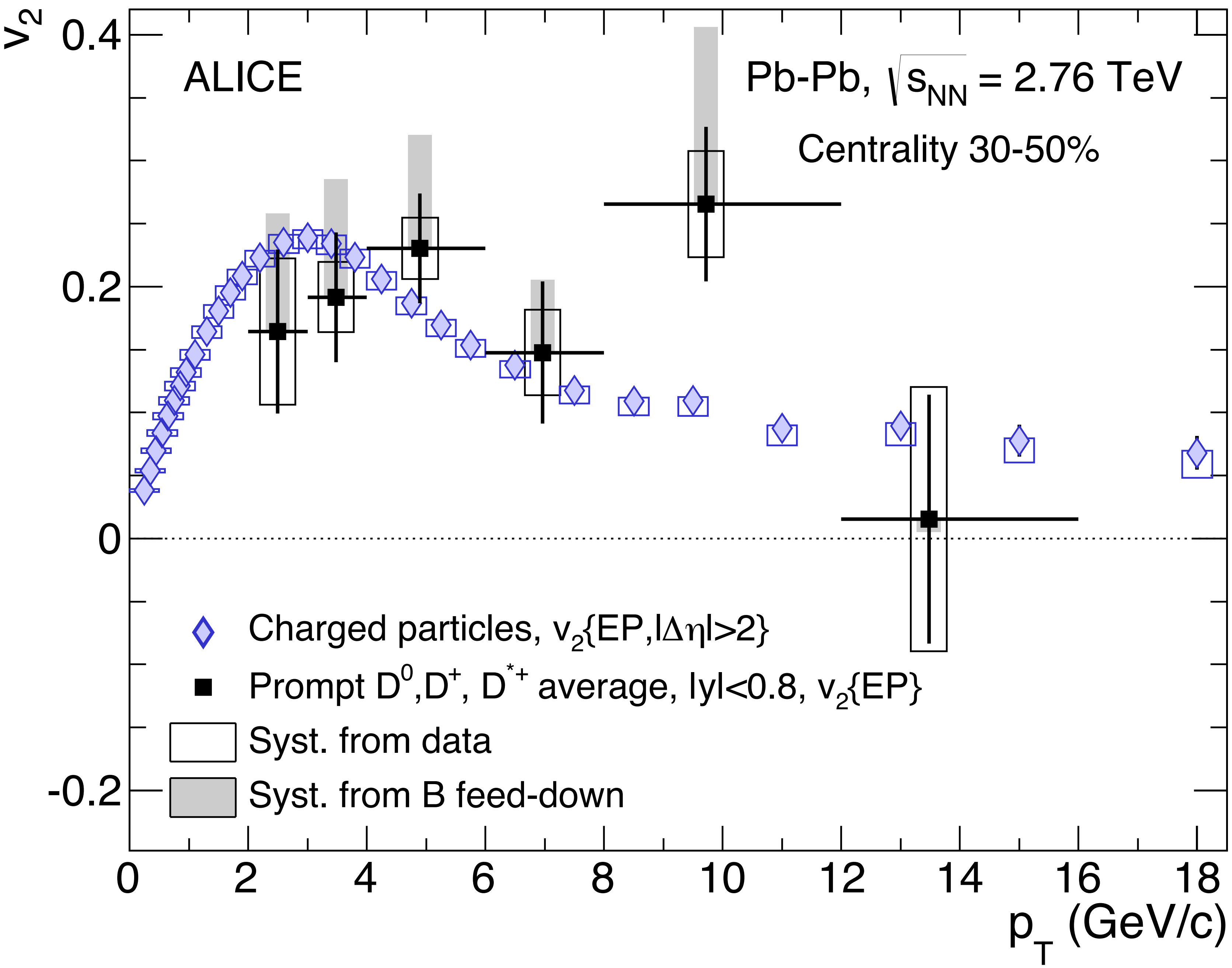
$$\eta/s|_{\mathrm{KSS}} = \frac{1}{4\pi} \approx 0.08$$

