High Energy Frontier - Recent Results from the LHC

University of Heidelberg WS 2012/13

Lecture 4

Higgs Physics I

Massive Vector Bosons in the SM

W and Z bosons are massive

$m_W = 80.40$ GeV

$m_Z = 91.18$ GeV

Masses of vector particles can be described by:

$$L = m_W^2 W_\mu W^\mu + m_Z Z_\mu Z^\mu$$

However, this Lagrangian is not gauge invariant under gauge transformations

long-standing problem...

Example of non-Gauge Invariance

$$m_\gamma^2 \, A_\mu A^\mu \;\rightarrow\; m_\gamma^2 \left(A_\mu + \frac{1}{e} J_\mu \alpha\right)\left(A^\mu + \frac{1}{e} J^\mu \alpha\right)$$

$$= \; m_\gamma^2 \left| A_\mu A^\mu + A_\mu \frac{1}{e} J^\mu \alpha + \frac{1}{e} J_\mu \alpha A^\mu + \frac{1}{e^2} (J^\mu \alpha)(J^\mu \alpha) \right|$$

equation can only be fulfilled if $m_\gamma = 0$

→ Gauge invariance requires massless gauge bosons!

Fermions in SM

3 generations of quarks and leptons represented by

- isospin doublets (left chirality) and singlets (right chiral states):

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

- isospin singlets (right chirality):

$$\begin{matrix} (\nu_e)_R & (\nu_\mu)_R & (\nu_\tau)_R & \quad & u_R & c_R & t_R \\ e_R & (\mu)_R & (\tau)_R & \quad & d_R & s_R & b_R \end{matrix}$$

Masses of fermions given by:

$$L = m_d \, d_L d_R + m_u \, d_L d_R + \ldots$$

violate gauge invariance in a similar way if $\Psi \rightarrow \Psi' = U \Psi$

Electroweak Unification

(Glashow, Salam, Weinberg Model)

$$H^{e.m.\,,\,weak} = \underset{\uparrow}{g}\, j_L W + \tfrac{1}{2}\, g'\, j^Y\, B$$

parity violating

SU(2)$_L$ x U(1)$_Y$ → SU(2)$_{weak}$x U(1)$_{e.m.}$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}$$

isospin_3

weak hypercharge

$$\begin{pmatrix} W^+ \\ W^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$$

→ electroweak symmetry breaking

What breaks the symmetry and makes the Z-boson massive?

Higgs mechanism

The problem with the gauge invariance can be solved by introducing a new scalar isospin doublet field Φ (instead of a mass constant) which respects gauge invariance.

$$L = \left(D_\mu \Phi\right)\left(D^\mu \Phi\right) + \ldots + \underset{\uparrow}{y_d} \phi \, d_L d_R + \underset{\uparrow}{y_u} \phi \, u_L u_R + \ldots \qquad \Phi = \begin{pmatrix} 0 \\ \phi \end{pmatrix}$$

Yukawa couplings

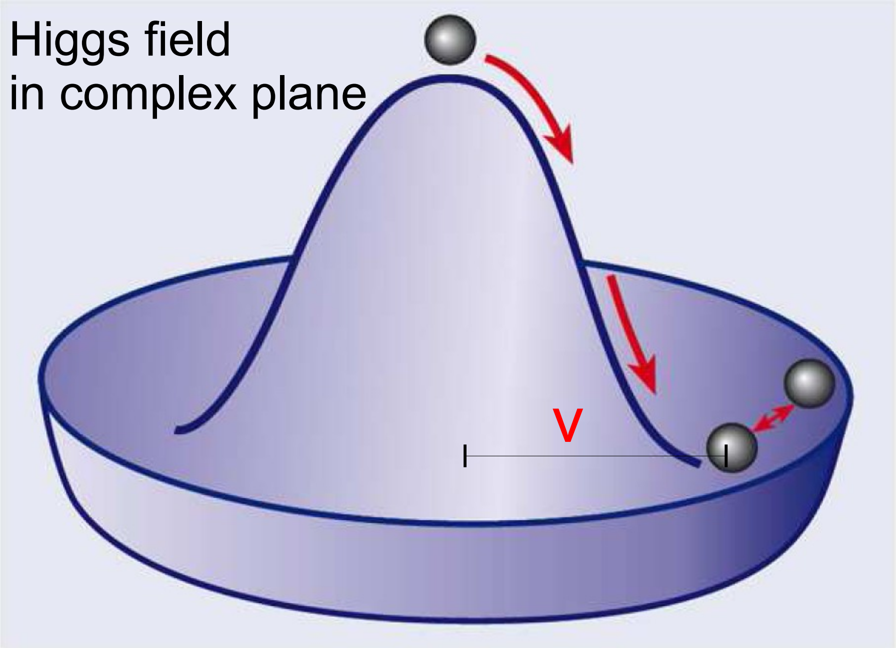
vector boson masses　　　fermion mass terms

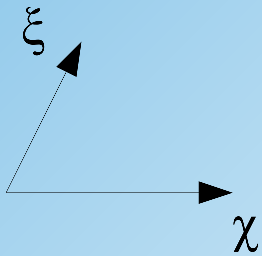
with the covariant derivative:

$$D_\mu = \partial_\mu + \left(i g \, W_\mu^j \sigma_j + i g' \, Y B_\mu\right)$$

This field creates **simultaneously** vector boson and fermion masses and breaks electroweak symmetry if the vacuum expectation values (vev) of the Higgs field is non-zero!

Higgs Field Parametrisation





choose minimum to be on the real axis (symmetry breaking):

$$\phi = v + \chi + i\xi$$

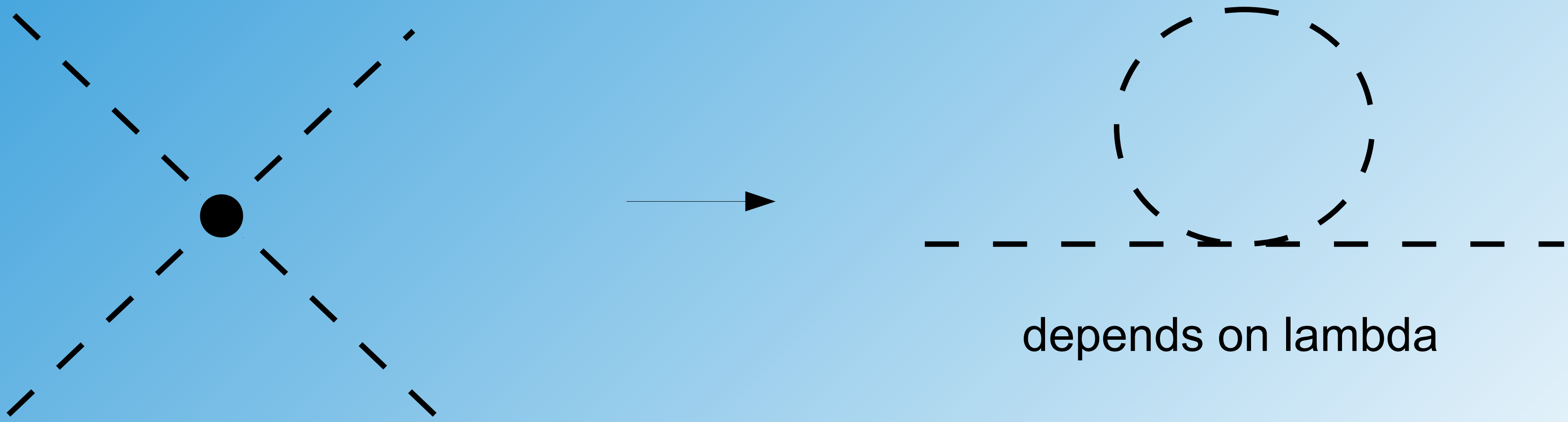
Mexican hat potential: $-\mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$

minimum if: $v^2 = \frac{-\mu^2}{2\lambda}$ (vev if lambda negative)

SM Lagrangian with Higgs Field

$$L=(D_\mu\Phi)(D^\mu\Phi)\ \underbrace{-\mu^2\Phi^*\Phi-\lambda(\Phi^*\Phi)^2}_{\text{Higgs self coupling}}+y_d\phi\, d_L d_R+y_u\phi\, u_L u_R+\ldots$$

Higgs self interactions



depends on lambda

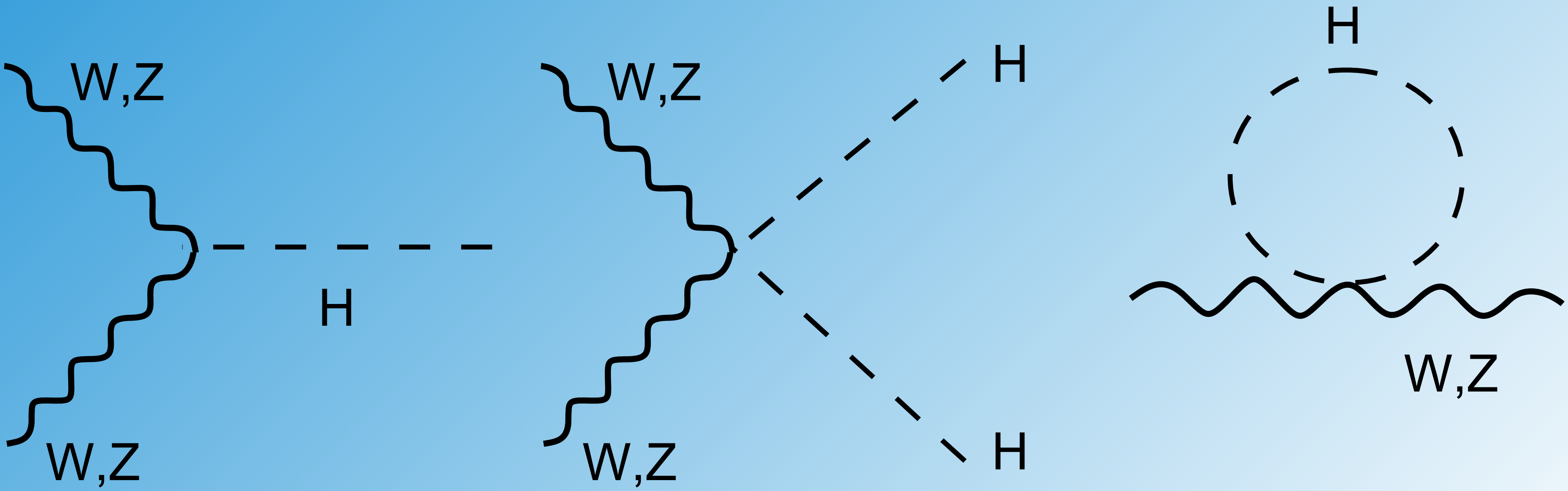
Higgs-Gauge Boson Interactions

$$L = \left(D_\mu \Phi\right)\left(D^\mu \Phi\right) - \mu^2\, \Phi^*\, \Phi - \lambda\, \left(\Phi^*\, \Phi\right)^2 + y_d \phi\, d_L d_R + y_u \phi\, u_L u_R + \ldots$$

Higgs vector boson interactions: $\tan \Theta_W = \frac{g'}{g}$ (Weinberg angle)

$$m_W^2 = \frac{1}{4} v^2\, g^2 \qquad m_Z^2 = \frac{v^2}{4}\left(g^2 + g'^2\right)$$

→ v=246 GeV



also here, coupling proportional to mass!

