Particle Production in p+p and A+A and the Maximum-Entropy Principle

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Outline

Goal:
Find a universal distribution which describes single-particle production in p+p and A+A based on the maximum-entropy method

- Introduction: the maximum-entropy method
- The light-cone plasma distribution
- Comparison with p+p and A+A data

Introduction (I): Number of States in a Phase Space Cell

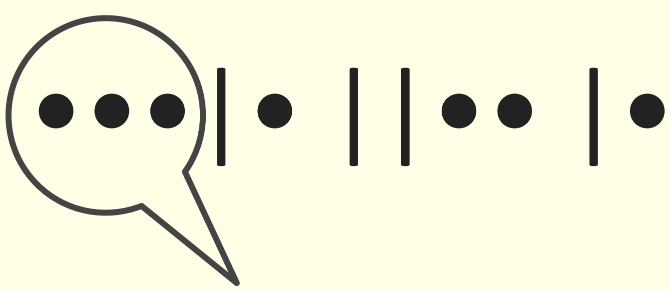
Consider gluons in a box

Number of quantum states in phase space cell d^3*r* d^3*p*
(phase space volume divided by h^3 times degeneracy g = 2×8):

$$G_{\vec{r},\vec{p}} = g \frac{d^3r \, d^3p}{(2\pi)^3} \qquad (\hbar = 1)$$

Number of possibilities to distribute *N* gluons on *G* quantum states:

$$\Delta\Gamma_{\vec{r},\vec{p}} = \frac{(G_{\vec{r},\vec{p}} + N_{\vec{r},\vec{p}} - 1)!}{(G_{\vec{r},\vec{p}} - 1)! \, N_{\vec{r},\vec{p}}!}$$



quantum state with 3 gluons

Introduction (II): Entropy for Bosons

Integration over the full phase space volume yields entropy *S*:

$$S = \sum \ln \Delta\Gamma_{\vec{r},\vec{p}}$$
$$= \sum G_{\vec{r},\vec{p}} \left[(1 + n_{\vec{r},\vec{p}}) \ln(1 + n_{\vec{r},\vec{p}}) - n_{\vec{r},\vec{p}} \ln n_{\vec{r},\vec{p}}\right]$$

Measure: $$\sum G_{\vec{r},\vec{p}} \ldots := \frac{g}{(2\pi)^3} \int d^3r \, d^3p \ldots$$

Mean occupation number: $$n_{\vec{r},\vec{p}} = \frac{N_{\vec{r},\vec{p}}}{G_{\vec{r},\vec{p}}}$$

Introduction (III):
Bose Distribution from the Max. Entropy Method

Use total energy of the gluon gas as constraint:

$$\sum G_{\vec{r},\vec{p}}\, E\, n_{\vec{r},\vec{p}} = \langle E \rangle \qquad \text{i.e.,} \quad \frac{g}{(2\pi)^3} \int d^3r d^3p\, E\, n_{\vec{r},\vec{p}} = \langle E \rangle$$

With 1/*T* as Lagrange multiplier the functional *S* + 1/*T* <*E*> needs to be maximized with respect to *n*. This yields the Bose distribution:

$$\frac{\delta\left(S + 1/T \cdot \langle E \rangle\right)}{\delta n_{\vec{r},\vec{p}}} = 0 \quad \Rightarrow \quad n_{\vec{r},\vec{p}} = \frac{1}{e^{E/T} - 1}$$

How can we apply this principle to describe particle production in p+p?

Maximum Entropy Method in p+p (I): Integrating out Coordinate Space

Light-cone variables: $p_+ = E^{\text{particle}} + p_z^{\text{particle}}, \qquad x_- = \frac{1}{2}(t-z)$

Scale with P_+: $x = p_+/P_+, \qquad P_+ = E^{\text{beam}} + p_z^{\text{beam}}$

Number of quantum states in a phase space cell: $$\hat{G}_{b_\perp, p_\perp, p_+, x_-} = g \frac{d^2 b_\perp d^2 p_\perp dp_+ dx_-}{(2\pi)^3}$$

Transverse coordinate space: $$\int d^2 b_\perp =: L_\perp^2$$

Sea quarks & gluons: $$\Delta x_- \approx 2\pi/\Delta p_+ \approx 2\pi/(xP_+) \rightarrow \frac{1}{2\pi}\int dp_+ dx_- \approx \int \frac{dx}{x}$$

This motivates $$G_{x,p_\perp} = g L_\perp^2 \frac{d^2 p_\perp}{(2\pi)^2}\frac{dx}{x}$$ (Note: d*x*/*x* = d*y*, *y* = rapidity)