in natural units

SI units: $\eta/s|_{\mathrm{KSS}} = \frac{\hbar}{4\pi k_B}$

Kovtun, Son, Starinets,
Phys.Rev.Lett. 94 (2005) 111601

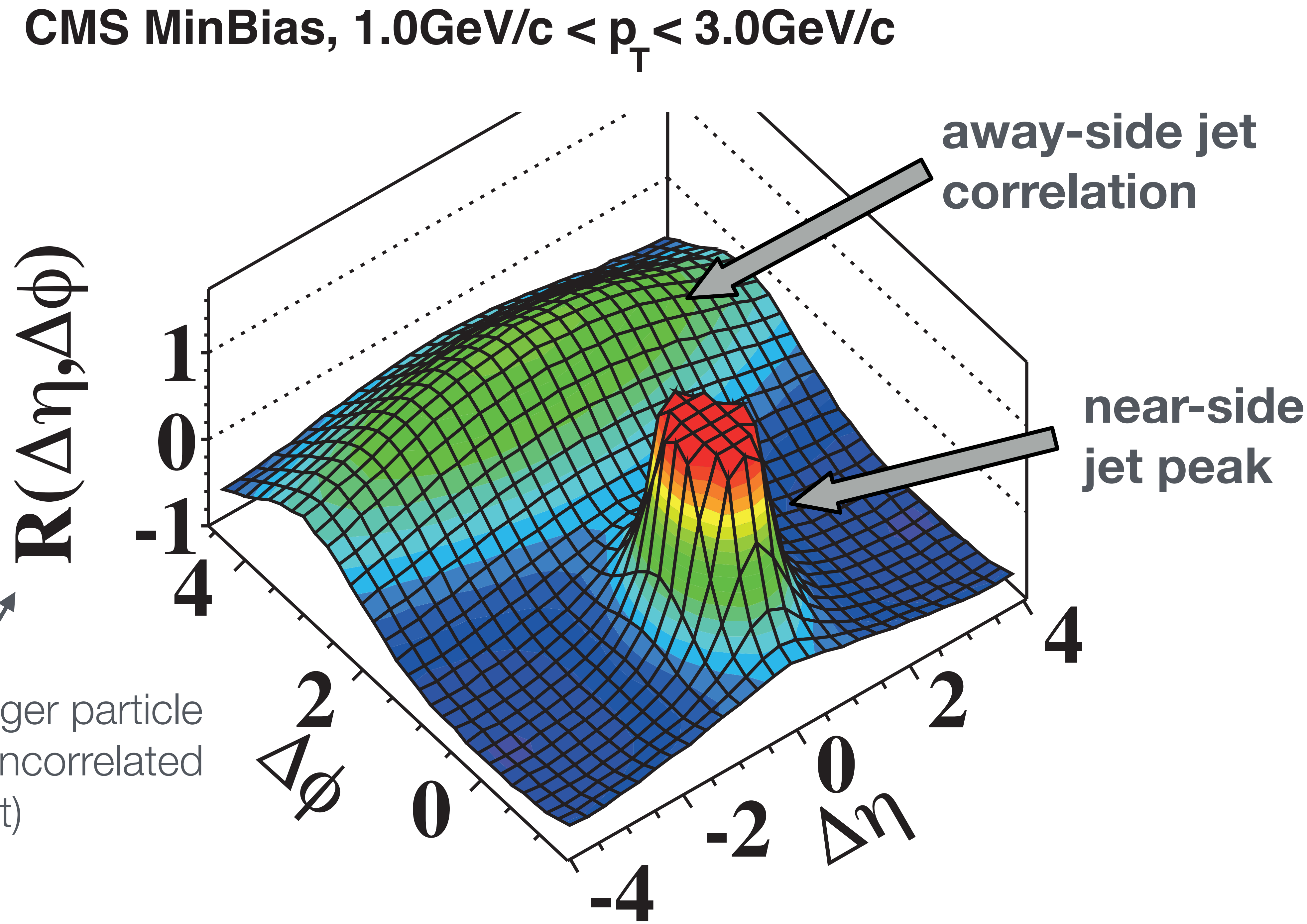
D meson v_2 in Pb-Pb: Heavy quarks seem to flow, too!



Given their large mass, it is not obvious that charm quarks take part in the collective expansion of the medium

Collective flow in small systems?

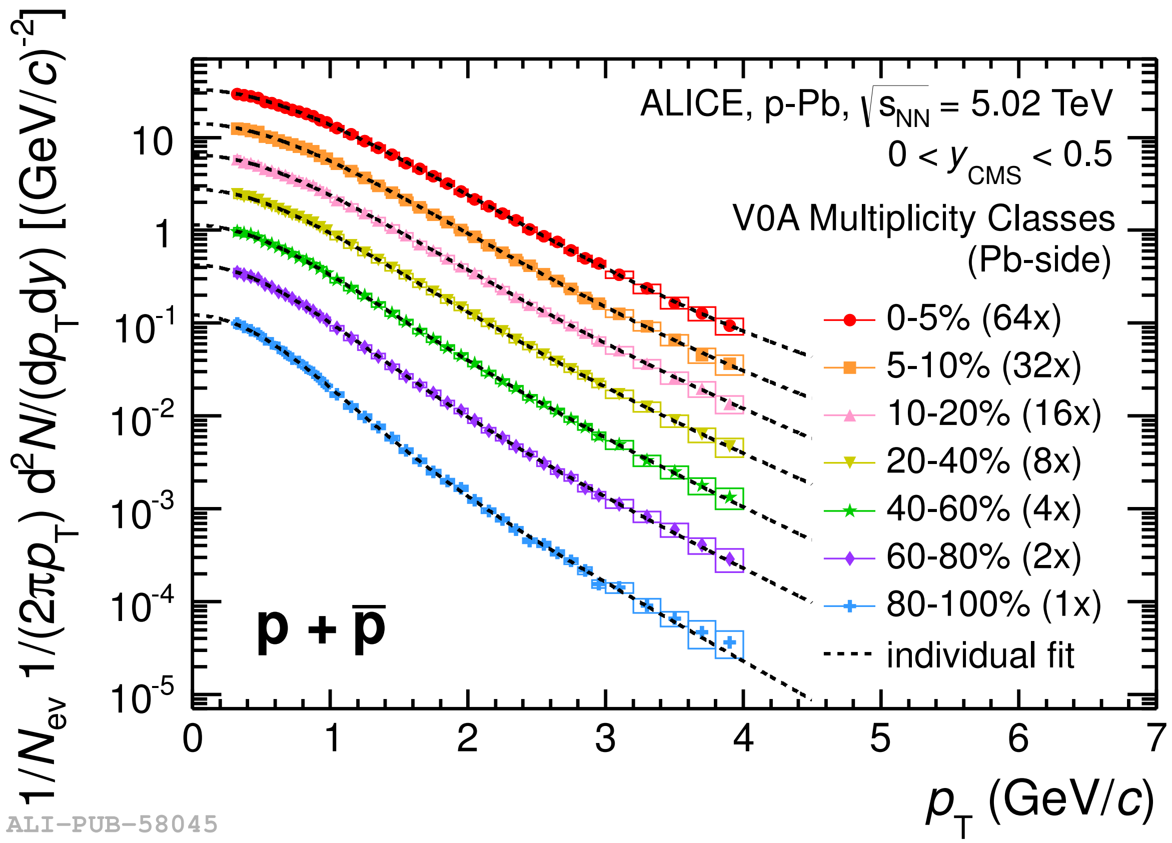
Collectivity in small systems: 2-particle correlation in pp at $\sqrt{s}$ = 7 TeV