Higgs-Fermion Interactions

Higgs-Fermion Coupling:

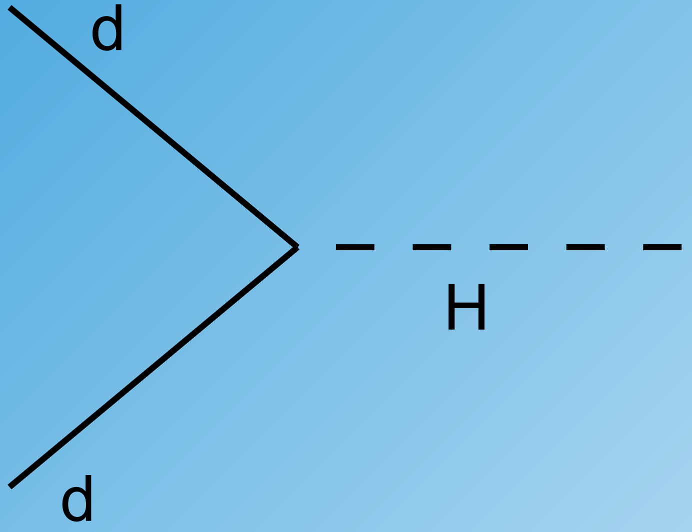
$$L_Y = - y_d \bar{L} \Phi d_R - y_u \bar{L} \tilde{\Phi} u_R$$

with $L = \begin{pmatrix} t \\ b \end{pmatrix}$, $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$,

$$\tilde{\Phi} = i \tau_2 \Phi^*$$

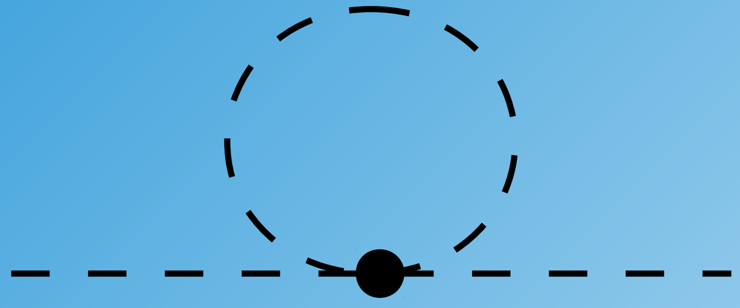
Higgs couples to masses:

$$m_{d,u} = \frac{y_{d,u} v}{\sqrt{2}}$$



coupling proportional to mass

Higgs Mass?



$\lambda\left(\Phi^{*}\Phi\right)^{2}$

coupling lambda is not known!

WW Scattering Amplitude

- Without Higgs boson, WW scattering violates unitarity at high energies

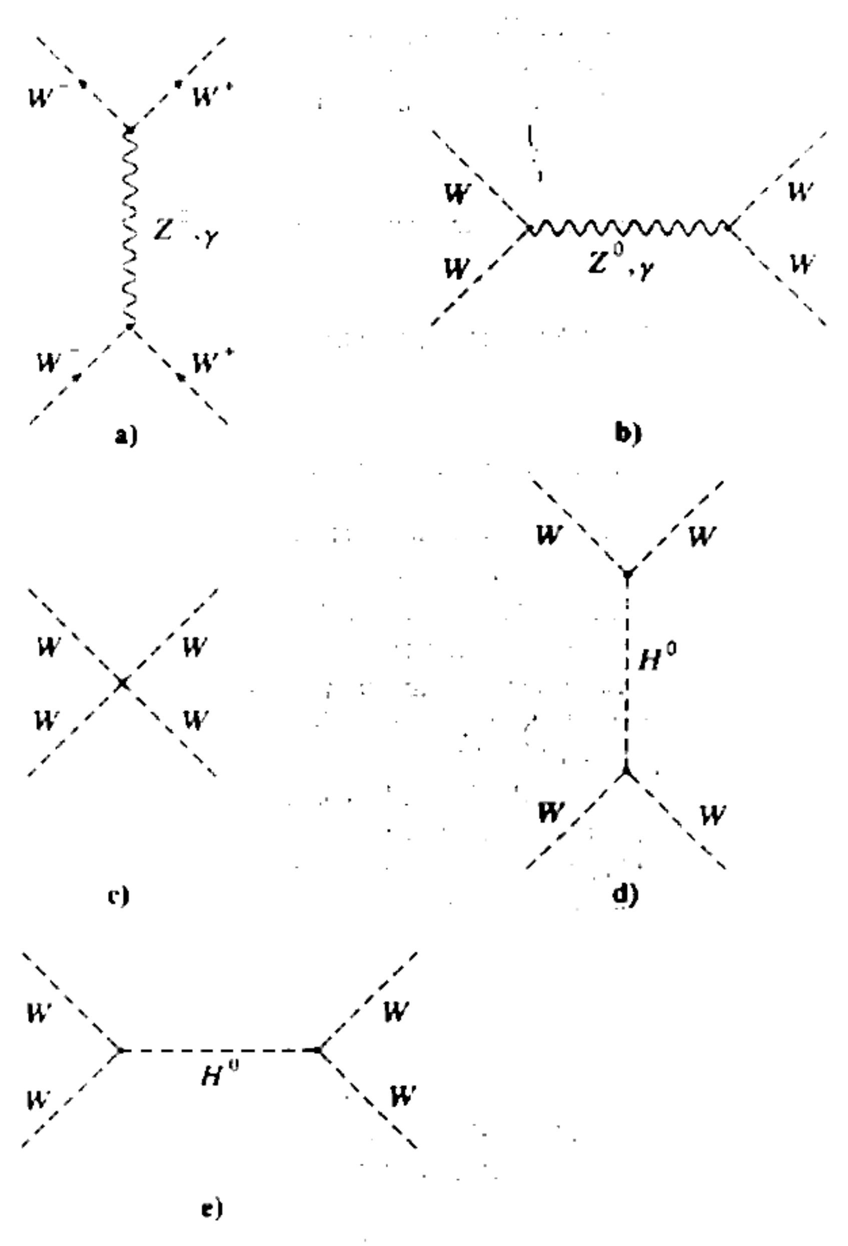
- A partial wave analysis yields :

$$|\Omega| \approx 2\,\frac{m_h^2}{v^2} < 8\pi$$

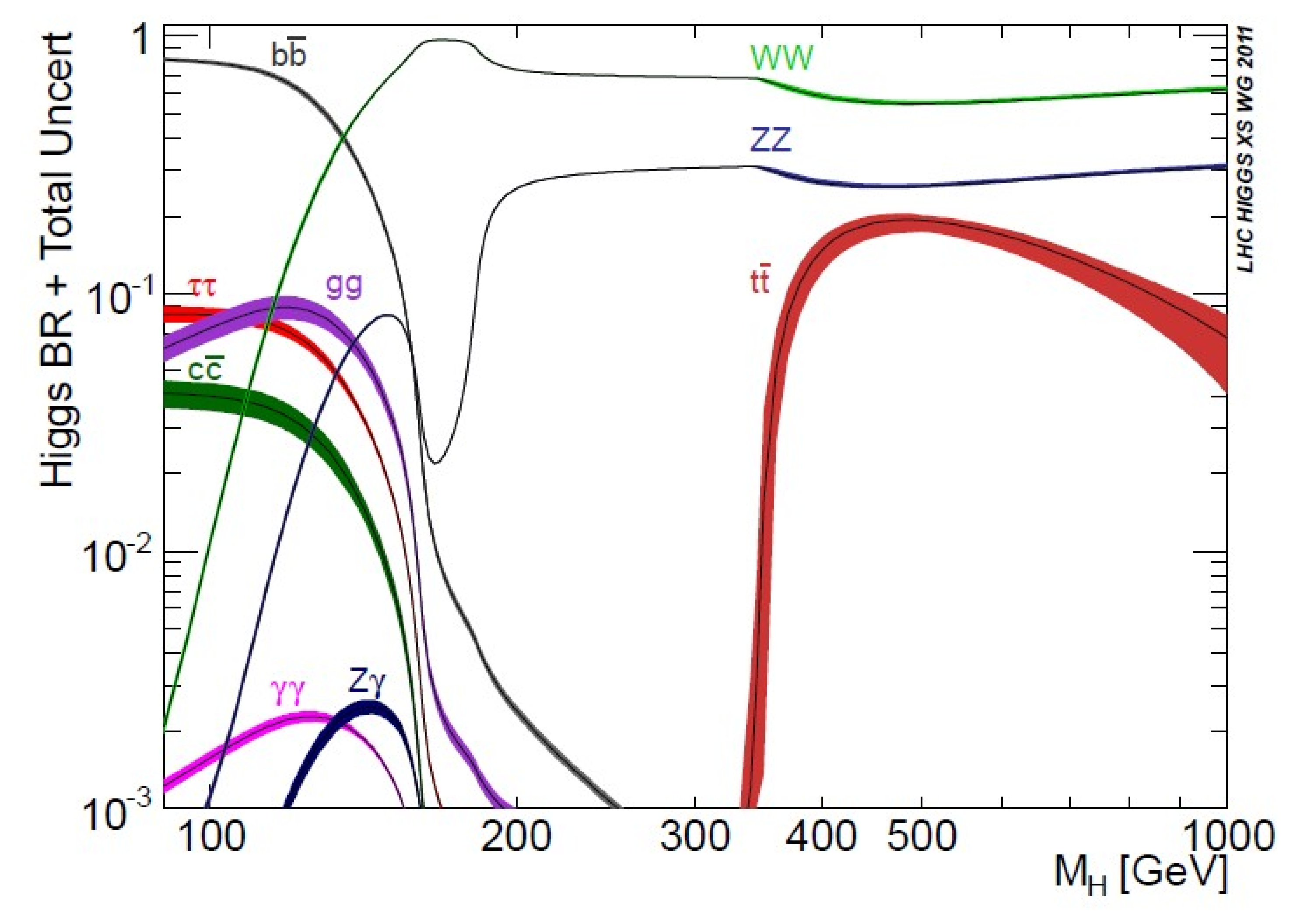
gives:

$$m_h < \left(4\pi\, v^2\right) \approx 870\ GeV$$

→ perturbative unitarity limit



Higgs Decays



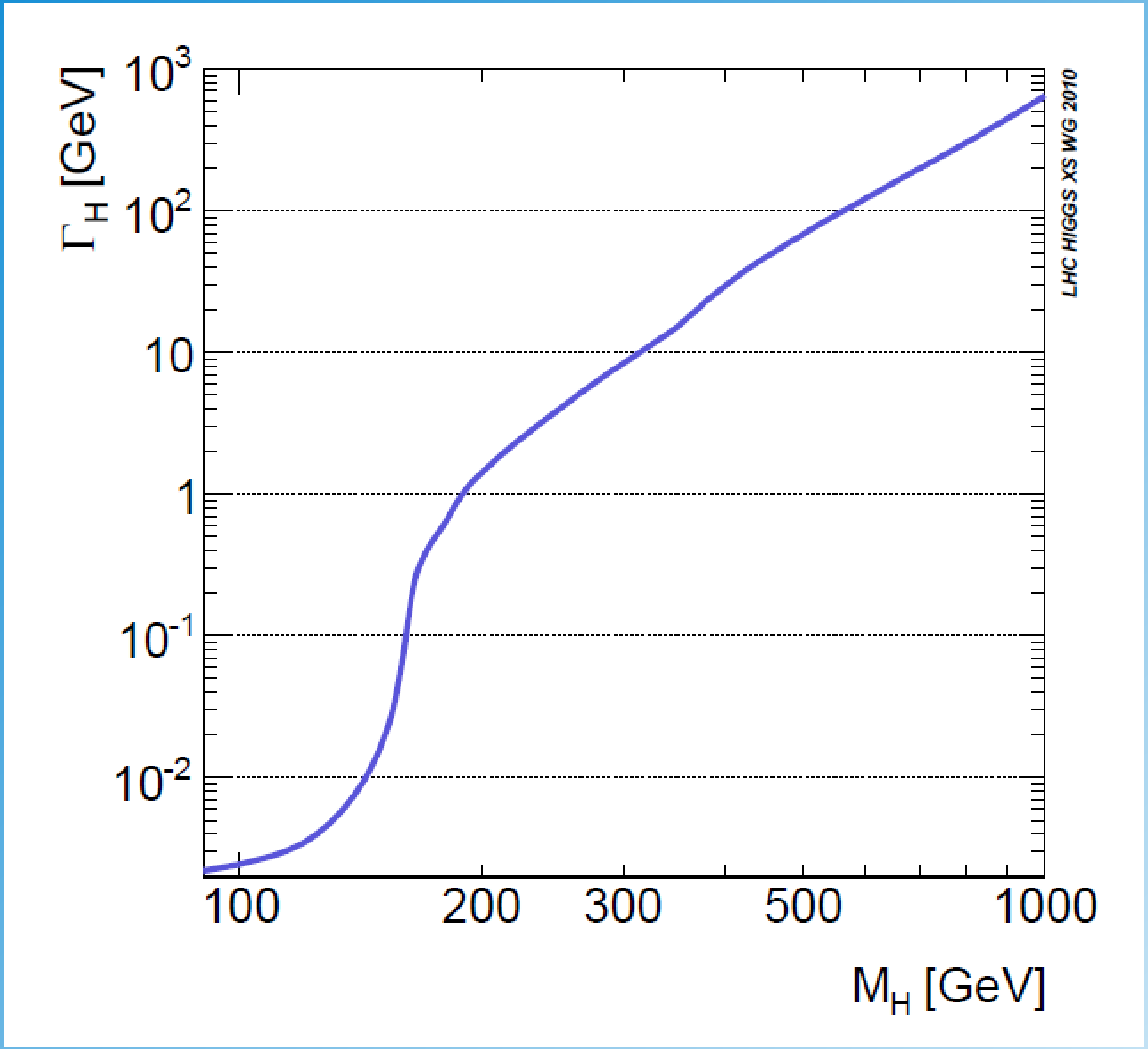
$$\Gamma(h \to l^+l^-) = \frac{G_F M_l^2}{4\sqrt{2}\pi} M_h \beta_l^3$$