Maximum Entropy Method in p+p (II): Constraints

1. Light-cone momentum fractions of the gluons sum up to unity (in one hemisphere)

$$\sum G_{x,p_\perp} x \, n_{x,p_\perp} = 1$$

2. Total transverse energy created in the collisions takes a certain value:

$$\sum G_{x,p_\perp} \, p_\perp n_{x,p_\perp} = \langle E_\perp \rangle$$

The maximum entropy distribution is found by solving:

$$\frac{\delta(S + \frac{1}{\lambda} \sum p_\perp n_{x,p_\perp} + w \sum x n_{x,p_\perp})}{\delta n_{x,p_\perp}} = 0$$

The Lagrange parameters are the transverse temperature λ and the softness *w*

The Light-Cone Plasma Distribution

$$n(x, p_\perp) = \frac{1}{e^{\frac{p_\perp}{\lambda} + xw} - 1}$$

λ: transverse temperature, *w*: softness

Example: Calculation of total gluon multiplicity

$$N/2 = \sum G_{x,p_\perp}\, n_{x,p_\perp} = \frac{gL_\perp^2}{(2\pi)^2} \int d^2 p_\perp \int_{x_{\min}}^{1} \frac{dx}{x}\, n(x, p_\perp)$$

$$x_{\min} = \sqrt{m_\pi^2 + p_\perp^2}$$

Transformation to Rapidity

The light-cone plasma distribution together with the measure generates a rapidity distribution of the following form:

$$\frac{dN}{dy d^2 p_\perp} = \frac{g L_\perp^2}{(2\pi)^2} \frac{1}{\exp\left[p_\perp \left(\frac{1}{\lambda} + \frac{w e^{|y|}}{\sqrt{s}}\right)\right] - 1}$$

When we compare to data we apply a simple form of parton-hadron duality and replace p_T by $m_T = \sqrt{(m^2+p_T^2)}$ where $m = m_{\text{pion}}$

Unlike in e^+e^- it cannot be expected in p+p that the total center-of-mass energy is available for particle production. This may be described by an effective energy:

$$\sqrt{s_{\text{eff}}} = \sqrt{s} - 2\langle E_{\text{leading}} \rangle \equiv K_{\text{eff}} \sqrt{s}$$

$K_{\text{eff}} \approx 0.5$ at RHIC
($\sqrt{s}$ = 200 GeV)
PRC, 74, 021902(R) (2006)

Parameters of the Light-Cone Plasma Distribution

λ, w, $L_\perp$ from the following three equations:

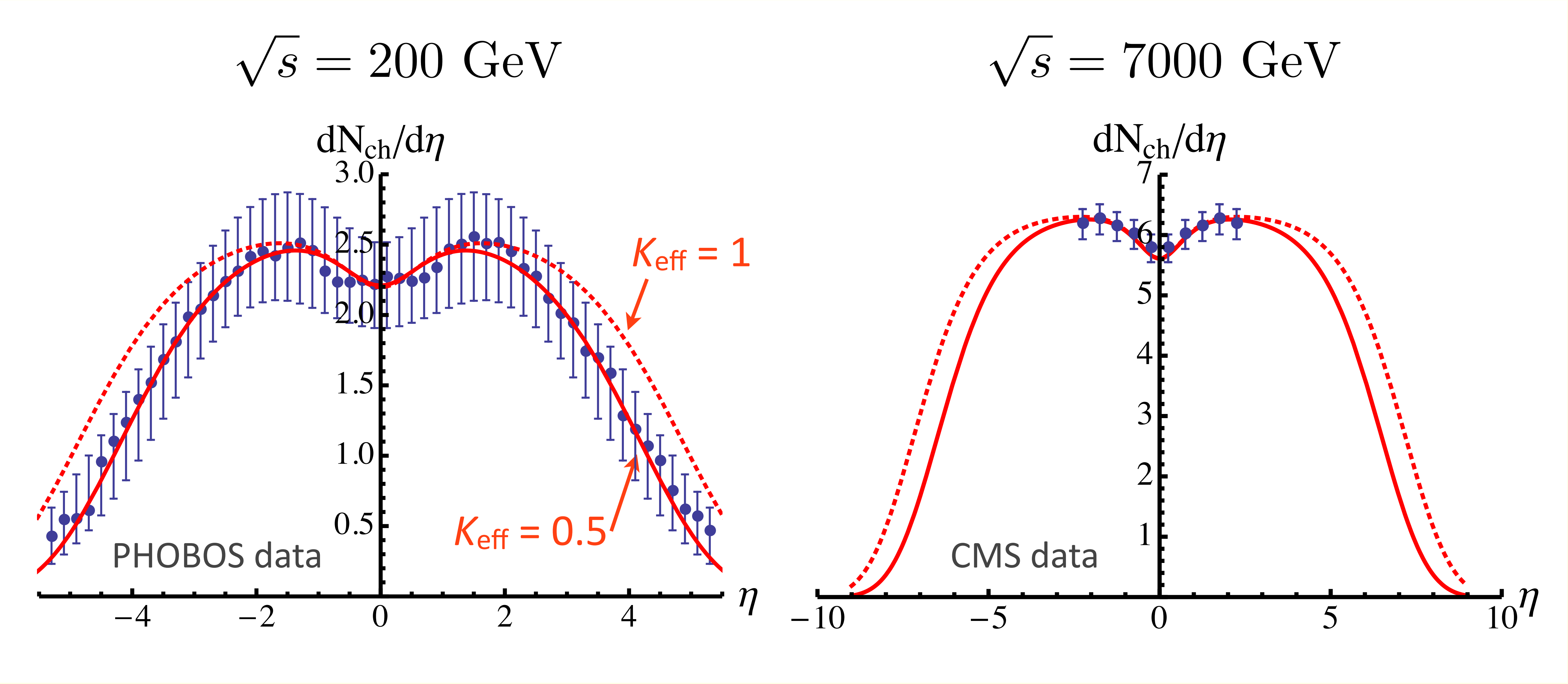
$$dN_{ch}/d\eta|_{\eta=0}(\text{theo.}) = dN_{ch}/d\eta|_{\eta=0}(\text{exp.})$$
$$\langle p_T \rangle_{\eta=0}(\text{theo.}) = \langle p_T \rangle_{\eta=0}(\text{exp.})$$
$$\sum G_{x,p_\perp} x \, n_{x,p_\perp} = 1$$