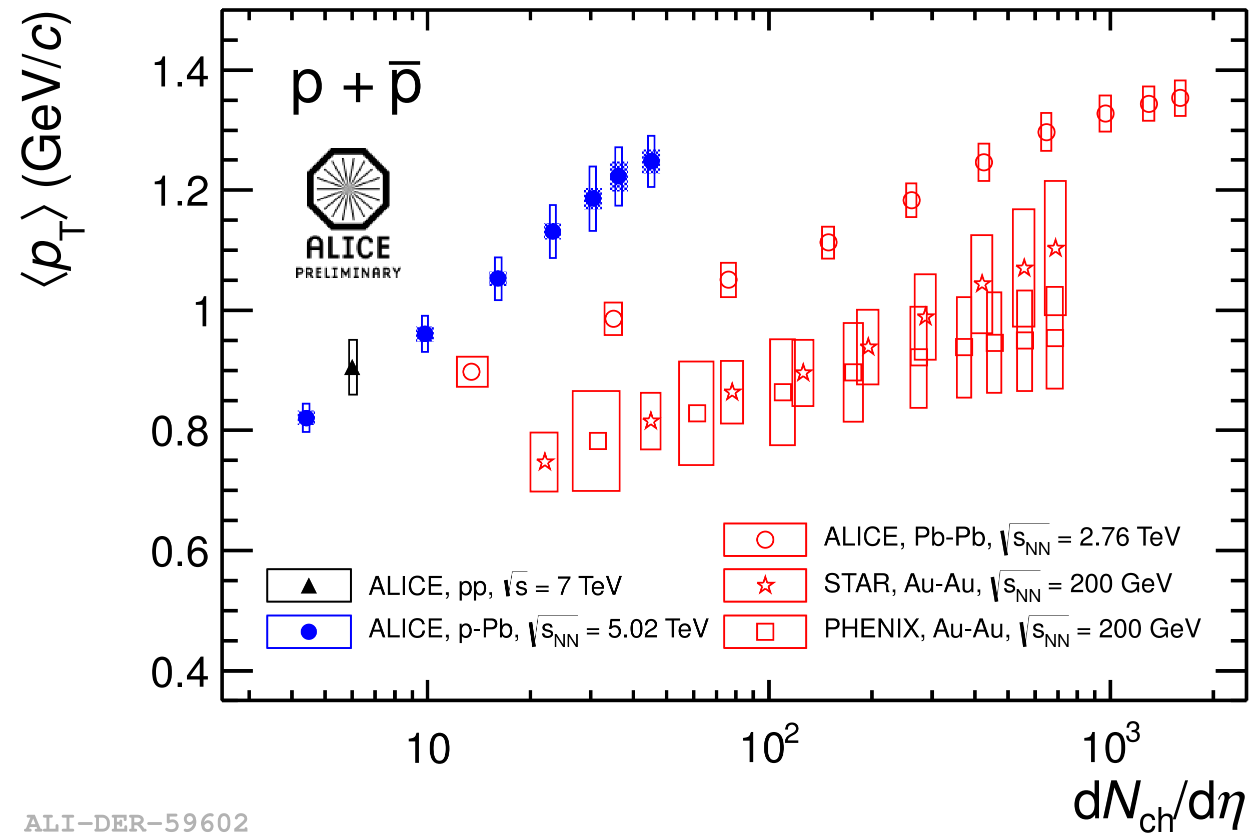


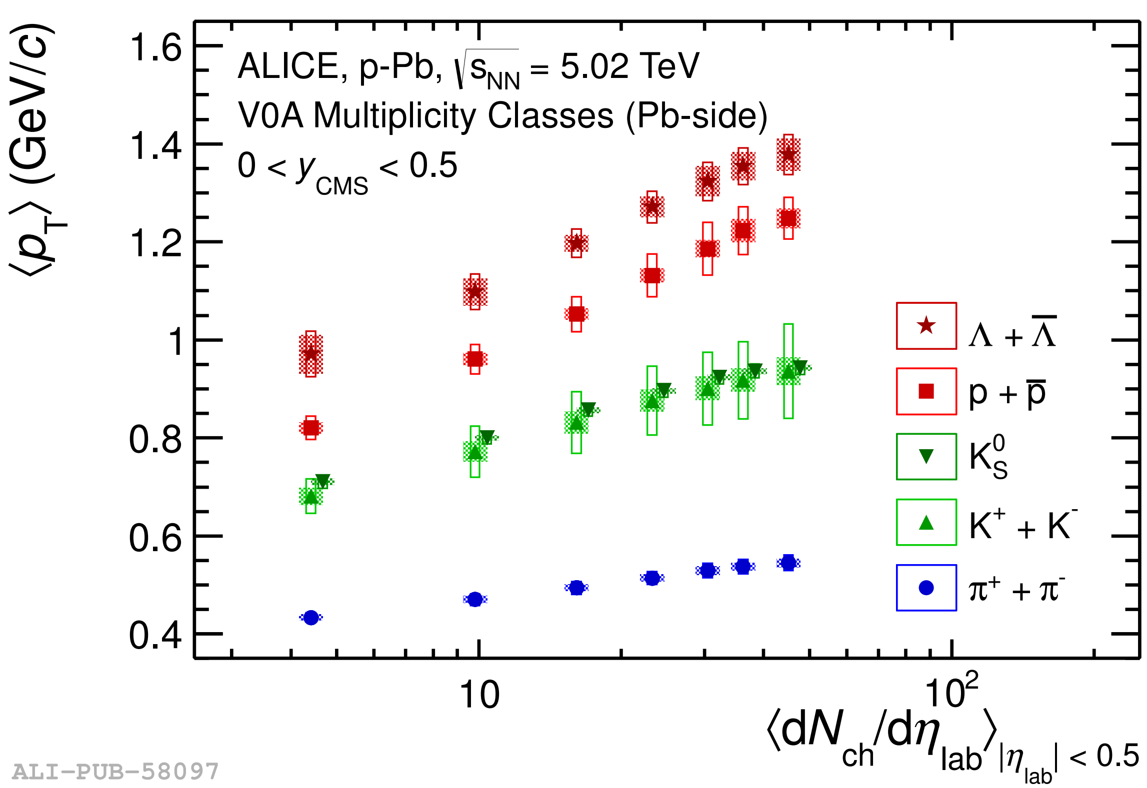
yield per trigger particle divided by uncorrelated (mixed-event) background

No indication for collective effects in minimum bias pp collisions at 7 TeV

Radial flow in p-Pb?







