Exercises for the lecture
„Moderne Methoden der Datenanalyse"

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Exercise 4: Simulation of a Photomultiplier

A photomultiplier is a device capable of detecting individual photons as illustrated in fig. 1. A photon strikes the photocathode, where there is a certain probability for it to eject an electron (called a photoelectron). The photoelectron is accelerated in an electric field towards an electrode (called dynode). In the collision with the first dynode, the photoelectron can liberate further electrons. These are accelerated towards the second dynode, where more electrons are produced. This continues through a series of stages until the electrons produced at the final dynode are collected. The number of electrons produced at the i-th dynode for each incoming electron can be modeled as a Poisson variable n_i with mean value ν_i, which in general can be different for each stage. Suppose the photomultiplier has N dynodes. The number of electrons n_{out} produced at the final stage for a single incident photoelectron has an expectation value

$$\bar{v}_{\mathrm{out}} = E[n_{\mathrm{out}}] = \prod_{i=1}^{N} \nu_i \tag{1}$$

Further information on photomultipliers can be found in:

- W. R. Leo, *Techniques for Nuclear and Particle Experiments*, Chapter 8, Springer Verlag, Heidelberg.
- K. Kleinknecht, *Detektoren für Teilchenstrahlung*, Chapter 4.1, Teubner, Stuttgart.

For the following exercises, copy the template program `pmt.cc` and the corresponding `Makefile` to your working directory.

Exercise 4.1

Write a Monte Carlo program to determine the distribution of the number of electrons n_{out} at the end of $N = 6$ dynodes produced by a single initial photoelectron.

For this, generate Poisson random numbers with the ROOT function `gRandom->Poisson(nu)` with $\nu = 3.0$ for each dynode. Run the program to simulate the passage of $M = 10000$ initial photoelectrons, one-by-one, through the detector.

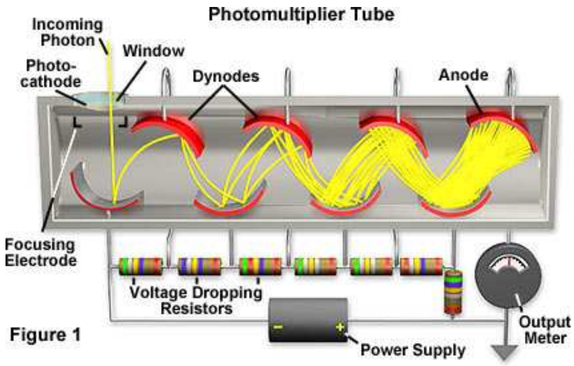

Figure 1: *Schematic drawing of a photomultiplier tube.*

Write the number of photoelectrons being emitted after each dynode stage into a histogram and, in particular, the number emitted from the final stage, n_{out}. Estimate the sample mean and variance using

$$\bar{n}_{\text{out}} = \frac{1}{M}\sum_{i=1}^{M} n_{\text{out},i} \tag{2}$$

$$V(n_{\text{out}}) = \frac{1}{M-1}\sum_{i=1}^{M}(n_{\text{out},i} - \bar{n}_{\text{out}})^2 \tag{3}$$

Compare the sample mean to the value from equation 1. Compare the sample variance (or standard deviation $\sigma(n_{\text{out}}) = \sqrt{V(n_{\text{out}})}$) to the value that one would obtain from a Poisson-distributed variable with mean ν_{out}. Form equation 3 in such a way that one loop is enough to calculate it. Explain qualitatively why the standard deviation of n_{out} is much larger than in the Poisson case.

Implementation proposal:

In the template, we suggest to implement Exercise 4.1 in the function `dynodes6` which takes five parameters: (1) the expectation value of the first dynode ν_1, (2) the expectation value of the other dynodes (2-6), (3) the number of dynodes in the PMT, (4) the number of experiments M, (5) a pointer to an array of histograms and (6) a pointer to a profile histogram (needed in Exercise 3). Further we suggest to implement the individual experiment in which one initial photoelectron is traced through the photomultiplier in the function `pmt`.

Exercise 4.2

Ideally, one would like the standard deviation of $\bar{n}_{\text{out}}$ to be as small as possible in order to determine as accurately as possible the number of photoelectrons emitted from the cathode (and thus estimate the number of photons entering the detector). In some applications, one would like to have the standard deviation small enough to distinguish between 1 and 2 photoelectrons and therefore one tries to have a relative resolution, i.e. the ratio of the standard deviation to the mean, less than unity. One way of achieving this is to increase the mean number of electrons produced at the first dynode. This can be done either by increasing the accelerating electric field or by using a dynode metal with a low work function, i.e. a high probability for secondary electron emission.

Repeat the simulation from Exercise 4.1 while increasing the mean number of electrons emitted by the first dynode to $\nu_1 = 6.0$. Estimate the ratio of the standard deviation to the mean of n_{out} for both values of ν_1. Explain qualitatetively why this gives a better resolution than in the case with equal ν_i. Why does it not help much to increase the gain of the dynodes in the later stages of the photomultiplier?

Exercise 4.3

Show that the mean of n_{out} is proportional to the number of electrons emitted by the first dynode. Use profile histograms to show this. How many electrons are emitted per electron of dynode one?

Exercise 4.4*

Extend your program to simulate $N = 12$ dynodes. It would take too much computing time to simulate the collision of each electron with each dynode. Instead, use the output of Exercises 4.1 and 4.2 for $N = 6$ with enough events to obtain a good estimate of the distribution of n_{out} (e.g. at least $M \sim 10^4$ events in a histogram with 50 bins from $0 \leq n_{\text{out}} \leq 5000$). Next, generate random numbers that follow this distribution using e.g. the acceptance-rejection method. For each electron obtained after the first six dynodes, generate in a similar way the number of electrons that it produces in the next six. For the first six stages, use a distribution of n_{out} based on $\nu_1 = 6.0$, $\nu_{2-6} = 3.0$; for the last six dynodes, take all $\nu_i = 3.0$. Repeat this Monte Carlo experiment 10,000 times.

Implementation proposal

The template contains a function `dynodes12` which can be used to implement the algorithm described above. `dynodes12` is meant to contain the code for one experiment.