Results for $K_{\text{eff}} = 0.5$:

$\sqrt{s}$ (TeV)	$\langle p_T \rangle\|_{\eta=0}$ (GeV/c)	$dN_{\text{ch}}/d\eta\|_{\eta=0}$	λ (GeV/c)	w	$L_\perp$ (fm)
0.2	0.39	2.2	0.183	3.442	1.344
0.9	0.45	3.48	0.216	5.362	1.250
2.76	0.50	4.56	0.252	6.806	1.28
7	0.56	5.65	0.288	8.214	1.20

Pseudorapidity Distributions in p+p Collisions at √s = 200 and 7000 GeV

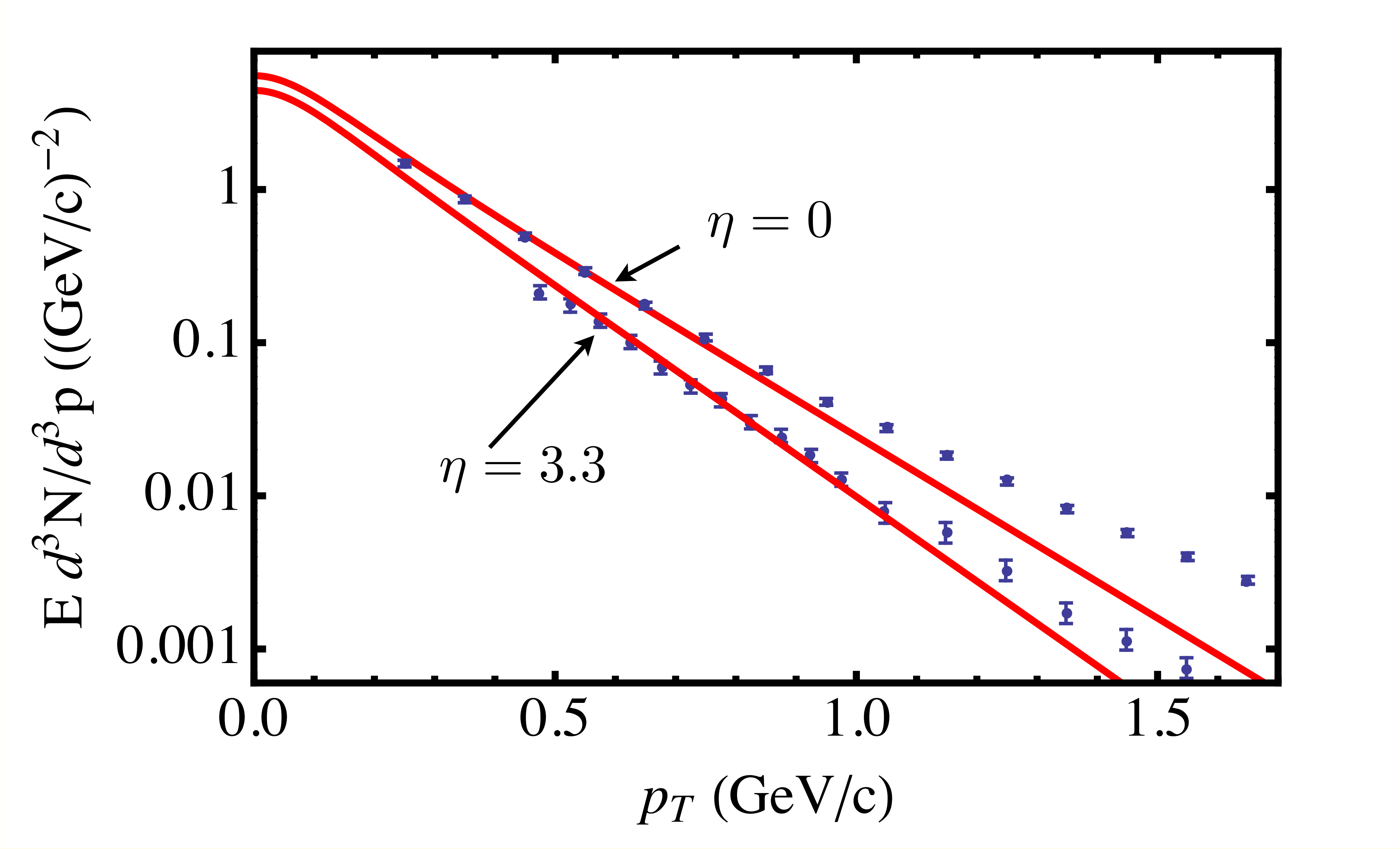
$$\frac{dN_{ch}^{AA}}{d\eta d^2 p_\perp} = \frac{2}{3}\sqrt{1 - \frac{m_\pi^2}{m_\perp^2 \cosh^2 y}}\frac{dN}{dy d^2 p_\perp}.$$