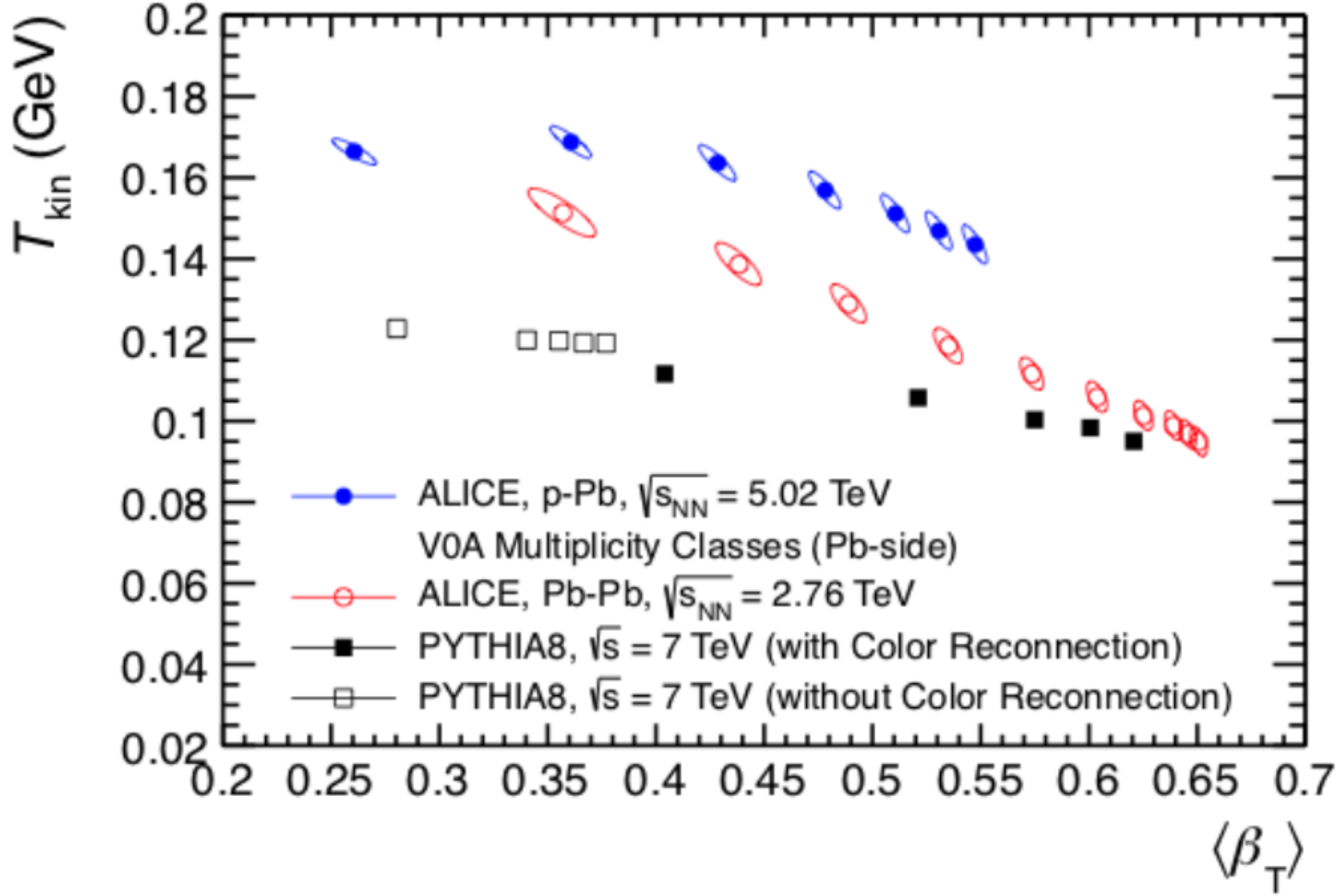
Shape of spectra changes with $dN_{ch}/d\eta$