$$\Gamma(h \to W^+W^-) = \frac{G_F M_h^3}{8\pi\sqrt{2}} \sqrt{1-r_W}(1 - r_W + \tfrac{3}{4}r_W^2)$$

$$\Gamma(h \to ZZ) = \frac{G_F M_h^3}{16\pi\sqrt{2}} \sqrt{1-r_Z}(1 - r_Z + \tfrac{3}{4}r_Z^2),$$

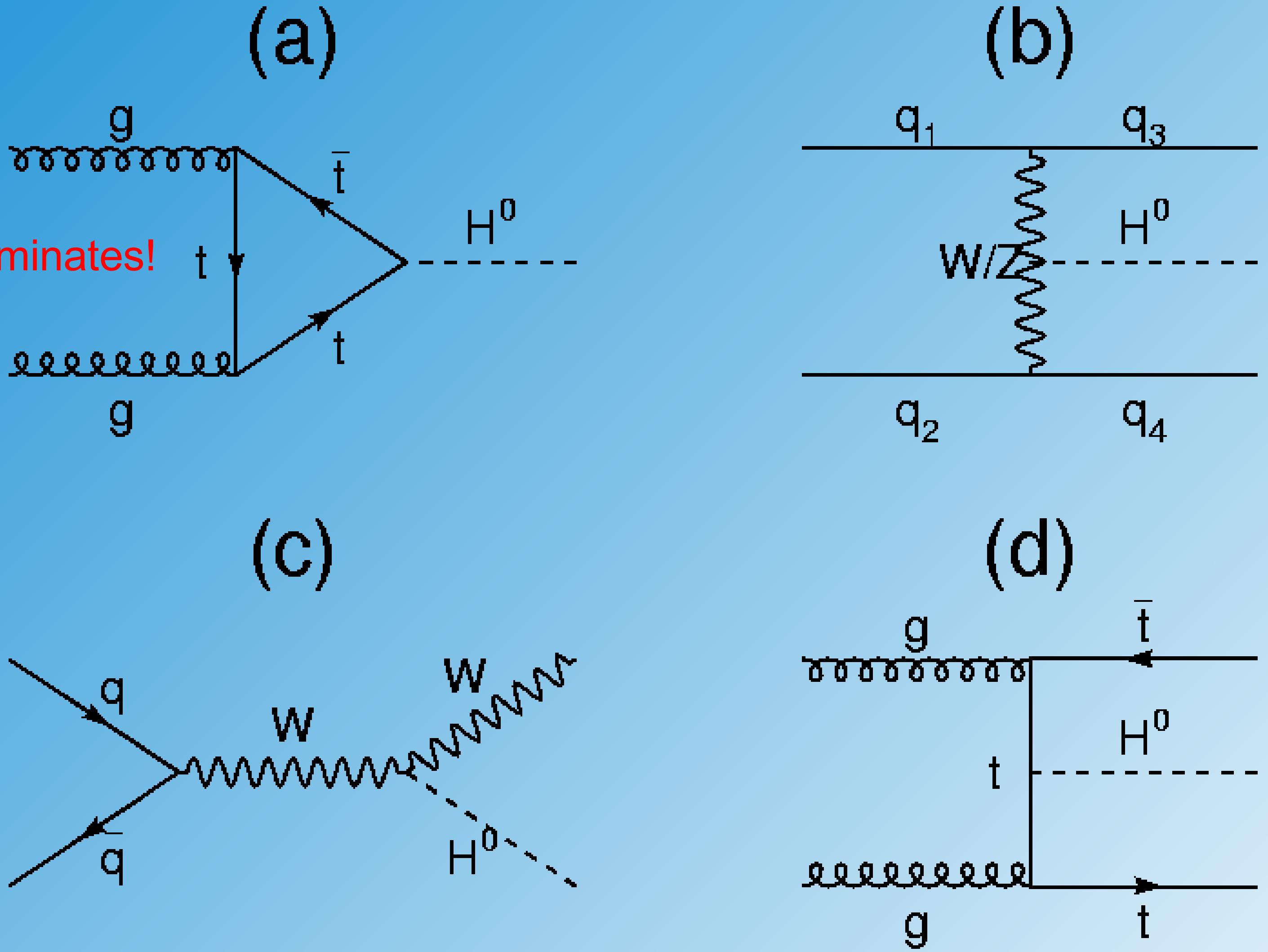
$$r_V \equiv 4M_V^2/M_h^2$$

Higgs Decay Witdh



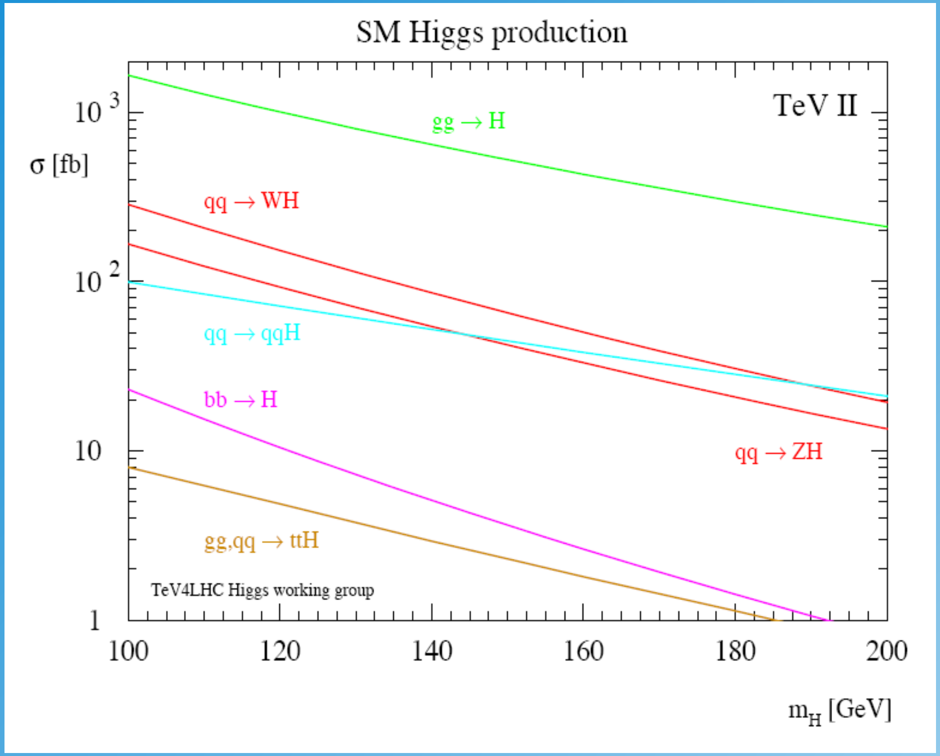
a heavy Higgs
would be very
broad!

Production of Higgs Bosons in Hadron Collisions

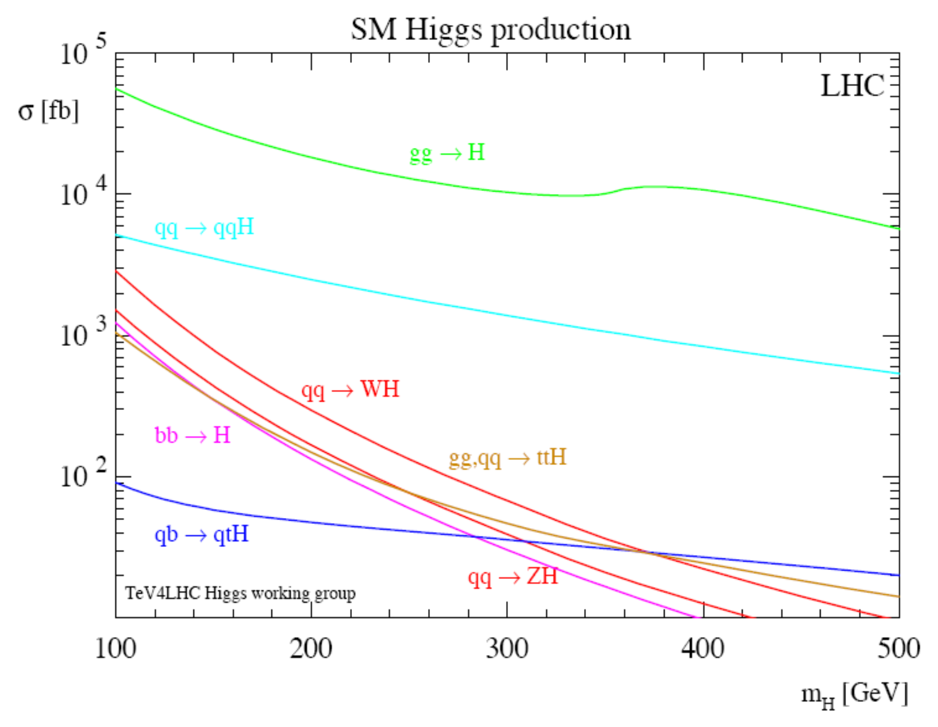


gluon fusion dominates!