Transverse Momentum Distributions at √s = 200 GeV

$\eta = 0: \quad (h^+ + h^-)/2$ from UA1, $p\bar{p}$ at $\sqrt{s} = 200$ GeV

$\eta = 3.3: \quad \pi^+$ from Brahms, pp at $\sqrt{s} = 200$ GeV



Light-cone plasma distribution describes the low-p_T part (soft physics)

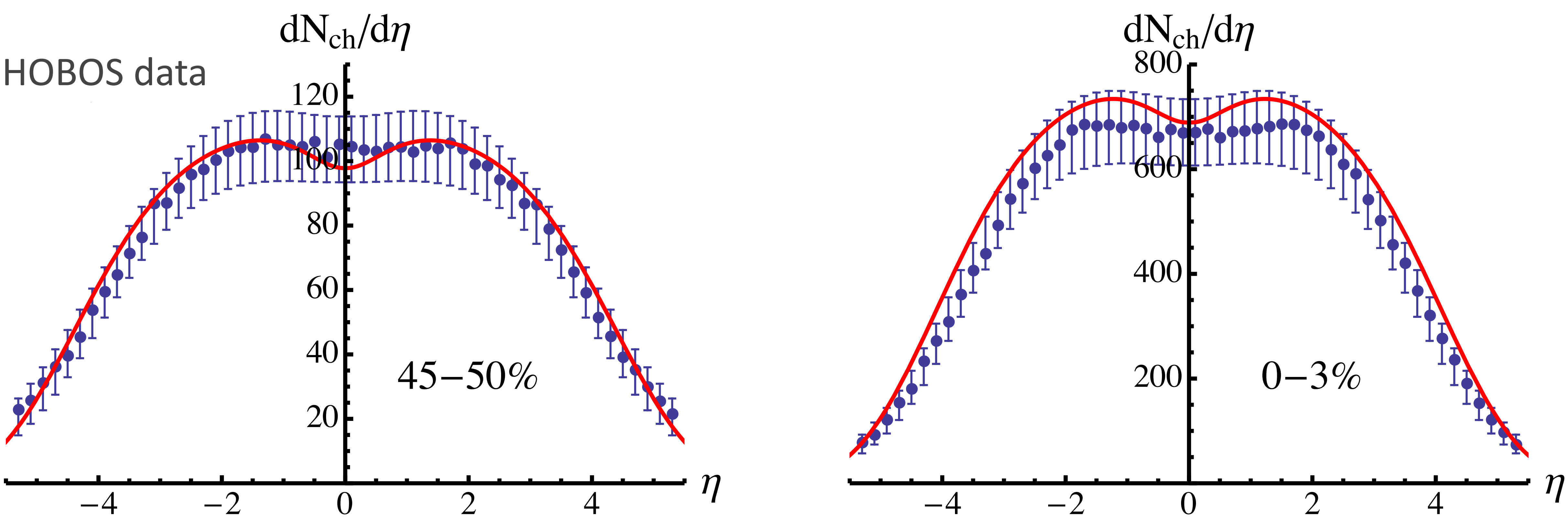
Generalization to A+A

N_{part} scaling + transverse momentum broadening (+ K_{eff} = 1):