Increase of $\langle p_T \rangle$ with $dN_{ch}/d\eta$

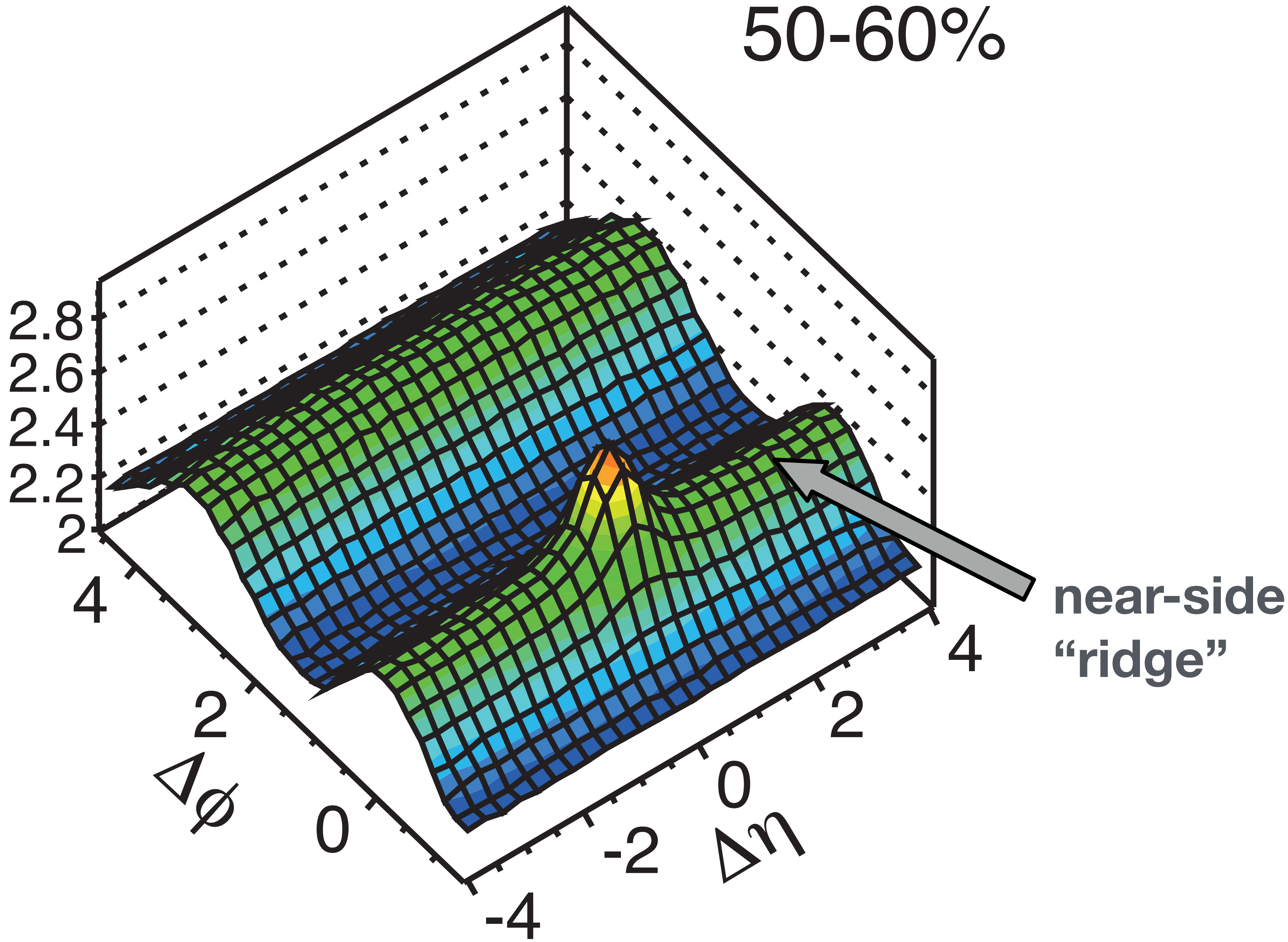
Effects which can be explained as resulting from radial flow