Higgs Production at Hadron Colliders

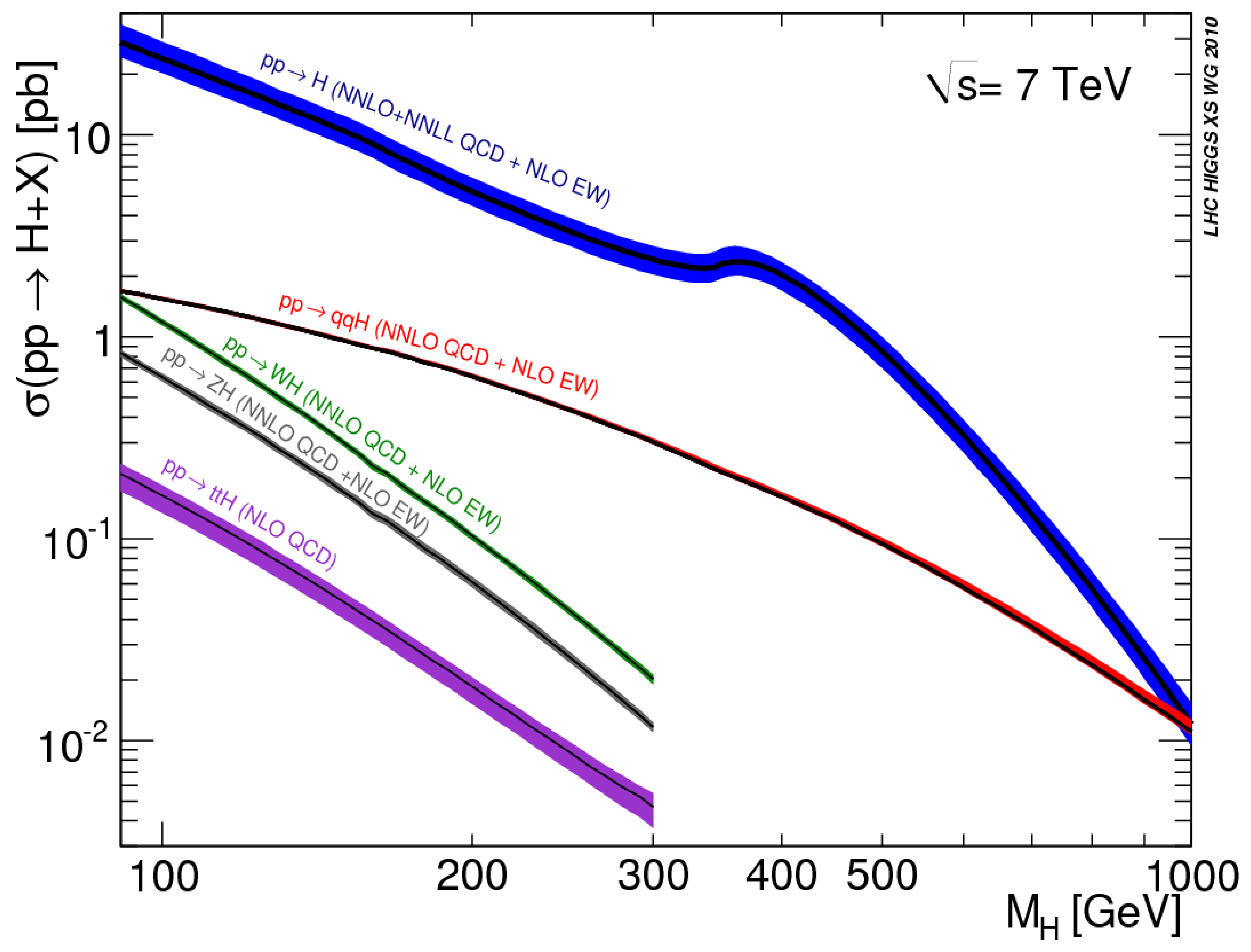


proton-antiproton at Tevatron $s^{1/2}$=2 TeV

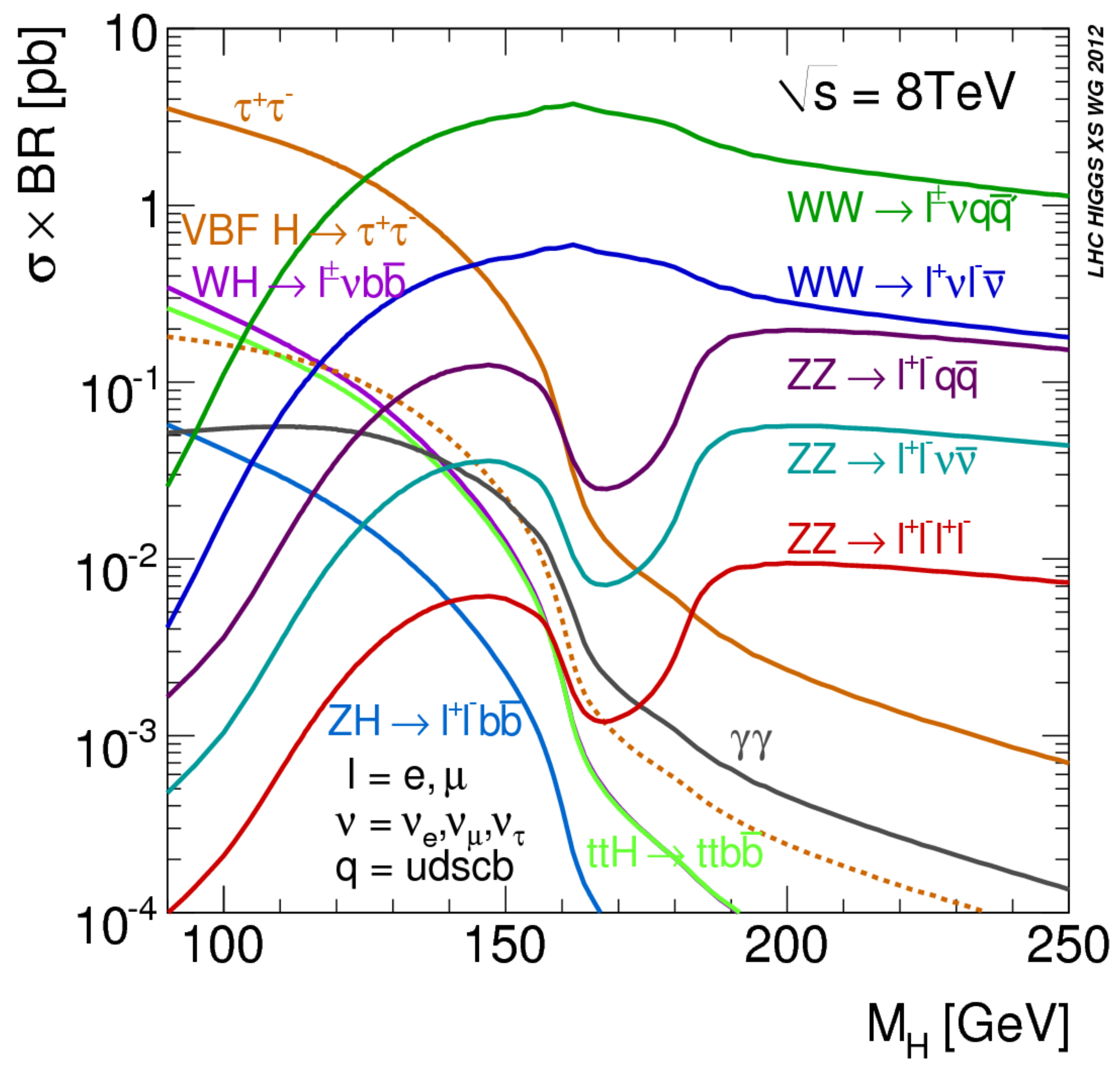


proton-proton at LHC $s^{1/2}$=14 TeV

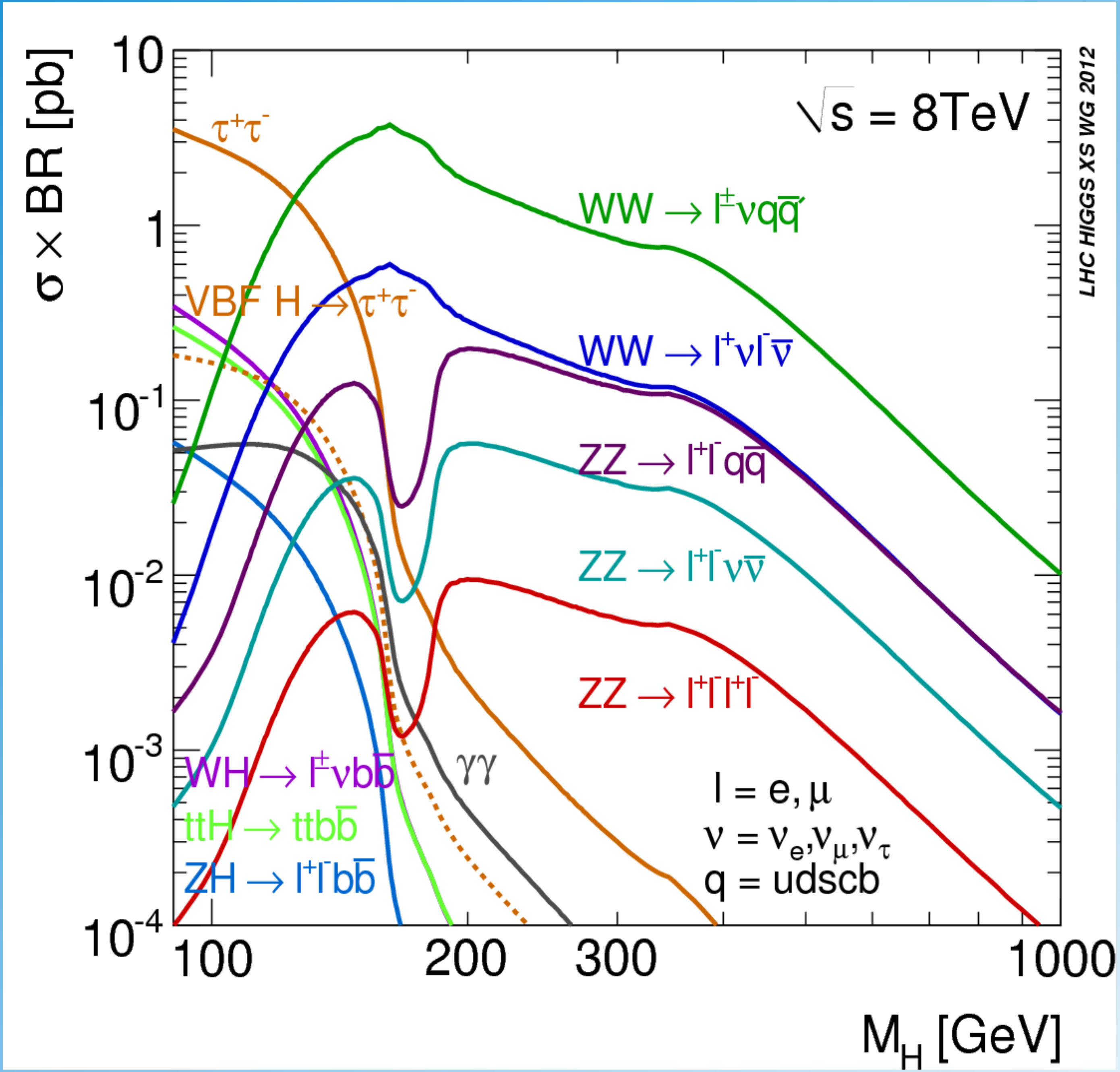
Higgs Production at Hadron Colliders



Higgs Search Channels



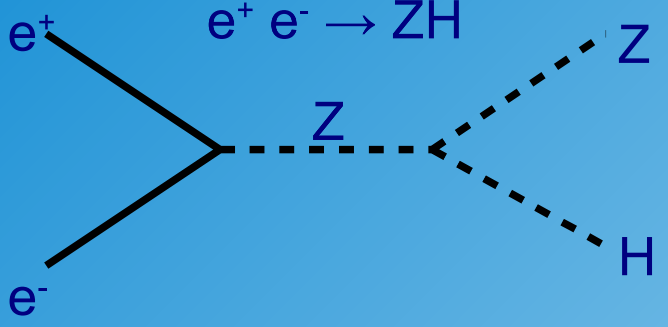
Higgs Search Channels

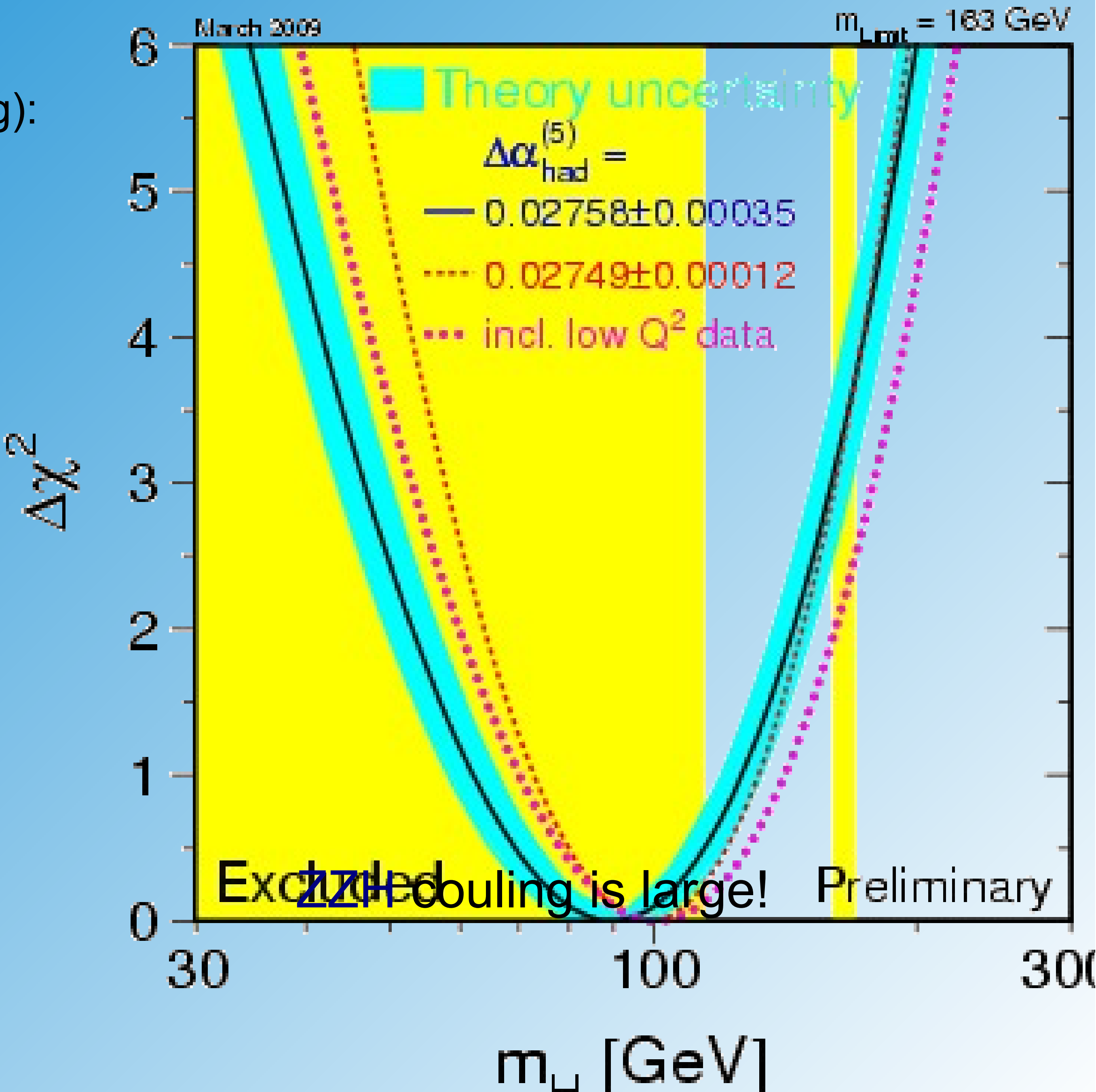


Situation before/without LHC

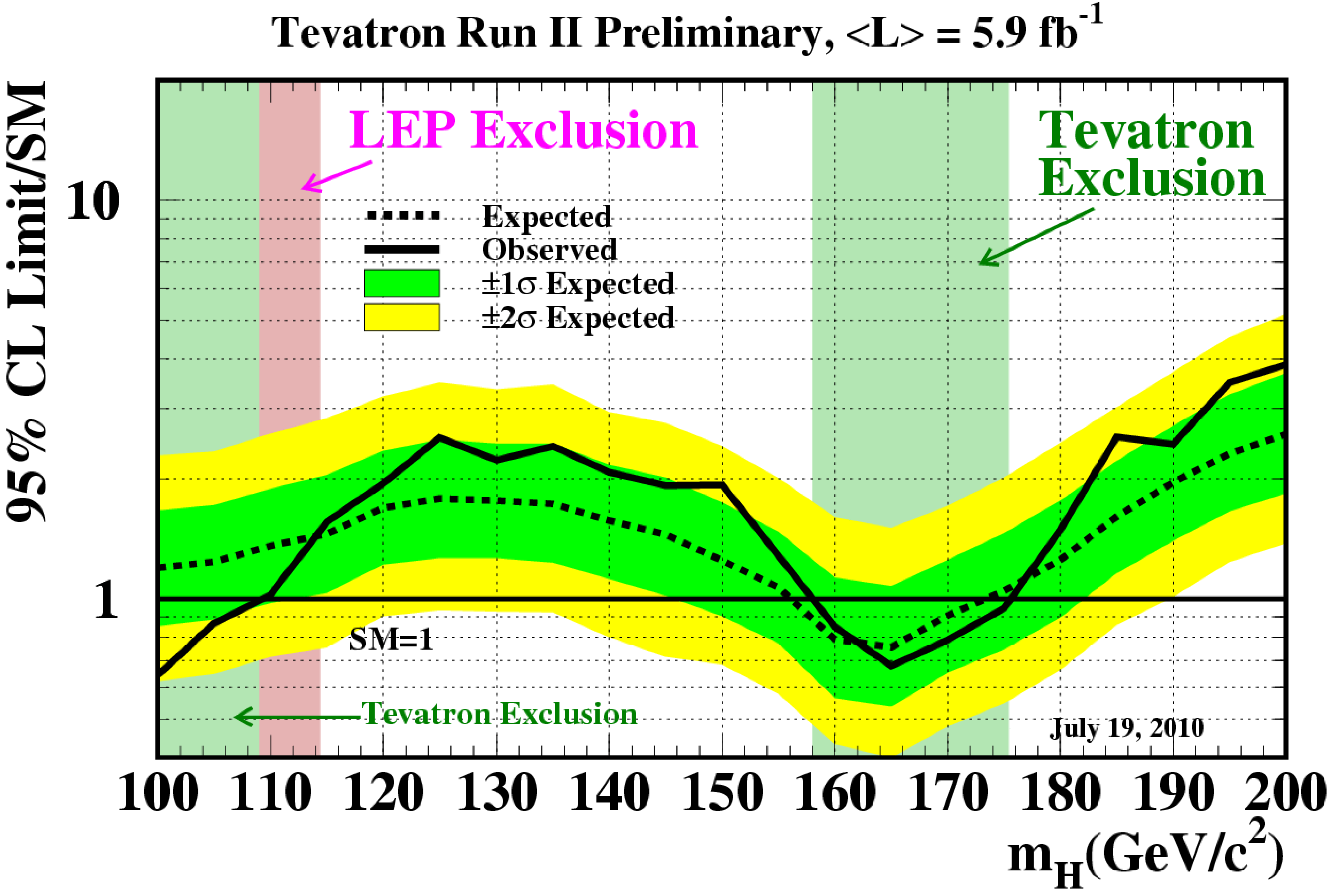
LEP Process (Higgs-Strahlung):

$e^+ e^- \rightarrow ZH$





ZZH couling is large!


Tevatron Run II Preliminary, <L> = 5.9 fb^{-1}
95% CL Limit/SM
10
1
LEP Exclusion
Tevatron Exclusion
Expected