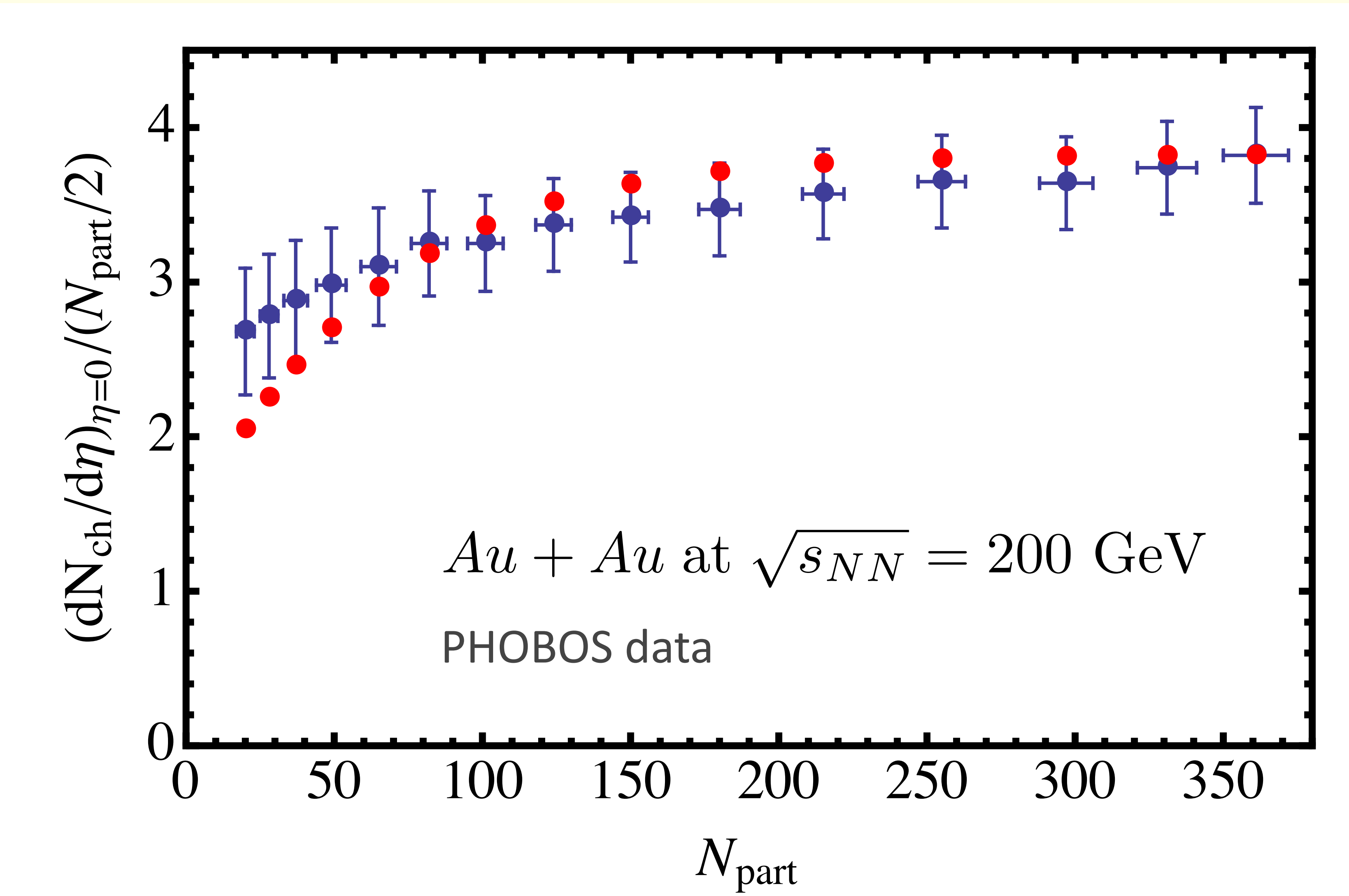
input: <p_T>(N_{part})

$$\frac{dN_{ch}^{AA}}{d\eta d^2 p_\perp} = \frac{N_{\mathrm{part}}}{2} \frac{2}{3} \sqrt{1 - \frac{m_\pi^2}{m_\perp^2 \cosh^2 y}} \frac{dN(\langle p_\perp \rangle)}{dy d^2 p_\perp}.$$