Results of blast-wave fits in p-Pb

ALICE, 1307.6796

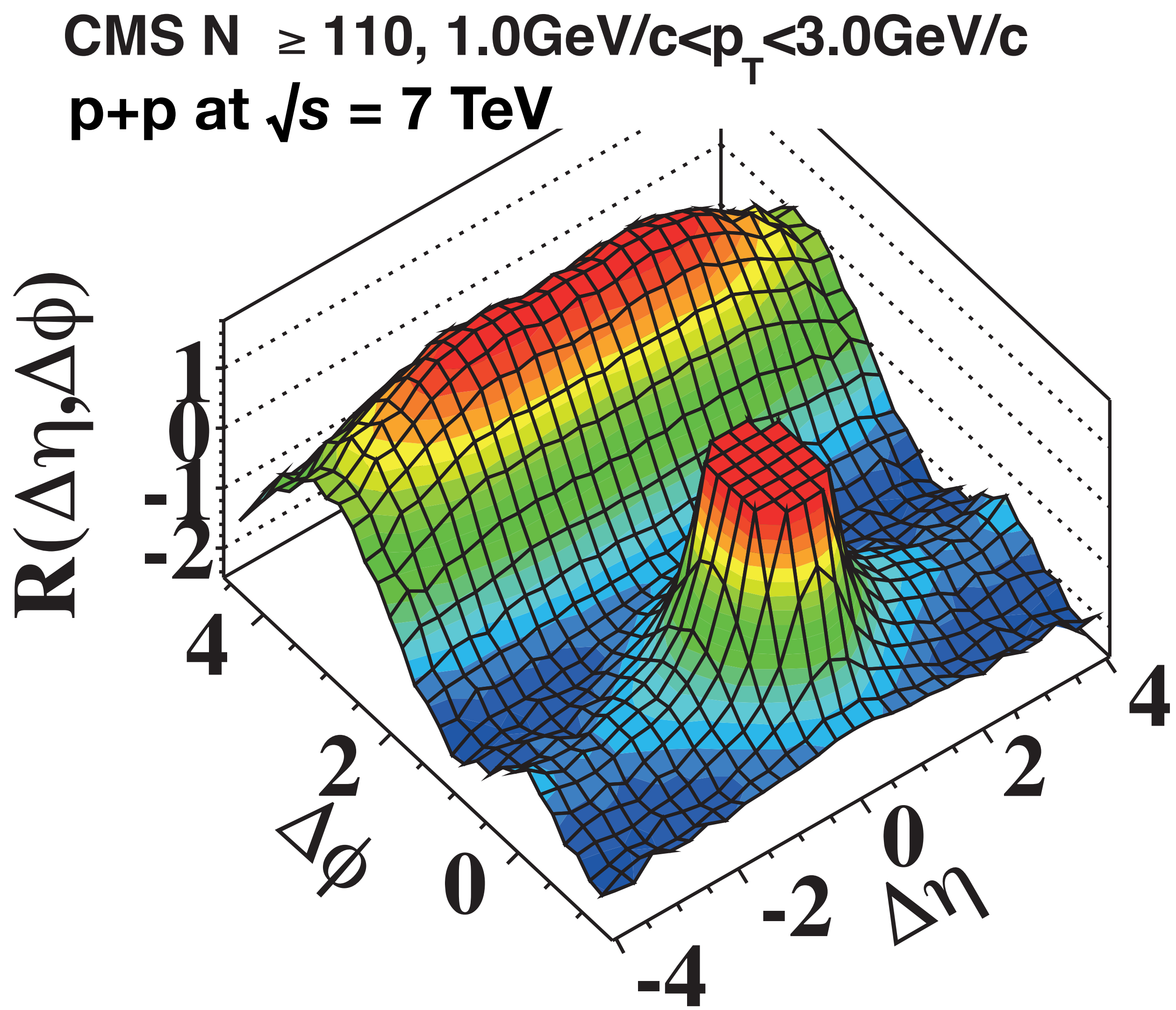


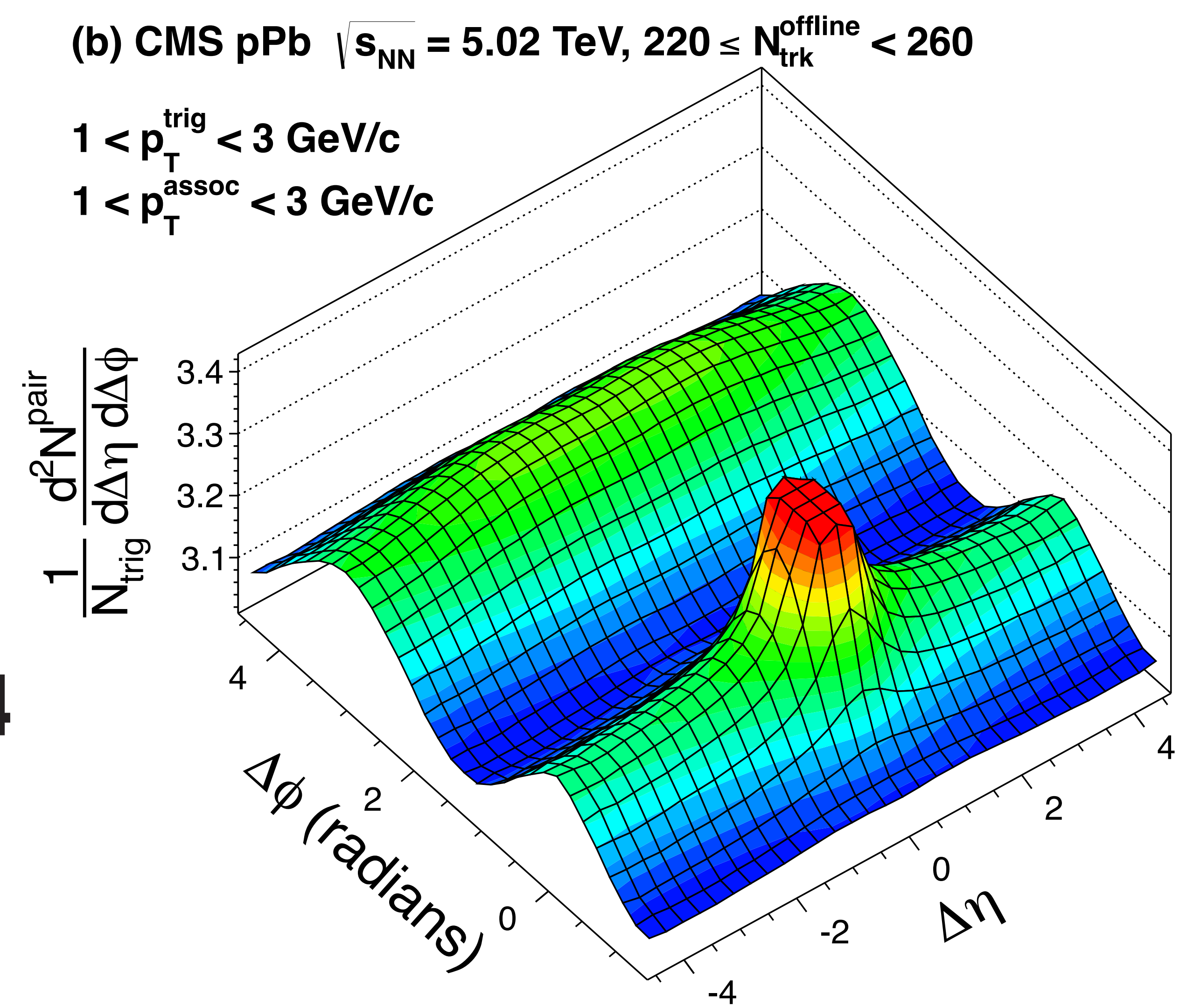
Collectivity in small systems: Two-particle correlations in Pb-Pb collisions



collective flow + jet correlations

Collectivity in small systems: Two-particle correlations in high-multiplicity pp and p-Pb

