Quark-Gluon Plasma Physics

7. Correlations and Fluctuations

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Correlations of identical bosons due to quantum interference

stochastic emission from extended source

consider 2 identical bosons (photons, pions, ...)

2 detectors in locations r_1 , r_2 observe identical bosons of momenta p_1 and p_2



cannot distinguish solid and dashed paths because of identical particles
for plane waves, the probability amplitude for detection of the pair is

$$A_{12} = \frac{1}{\sqrt{2}} \left[e^{ip_1(r_1 - x)} e^{ip_2(r_2 - y)} + e^{ip_1(r_1 - y)} e^{ip_2(r_2 - x)} \right]$$

with 4-vectors p,r,x,y (to be general for nonstatic source)

square of amplitude: intensity ____ "intensity interferometry"

probability $A_{12}^2 \propto (1 + \cos(\Delta k \cdot \Delta x))$

is enhanced for momenta close to each other due to Bose-Einstein statistics

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Hanbury-Brown Twiss effect

technique of intensity interferometry developed by Hanbury-Brown and Twiss in astrophysics as a means to determine size of distant objects R. Hanbury-Brown and R.Q. Twiss, Phil. Mag. 45 (1954) 663 radiowaves to determine size of galaxies (Cygnus, Cassiopeia) - this could be done with classical Michelson (amplitude) interferometry and Nature 178 (1956) 1046

visible light to determine the size of stars (Sirius)



correlated signal varies on characteristic length scale: $D=\lambda \frac{K_{star}}{L}$ varying separation of detectors one learns the apparent

with detector separation of the order of meters, star size of order 10⁶ km 1 light year away can be determined by intensity interferometry with visible light

Angular diameter of Sirius from HBT Correlations



Angular diameter R/L of Sirius from intensity interferometry: 3.1.10⁻⁸ rad

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HBT interference in particle physics

when phase space volume smaller than $\Delta p_x \Delta x \approx \hbar$ is considered, chaotic system of identical noninteracting particles exhibits quantum fluctuations following Bose-Einstein (or Fermi-Dirac) statistics

first observation in particle physics: pions with small relative momenta G. Goldhaber, S. Goldhaber, W.Y. Lee, A. Pais, Phys. Rev. 120 (1960) 300 pion correlations after a pbarp annihilation



look at angular opening between like- and unlikesign pions in events with 4 charged pions at sqrt(s) = 2.1 GeV

radius ρ of volume of strong interaction in Compton wavelengths of pion $\frac{\hbar c}{mc^2} = 1.4\,\mathrm{fm}$

found to be 0.5 - 0.75 i.e. 0.7-1.0 fm

HBT interferometry in nuclear and particle physics

after discovery of GGLP systematically used as tool to determine source size in particular in collisions of high energy nuclei

useful references: G.I. Kopylov, M.I. Podgoretsky, Sov. J. Nucl. Phys. 15 (1972) 219 and ibid 18 (1974) 336 and PLB50 (1974) 472 theory of interference of identical pions, lifetime of source E. Shuryak, Phys. Lett. B44 (1973) 387 introduces time, i.e. duration of emission

excellent review by: G. Baym, Acta Phys. Polon. B29 (1998) 1839

2-particle correlation function

typical particle physics experiment: vary momenta at fixed location of detector

$$C_2(\vec{p}_1 - \vec{p}_2) = \frac{d^6 N/d\vec{p}_1 d\vec{p}_2}{d^3 N/d\vec{p}_1 \cdot d^3 N/d\vec{p}_2} = 1 + f(\vec{p}_1 - \vec{p}_2)$$

get uncorrelated denominator typically from event mixing



in heavy ion collisions typical dimensions 1-10 fm, → leads to interference at momentum differences of 20-200 MeV/c

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for a stochastic source we get a 2-particle correlation function

$$C_2(q) = P_{12} = \int d^4x d^4y \mid A_{12} \mid^2 \rho(x)\rho(y)$$

with a 4-momentum difference $q \equiv p_1 - p_2$ and momentum independent source functions $\rho(x), \rho(y)$ transformation to relative coordinates and integration leads to

$$C_2(q) = 1 + |\tilde{\rho}(q)|^2$$
$$\tilde{\rho}(q) = \int d^4 x \rho(x) e^{-iqx}$$

Example: Gaussian space-time density distribution $\rho(x)$

$$\begin{split} \rho(x) &= c \, \exp(-\frac{|\vec{x}|^2}{2r_0^2} - \frac{t^2}{2\tau_0^2}) \\ \longrightarrow \quad \tilde{\rho}(q) &= c' \, \exp(-\frac{|\vec{q}|^2 r_0^2}{2} - \frac{q_0^2 \tau_0^2}{2}) \\ \text{and} \quad C_2(q) &= 1 + \exp(-r_0^2 |\vec{q}|^2 - \tau_0^2 q_0^2) \quad \text{with norm} \ \tilde{\rho}(0) = 1 \end{split}$$

note that the 3d rms radius is $r_{rms} = \sqrt{3}r_0$

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Coupling of spatial information and time

time and space information are coupled, i.e. q_0 and q(vector) are not independent in the pair rest frame $q^* = (0, \vec{q^*}) = (0, 2\vec{k}^*)$ with $\vec{k}^* = \vec{p}_1^* = -\vec{p}_2^*$