PHOBOS data



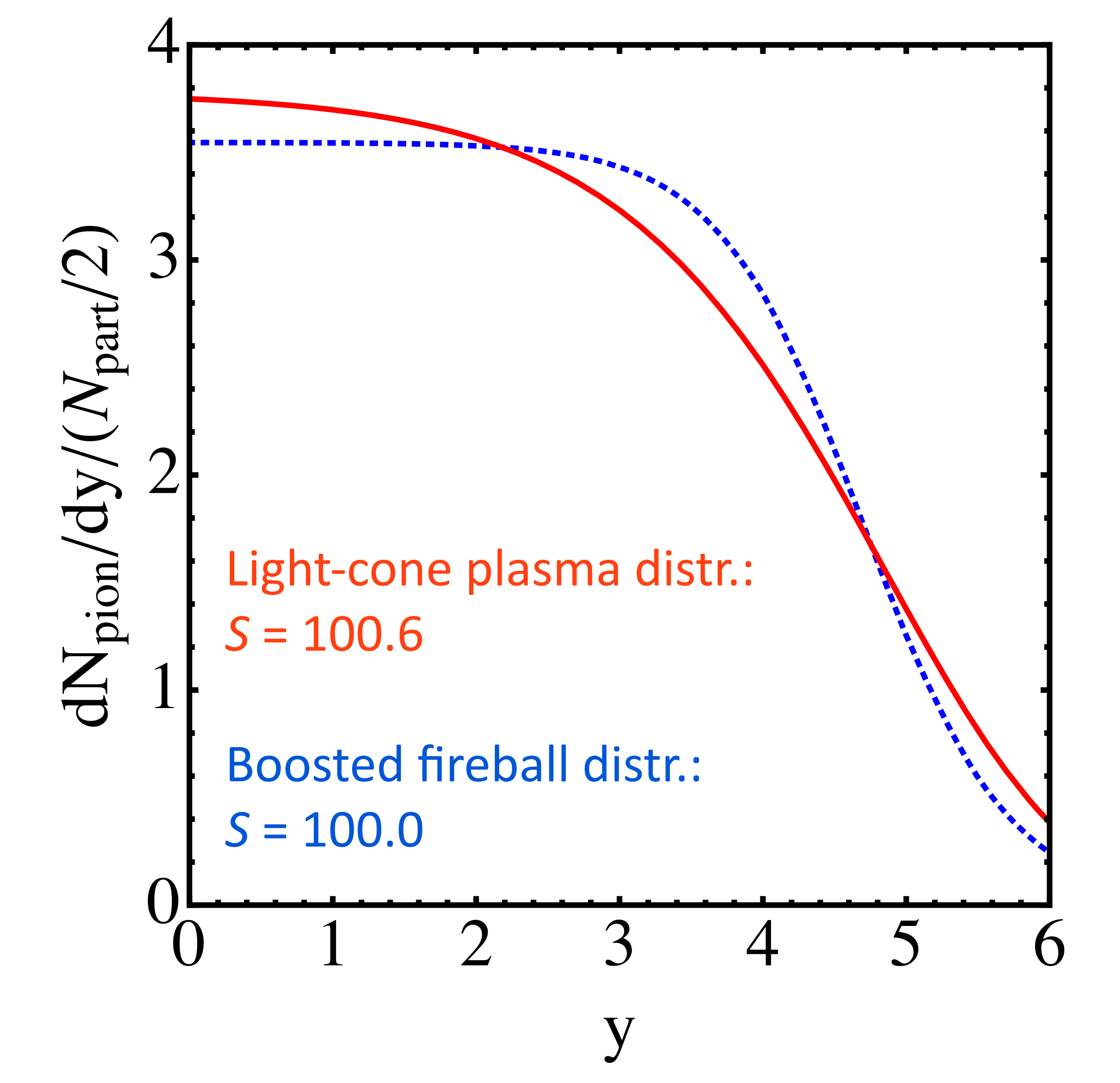
Centrality Dependence of dN_{ch}/dη in Au+Au at $\sqrt{s_{NN}}$ = 200 GeV



$L_\perp$ = 1.12 fm fixed
(in central collisions)

λ and w calculated from conservation of light-cone momentum sum rule and $\langle p_T \rangle(N_{\text{part}})$

Entropies of the Boosted Fireball Distribution and the Light-Cone Plasma Distribution