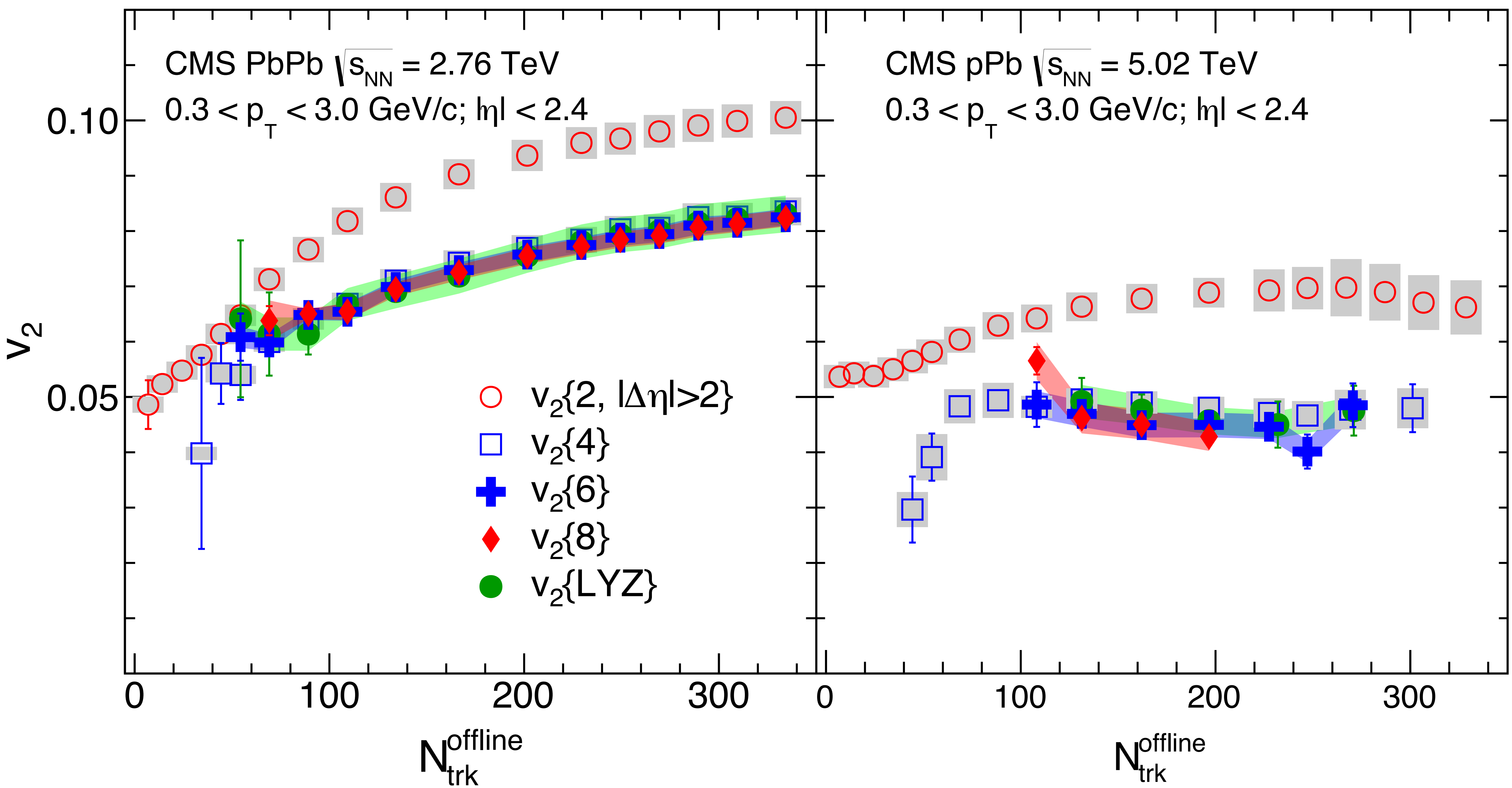




Flow-like two-particle correlation become visible in high-multiplicity pp and p-Pb collisions at the LHC

Comparison of v_2 in Pb-Pb and p-Pb for the same track multiplicity

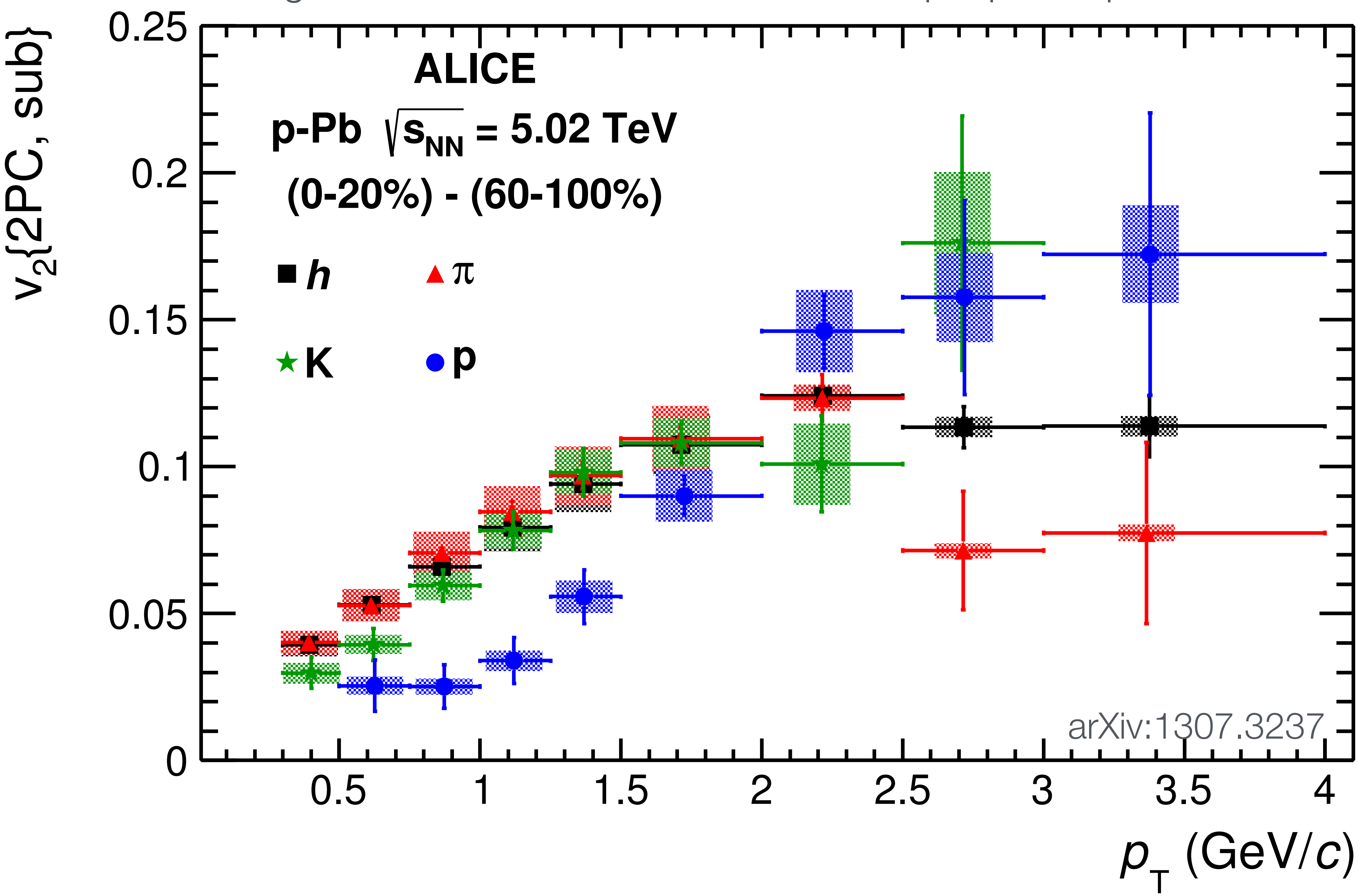
CMS, arXiv:1502.05382v2



- $v_2\{8\}$ measured: v_2 in p-Pb is a genuine multi-particle effect
- v_2 in p-Pb only slightly smaller than in Pb-Pb