Observed
±1σ Expected
±2σ Expected
SM=1
Tevatron Exclusion
July 19, 2010
100
110
120
130
140
150
160
170
180
190
200
m$_H$(GeV/c^2)

Statistics and Limit Setting

chi² fit:

$$\chi^2 = \sum_i \frac{(y_i - \mu_i)^2}{\sigma_i^2}$$

y_i measurement

μ_i model prediction (nuisance parameter)

σ_i uncertainty (statistical and systematical)

chi² fit with correlated errors:

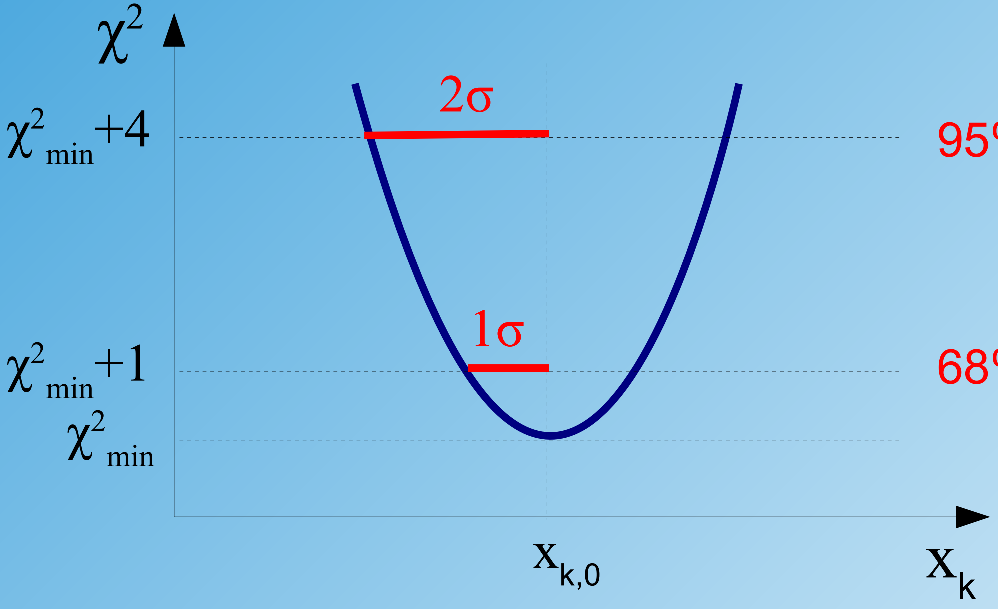
$$\chi^2 = \sum_i \sum_j (y_i - \mu_i) cov_{ij}^{-1} (y_j - \mu_j)$$

cov_{ij} covariance (error) matrix

Parameter Fit:

$$\mu_i = \mu_i(x_1, x_2, \ldots, x_n)$$

x_k model parameter



95% confidence level

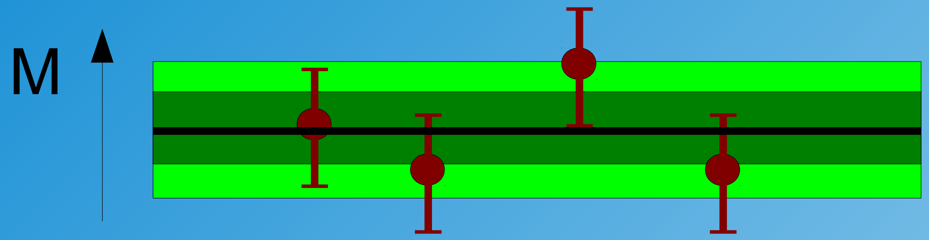
68% confidence level

good fit if:

$\chi^2_{\min}$ / degrees of freedom ~ 1

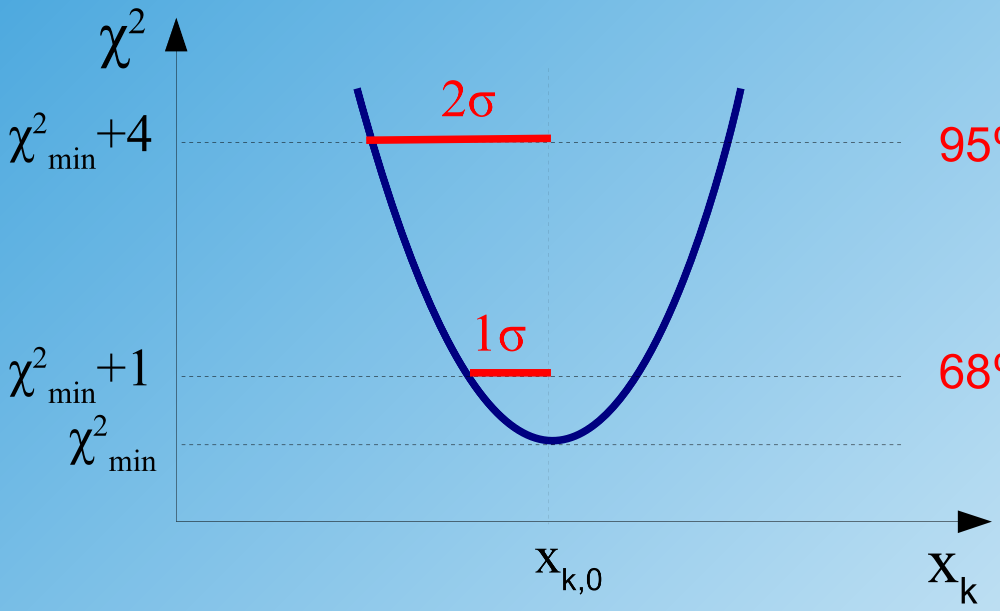
Example Fit

Measurement of some mass (1-Parameter fit) from 4 experiments:



$+2\sigma$ (95%)
$\pm 1\sigma$ (68%) confidence level bands
-2σ (95%)

(could also be a cross section)



95% confidence level

68% confidence level

good fit if:
χ^2_{min} / degrees of freedom ~ 1

Probability Densities

for the above example a Gaussian probability density was used

Gaussian (normal)distribution:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \rightarrow \quad P(a<x<b) = \int_a^b f(x)\, dx$$

- used for systematic uncertainties (symmetric)

Poisson distribution:

$$P(N) = \frac{e^{-\mu}\, \mu^N}{N!}$$

- used for statistical uncertainties

Note, Poisson distribution approaches Gaussian distribution for large μ

Limit Setting Philosophies

Bayesian Method:

- based on the experiment posterior, exclusion limits are calculated
- low probability models are excluded
- probabilities are assigned to models using a prior

“natural method” but choose of prior is arbitrary

Frequentist Method:

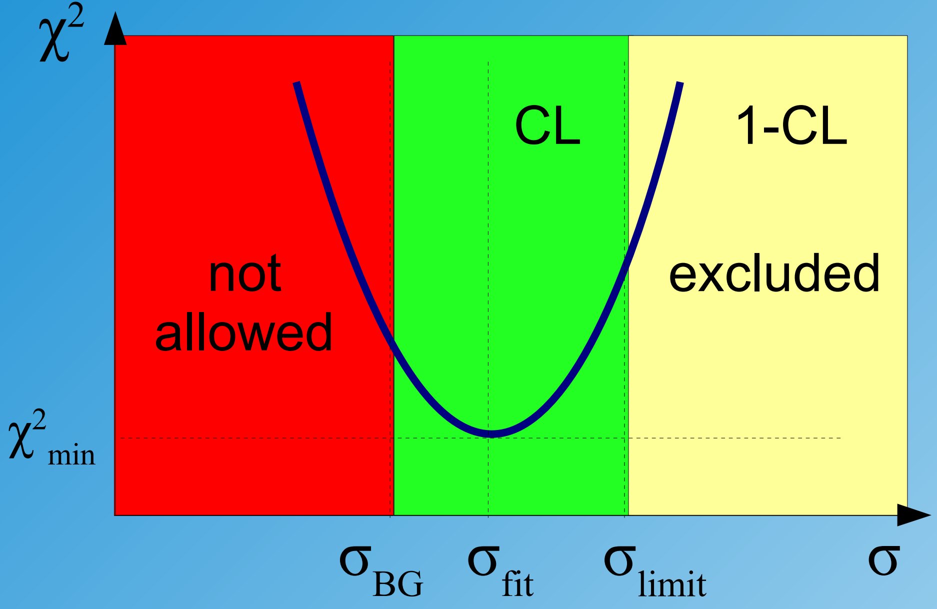
- based on Monte Carlo toy experiments probabilities are assigned to all possible experimental outcomes
- exclusion limit is set by that model which excludes this experimental outcome with certain confidence interval

computationally expensive and might give unphysical results
(e.g. negative cross sections)

Bayesian Method

Model: $N = N_{BG} + N_{Sig}$ (background + signal)

$\sigma = \sigma_{BG} + \sigma_{Sig} = N/L$ choose cross section as “prior”



Probability:

$$P \propto e^{-\frac{1}{2}\left(\chi^2(\sigma)-\chi^2_{fit}\right)}$$

additional constraint:

$\sigma \geq \sigma_{BG}$ because $\sigma_{Sig} \geq 0$

$$CL = \frac{\int_{\sigma > \sigma_{BG}}^{\sigma_{CL}} P(\sigma)\, d\sigma}{\int_{\sigma > \sigma_{BG}}^{\infty} P(\sigma)\, d\sigma}$$

CL = confidence level

Choice of Prior

- Cross sections depend on couplings
- Alternatively, choose coupling as prior

$$\sigma_{Sig} = \sigma_0\, \alpha^2$$

$$\frac{d\sigma}{d\alpha} = \frac{d\sigma_{Sig}}{d\alpha} = 2\sigma_0\, \alpha$$

$$CL = \frac{\int_{\alpha>0}^{\alpha_{CL}} P(\alpha)\, d\alpha}{\int_{\alpha>0}^{\infty} P(\alpha)\, d\alpha} = \frac{\int_{\alpha>0}^{\alpha_{CL}} P(\alpha)/\alpha\, d\sigma}{\int_{\alpha>0}^{\infty} P(\alpha)/\alpha\, d\sigma} \neq \frac{\int_{\sigma_{Sig}>0}^{\sigma_{CL}} P(\sigma)\, d\sigma}{\int_{\sigma_{Sig}>0}^{\infty} P(\sigma)\, d\sigma}$$

Results depends on choice of prior!

Frequentist Method

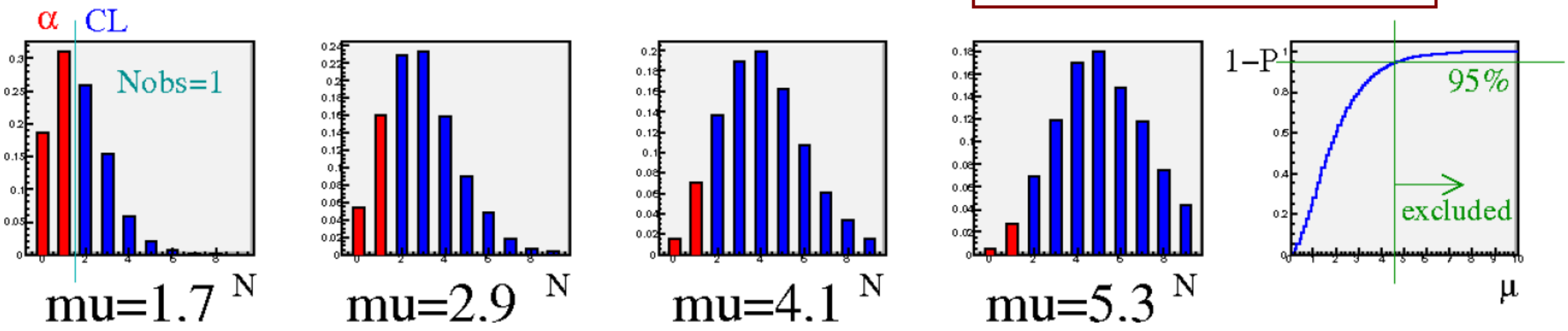
S.Schmitt

- Frequentist limit: exclude all theories which produce the data at small probability α less than 1-CL (typically: CL=0.95)

$$\alpha = P_{\mu}(N \leq N_{\text{obs}}) < 1 - \text{CL}$$

α: also called p-value

Frequentist limit:
sum (integrate)
over observations
up to N_{obs}
Repeat for each model



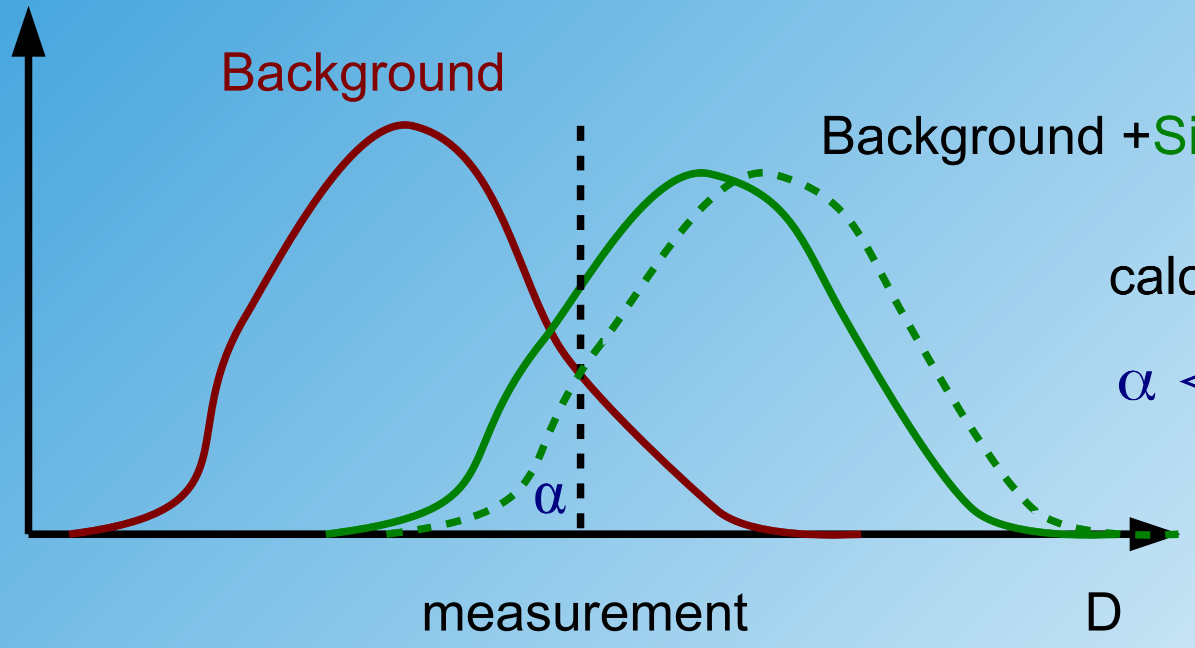
➜ set limit with 95% confidence level for mu=4.6
➜ experiment has a 5% probability to happen

Frequentist Method

In case of many observables x_k a combined discriminator variable is often defined: $D = D(x_{1,} x_{2,} x_{3,} \dots X_k)$

- large discriminator means high probability
- small discriminator means low probability

Often, the output from artificial neural nets or other multivariate methods is used as discriminator variable