Boosted fireballs:
superposition of stationary isotropic fireballs in rapidity

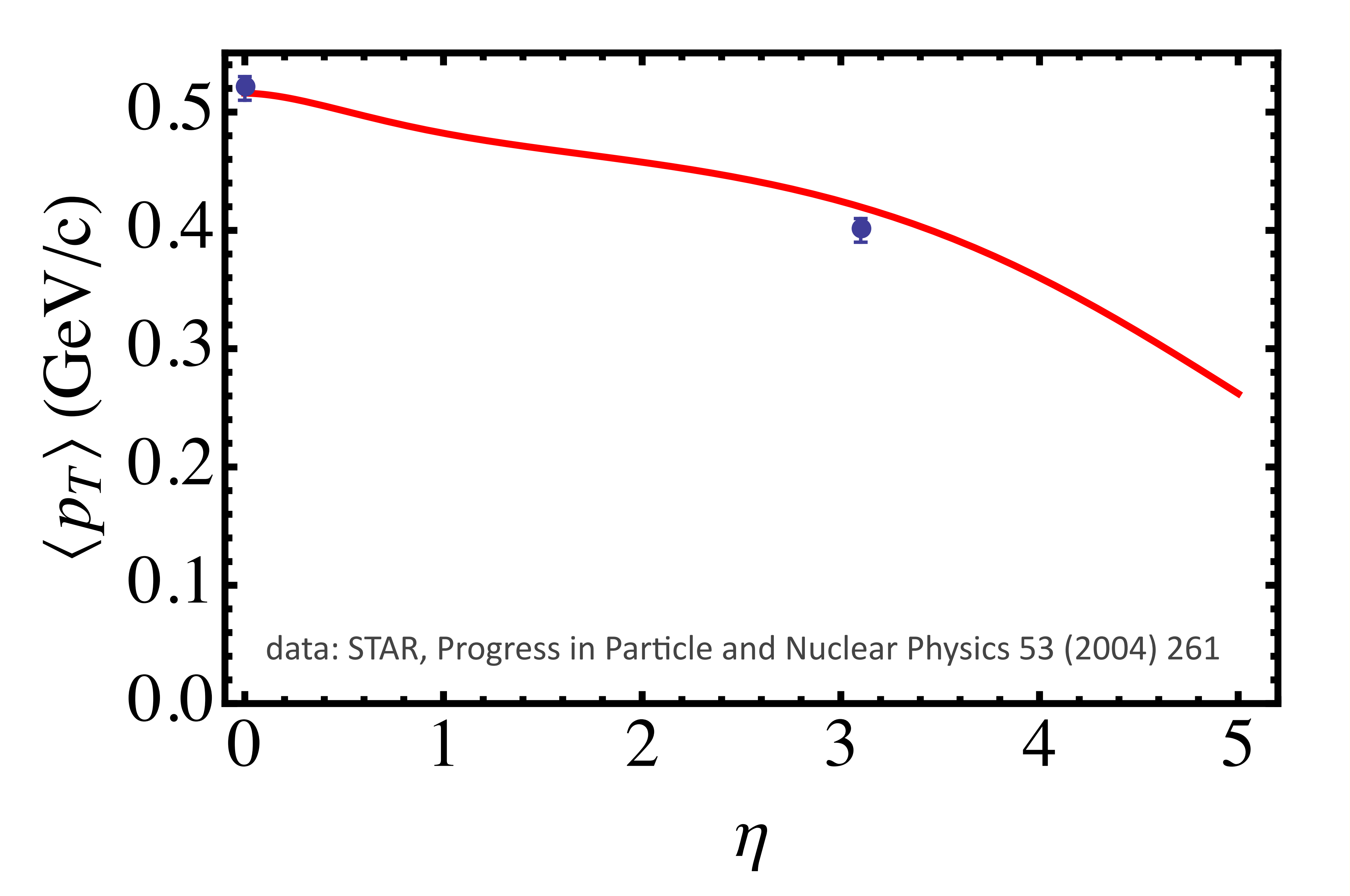
$$\frac{dN_0^{\text{bt}}(y, p_\perp)}{dy d^2 p_\perp} = \frac{16V}{(2\pi)^3} \frac{E_p}{e^{E_p/T} - 1}$$

$$\frac{dN^{\text{bt}}(y, p_\perp)}{dy d^2 p_\perp} = \int_{-\eta}^{\eta} du \frac{dN_0^{bt}(y - u, p_\perp)}{dy d^2 p_\perp}$$

Schnedermann, Sollfrank Heinz,
PRC 48 (1993) 2462-2475
Braun-Munzinger, Stachel, Wessel, Xu,
Phys.Lett. B344 (1995) 43-48

Entropy of the Light-Cone Plasma only slightly higher (percent effect)