Collectivity in small systems:
Mass ordering in p-Pb collisions

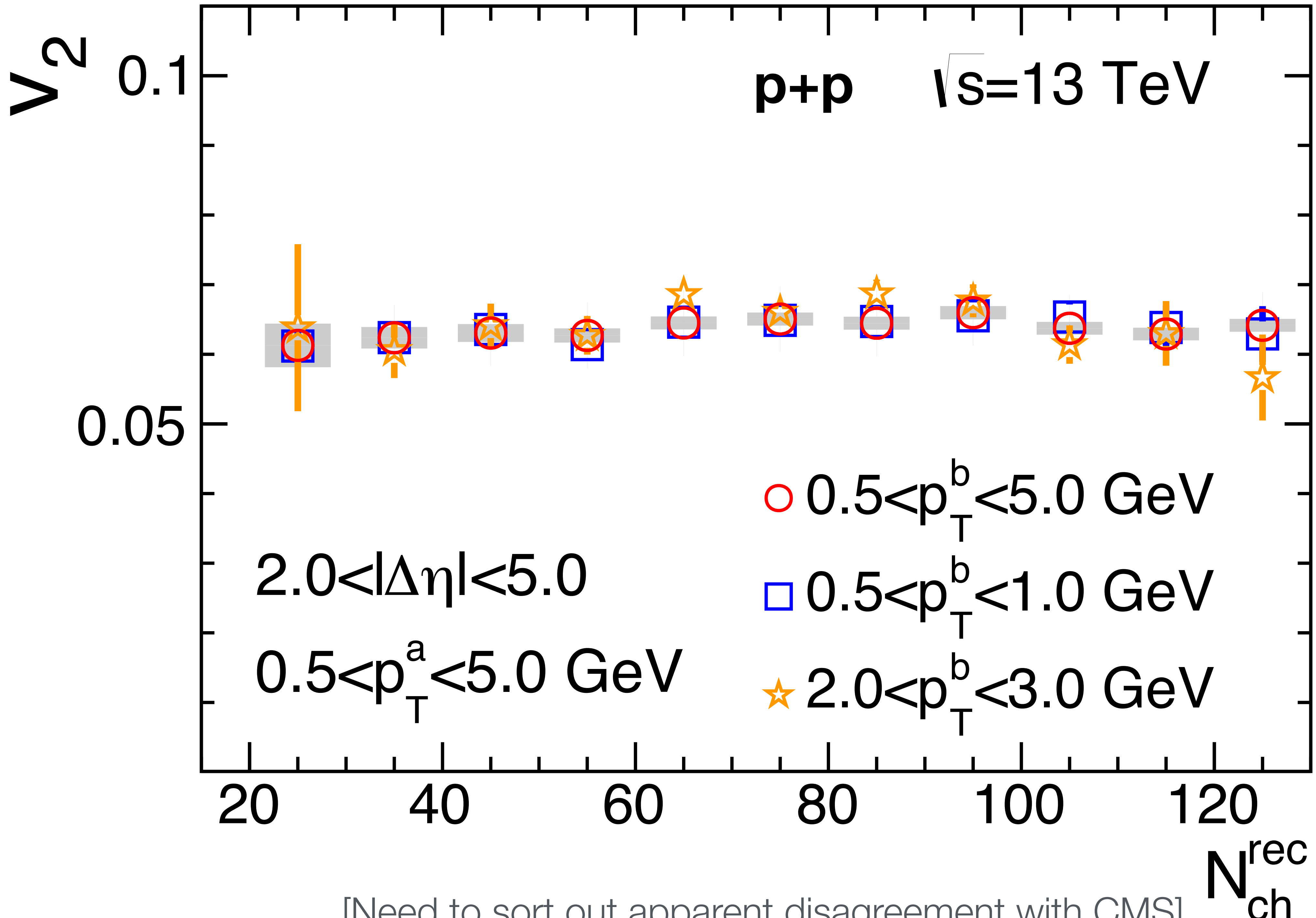
v_2 from fit of two-particle correlation, jet-like correlation removed by taking the difference between central and peripheral p-Pb collisions



Consistent with hydrodynamic expansion of the medium als in p-Pb

Elliptic flow not only in high multiplicity pp collisions?

ATLAS, 1509.04776



[Need to sort out apparent disagreement with CMS]

Summary/questions space-time evolution

- Hydrodynamic models provide an economic description of many observables (spectra, flow)
- Shear viscosity / entropy density ratio in Pb-Pb at $\sqrt{s_{NN}}$ = 2.76 TeV from comparing hydrodynamic models to data:

$$(\eta/s)_{\mathrm{QGP}} \approx 0.2 = 2.5 \times \left.\frac{\eta}{s}\right|_{\mathrm{min,KSS}} = 2.5 \times \frac{1}{4\pi}$$

- Appropriate theoretical treatment of thermalization and matching to hydrodynamics?
  - Strong coupling or weak coupling approach?
  - Weak coupling: Applicable at asymptotic energies, but still useful at current $\sqrt{s_{NN}}$
  - Strong coupling (string/gauge theory duality), see e.g. arXiv:1501.04952: Fast thermalization of the order of $1/T$, but too much stopping?
- Does one need hydrodynamics to explain collective effects in small system (pp, p-Pb)?