(in general
$$\vec{k}^* = rac{m_2 \vec{p}_1^* - m_1 \vec{p}_2^*}{m_1 + m_2}$$

if \tilde{v} is the velocity of the pair in the overall cm frame, $\gamma^2 = \frac{1}{1 - v^2/c^2}$

and $\vec{k}_T^* = \vec{q}_T/2$, $k_L^* = q_L/2\gamma$ (here T and L refer to the direction of the pair) and $\vec{v} \cdot \vec{q} = v q_L$ but $\frac{\vec{v}}{c} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$ $\rightarrow \vec{v} \cdot \vec{q} = E_1 - E_2 = q_0$ $\rightarrow C_2(q) = 1 + exp(-4r_0^2k_T^{*2} - 4\rho^2\gamma^2k_L^{*2})$

where $\rho^2 = r_0^2 + (v\tau_0)^2$

in direction of the pair, spatial and temporal information cannot be distinguished for identical particle

Coulomb correction

in addition to quantum correlations 2 identical pions also experience correlations due to electromagnetic interaction, i.e. Coulomb repulsion

for 2 pions from a point source this problem was already solved for beta-decay (Gamov function)



 $\sigma (GeV/c)$

3-dimensional correlations functions in pp and heavy ion collisions

go from momentum difference in cartesian coordinates to so-called Bertsch-Pratt variables (G. Bertsch, Phys. Rev. C37 (1988) 1896) define z-axis as beam axis

$$q_{long} = \vec{q} \cdot \hat{e}_z = q_z$$

and in transverse direction distinguish direction parallel to pair momentum



Example: raw measured $\pi\pi$ correlation function

CERES/NA45 at CERN SPS - central 158 A GeV/c PbAu collisions



characteristic width order 30 MeV/c

fit function of form $C_2 = 1 + \lambda exp(-R_{long}^2 q_{long}^2 - R_{side}^2 q_{side}^2 - R_{out}^2 q_{out}^2)$ to the exp. correlation function with correlation strength $\lambda = 1$ for chaotic source and 0 for fully coherent source

Dynamical source with space-momentum correlations

collective expansion introduces space-momentum correlations leading to an apparent reduction in source radius

simple example:

source with T=0 emitting particles radially with a Hubble-like velocity profile



the apparent source from 2-particle correlations looks pointlike! only particles emitted from the same space point have the same momentum, i.e. a small momentum difference



Dynamical source with space-momentum correlations

effect of hydrodynamic expansion on radius parameters extracted from 2-pion correlations

S. Pratt, Phys. Rev. Lett. 53 (1984) 1219 A.N. Makhlin, Y.M. Sinyukov, Z. Physik C39 (1988) 69 Y.M. Sinyukov, Nucl. Phys. A498 (1989) 151c

<u>picture</u>: thermal source that expands hydrodynamically with velocity v(r) and corresponding $\beta(r)$ and $\gamma(r)$

look at 2-pion correlation function for different slices in pair transverse momentum

$$\vec{k} = \frac{1}{2}(\vec{k}_1 + \vec{k}_2)$$
 or also $k_t = \frac{1}{2}(k_{t,1} + k_{t,2})$

find that R(k) decreases with k, as ratio of energy of collective expansion over thermal energy increases

$$R(k) = R[(y \tanh y)^{-1} - \sinh^{-2} y]$$

with $y = k\gamma\beta/T$

faster pions are more likely to be emitted near the point on spherical shell expanding with velocity β in direction of k (S.Pratt)



First successful hydrodynamic description of HBT data



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Velocity of source emitting pions relative to pair velocity



R_{long} - Longitudinal Expansion of Fireball

due to finite T, pions are correlated over an interval in rapidity and due to expansion this grows linearly with duration of expansion R_{long} is longitudinal correlation length

$$R_{long} \approx \tau \sqrt{T_f/m_t}$$
 Y.Sinyukov

thermal velocity

duration of expansion (lifetime) τ

of the system can be estimated from the transverse momentum dependence of R long

 $\tau = 6 - 8 \text{ fm/c}$ for $T_f = 120 - 160 \text{ MeV}$ CERES Pb-Au Nucl. Phys. A714 (2003) 124



Hubble plot of nuclear fireball

Freeze-out volume vs. beam energy

estimate freeze-out volume
$$V_f$$
: $V_f = (2\pi)^{3/2} R_{side}^2 R_{long}$



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Densities at chemical and thermal freeze-out



volume appears only to grow 30 % between chemical and thermal freeze-out get from the 2 densities, the velocity and final radii: isentropic expansion for 0.9-2.3 fm $T_f = 158 - 132$ MeV, rate of cooling near T_c : (13 ± 1) % per fm/c

ALICE PbPb collisions at the LHC at $\sqrt{s_{nn}} = 2.76$ TeV



kt dependence shows shape

typical for hydrodynamically expanding source rather similar to RHIC data but radii are larger Rout/Rside dropping below 1 reproduced reasonably well by Krakow and Kiev hydro models

HBT radii much bigger than 1-d charge radii of colliding nuclei

Updated energy dependence of HBT radii

Phys. Lett. B696(2011)328



 $<k_t> = 0.3 \text{ GeV/c}$

 significant extension of reach of world data by ALICE at higher energies radii grow as cube root of multiplicity indicating freeze-out at constant density
hydro models reproduce growth reasonably well

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freeze-out volume and duration of expansion

coherence volume $V = (2\pi)^{3/2} R_{side}^2 R_{long}$



ALICE, Phys. Lett. B696(2011)328

Volume in pp and heavy ion collisions different



PbPb coll.scale with cube root of multiplicity i.e. freeze-out at constant density for different centralities and energies

pp falls on different curves, i.e. at a given multiplicity pp physics different

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