< p_T > vs η in central Au+Au at √s = 200 GeV

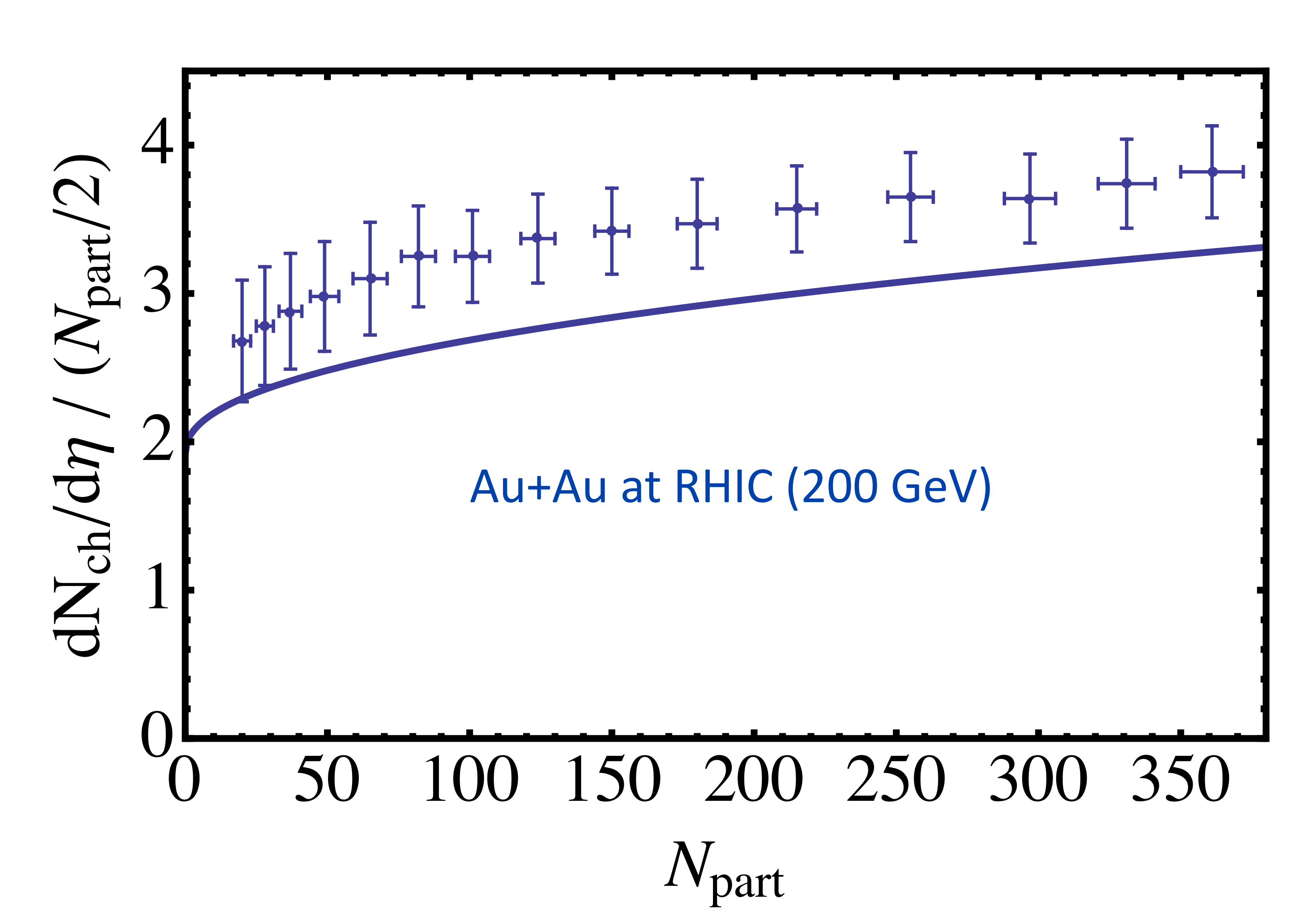


Decrease of < p_T > with η described by light-cone plasma distribution

An Extension of the Maximum Entropy Model: Increase of <p_T> with Centrality in A+A from Multiple Scattering

$$\langle \Delta p_\perp^2 \rangle = \langle \langle \sigma \cdot p_\perp^2 \rangle_g \rangle \cdot \langle T(b) \rangle \cdot \langle z^2 \rangle_{\pi/g}$$

- $\langle \Delta p_\perp^2 \rangle$: increase of p_T^2
- $\langle \langle \sigma \cdot p_\perp^2 \rangle_g \rangle$: parton cross section weighted with p_T^2
- $\langle T(b) \rangle$: path length ρ·*L* in the cold nucleus
- $\langle z^2 \rangle_{\pi/g}$: converts gluon p_T to pion p_T



- Parameterization of mean p_T replaced by a microscopic calculation
- Work in progress, currently ~ 10% below RHIC data

Conclusions

- There is a universal maximum-entropy distribution satisfying the constraints of a given transverse energy and unity-sum of parton light-cone momentum fractions

- It is non-thermal and has two parameters:
transverse temperature λ and softness w

- Entropy only slightly larger than for boosted fireballs (percent effect)

- It provides a good fit to the pp data at RHIC and LHC using a reasonable overlap area $L^2 \approx (1\ \mathrm{fm})^2$ and the correct number of degrees of freedom

- Extension to A+A: Participant scaling + p_T broadening