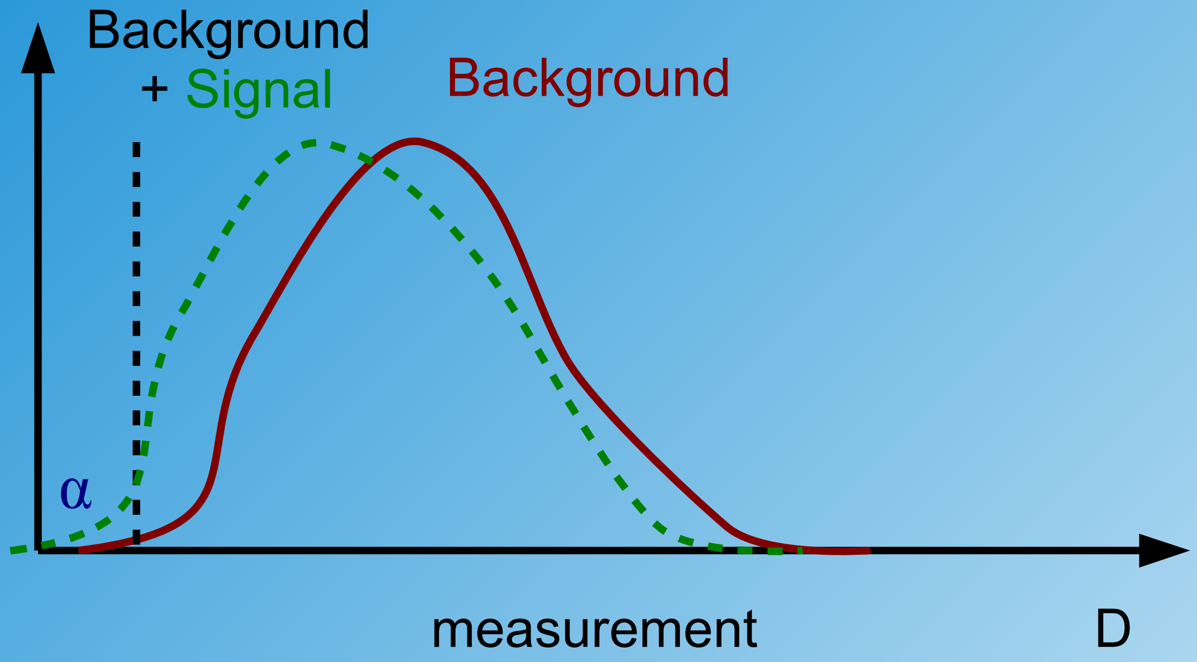
calculate smallest signal with:

$\alpha < 1 - CL$

Problem with Frequentist Method

Problem in case of a very small measurement value with P(BG)< (1-CL)

- would require a negative signal cross section:



unphysical solution!

CL_S Method

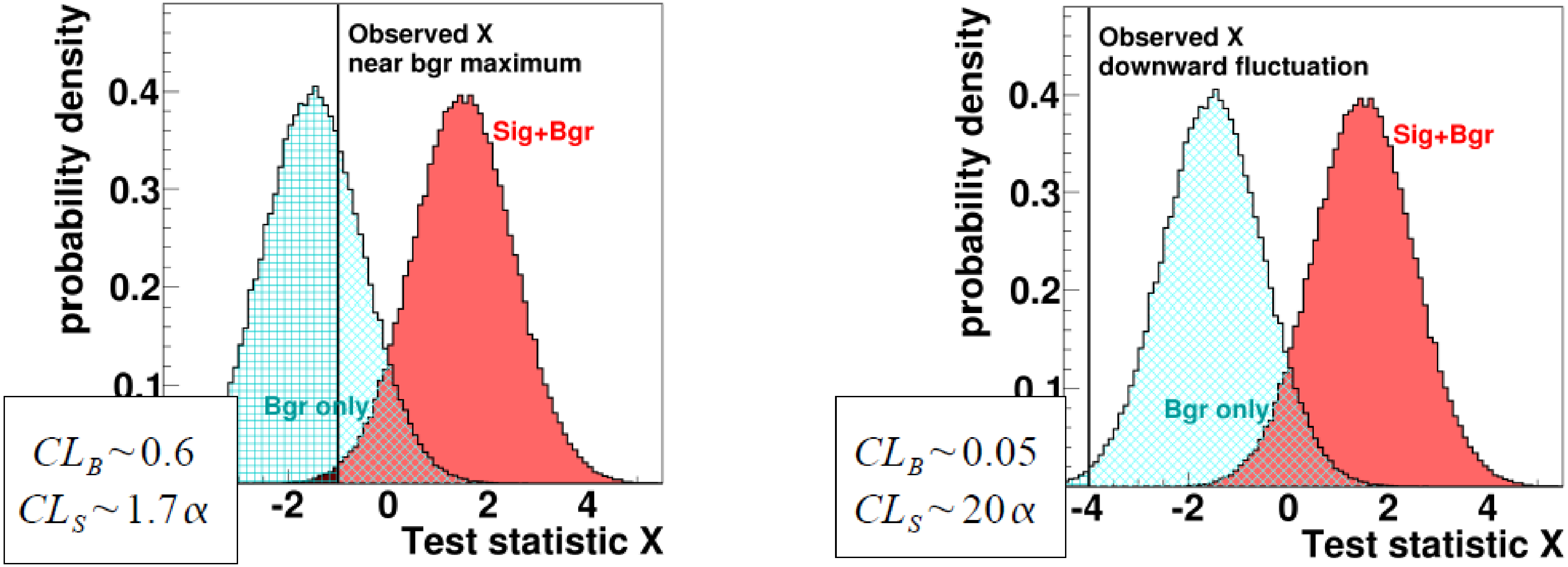
- Use ratio of two probabilities CL_S instead of α to test against CL

$$CL_{SB} = \alpha = \int_{X<X_{obs}} P(X|\text{signal}+\text{bgr})\,dX$$

$$CL_B = \int_{X<X_{obs}} P(X|\text{bgr})\,dX$$

$$CL_S = \frac{CL_{SB}}{CL_B}$$

- Standard model has CL_S=1 and is never excluded



$CL_B \sim 0.6$
$CL_S \sim 1.7\alpha$

$CL_B \sim 0.05$
$CL_S \sim 20\alpha$

S.Schmitt

CL_S > CL_{SB} by definition!

Another Example CL_s Method

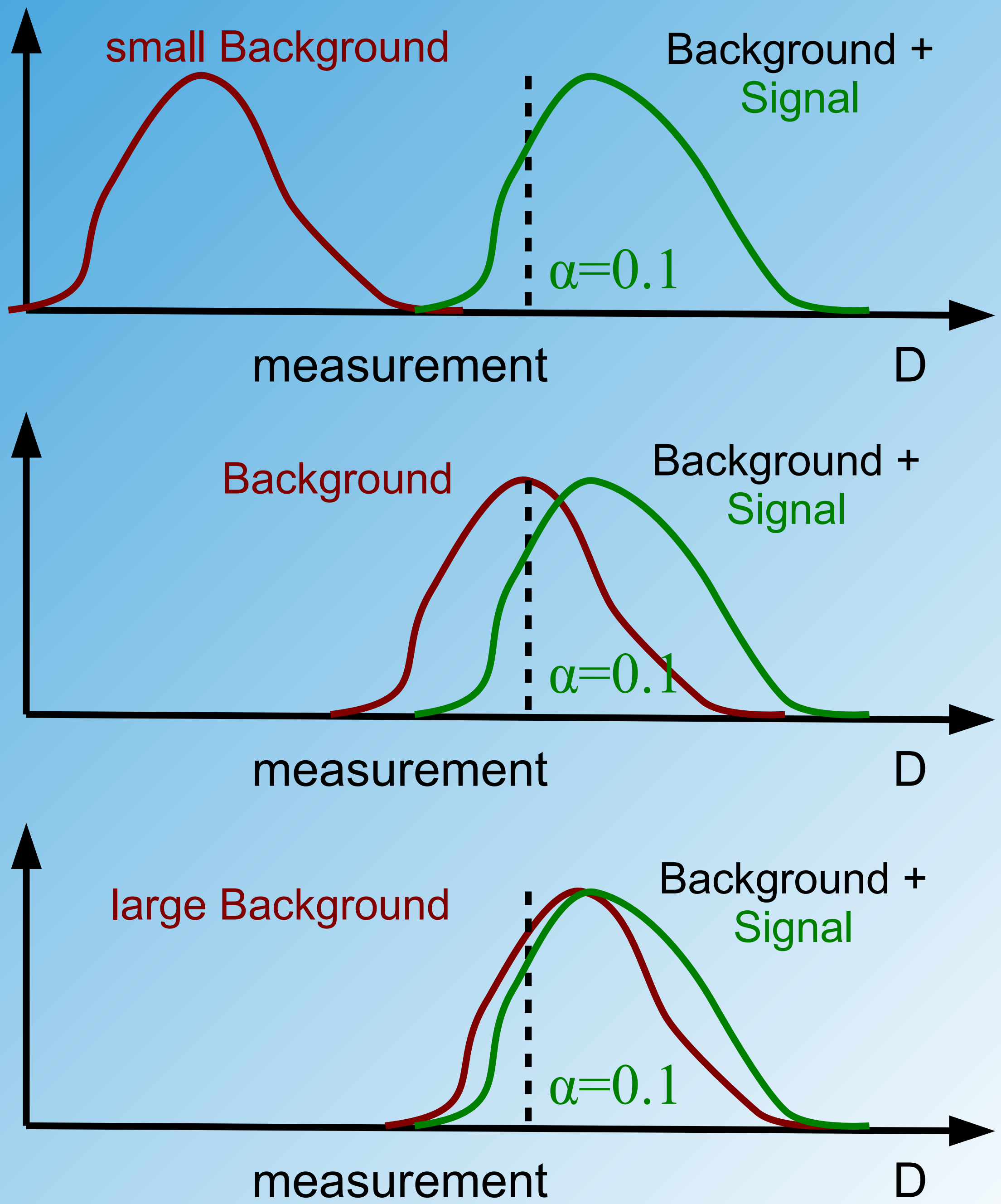
Definition:

$$CL_S = \frac{CL_{SB}}{CL_B}$$

$$0.1 = \frac{0.1}{1.0}$$

$$0.2 = \frac{0.1}{0.5}$$

$$0.5 = \frac{0.1}{0.2}$$

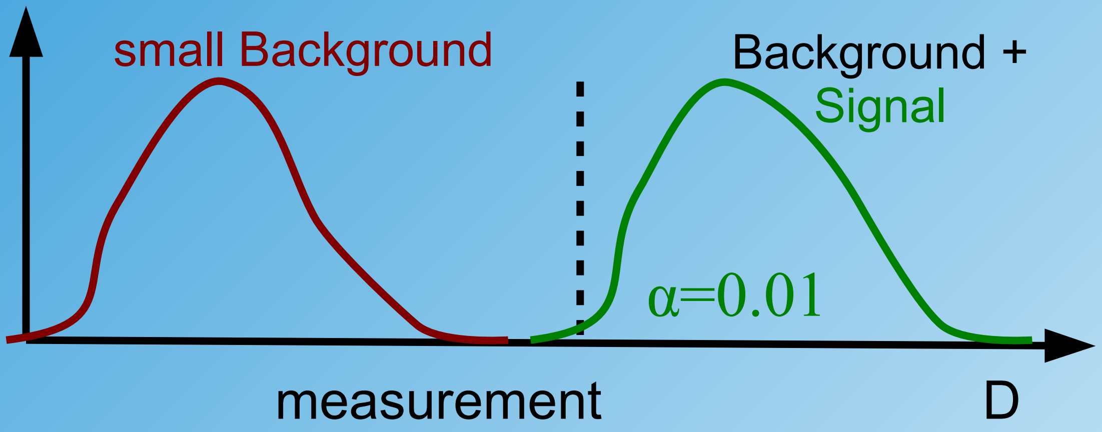


Another Example CL_S Method

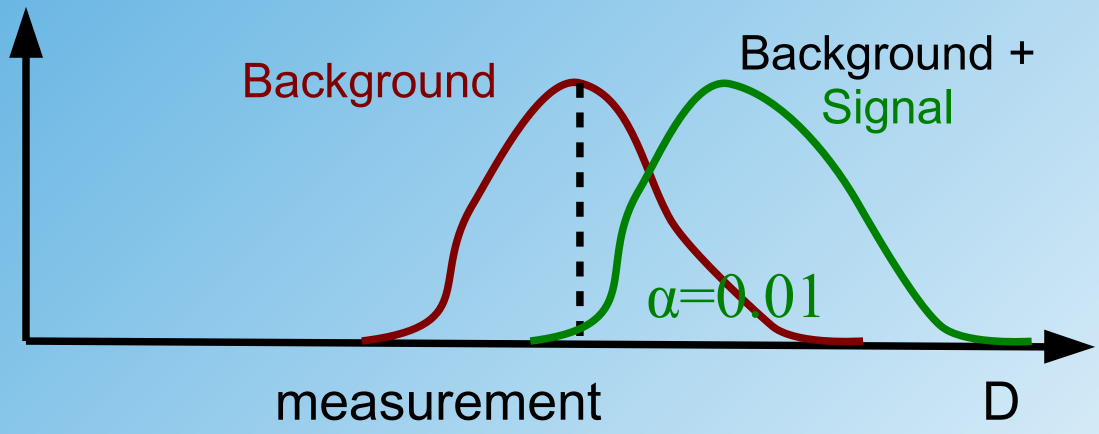
Definition:

$$CL_S = \frac{CL_{SB}}{CL_B}$$

$$0.01 = \frac{0.01}{1.0}$$

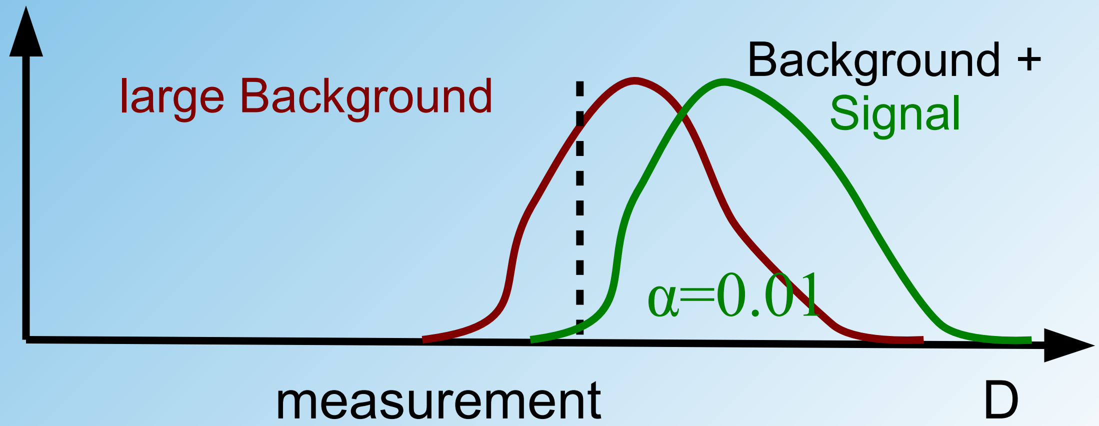


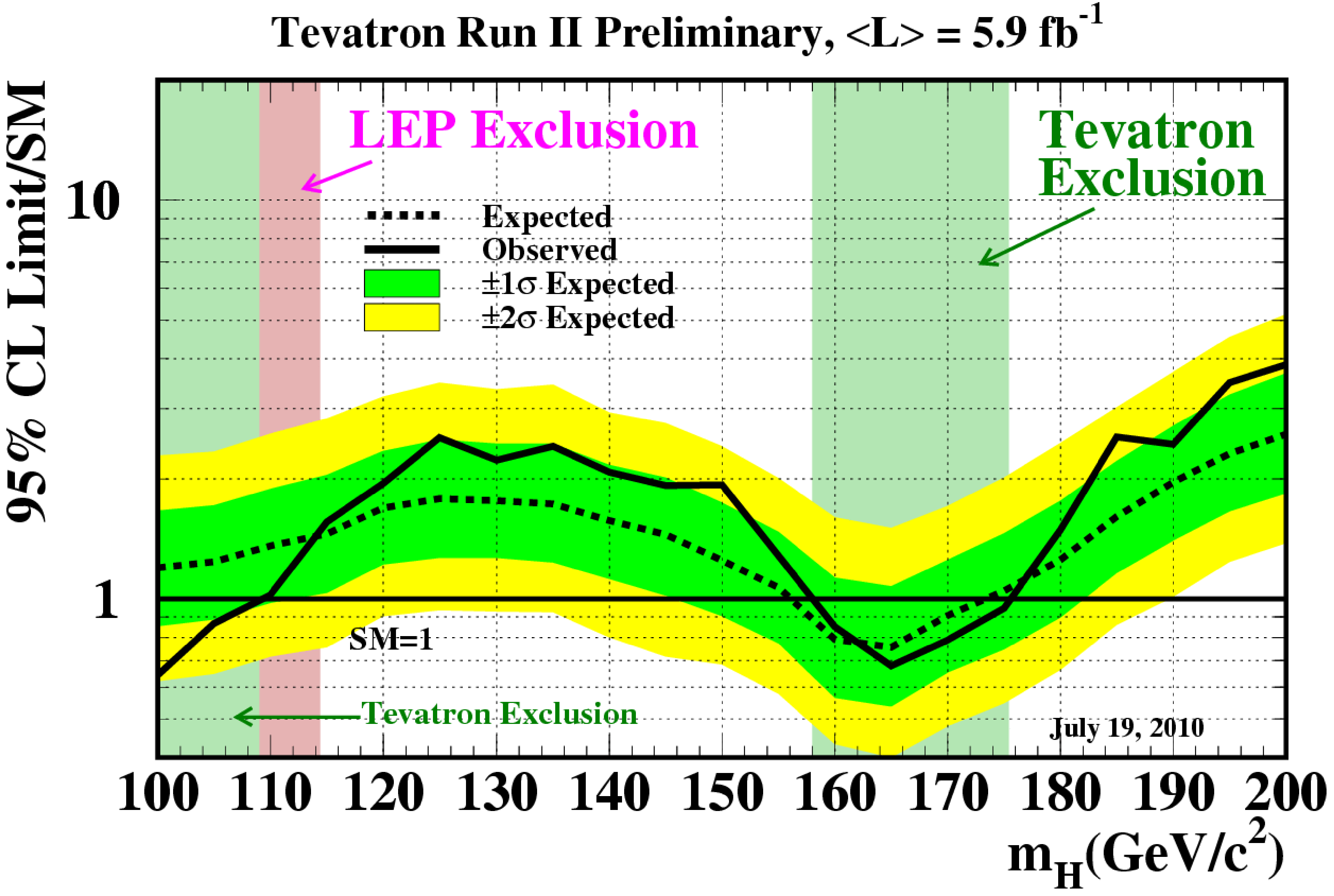
$$0.02 = \frac{0.01}{0.5}$$



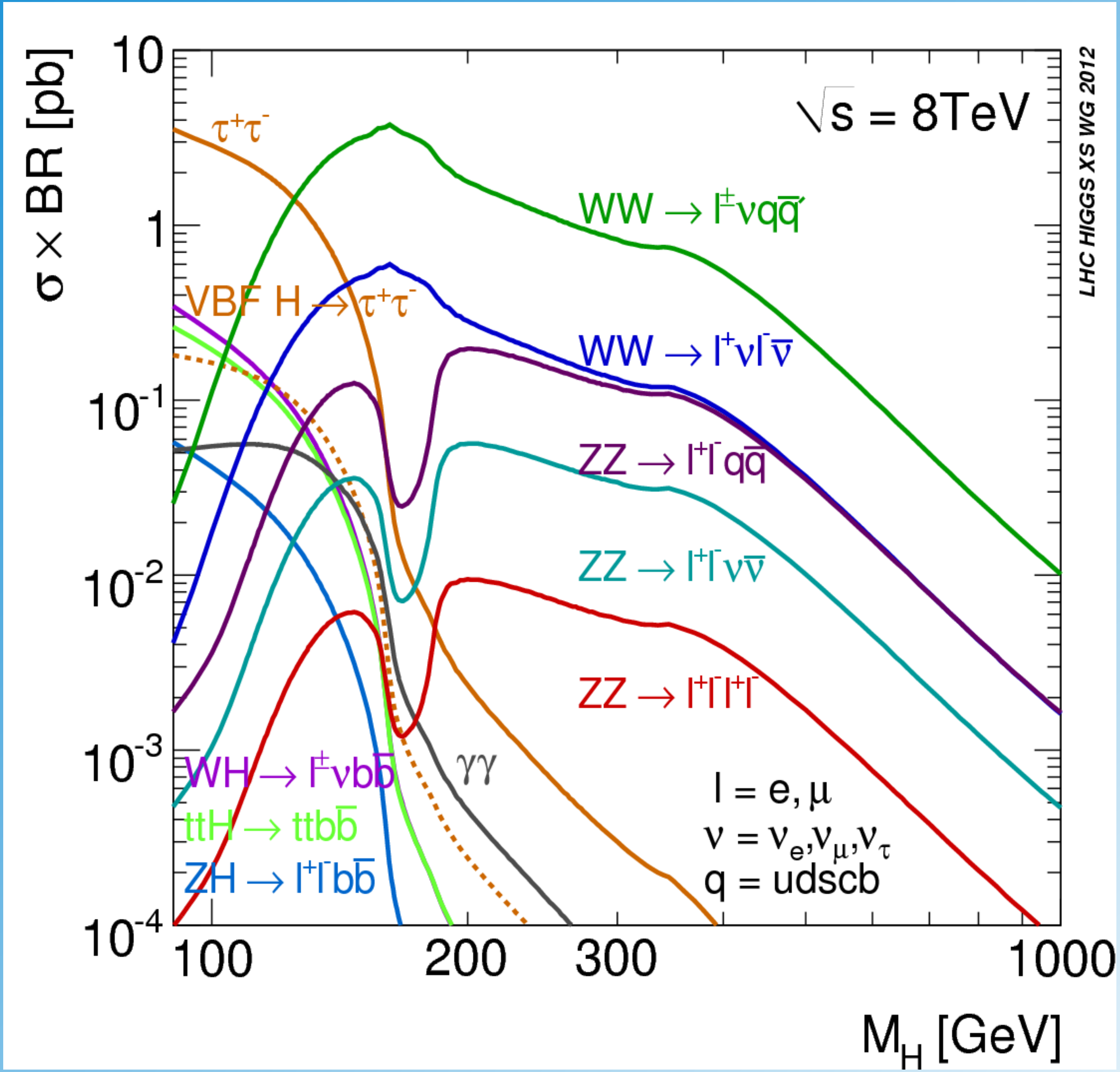
$$0.05 = \frac{0.01}{0.2}$$

excluded at 95% CL




Tevatron Run II Preliminary, <L> = 5.9 fb^{-1}
95% CL Limit/SM
LEP Exclusion
Tevatron Exclusion
Expected
Observed
±1σ Expected
±2σ Expected
SM=1
Tevatron Exclusion
July 19, 2010
10
1
100
110
120
130
140
150
160
170
180
190
200
m$_H$(GeV/c^2)

Discussion!


$\sigma \times$ BR [pb]
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
100
200
300
1000
M_H [GeV]
$\sqrt{s}$ = 8TeV
LHC HIGGS XS WG 2012
$\tau^+\tau^-$
VBF H $\rightarrow \tau^+\tau^-$
$WW \rightarrow l^\pm \nu q\bar{q}'$
$WW \rightarrow l^+ \nu l^- \bar{\nu}$
$ZZ \rightarrow l^+ l^- q\bar{q}$
$ZZ \rightarrow l^+ l^- \nu\bar{\nu}$
$ZZ \rightarrow l^+ l^- l^+ l^-$
$\gamma\gamma$
$WH \rightarrow l^\pm \nu b\bar{b}$
$ttH \rightarrow tt b\bar{b}$
$ZH \rightarrow l^+ l^- b\bar{b}$
$l = e, \mu$
$\nu = \nu_e, \nu_\mu, \nu_\tau$
q = udscb